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GUIDED MISSILES AND PILOTLESS AIRCRAFT

A Report of the AAF Scientific Advisory Group

by

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The AAF Scientific Advisory Group was activated late in 1944 by General of the Army H. H. Arnold. He secured the services of Dr. Theodore von Karman, renowned scientist and consultant in aeronautics, who agreed to organize and direct the group.

Dr. von Karman gathered about him a group of American scientists from every field of research having a bearing on air power. These men then analyzed important developments in the basic sciences, both here and abroad, and attempted to evaluate the effects of their application to air power.

This volume is one of a group of reports made to the Army Air Forces by the Scientific Advisory Group.

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H. L. DRYDEN

PART I

PRESENT STATE OF THE GUIDED MISSILE ART

23 APRIL 1945

INTRODUCTION

There is practically a universal belief among laymen, scientists, and military leaders that the development of guided missiles is in its infancy. The state of the art is often compared with that of aircraft design in the first World War, and it is fully expected that great advances will be made before another war. In fact the brief experiences in the tactical use of guided missiles in this war indicate that another war will probably be opened by the descent in large numbers of missiles launched from distances perhaps of the order of from 1000 to 3000 miles on an unsuspecting and unprepared country. It is vital to the future defense of our nation that research and development in this field be continued not only so that adequate countermeasures may be developed against enemy missiles, but also that we may have available the best weapons of this type in the world.

Our military leaders are fully aware now of the necessity of pushing developments of guided missiles, and almost frantic efforts are being made to compress within a few months developments which ordinarily take years. Much of the necessary fundamental research information is not and will not be available for another year, and experience has shown that development of even the simpler forms of missiles has required about two years. Until the technical possibilities are more fully explored by design and field test of experimental missiles of various types, there will be much confusion as to the military requirements which should be set forth. It is especially important that field commanders be sympathetic to actual combat trials of missiles now available so that the military possibilities may be more fully explored. Opportunities for such tests have disappeared with the coming of peace.

In estimating future possibilities and planning future research it is helpful to survey the present state of the art. That is the aim of this report. Because of the large number of projects under way, the rapid pace of development, and the secret classification, it is difficult for any one person or agency to obtain accurate and up-to-date information on every project. Taken as a whole, however, it is believed that the picture presented is adequate to serve as a basis for future planning.

OPERATIONAL USE OF GUIDED MISSILES BY GERMANY

The current interest in guided missiles is stimulated mainly by their operational use by the Germans, beginning in August, 1943. In that month the first use was made of PC 1400 FX, a high-angle armor-piercing bomb remotely controlled by radio in the releasing aircraft by an operator who watches the bomb visually. Additional fins, really rudimentary wings, were added to give greater maneuverability.

On 1 October 1943, the first use was made of Hs-293, a glide bomb also remotely controlled by radio on the basis of visual information. This bomb was accelerated by a liquid-fuel rocket for 12 to 15 seconds after release to provide a more favorable geometrical situation for visual control.

Neither of these missiles was extensively used, apparently because air superiority appears to be required for their successful use. The accuracy was none too good. Of 28 FX bombs used, five were hits resulting in one ship sunk and four damaged. Some 159 Hs-293 missiles were dropped to sink five ships, damage two seriously, and inflict minor damage on four. These results would appear to indicate that direct-sight methods are not likely to lead to high accuracy.

On the night of 12 June 1944, attacks were begun on London with the V-1, a winged missile propelled by an aeropulse motor and ground-launched from special ramps in France, Belgium, and Holland. Between this date and Septemer 3, 8205 missiles were launched, of which 5471 crossed the English coast. Of those missiles crossing the coast 2354 landed, causing 5476 deaths, 15,918 serious injuries and 29,812 slight wounds. About 23,000 houses were destroyed and 1,104,000 damaged. In spite of much controversy about the effectiveness of this type of missile as compared to conventional bombers to which the large number of man-hours might have been devoted, the results obtained in the attack on London definitely herald a new type of aerial attack. Whether pilotless aircraft will completely replace conventional manned aircraft for bombing attacks or merely supplement them, a new field of development has been opened and the results obtained after a few years development will stand in the same relation to the present results as the performance of present aircraft to those of the last war.

On 8 September 1944, attacks were begun with the large fin-stabilized rocket V-2. This rocket reaches a speed of 5000 ft/sec, attains an altitude of from 50 to 60 miles, and reaches its point of impact 200 miles away about five minutes after launching. The launching weight is 12.2 T of which only 1620 lb is explosive load. Between 8 September and 23 December, 380 fell on England and 1120 on the continent, an average of about 13 launchings per day. There are no effective countermeasures, once the missile has been launched. The design of the V-2 is a great technical achievement, but the missiles are expensive in man-hours and carry only a small fraction of their weight as explosive load.

Neither the V-1 nor V-2 is controlled from the ground after release, each carrying an autopilot. The accuracy is not high, the circular probable error being about 5 miles at a range of 130 miles for V-1, and 10 miles at a range of 200 miles for V-2. In spite of this inaccuracy considerable military damage had been produced.

AMERICAN MISSILES REACHING COMBAT EVALUATION STAGE

No American missiles have been used yet in any considerable number in combat but a few have reached the combat evaluation stage.

The glide bomb GB-1 developed by the Army Air Forces was used in certain operations in Normandy. This weapon is an unpowered glider controlled by an autopilot to maintain a straight course. The pay load is a 2000-lb GP bomb. In one operation 116 were dropped of which 48 landed within the target area. The errors are estimated as of the order of 1/2 to 1 mile in range and 1/8 mile in azimuth when launched at 20 miles slant range. There are no plans for further combat use.

The glide bomb GB-4 developed by the Army Air Forces has received a few tests under combat conditions. This bomb is remotely controlled by radio, and equipped with television apparatus to repeat back to the operator the picture seen from the nose of the bomb. There are no plans at present for combat use.

The 1000-lb Azon or VB-1 developed by Division 5 of the National Defense Research Committee has received some combat use. This missile is a high-angle bomb remotely controlled by radio (in azimuth only) by an operator who follows the motion of the bomb, visually assisted by a flare on the bomb. Excellent results have been obtained against bridges, railways, and roads.

The missiles of the TDR series, which are pilotless aircraft of more or less conventional aircraft design developed by the Navy Bureau of Aeronautics, have received combat evaluation, but no further combat use is planned.

The radar-homing missile, Bat (SWOD Mark 8 Mod 0), developed by Division 5 of the National Defense Research Committee and the Navy Bureau of Ordnance is scheduled for limited combat use.

Other missiles for which development is substantially completed are the 1000-lb Razon or VB-3, the 2000-lb Azon or VB-2, and the heat-homing missile VB-6, all developed by Division 5 of the National Defense Research Committee for the Army Air Forces. The Army Air Forces have substantially completed development of a Chinese copy of V-1, known as JB-2.

It is highly desirable that combat tests be made of all types of missiles developed in this country, regardless of whether extensive combat use is contemplated, in order that development agencies may obtain information to aid in the future development of missiles.

TABLE 1

Classification of Missiles according to Method of Launching and Propulsion

Method of Launching	Method of Propulsion	Power Plant	Aerodynamic Lift	Resulting Missile	Examples of Guided Missiles
Surface	Projected	Gun	No	Bullet or shell	—
"	Accelerated	Rocket	Yes	Winged rocket	—
"	"	"	No	Fin- or spin-stabilized rockets	V-2, Ord-Cit, JB-3
"	Self-propelled	Reciprocating Engine and propeller	Yes	Pilotless aircraft of conventional type	Willie XBQ TD3R-1 TDR-1
"	"	Gas Turbine and propeller	Yes	Pilotless aircraft with gas-turbine drive	—
"	Projected or accelerated, then self-propelled	Aeropulse	Yes	Pilotless aircraft or flying bomb	V-1, JB-2, JB-4
"	"	Turbojet	Yes	"	JB-1, JB-7
"	"	Ramjet	Yes	"	Tuve Project B
Aircraft	Gravity	Gravity	Yes	Bomb-carrying glider	GB series Pelican-Bat series Glomb
"	"	"	No	Ordinary bomb Special Bomb	VB-1 to 8, Dove (VB-9 to 12), Roc., PC 1400 FX
"	Projected	Gun	No	Bullet or shell	—
"	Accelerated	Rocket	Yes	Winged rocket	Hs-293 Cargoyle, Gorgon
"	"	"	No	Fin- or spin-stabilized rocket	JB-5, JB-6, Lark
"	Self-propelled	Reciprocating engine	Yes	Pilotless aircraft or flying bomb	German weapon
"	"	Gas Turbine and Propeller	Yes	"	—
"	"	Aeropulse	Yes	"	JB-2, V-1
"	"	Turbojet	Yes	"	Gorgon
"	"	Ramjet	Yes	"	—

CLASSIFICATION OF MISSILES ACCORDING TO METHOD OF LAUNCHING AND PROPULSION

In surveying the present state of the guided-missile art it is convenient to study the missiles now developed or under development according to several schemes of classification. The tactical user of the missile is interested in the kind of target for which the missile is suitable (i.e., aircraft, ship, or land target), the launching method and method of propulsion (whether from ground, ship, or airplane), and the range, speed, over-all weight and pay load of the missile. He is interested also in the logistics of production and supply, the number and training of the service crews, and many other matters which will not be discussed in this report.

Table 1 shows a classification according to method of launching and method of propulsion. Missiles may be launched either from a surface installation on land, on a ship, or from aircraft. They may be projected from a gun, accelerated for a certain time interval by means of rocket motors, or self-propelled. Especially for ground launching, the missile may be projected or accelerated for a time and then be self-propelled. For launching from aircraft, the missile may merely be released without projection, acceleration, or propulsion at the speed and altitude imparted to it by the aircraft.

The available types of power plants are gravity for missiles dropped from aircraft, guns, rockets, reciprocating engines with propeller, gas turbines with propeller, aeropulse motors, turbojet motors, and ramjet motors. Of these power plants all are in use except gas turbines with propeller and ramjet motors. The latter seem especially applicable to missiles travelling at supersonic speeds because of their apparent simplicity and light weight. Much research is in progress but as one engineer stated recently the ramjet motor is at the present moment a fiction. There is, however, little doubt that within a year or two research will lead to an operating ramjet motor.

From the propulsion point of view there are important differences between those missiles which rely on momentum to extend their range before the force of gravity brings them to the ground, and those which depend for support on aerodynamic lifting forces. Hence this element has been included in the classification.

This classification leads to missiles of the following types: bullets or shells, winged rockets, fin- or spin-stabilized rockets, bombs, winged bombs, and pilotless aircraft or winged self-propelled bombs. There are no known projects for guiding bullets or shells. There are projects for guided missiles of all the other types. Until recently the American effort was devoted mainly to glide bombs both with and without wings. The greatest interest now appears to be in self-propelled guided missiles. It is generally believed that pilotless aircraft of conventional type with reciprocating engines and propeller are too slow and vulnerable to be useful. Hence rockets both with and without wings and pilotless aircraft with jet propulsion are receiving greatest attention in present planning.

CLASSIFICATION OF MISSILES ACCORDING TO SOURCE OF CONTROL AND NATURE OF INTELLIGENCE

The usual aim of American designers of guided missiles has been to obtain increased accuracy as compared to unguided missiles, but the German use of the V-1 and V-2, which are less accurate than conventional bombing from aircraft, has suggested that accuracy is not the sole consideration. Nevertheless accuracy will always be an important objective. In the opinion of the writer, guided missiles will not come into wide use until the accuracy is substantially better than that obtainable by unguided missiles.

Table 2 shows a classification of guided missiles according to source of control and nature of intelligence. The missiles receiving greatest combat use, V-1 and V-2, are guided by autopilots which receive no intelligence from the target directly or indirectly. From the known location of the target with respect to the launching point and estimates of wind drift, the necessary settings of the autopilot are computed and made before launching. The operator has no control after launching and the missile does not receive any form of information directly from the target. For the V-1 the accuracy depends partly on the accuracy of the autopilot but mainly in the present state of the art on the accuracy of the allowance for wind drift. The circular probable error obtained is of the order for four percent of the range. It seems unlikely from analyses made by various groups that this error could be reduced below two percent by any improvement in the autopilot or weapon, so long as the speed is as low as 400 mph. The wind-drift errors decrease with increasing speed of the missile. For the V-2 type the wind-drift errors are negligible and the accuracy depends on the accuracy with which the autopilot sets the angle of pitch and the speed and is hence capable of considerable improvement.

The Japanese have often used piloted aircraft as "suicide" missiles with good accuracy.

The use of a remote human pilot has been very attractive because human judgment is introduced into the problem at all stages. In simplest form the remote human pilot views the missile and target at all times, usually assisted by a flare on the missile, and exercises remote control by radio. The best results have been obtained with high-angle bombs where the range is not so great as for glide bombs, and where the effects of haze are less. Any target which can be seen visually can be attacked. The use of the weapon is, however, limited to daytime and good visibility. The accuracy is dependent to a considerable degree on freedom from disturbance by flak and enemy fighters.

Because of the difficulty of seeing at long distances, the next development is to place effectively the remote pilot in the missile by introducing a television transmitter in the missile to repeat back to the operator what the missile "sees." New technical problems are introduced since most missiles do not "look" in the direction

TABLE 2

Classification of Guided Missiles according to Source of Control and Nature of Intelligence

Source of Control	Nature of Intelligence	Physical Property of Target Utilized	Method of Target Discrimination	Examples
Autopilot	None	Map location	Human judgment	V-1, V-2, Ord-Cit, JB-2, GB-1, JB-5, JB-6, JB-4
Human Pilot in Missile	Visual	Visual aspect	Human judgment	Japense suicide bombers
Remote Human Pilot	Visual	Visual aspect	Human judgment	PC 1400 FX, Hs-293, GB-8, GB-9 VB-1, VB-2, VB-3, VB-4, VB-12, Gargoyle
" " "	Repeat-back of television information	Emission and reflection of light	Human judgment	GB-4, GB-10, Gorgon, Glomb TDR-1, TD3R-1, VB-7, VB-8, VB-10
" " "	Repeat-back of radar information	Emission and reflection of radar signals	Human judgment aided by range and directional information	—
" " "	Radio or radar location of missile	Map location, reflection of radar signals	Human judgment, aided by range and directional information	Later Ord-Cit, JB-2 Willie
Homing	Optical	Emission of light (flares or searchlight)	Intensity and direction	GB-5D VB-5
"	"	Light contrast or discontinuity	Intensity and direction	GB-5A GB-5C
"	"	Visual pattern as seen by animal (pigeon or cat)	Training of animal	General Mills
"	Heat	Heat radiation	Intensity and direction	GB-6A, VB-11, VB-6, Dove
"	Radar	Emission of radar signals	Intensity and direction	GB-7C
"	"	Radar reflection, transmitter in mother ship	Intensity, direction and range	GB-7 Pelican, JB-3 (?)
"	"	Radar reflection, transmitter in missile	Intensity, direction and range	Bat, GB-7B
"	Acoustic	Emission or reflection of sound	Intensity and direction	—
Beam-guided	Radar beam	Radar reflection	Human judgment, aided by range and directional information	Tuve Project B

in which they are moving. In tests a circular probable error of 200 feet has been obtained. Difficulties have been encountered in locating the target because of the limited resolution and field of view. This type of control and intelligence is applicable to any type of target in the daytime in good visibility. Equipment of great sensitivity has been developed which gives good images at very low light levels, and permits operation at dawn and dusk if haze is not present.

Instead of television repeat-back, radar repeat-back could be used. Targets would then be limited to those giving good radar reflections, but operation would be possible at night, and through fog, i.e., entirely independent of weather. No missiles using this type of intelligence have yet been developed.

A method which is now being actively investigated, and which uses a remote human pilot is that which tracks the missile continuously by radar and pilots the track on a map on which the target location is known. An operator then controls the missile by radio so that the track intercepts the target and the missile begins to dive into the target at the proper time to strike the target. There has been insufficient experience with this method to establish experimentally the value of the circular probable error to be expected.

Missiles which automatically seek or home on the target have always appeared attractive. Such a missile can be successful only against targets which stand out from the background in some way. The property of the target most commonly utilized is its emission or reflection of electromagnetic radiation. The three major divisions of the electromagnetic spectrum of practical use for this purpose are described by the terms light, heat, and radio.

Searchlights and flares are suitable targets for optical homing missiles which respond to light emission. All optical homing missiles are useful only under conditions of good visibility. On ships or other isolated targets which show marked optical contrast with the background, a missile responding to light contrast or discontinuities in the optical pattern would be operative. Although some missiles of these types have been constructed and tested, the information available is insufficient to determine the circular probable error.

There have been one or two proposals to use animals as intelligence devices by training them to select a particular target from an image of the ground as seen from the missile. None of these proposals was carried to the point of test in an actual missile.

Heat-homing missiles are applicable to certain types of targets, for some targets only under favorable weather conditions. When used against land targets it will usually be necessary to make a thermal reconnaissance survey to determine whether the targets are sufficiently isolated from the background and are in fact the warmest areas. Industrial plants, containing large heat sources, and ships should be suitable targets.

Radar-homing missiles are not dependent on favorable weather; they may be used at night and through fog. The equipment which has been developed to date requires the operator to select and lock on a target before release thus permitting human judgment to enter into the target selection. Range discrimination is used in addition to direction and intensity. Ships are the most favorable targets, although there is some possibility of use against land targets giving isolated radar echoes.

Two radar-homing systems, the RHB type and the SRB type, have been developed. In the RHB type the missile carries a radar receiver. The target is illuminated by radar impulses from a transmitter on the aircraft carrying the missile. In the SRB type the missile is self-contained, carrying both transmitter and receiver. In each case a range gate is locked on the selected target before release, and the missile continues to track the target in range. There is no technical reason preventing the development of equipment which can be released blind, with automatic provision for searching in range and selecting according to some preset plan, for example, the largest echo, the third echo as regards range, etc. The removal of human judgment from the target selection would be disadvantageous, but the range of operation of the equipment could be greatly extended.

Acoustic homing missiles have often been suggested, but the investigations made to date offer little hope of obtaining useful ranges because of the large self-noise of the missile produced by its travel through the air.

There have been many proposals to guide missiles by means of directed optical or radar beams. Work is in progress on a method involving radar tracking of the target, computing the collision course, and setting a radar beam along the collision course. The missile is to contain equipment to keep it within the beam, whose position might change during the flight of the missile as a result of evasive action of the target.

RANGE, SPEED, WEIGHT AND PAY LOAD OF CURRENT MISSILE PROJECTS

The various types of guided high-angle bombs have ranges and speeds of the same order as standard bombs at the same altitude and air speed. They are mainly built around standard bombs such as the 1000-lb or 2000-lb GP bombs in American missiles and the 1400-kg AP bomb in the German missile. The maximum amount of guiding possible is from 500 to 3000 ft for different missiles.

The glide bombs can travel to ranges equal to seven or eight times the altitude of release, but to allow for adverse winds and a reserve of control it is best to consider their useful range as about five times the altitude of release. Their speeds are from 200 to 400 mph. The pay loads in most current projects are standard GP bombs, 500 lb, 1000 lb, 2000 lb, and in one instance (a towed glider) 4000 lb. The gross weight varies from 700 to 2600 lb for gliders carried by aircraft to 7138 lb for a towed glider.

Information concerning accelerated or self-propelled missiles is given in the following table:

TABLE 3

Designation	Total weight lb	Weight of fuel lb	Pay load lb	Speed mph	Range miles	Status
			Rockets without wings			
V-2	24,400	18,540	1,620	3,500	200	In use
Ord-Cit						
Private	500	185 (?)	—	750	11	Development tests
Ord-Cit						
Corporal	10,000	?	?	2,250	80	Design
JB-3	600	?	150	600	?	Design
JB-5	850	?	500	250-500	4(?)	Design
JB-6	75	?	5	Supersonic	4(?)	Design
			Rockets with wings			
Hs-293	2,000	?	1,322	225-400	11	In use
Gargoyle	1,517	?	1,000	200-690	5	Early test
			Pilotless aircraft			
Willie	?	?	20,000		3,000	Combat evaluation
XBQ	?	?	2,000 or 4,000	180-230	1,500-3,200	Development tests
TD3R-1	10,500	?	2,000	185	900	Development tests
TDR-1	6,300	?	2,000	155	470	Development tests
V-1 and JB-2	5,000	900	2,000	340-450	150	In use
JB-1	7,084	?	3,400	350-450	400	Early test
JB-4	3,000	665	1,000	400-450	75	Early test
JB-7	9,700	1,750	4,000	550	400	Design
Tuve Project B	2,000	?	600	1,800	20	Design
Gorgon	971	?	100	490	25	Early test

It will be noted from the table that all of the self-propelled missiles actually in use are of German origin. The studies planned of rockets without wings cover a wide range of weights and speeds. Though not listed, it is understood that the Ord-Cit project will study rockets with wings for moderately long ranges thus filling one obvious gap in the table. The studies of pilotless aircraft include only one supersonic missile for a fairly short range (20 miles). The possibilities of supersonic pilotless aircraft for longer ranges should be further investigated.

STATUS OF RESEARCH ON FUNDAMENTAL ASPECTS OF MISSILE DEVELOPMENT

As is very often the case in technical development, research on many aspects of guided missiles has proceeded more slowly than actual design and experimentation on complete missiles. Research on missiles has been sporadic and incidental to particular projects, consisting generally of trouble shooting and correction of specific defects in performance. There has recently been established a special committee on guided missiles by the National Advisory Committee for Aeronautics whose principal function is to formulate a program of research to be undertaken by the National Advisory Committee for Aeronautics. The committee includes members from the Army, Navy, and National Defense Research Committee as well as from the staff of the Committee. Through this membership it is hoped that there may be an exchange of information and thus informal coördination of related research undertaken directly by all agencies.

Division 5 of the National Defense Research Committee has carried out much research on intelligence devices, radio links, and servomechanisms in addition to developing specific missiles. The work has been carried out by numerous contractors, both industrial and university laboratories. The military services have also contracted for numerous specific developments.

Aerodynamic and power-plant problems in connection with most of the existing missile projects have sooner or later been referred to the National Advisory Committee for Aeronautics, largely to obtain the use of the Committee's large wind tunnels and other test equipment, and also to benefit from suggestions of members of the Committee's staff.

In addition to those research problems which are easily foreseen, actual experience in design and use of the simpler guided missiles reveals other problems not so easily foreseen. The research problems may be grouped in five fields, aerodynamics, power plants and propulsion, autopilots and servomechanisms, intelligence devices, and systems coordination.

Missile design makes use of all available aerodynamic knowledge, but the interest in high-speed missiles places increased emphasis on transonic and supersonic aerodynamics. Equipment for aerodynamic research at high speeds is at present very limited, but new installations have been authorized which will be in operation within one year. Reserach at these speeds is also of interest to aircraft designers, and little comment need be made on programs of research to determine lift, drag, stability, and control methods in transonic and supersonic regions. Some of the aerodynamic problems suggested by missile experience are as follows:

(1) The body required to contain pay load, fuel, and intelligence equipment is likely to be much larger in proportion to the wings required for sustentation at high speeds than for normal aircraft in which take-off and landing requirements must be met. Thus aerodynamic data should be collected on bodies with wings of

small area corresponding to high-wing loadings. For structural reasons and for convenience in handling, wings of small aspect ratio will be of interest.

(2) Every group experimenting with missiles learns by experience that equilibrium about the roll axis is difficult if not impossible to obtain by purely aerodynamic means. There are aerodynamic stabilizing moments about the pitch and yaw axes, such that out-of-trim moments simply cause the missile to fly at an angle of yaw or pitch. An out-of-trim moment about the roll axis produced by lack of symmetry causes the missile to roll continuously at a rate determined by the damping moment in roll. Equilibrium can be obtained either by the use of ailerons controlled by an autopilot or by a rudder controlled by an autopilot to yaw the missile until the rolling moment is zero. Research is needed on unconventional arrangements to determine whether equilibrium of moments can be obtained by purely aerodynamic means.

(3) A missile which travels at an angle of pitch or yaw is subject to bore-sighting errors since the compass, gyro, or intelligence unit is normally aligned with the axis of symmetry of the missile and the maneuverability of the missile is limited. Research on methods of reducing or correcting for bore-sight errors is needed.

(4) In a homing missile, a control which changes the angle of attack or yaw as the control is applied, will be subject to variable bore-sighting errors. Research on methods of control of the trajectory without changing the angle of attack or yaw is desirable. In popular language, a homing missile should be designed so that it looks in the direction of travel of its center of gravity.

(5) In self-propelled missiles, there is need for information on the aerodynamic characteristics of bodies suitable for internal installation of ramjet, aeropulse, or turbojet.

(6) With ramjet, aeropulse, or turbojet propulsion, there are many problems of internal aerodynamics connected with the design of ducts, diffusers, and nozzles at supersonic speeds, and the control of shock waves.

The great interest for the future is in self-propelled missiles, and hence research on power plants and methods of propulsion are of great importance for future development. While conventional aircraft with propeller drive are available for subsonic speeds, the weight and complexity of the power plant makes its use for missiles unsatisfactory. The range of conventional aircraft is very great, but the speed is low and the cost high. Whether similar considerations will make a turbojet power plant also unsuitable for an expendable missile can be decided only after further experience with turbojets. Certainly the ramjet and aeropulse seem more suitable (especially the former), although no actual ramjet has yet propelled a missile, and considerable research will probably be necessary to achieve this result. The fundamental problems are those relating to the efficient combustion of fuel in a small space, in an air stream of high speed, and over a wide range of air-fuel ratio, temperature and pressure. The stability of the combustion is an important consideration, especially in relation to the reactions between the internal flow in the ramjet, and the external flow around the missile. A further research problem is the determination of the relation between the efficiency and the thrust of a unit of given size. The maximum thrust is not likely to occur simultaneously with maximum efficiency, and the relation between the two must be known if a proper engineering compromise is to be made in design.

An unmanned missile must be provided with a substitute for the human pilot. The intelligence device, which will be discussed later, takes the place of the human pilot's intelligence and judgment. An autopilot is needed to detect changes in altitude and flight path produced by gusts, and to adjust automatically the control surfaces to give steady flight in the absence of signals from the intelligence device and in the presence of wind gusts. The autopilot substitutes for the sensory organs of the pilot. Finally a servomechanism is required to apply forces to the control surfaces in accordance with the signals from the autopilot. Some of the research problems relating to autopilots and servomechanisms are as follows:

(1) Reduction of lag. Missiles usually have much higher wing loadings than airplanes, hence wings of smaller span. The weight is usually concentrated near the longitudinal axis. The natural frequencies of oscillation in pitch and yaw, and especially the rate of response in roll, are such as to require the smallest possible lag in the autopilot and servomechanism.

(2) Development of antihunt or correcting elements to compensate for the small lag unavoidably present.

(3) Development of computers to correct for wind drift.

(4) Comparison of various types of autpilots on flight test tables or other simulators of missile flight.

Numerous electronic intelligence devices have been developed, most of them capable of only primitive judgment as compared to a human brain. The optical and heat-homing devices usually home on the center of gravity of the radiation pattern within their field of view. The radar devices have in addition discrimination in range, and in their present form introduce a human operator to make the initial selection in range. There is no doubt that more complicated feats of judgment can be performed. The research problems in this field are many, and range from target surveys (to determine the feasibility of various types of target selection by electronic, optical, heat, or other devices) to the development of specific devices of appropriate sensitivity and field of view. There is need for much closer contact between research workers on intelligence devices and flight research on missiles.

There are still great advantages in having the intelligence of a human operator within the link, even if he only supervises and corrects the action of a mechanism. Many of the proposed methods of control with human links have never actually been tried, and should be made the subject of research.

The major problem in missile design is the coordination of all elements to give stable operation or what may be called "systems coordination." So much is unknown about this subject that a research approach is essential. Three lines of attack are possible and some development has already been accomplished. These are as follows:

(1) Development and check of analytical methods of computing the over-all performance. Considerable progress has been made at the Servomechanisms Laboratory of the Massachusetts Institute of Technology and at the Langley Memorial Laboratory of the National Advisory Committee for Aeronautics.

(2) Free-flight tests of missiles instrumented to give information about all necessary elements of operation of the complete system. Some of the missile projects have obtained data of this character.

(3) Development of flight-test tables and flight simulators so that system checks can be made in the laboratory with most of the actual components. Division 5 of the National Defense Research Committee has sponsored many developments of this character, such as electronic simulators of Azon, Razon, and Roc flights, the flight-test table developed at the Servomechanisms Laboratory of the Massachusetts Institute of Technology, and the electromechanical model of the pitch control on the Pelican project.

Plans are under way for the establishment of special facilities for missiles research in free-flight by the National Advisory Committee for Aeronautics.

Before the design of missiles can be made a more or less straightforward matter of engineering design, the research on fundamental aspects of missile development will have to be extended far beyond the boundaries of information now available. The design of a guided missile is now actually more complicated technically than that of an aircraft because of the absence of the necessary fundamental information.

PART II

AUTOMATIC CONTROL OF FLIGHT

By

W. H. PICKERING

PART II

AUTOMATIC CONTROL OF FLIGHT

7 DECEMBER 1945

SUMMARY

This report discusses some of the problems of the design of automatic control systems, particularly for pilotless airplanes and controlled missiles. Some suggestions for future developments are outlined. It is pointed out that major emphasis should be on the solution of the antiaircraft missile problem. Once this defensive weapon has been controlled, the control techniques can be readily adapted to future offensive weapons.

INTRODUCTION

As the speed of airplanes increases and as the length of airplane flights increases, the necessity for some automatic device to fly the plane becomes of paramount importance. Furthermore, in the warfare of the future, pilotless airplanes and controlled missiles are bound to be of great importance. These also require automatic flight control to give them the necessary stability for proper interpretation of guiding signals.

Automatic pilots have been used for some ten years, and indeed some excellent devices are now available. Let us consider the basic requirements of an automatic pilot for airplanes.

(1) The device must contain at least one reference axis fixed in direction in space, and one reference axis giving the true vertical (or horizontal) at all times.

(2) The motion of the plane relative to these axis, when translated into changes of yaw, pitch, and roll, must be measured.

(3) The controls must be actuated by these measured quantities in such a way as to neutralize the changes.

(4) There must be provision for the human pilot to change the heading of the plane along any of the three axes, either by direct overriding of the automatic control or by adjusting the settings of the automatic control.

(5) The motion of the plane, when under automatic control, must be stable and nonoscillatory.

If we consider the problem of the pilotless airplane or controlled missile, then the automatic control may be asked to perform several other functions. For example:

(1) Receive intelligence from a radio signal, or some other electrical device, and interpret this intelligence to change the settings of the automatic pilot.

(2) Perform some readjustments of the automatic pilot at certain time intervals.

(3) Measure the velocity of the plane or missile and use this information in the adjustment of the automatic pilot.

(4) Control the engine or a war-head fuse or a homing mechanism in response to some signal, either internal or external.

In the light of these and several other possible functions, and in view of the fact that there may be serious space, weight, and acceleration limitations, it is apparent that the automatic control for a pilotless device will be a much more difficult problem than for a piloted airplane. Therefore, although satisfactory automatic pilots are now in use, it must not be assumed that automatic control of pilotless planes and missiles is an accomplished fact. Indeed, this field will be of extreme importance for future research. Automatic control will make the fantastic weapons of the next war possible. We now know the answers to only the simplest control problems. We must, by experimental research, find the solutions for the supersonic, long-range weapons of the future.

SOME STABILITY CONSIDERATIONS

In order to simplify the discussion, a system with one degree of freedom will be considered. After the system is displaced from its position of equilibrium, we may write a differential equation for the motion in the form:

$$A\ddot{\chi} + B\dot{\chi} + C\chi = O$$

where χ is a measure of the displacement, and the coefficients A, B and C are normally all positive. As is well known, the resulting motion is damped, and is nonoscillatory if:

$$B^2 > 4AC$$

Let an automatic control device be added to the system, and first let us suppose that the restoring force produced by the automatic device is proportional to the displacement χ. Then we have

$$A\ddot{\chi} + B\dot{\chi} + (C + C')\chi = O \qquad (1)$$

And, if $C'\chi$ is large enough, it is obvious that a nonoscillatory motion will be changed to an oscillating motion.

In any physical problem the situation would actually be worse, because there would be a time delay, Δ t, between the application of the intelligence to the automatic control and the appearance of the restoring force. Hence we should write the restoring force as:

$$C'\chi(t - \Delta t) = C'\left(\chi - \Delta t \frac{d\chi}{dt}\right)$$

so that the equation becomes:

$$A\ddot{\chi} + \left(B - C'\Delta t\right)\dot{\chi} + \left(C + C'\right)\chi = 0 \qquad (2)$$

And we see that not only may the motion become oscillatory, but also it may become unstable if:

$$B - C'\Delta t < 0.$$

Stated in words, we have the following conclusions:

The application of an automatic control giving a restoring force proportional to the displacement of the system from its position of equilibrium will result in an originally stable system becoming unstable if (a) the magnitude of the controlled restoring force constant is too large, and (b) the time delays in the control system are too long. In all cases the control exerts a destabilizing effect on the system. Clearly such a control is undesirable.

To improve the situation, let the automatic control give a restoring force proportional not only to the displacement of the system, but also to the first derivative of this displacement. The equation is then:

$$\left(A - B'\Delta t\right)\ddot{\chi} + \left(B + B' - C'\Delta t\right)\dot{\chi} + \left(C + C'\right)\chi = 0 \qquad (3)$$

And we have again stable, damped motion if:

$$B + B' - C'\Delta t > 0 \qquad (4)$$

$$\text{and } \left(B + B' - C'\Delta t\right)^2 > 4\left(A - B'\Delta t\right)\left(C + C'\right). \qquad (5)$$

Here we assume the same time delay in both the C' and B' functions, and it is asusmed that the control action is not sufficient to reverse the sign of the second derivative term.

This type of control is much more satisfactory. In most practical cases it is necessary to add the derivative term to prevent hunting or instability. It is called "rate control," or sometimes, incorrectly, "anticipation control."

Some practical control systems also add a control force proportional to the second derivative of the displacement or to the integral of the displacement.

It is interesting to consider a system in which $C' \equiv O$ and only B' is present. Clearly the system is stable, and the stability is greater with control than without it. Such a system is useful where a very simple control is required. It has the disadvantage that there is no fixed value for the position of equilibrium. For example, if it were used to control the azimuth heading of an airplane, the flight would be stable, but would tend to drift away from the correct heading. This is a case where it could not be used, but it does have applications to controlled missiles, where the flight is of short duration or is being continuously corrected by external means.

The preceding discussion is obviously oversimplified in terms of airplane control. Clearly additional degrees of freedom and coupling between them will make the situation very complex. However, we can generalize our conclusion and state that:

An automatic control will have a stabilizing effect on the motion only if it contains provision for a restoring force proportional to the first derivative of the error with respect to time. The larger the time delays in the system, or the stiffer the control, the greater must be this force.

It should be pointed out that such a control system may make an unstable system stable. In all cases, the stability of the system with control will be better than without control. It is essential, then, that all airplanes and missiles using automatic control be investigated for stability with the control operating. In many cases the natural period and the damping of the system will be almost completely determined by the characteristics of the control.

ELEMENTS OF AN AUTOMATIC CONTROL

In Fig. 1, the essential components for a complete automatic control are shown diagrammatically.

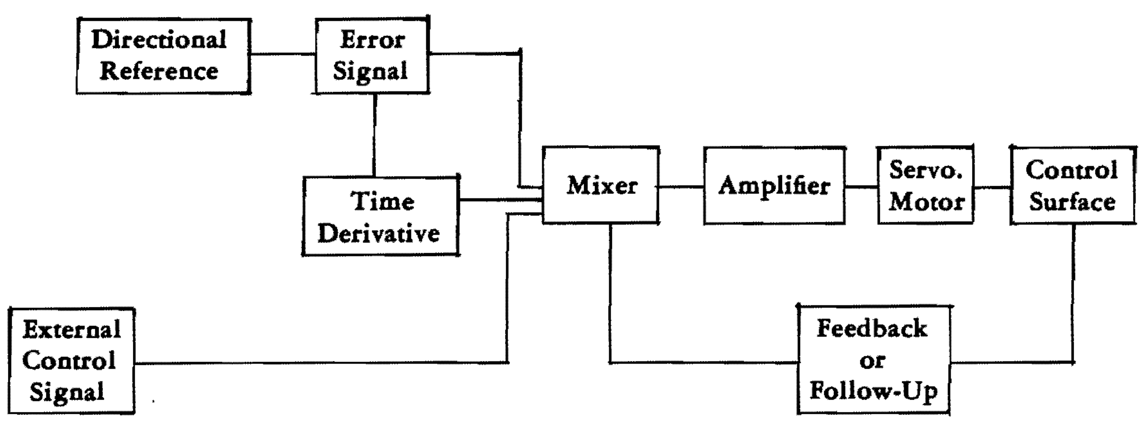

Figure 1

The designations are obvious, but it is necessary to discuss the feedback or follow-up part of the circuit.

The purpose of this feedback is to make the motion of the control surface proportional to the error signal. Without feedback a small error signal would result in the control surface being driven violently into the hard-over position. Such a system, sometimes called a "bang-bang" control, is not necessarily undesirable and has been used in some controlled missiles. Investigations of the stability of these systems have been made in Germany at DFS and DVL.*

When feedback is present, the signal into the amplifier is large enough to run the servomotor at full speed until the control surface has been moved through an angle

* "Survey of Facilities in Germany for Development of Guided Missiles," Part IV, Klemperer; also Report WBK/292.

determined by the feed-back loop. At this angle the feed-back signal just balances the input signal and the motor stops. The principle is familiar enough in electrical circuits and indeed it is instructive to analyze the system from the point of view of amplifier theory.

Let the complete system be regarded as a device in which an input signal D_i produces an output signal D_o given by: $D_o = A\ D_i$, where A in general is not a pure number, but involves a time lag. Let the feed-back loop introduce a signal βD_o at the input. Then with feedback present

$$D_o = A\ (D_i + \beta D_o)$$

$$\therefore D_o = \frac{A}{1 - A\beta} \cdot D_i \qquad (6)$$

and again β is not necessarily a pure number.

If D_i is supposed sinusoidal with time, then we can use the complex numbers of the electrical engineers and take over their results. The most important are the following:

The system is stable provided the plot of $A\ \beta$ in the complex plane with the frequency ω as parameter, does not include the point 1.0.

Negative feedback, i.e., $|1\text{-}A\beta| > 1$ decreases the effective amplification of the system, but increases its stability against internal changes. If $|A\beta| \gg 1$ then the over-all amplification becomes simply $(-^1/\beta)$ and is independent of the value of A.

With a mechanical system we may frequently regard the time lags as constant. This means that, in terms of a sinusoidal frequency ω, the phase angle equivalent to the time delay is proportional to ω. Therefore $A\ \beta$ will alternate between positive and negative values as ω increases. Consequently the system will tend to be unstable unless A is so chosen as to decrease rapidly as ω increases. It is easy to show that simply increasing the magnitude of β may be enough to make a stable system unstable.

Thus, there are two possibilities of making the controlled system unstable. The first, given by equation (3), is the instability of the entire system because of too large a spring constant or too long time delays in the control. The second, given by equation (6), is the internal instability of the amplifier and servo system. The first gives rise to a relatively long-period oscillation of the complete body with the controls apparently acting normally. The second is a rapid oscillation of the control surfaces with the accompanying body oscillations of small amplitude.

The second result given above is important because it makes the characteristics of the system dependent on the feedback β and not on the amplifiers and servos. This feedback is frequently a simple cable connection from control surface to mixer and is hence as reliable and as constant as possible. Furthermore, if it is desired to change the control characteristics, as for example to compensate for a change of speed or air density, the feedback loop is the best place to make the change.

The feedback is usually designed to give a signal proportional to the angular deflection of the control surface. This is called position feedback or position follow-up. However, it should be pointed out that such an arrangement is not necessarily the best. It will produce a constant control surface deflection for a constant error signal, regardless of velocity or air density. In a controlled missile, where these quantities may be varying over very wide ranges, the accompanying changes in the dynamical characteristics of the missile may be very serious. One solution has already been mentioned, to make β a function of velocity or density. Another possibility is to make the feedback signal proportional to control surface force instead of position. We then have "force feedback."

To sum up, the important parameters in the control system, as far as stability is concerned, are (1) the time delays in the system, (2) the relative amounts of error and error derivative signals applied to the amplifier, and (3) the nature and magnitude of the feedback or follow-up signal.

In a piloted airplane adjustments of these parameters, usually (2), can be made during flight, to keep the automatic control operating smoothly. In the pilotless airplane or missile it may be necessary to make such adjustments either by signals over the control link, or by some internal means. Unfortunately complete aerodynamic data on the missiles is usually missing. The control must then be designed as best it may and test flights with telemetering equipment relied upon to give the final design.

COMPONENTS OF CONTROL SYSTEMS

In considering future developments in automatic controls, there are two fields to be investigated.

(1) Improvement in components of existing automatic pilots and development of new components.

(2) Adaptation of control techniques and components to pilotless airplanes and missiles.

It can be asserted that the theoretical principles necessary for an understanding of control problems are well understood. The research problems are primarily experimental, except in so far as it is necessary to include aerodynamical information in the design equations of the control system. The point cannot be too strongly stressed that the whole control problem must be treated as a unit in any plane or missile relying on automatic control. In the past the control has been added after the plane has been designed, and the control system characteristics calculated to fit the known flight characteristics. Although this will result in stable flight, it does not follow that it is the best solution. It is assuming that the automatic pilot has the same reactions as the human pilot, surely a poor assumption.

All automatic pilots must start with a reference direction. It is universal practice to use a gyroscope to fix this direction. The modern gyroscope is a very satisfactory device, albeit rather expensive to make, and subject to difficulties from dust and dirt. If used for very long times or while moving over an appreciable part of the earth's surface, a primary reference such as the magnetic compass or the average position of a pendulum must be used to correct the gyroscope. However, it is difficult to see what could take the place of the gyroscope as an internal reference direction. Even in controlled missiles the gyroscope would appear to be the best solution. Here, of course, special types of gyroscopes will be necessary. The specifications may include the following:

(1) Ability to stand high accelerations, perhaps of the order of 100 g.

(2) Ability to keep their proper orientation through all possible maneuvers of the missile.

(3) Ability to stand excessive vibration or noise.

At the same time they should be constructed for single use, so that some economies in design are possible, although the first requirement apparently contradicts this statement.

Some recent work in Germany by Stinshoff at DFS has indicated that noise or vibration introducing forces through the bearings may noticeably affect the motion of a gyroscope. This may become an important factor in supersonic missiles and should be investigated further.

Motive power for gyroscopes is either electric or pneumatic, with the present trend toward electric operation. This has the advantage of constant speed and relatively high available power. It has the disadvantage of requiring electric connections to the inner gimbal ring, if not to the rotor.

Some guided missile applications may more conveniently use pneumatic gyros. In this connection the German allegedly developed a gyro which was driven by the gas from a burning powder charge. This supplied a very convenient form of energy to operate the gyro during the single short flight of a missile.

Before leaving the subject of gyros, brief mention will be made of "rate" gyros. It is well known that if the axis of rotation of a gyroscope is rotated about an axis normal to the gyro axis, a precessional torque will be set up whose magnitude is proportional to the rate of rotation. A gyroscope designed to utilize this torque is known as a rate gyro. Rate gyros are much simpler mechanically than free gyros. They can be used to give the time derivative signal needed in the control system, or, as previously pointed out, they can serve as the main reference in a control system, particularly in controlled missiles.

A substitute for the gyroscope would have to be a device which is simpler to build and more reliable in operation than existing gyroscopes. Furthermore, it would have to be possible to obtain error signals from it without disturbing its reference direction. The only device at present showing much promise is a Foucault pendulum. In practice this would take the form of a vibrating reed. Such a device might conceivably be used in place of a rate gyro without much further development. However, the simplicity of the rate gyro makes the substitution unnecessary, except possibly in guided missiles. Future development of the pendulum may make it a serious competitor of the free gyro.

The design of the remainder of the control system is dependent on the kind of power to be used. Pneumatic, electric and hydraulic systems have all been built. The present trend is toward all-electric systems. This is probably best for piloted airplanes, particularly very large airplanes. However, again we find the controlled missiles introducing some special requirements: (1) limited space and weight for a power supply; (2) operation for a short time only; (3) no radio interference, in spite of limited shielding; (4) operation at extreme altitudes; and (5) operation with minimum time delays. Electric systems are not necessarily the best to fulfill these requirements.

Most of the control power is consumed at the servomotor, and indeed, in missiles of the V-2 type this power may exceed that required by a large airplane. Electric motors capable of delivering the power, even over short periods of time, are bound to have a relatively large ratio of moment of inertia to torque and consequently will be sluggish in operation. On the other hand, hydraulic or pneumatic motors, of either the piston or the rotor types, are much faster acting and can be controlled positively and certainly by very small amounts of power.

The primary sources of power in these cases are normally electric storage batteries, compressed air, and high-pressure liquid, usually with an electric motor driving a pump to maintain the pressure. On either a weight or a volume basis, the storage battery is far more efficient than the compressed gas. The hydraulic system with a pump also obtains power from the storage battery, the pump serving to smooth out the power drain and to avoid the use of electric servomotors.

An alternative power source for the pneumatic system would be to use the gas produced in the combustion of solid or liquid fuels. Zippermayer in Austria has operated a small turbine in this way. This would avoid the necessity for a large high-pressure storage tank and would make the pneumatic system more comparable to the hydraulic system. It could then compete favorably with either hydraulic or electric systems. The space and weight requirements are difficult to estimate, but they are probably comparable with the storage battery.

The complete system is not necessarily operated from one type of power. Electric amplifiers followed by hydraulic servomotors are common. Both hydraulic and pneumatic mixers and amplifiers can be built. Differentiation of the error signal may be mechanical or electrical. Considering the problems of construction, packaging, and adjusting, an electric system is the most versatile and is, therefore, the best solution for most guided missiles. This statement is made with full cognizance of the fact that failure of a single vacuum tube or other part will put the system completely out of operation. Pneumatic and hydraulic systems are equally vulnerable to leaks and dirt. In all cases good engineering design can minimize the probability of failure. However, this should not be construed as meaning that all research should be concentrated on electric systems. As a matter of fact, some very elegant pneumatic systems have been built. They have a minimum of parts and can be made surprisingly versatile.

As indicated above, the servomotor for a controlled missile should probably be hydraulic or pneumatic. The motor is the chief cause of time delays in the system and therefore, must be designed to be as fast as possible in its operation. It must have a very small inertia, so that it does not coast after power is removed. High-speed missiles may require the motor to operate the controls from hard left to hard right in about 0.1 second.

If a pneumatic system is used the motor must be of the rotary type. With a piston moving in a cylinder, the compressibility of the air would give a very undesirable cushioning effect.

During the war there has been a tremendous application of servo systems, not only to the problem of flight control, but also to fire control, radar, and numerous other control problems. Much of the research in this field has been aimed at improving the servomotors as far as power output and speed of response were concerned, and designing the amplifiers necessary to control the motors. One very useful development was the amplidyne, which gave a tremendous power amplification with a minimum of parts and space. These servo developments should not be overlooked in future plans for automatic pilots, particularly for piloted airplanes. The "one-shot" pilotless planes and missiles will probably always need systems especially designed to their needs.

"BANG-BANG" CONTROL

It has already been mentioned that stable control is possible with a system in which the control surfaces are continually oscillating between their extreme positions. This type of control has been used to stabilize the Azon bomb in roll, but it is not very popular in this country. In Germany, on the other hand, it was used very widely. The basic idea seems to have been due to Dr. Max Kramer, who devised the oscillating "spoiler." The advantages of the system are as follows: (1) The spoiler requires practically no power to operate it; (2) the time of response can be made very small; and (3) the operating mechanism is much simpler than the conventional servo system.

A novel type of spoiler was proposed by the Aachen group. Air taken in at the front of an airfoil was discharged on one side or the other near the trailing edge. The resulting disturbance in the air flow provided the desired lateral force. Wind-tunnel tests were made and showed considerable promise. From a control point of view this would constitute an excellent system, as the only power required would be that to operate the valves controlling the flow of air through the airfoil.

In view of the German experience with spoilers, further research on the technique should be initiated, particularly for the smaller controlled missiles, and for those not needing large lateral control forces.

EXTERNAL CONTROL SIGNALS

In the systems so far discussed, the signal actuating the control is assumed to originate at a gyroscope which forms an intimate part of the control loop. The exact technique by which motion of the gyroscope is converted into an equivalent electric or pneumatic signal has been omitted as being a technical problem of practical interest, but adequately solved. However, other signals must be added to the amplifier input; for example, there is the problem of banking during a turn. This requires a roll signal when an azimuth correction is made. Some automatic pilots do this automatically; in a pilotless plane of conventional design it would be essential. Likewise, changes of air speed and air density affect the dynamical constants of the airplane. With suitable instruments the speed and density can be measured and the results fed into the automatic pilot as a change in the control constants. The procedure is relatively straightforward, particularly with electric systems.

The controlled missiles will normally receive flight instructions from a radio signal. These will again appear as electric, or possibly pneumatic, signals which will be fed into the correct control circuit. A variety of methods for selecting the correct circuit are possible. Perhaps the most popular is that depending on the choice of a particular audiotone or combination of tones.

Future missiles may be guided to the vicinity of the target by a remote radio signal, and then turned over to a homing mechanism. Again several techniques are possible, the only requirements being that the homing mechanism be put into the circuit in place of the radio, and that the correct type of signal be provided.

It should be emphasized in this discussion that the signals required to control the automatic pilot have very little energy so that there is no particular problem in taking them from a radio receiver or a sensitive homing device.

An interesting type of external control might be used on a missile of the V-1 type. This would consist in making it fly an erratic or jinking course to avoid being shot down. The maneuvers could be initiated by a clockwork mechanism which put in a predetermined program of turns.

On very long pilotless flights the problem of altitude control will come up. The gyroscope alone cannot be depended upon to fly the plane at a constant altitude; the accuracy required to keep within say 1000 feet in 100 miles is too great. Therefore, an altimeter must be used. Either radio or barometric types could be used, with the barometric preferred for simplicity. The reading of the altimeter will be converted into an electric signal and made a part of the correct control loop. If necessary, the flying altitudes could easily be set to a predetermined program.

One final comment on external control: Once the principle of the overriding signal readjusting the automatic control has been accepted, it is interesting to carry it to the logical conclusion, completely automatic flight from airport to airport. There is no inherent difficulty in this, and indeed the British have demonstrated a "black box" which will fly an airplane from take-off to landing.

CONTROL SYSTEMS FOR MISSILES

Various aspects of the controlled missile problem have already been discussed. We shall now consider the complete systems. For this purpose we may consider the following types of missiles: (1) very long-range missiles of the V-2 type; (2) very long-range missiles of the V-1 type; (3) antiaircraft missiles; (4) controlled bombs; and (5) air-to-air missiles.

The V-2 type of missile needs an automatic pilot to get it started on the correct trajectory. In addition it should have a means of stopping the thrust at the correct instant, or else provision to map its course during the flight, either on the ground or in the missile, and apply suitable corrections. It is interesting to note that the Germans claim that under good conditions, an accuracy of one in 1000 is possible with their system of controlling the missile up to the point of the end of burning.

The V-1 will operate with a standard automatic pilot with altitude control and possibly programmed jinking or overriding radio control. There is also the possibility of adding television reporting to the ground or some type of homing.

The antiaircraft missiles present the most difficult control problem. There is usually a high initial acceleration, the lateral acceleration produced by the controls must be large, and the trajectory must be under very tight control, either from the ground or from a homing head. In the past war neither side produced a satisfactory antiaircraft controlled missile.

The basic control elements will depend upon the wing design and the type of trajectory. For example, the Germans developed a polar coordinate control, "Burgund."* Here the rocket was to be a "beam climber," remaining on the line of sight from ground to target, and the corrections to the flight were in terms of the polar coordinates of the apparent deviation from the true trajectory as observed on the ground. Only elevators and roll control were provided on the rocket. By signaling the roll position, as well as the elevator position, it could be maneuvered in any direction.

With a cross-winged missile which does not need to bank in order to make a turn, the problem is quite different.

In some cases a gyroscope in the missile can be "asked" to remember which way is up and to connect the elevator and rudder signals to the appropriate control surfaces.

These missiles will all contain a basic automatic pilot to enable them to fly a straight course. However, as stated earlier, they may be controlled from rate gyros instead of free gyros, because of the tight external control.

Controlled bombs offer a simpler application of homing techniques than the antiaircraft missiles. Something akin to the Baka bomb, with a good homing head, would be an excellent weapon against shipping. The control requirements are simply those of the automatic pilot, with provision for overriding by signals from the homing device.

Air-to-air missiles have been given a separate heading largely because of the German X-4 wire-controlled missile. This apparently impractical idea works, and, therefore should not be neglected in a consideration of short-range missiles. It has the advantage of requiring no radio link, thereby simplifying the missile equipment and being absolutely free from jamming. It has the disadvantage that, if used over friendly territory, it leaves miles of steel wire over the countryside to cause trouble.

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* "Burgund control for "Schmetterling." Intelligence Report GDM-1 Intelligence Branch T/1 OC Signal O ETO USA.

CONCLUSIONS

Automatic pilots for conventional piloted airplanes have reached the stage of development where future research will probably be aimed primarily at engineering improvements, unless something is discovered that will satisfactorily replace the gyroscope. If larger and faster planes need more power in the controls, then we may expect to see the amplidyne type of servo system used. In any case, the relative merits of electric and hydraulic servos, and indeed, entire systems, will continue to be debated. Pneumatic systems and servos might come back into favor, but it is not very probable.

The biggest problems in automatic control lie in the missile field, particularly the antiaircraft missile. When we have successfully solved the problem of a missile which will reliably shoot down a supersonic bomber, most other control problems will look easy. In this development, the special requirements on the control will make it quite unlike the automatic pilot. Electric, pneumatic, and hydraulic systems are all possible, but the requirements of high speed of response and high power seem to indicate that the servomotor shall not be electric. Rate gyros may be used instead of free gyros, particularly where a high take-off acceleration occurs. Special sources of power, such as the use of the products of combustion to drive pneumatic servos, may be developed. Special techniques for interpreting external control signals and feeding them into the control loop are necessary. And finally, of course, in addition to the control itself, there are the problems of the ground computing mechanism, the radio link, and the homing mechanism.

When missiles or airplanes are being designed for automatic control, it is necessary that the characteristics of the automatic system be included in all the calculations of stability and performance. Automatic control can make an unstable system stable, or vice versa. In other words, the natural period and damping of an airplane under tight automatic control is almost entirely determined by the characteristics of the control system.

A logical development of automatic control with external intelligence is the airplane that lands automatically. This is technically possible, has been demonstrated, and certainly should not be overlooked in plans for future research.

RECOMMENDATIONS

1. Automatic control systems are an integral part of future planes and missiles and must be treated as such, even in the early stages of design.

2. Studies and calculations of the dynamical stability of controlled systems are particularly necessary for the controlled missiles. Artificial missiles, or simulators, are a vital adjunct to such study.

3. The solution of the guided antiaircraft missile problem should receive the highest priority.

4. The search for a substitute for the gyroscope, particularly for use in missiles, should be pressed.

5. Investigations of high-speed, low-inertia, high-power servomotors should continue. Hydraulic and pneumatic motors must not be overlooked in favor of electric motors.

6. Sources of power for operating missile servos must be investigated, particularly the possibility of using the products of combustion to pressurize hydraulic systems or to operate pneumatic systems.

7. Research on conventional automatic pilots should be directed toward engineering improvements and should include the problem of designing to minimize pilot fatigue.

8. Automatic flight control should be developed to the end of obtaining a "black box" which will not only fly the airplane on its correct course, but which will also find the airport and land the plane.

PART III

THE LAUNCHING OF A WINGED MISSILE FOR SUPERSONIC FLIGHT

By

H. S. TSIEN

PART III

THE LAUNCHING OF A WINGED MISSILE FOR SUPERSONIC FLIGHT

8 APRIL 1945

STATEMENT OF THE PROBLEM

Recent investigations of the Scientific Advisory Group of the AAF Headquarters have shown that the range of a jet-propelled supersonic winged missile could be of the order of one to two thousand miles, if the lift over drag ratio of the missile can be made to be higher than three. With the present available data on the supersonic flows over shell bodies and airfoil, it is estimated that such values of the lift-drag ratio can be achieved at a flight Mach number of 1.5 to 2.0. The analysis further shows that for thermal-jet-propelled missiles using atmospheric air there is no particular avdantage in going to extreme altitudes.

Since the performance of such winged missiles is studied on the basis of straight level flight without any consideration of how the flying altitude and the flying velocity are reached, it is natural to investigate the launching problem as a sequel to the calculation. In other words, *the problem is to estimate the time and the fuel weight necessary to reach a certain altitude and velocity from ground level and from rest.*

METHOD OF SOLUTION

There are infinitely many varieties of the flight path to reach a given altitude and a given speed. The present method of launching V-1 type flying bombs is to accelerate the missile along essential level track till the missile is airborne and further acceleration and climb are carried out in a flight path of very small inclination as shown in Fig. 1. To apply the same launching method to the supersonic winged missiles under consideration has several disadvantages: Due to the fact that the missile is designed for efficient operation at a Mach number of 1.5 or 2, the wing area, is very small or the wing loading in the lb/sq ft is very high. Since the maximum lift coefficient of any airfoil is approximately equal to unity, the high wing loading results in a very high

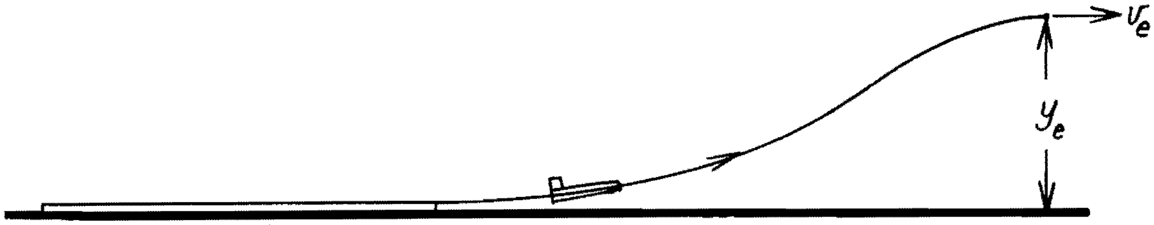


Figure 1 — Launching Path of V-1 Flying Bomb

take-off speed, or the speed at which the missile is airborne. Therefore, if the missile is to be launched similar to the present V-1 flying bomb, the length of the launching track would be excessively long. For instance, with a take-off of 700 ft/sec, and a launching time of 1 sec, the track length would be 350 ft. With the same take-off speed, but with the launching time increased to 2 sec, the track length would be 700 ft. The acceleration of the first case is 22 g and the second case 11 g. Therefore, if one assumes such long tracks are practical, the extremely large acceleration will lead to structural difficulties. Furthermore, after the missile is airborne, it still has to accelerate to pass the sonic velocity and reach the supersonic flight speed. It is known that the aerodynamic characteristics of a winged body are very complicated, and subject to drastic changes at the transonic speeds. Hence, the problem of controlling and stabilizing the missile during this transition region in flight velocity can be very difficult, if not impossible.

An alternate method of launching is patterned after the V-2 rocket missile. In this scheme, the missile is set up in a vertical position and launching takes place initially in a vertical flight path, as shown in Fig. 2. The acceleration is kept to approximately

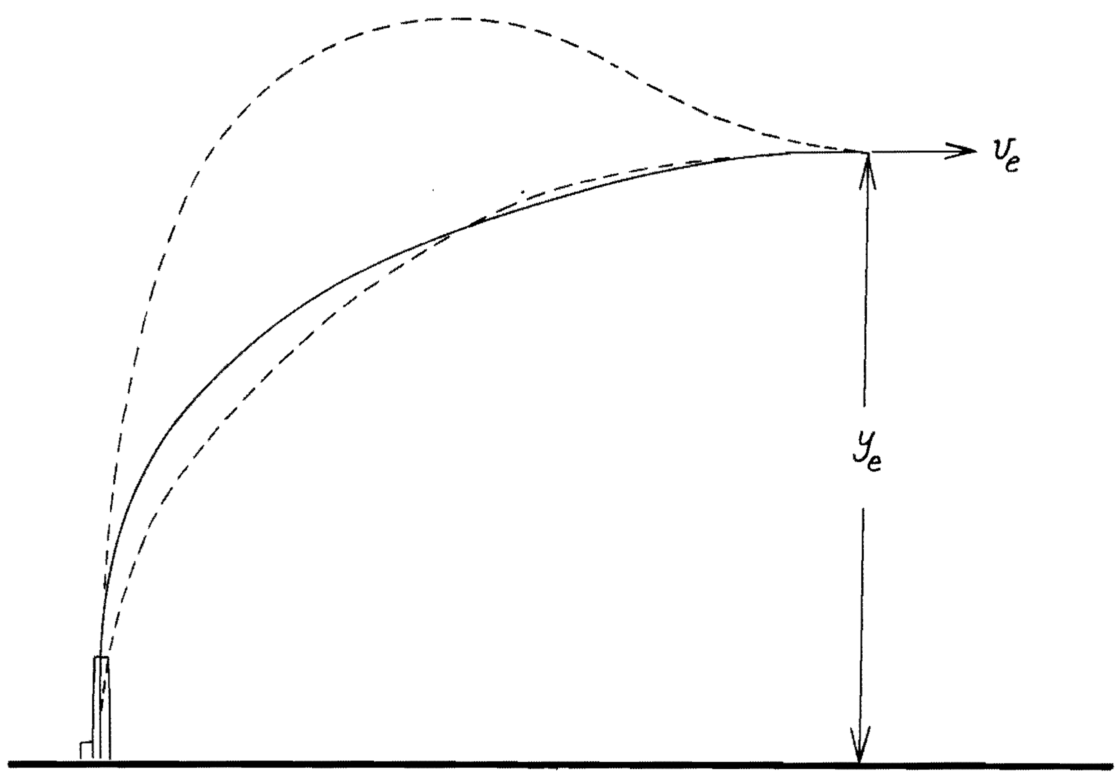


Figure 2 — Proposed Launching Path for a Supersonic Missile

1 g at the beginning, and is gradually increased to 2 g at the end of the launching path. Thus, the structural difficulty due to high acceleration is avoided. Furthermore, since a launching track is not necessary, the installation is highly mobile. As the sonic flight velocity is passed very rapidly in an almost vertical trajectory, the difficulty of stabilization and control is minimized. In fact, the successful performance of the rocket missile indicates that this method of launching is entirely feasible.

The present calculation is carried out under the assumption of such a launching scheme. However, a few additional assumptions are made in order to simplify the numerical work; they are:

(a) The missile is so controlled that the change of the angle of inclination θ (Fig. 3) is related to the time t by

$$\theta = \frac{\pi}{2} - \omega t^2 \qquad (1)$$

where ω is a constant. Thus, the missile has a uniform angular acceleration.

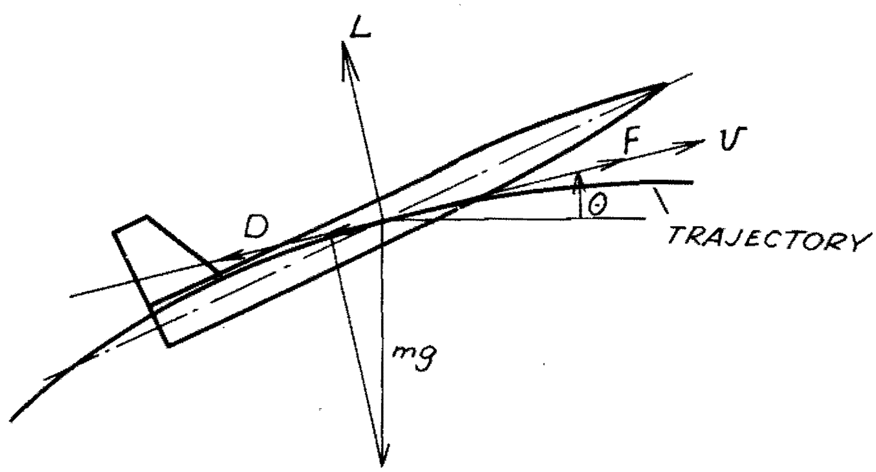


Figure 3 — Force Balance for Trajectory Calculation

(b) The thrust F is assumed to be obtained from a rocket motor or a system of rocket motors so that its value is a constant.

(c) The drag D is assumed to be composed of two parts and

$$D = D_0 + \epsilon L \qquad (2)$$

where L is the lift force and ϵ is a constant. D_0 is again assumed to be a constant or a constant fraction of the thrust F.

The significance of these assumptions will be discussed presently. The assumption (a) means the selection of a particular family of launching paths as the subject of investigation, i.e., the family of flight paths with constant angular acceleration. The best launching path has to be determined, of course, under less restricted conditions provided that the initial trajectory is vertical, and the final velocity and altitude agree with the required values. For instance, all the dashed curves in Fig. 2 could be used as the launching path and the best path is the one which requires the least amount of rocket fuel. However, it is believed that the variation of the fuel consumption for different paths will not be large and for the present purpose of estimating rather than designing the launching of the missile, the simplifying assumption (a) is justified.

The assumption (b) means negligence of the variation of thrust with altitude and the adoption of a constant propellant rate for the rocket motors. This assumption is usually made and is a reasonable choice.

The assumption (c) is the result of several considerations. Actually the drag due to lift is not a constant but is a function of both flight Mach number and the lift coefficient. The assumption of a constant ratio means that an average value for the whole range of flight paths is taken. This is, of course, an approximation. However, with the present lack of aerodynamic data at the transonic speeds, the use of such an average value is not beyond the accuracy of the method, and should be sufficient for the present purpose. The profile drag D_o of the missile is taken to be constant. Again the averaging method is adopted. However, although the speed of the missile is increasing very rapidly with altitude, while the density of air is *decreasing* with altitude, the drag, being an increasing function of speed and of air density, does not vary as rapidly as was first expected. Furthermore, due to the assumption of constant ratio of D_o/F, the profile drag is increased with increase in thrust. Since an increase in thrust means that faster acceleration and thus higher speed is reached at lower altitude where the air density is larger, the resultant higher drag is accounted for by such an assumption.

RESULTS OF CALCULATION

The detailed mathematical analysis of this problem with the assumptions given in the previous section is explained in the Appendix. Here only the result will be discussed. As a concrete example, the effective exhaust velocity of the rocket is taken as 6400 ft/sec, a value easily obtained by the present rocket propellant. The ratio of D_o/F is taken as 0.1, i.e., 10% of the thrust of the rocket is lost to the profile drag. The drag due to lift is taken as $\frac{2}{\pi}$ times the lift. This value is also quite conservative. The result of the calculation is shown in Fig. 4. At the end of the launching the flight path is assumed to be horizontal, so that it joins smoothly with the level flight part of the trajectory. In Fig. 4, the altitude at the end of the launching path is plotted against the velocity at the end of the path. The parameters are the total time T of launching flight and the weight fraction ζ of the rocket propellant required to the initial total weight of the missile with the rocket power plant. The curves of constant weight fraction ζ have a peak value in altitude which corresponds to the most efficient usage of the rocket, characterized by a certain time of flight.

To show the use of this chart, take $\zeta = 0.3$, then the highest value of altitude reached is 10,800 ft with velocity 2200 ft/sec. The time of flight is then approxi-

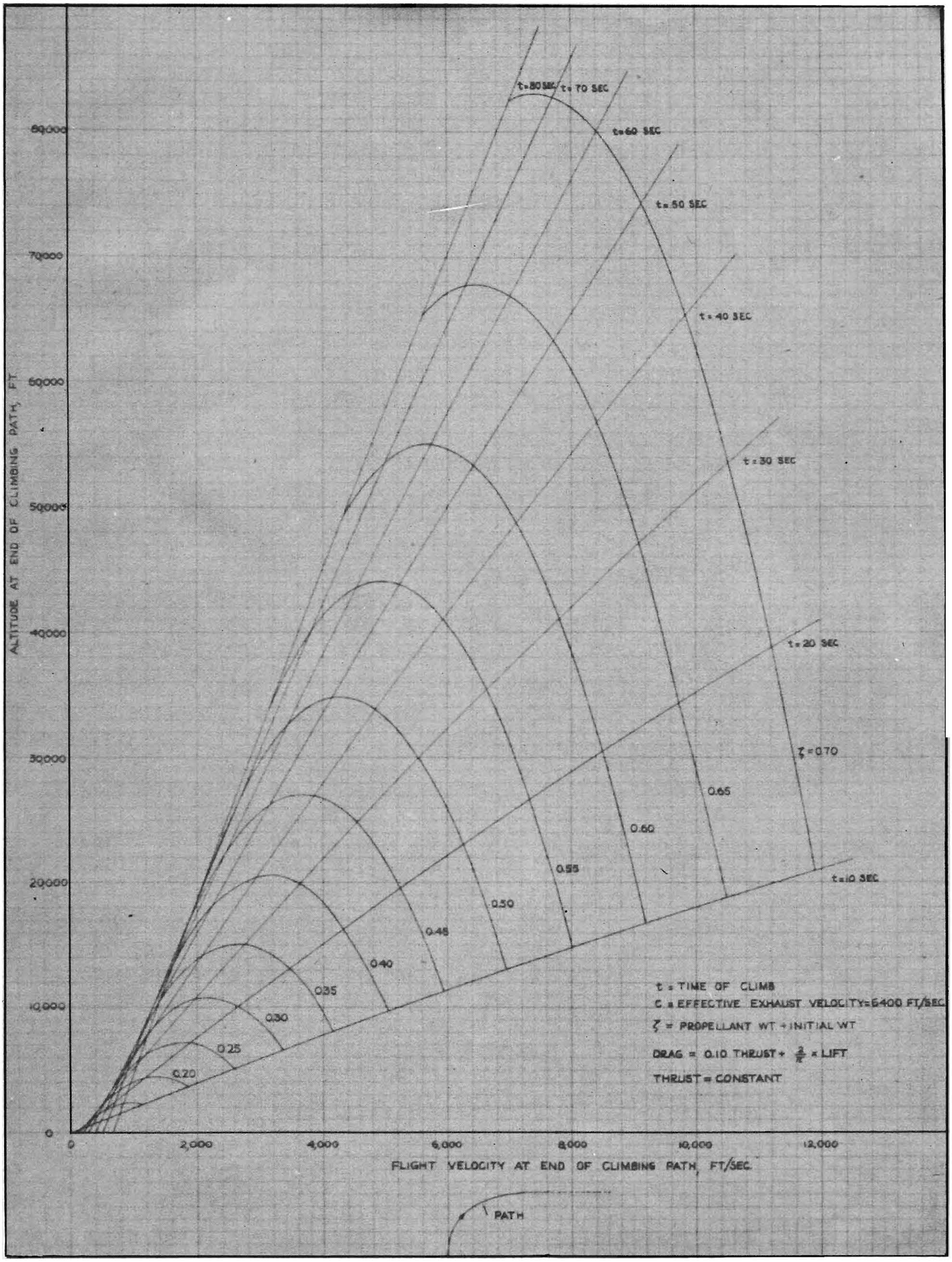


Figure 4 — Chart for Propellant Weight Fraction ζ

mately 27 sec. If the initial total weight of the missile is W_o, the propellant weight is $0.3\ W_o$. The propellant rate is $0.3\ W_o/27$. The thrust of the rocket is then

$$F = \frac{0.3W_o}{27} \cdot \frac{6400}{32.2} \tag{3}$$

$$\text{Hence } \frac{F}{W_o} = 2.21$$

If the time of flight is lengthened, the ratio of thrust to the initial weight will be decreased. If the time of flight is shortened, the ratio of thrust to the initial weight will be increased. Hence, the optimum value of the time of flight really corresponds to the optimum value of acceleration. The above calculated optimum thrust-weight ratio checks very closely with the optimum weight ratio for sounding rockets and V-2 type rocket missiles, and thus may be considered as a substantiation of the various simplifying assumptions used in the present analysis.

Since the end altitude of 10,800 ft and the end velocity of 2200 ft/sec correspond very closely to the values required for the supersonic missile under consideration, it may be profitable to pursue further the calculation. If the main power plant for the propulsion of the missile in level flight is turbojet, the propellant pumps for the launching rocket can be driven through the turbojet shaft. It is then reasonable to assume the weight of the rocket power plant to be 10% of the rocket propellant weight, or $0.03\ W_o$. This additional weight will be mostly in the propellant tanks, and in an actual design will be attached to the missile, and will be dropped at the end of the launching flight. Therefore, at the beginning of the level flight, the weight of the missile proper is $W_o - 0.30W_o - 0.03W_o$ or $0.67W_o$. If the weight of the missile proper is denoted by W_1, then for reaching 10,800 ft altitude and 2200 ft/sec velocity and the end of launching, the initial weight is $1.49W_1$, the propellant weight is $0.457W_1$, and the weight dropped $0.046W_1$.

From the weight ratios calculated above, it is seen that the system of the launching rocket and the missile proper is not unlike that of a step rocket. If the rocket-propellant consumption could be lowered, there would be a considerable reduction in the initial weight of the complete missile. Hence, here again the situation is similar to that of a long-range rocket missile, and the emphasis on rocket research should be directed to the increase in the specific impulse of the propellant for lowering the necessary propellant weight.

APPENDIX

MATHEMATICAL ANALYSIS OF THE LAUNCHING PATH

Let m be the mass of the missile at the time instant t when the velocity is v, the inclination of the trajectory is θ, the lift is L and the drag is D. The force balance is shown in Fig. 3. The differential equations of motion are then

$$m \frac{dv}{dt} = F - D - mg \sin\theta \tag{4}$$

$$mv\frac{d\theta}{dt} = -\ mg\cos\theta + L \tag{5}$$

According to Eq. (1) $\qquad \theta = \frac{\pi}{2}\,\omega t^2$

Therefore, $\qquad \sin\theta = \cos\omega t^2 = 1 - \frac{1}{2!}\omega^2t^4 + \frac{1}{4!}\omega^4t^8 - \frac{1}{6!}\omega^6t^{12} + \ldots \qquad (6)$

$$\cos\theta = \sin\omega t^2 = wt^2 - \frac{1}{3!}\omega^3t^6 + \frac{1}{5!}\omega^5t^{10} - \frac{1}{7!}\omega^7t^{14} + \ldots \tag{7}$$

The thrust F is a constant, but the mass m is a variable due to the continuous discharge of rocket propellant. If κ is the rate of mass discharge, them $m = m_o - \kappa\tau$ where m_o is the initial mass. By eliminating D by Equations (5) and (2), the equation for v can be written as

$$e^{\epsilon\omega t^2}\frac{d}{dt}\left(e^{-\epsilon\omega t^2}v\right) = \frac{F-D_o}{m_o}\,\frac{1}{1-\frac{\kappa}{m_o}t} - g\Big[1 + \epsilon\omega t^2 - \frac{1}{2!}\omega^2t^4 - \frac{1}{3!}\epsilon\omega^3t^6 + \frac{1}{4!}\omega^4t^8$$
$$+\frac{1}{5!}\epsilon\omega^5t^{10} - \frac{1}{6!}\omega^6t^{12} - \frac{1}{7!}\epsilon\omega^7t^{14}\ldots\ldots\Big] \tag{8}$$

If v_o is the initial velocity, or the velocity v at t = o, the integral of Equation (8) is

$$v = e^{\epsilon\omega t^2}\Big[\frac{F-D_o}{m_o}\int_o^t\frac{e^{-\epsilon\omega\xi^2}d\xi}{1-\frac{\kappa}{m_o}\xi} - g\int_o^t e^{-\epsilon\omega\xi^2}\Big(1 + \epsilon\omega\xi^2 - \frac{1}{2!}\omega^2\xi^4 - \frac{1}{3!}\epsilon\omega^3\xi^6 + \frac{1}{4!}\omega^4\xi^8$$
$$+\frac{1}{5!}\epsilon\omega^5\xi^{10} - \frac{1}{6!}\omega^6\xi^{12} - \frac{1}{7!}\epsilon\omega^7\xi^{14} + \ldots\Big)d\xi + v_o\Big] \tag{9}$$

The first integral in the above expression can be evaluated by expanding the factor $\left(1-\frac{\kappa}{m_o}\xi\right)^{-1}$ into a series and then calculating the integral term by term. The result of calculation is given in the following equation for v where $\eta = \sqrt{\epsilon\omega}$:

$$v = \frac{F-D_o}{m_o}\Bigg\{\frac{\sqrt{\pi}}{2}\,\frac{1}{\eta}\Big[1 + \frac{1}{2}\Big(\frac{\kappa}{m_o\eta}\Big)^2 + \frac{3}{4}\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \frac{15}{8}\Big(\frac{\kappa}{m_o\eta}\Big)^6 + \ldots\Big]e^{\epsilon\omega t^2}Er(\eta t)$$
$$-\frac{1}{2\eta}\Big(\frac{\kappa}{m_o\eta}\Big)\Big[\Big\{1+\Big(\frac{\kappa}{m_o\eta}\Big)^2 + 2\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\Big\} + \Big\{\Big(\frac{\kappa}{m_o\eta}\Big)^2 + 2\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\Big\}(\eta t)^2 + \Big\{2\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\Big\}\frac{(\eta t)^4}{2} + \ldots\Big]$$
$$-\frac{1}{2\eta}\Big(\frac{\kappa}{m_o\eta}\Big)^2(\eta t)\Big[\Big\{1 + \frac{3}{2}\Big(\frac{\kappa}{m_o\eta}\Big)^2 + \frac{15}{4}\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\Big\} + \Big\{\Big(\frac{\kappa}{m_o\eta}\Big)^2 + \frac{5}{2}\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\Big\}(\eta t)^2$$
$$+\Big\{\Big(\frac{\kappa}{m_o\eta}\Big)^4 + \ldots\ldots\Big\}(\eta t)^4 + \ldots\ldots\Big]\ldots\ldots\ldots\Bigg\}$$

$$-g\Bigg\{\frac{\sqrt{\pi}}{2}\,\frac{1}{\eta}\,e^{\epsilon\omega t^2}Er(\eta t)\Big[\frac{3}{2} - \frac{11}{16}\,\frac{1}{\epsilon^2} + \frac{133}{256}\,\frac{1}{\epsilon^4}\ldots\ldots\Big] - t\Big[\frac{1}{2} - \frac{11}{16}\,\frac{1}{\epsilon^2} + \frac{133}{256}\,\frac{1}{\epsilon^4}\ldots\ldots\Big]$$
$$+t\Big[\frac{11}{24}\,\frac{1}{\epsilon^2} - \frac{133}{384}\,\frac{1}{\epsilon^4} + \ldots\Big](\eta t)^2 + t\Big[\frac{1}{12}\,\frac{1}{\epsilon^2} - \frac{133}{960}\,\frac{1}{\epsilon^4}\ldots\Big](\eta t)^4$$
$$-t\Big[\frac{19}{480}\,\frac{1}{\epsilon^4}\ldots\ldots\Big](\eta t)^6 - t\Big[\frac{1}{240}\,\frac{1}{\epsilon^4}\ldots\ldots\Big](\eta t)^8 + \ldots\ldots\Bigg\}$$
$$+ v_o\,e^{\epsilon\omega t^2} \tag{10}$$

In the above expression, Er(z) denotes the error function defined by

$$Er(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-s^2} ds \tag{11}$$

The altitude y at any instant is given by

$$y = \int_0^t v \sin\theta dt = \int_0^t v \left[1 - \frac{1}{2'} \omega^2 t^4 + \frac{1}{4'} \omega^4 t^8 \ldots\ldots\right] dt \tag{12}$$

By using the expression for v given in Equation (10), y_e can be written as

$$y_e = \frac{F - D_o}{m_o \eta^2} I - \frac{g}{\eta^2} J + \frac{v_o}{\eta} K \tag{13}$$

where I, J, K, are complicated functions given by the following equations.

$$G(z) = \frac{1}{2} \sum_1^\infty \frac{z^{2n}}{n\left[\Gamma\left(n + \frac{1}{2}\right) / \Gamma\left(\frac{3}{2}\right)\right]} \tag{14}$$

$$F(z) = z \sum_0^\infty \frac{z^{2n}}{(2n+1)\, n'} \tag{15}$$

$$I = \left[1 + \frac{1}{2}\left(\frac{\kappa}{m_o\eta}\right)^2 + \frac{3}{4}\left(\frac{\kappa}{m_o\eta}\right)^4 + \frac{15}{8}\left(\frac{\kappa}{m_o\eta}\right)^6 + \ldots\right] \Big\{ G\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{3}{4\epsilon^2}\Big[\sqrt{\frac{\epsilon\pi}{2}} e^{\frac{\epsilon\pi}{2}} \frac{\sqrt{\pi}}{2} Er\left(\sqrt{\frac{\epsilon\pi}{2}}\right) \left\{\frac{1}{3}\left(\frac{\epsilon\pi}{2}\right) - \frac{1}{2}\right\}$$

$$+ \left(\frac{\epsilon\pi}{2}\right)\left\{\frac{1}{4} - \frac{1}{12}\left(\frac{\epsilon\pi}{2}\right)\right\} + \frac{1}{2} G\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\Big] + \frac{7}{48\epsilon^4}\Big[\sqrt{\frac{\epsilon\pi}{2}} e^{\frac{\epsilon\pi}{2}} \frac{\sqrt{\pi}}{2} Er\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\left\{\frac{1}{7}\left(\frac{\epsilon\pi}{2}\right)^3 - \frac{1}{2}\left(\frac{\epsilon\pi}{2}\right)^2 + \frac{5}{4}\left(\frac{\epsilon\pi}{2}\right) - \frac{15}{8}\right\}$$

$$+ \left(\frac{\epsilon\pi}{2}\right)\left\{\frac{15}{16} - \frac{5}{16}\left(\frac{\epsilon\pi}{2}\right) + \frac{1}{12}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{1}{56}\left(\frac{\epsilon\pi}{2}\right)^3\right\} + \frac{15}{8} G\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\Big] \ldots\ldots \Big\}$$

$$+ \frac{1}{2}\left(\frac{\kappa}{m_o\eta}\right)\left[1 + \left(\frac{\kappa}{m_o\eta}\right)^2 + 2\left(\frac{\kappa}{m_o\eta}\right)^2 + \ldots\ldots\right] \Big\{ F\Big(\sqrt{\frac{\epsilon\pi}{2}} - \frac{1}{2\epsilon^2}\Big[\frac{3}{4} F\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{1}{2} e^{\frac{\epsilon\pi}{2}} \sqrt{\frac{\epsilon\pi}{2}} \left\{\frac{3}{2} - \left(\frac{\epsilon\pi}{2}\right)\right\}\Big]$$

$$+ \frac{1}{24\epsilon^4}\left[\frac{105}{16} F\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{1}{2} e^{\frac{\epsilon\pi}{2}} \sqrt{\frac{\epsilon\pi}{2}} \left\{\frac{105}{8} - \frac{35}{4}\left(\frac{\epsilon\pi}{2}\right) + \frac{7}{2}\left(\frac{\epsilon\pi}{2}\right)^2 - \left(\frac{\epsilon\pi}{2}\right)^3\right\}\right] + \ldots\ldots \Big\}$$

$$- \frac{1}{2}\left(\frac{\kappa}{m_o\eta}\right)\sqrt{\frac{\epsilon\pi}{2}}\Big[\left\{1 + \left(\frac{\kappa}{m_o\eta}\right)^2 + 2\left(\frac{\kappa}{m_o\eta}\right)^4 + \ldots\ldots\right\}\left\{1 - \frac{1}{10\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^2 + \frac{1}{316\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^4 \ldots\ldots\right\}$$

$$+ \left\{\left(\frac{\kappa}{m_o\eta}\right)^2 + 2\left(\frac{\kappa}{m_o\eta}\right)^4 + \ldots\right\}\left\{\frac{1}{3}\left(\frac{\epsilon\pi}{2}\right) - \frac{1}{14\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^3 + \frac{1}{264\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^5 + \ldots\ldots\right\}$$

$$+ \left(\frac{\kappa}{m_o\eta}\right)^4\left\{\frac{1}{5}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{1}{18\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^4 + \frac{1}{312\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^6 + \ldots\ldots\right\} + \ldots\ldots\Big]$$

$$- \frac{1}{2}\left(\frac{\kappa}{m_o\eta}\right)^2\left(\frac{\epsilon\pi}{2}\right)\Big[\left\{1 + \frac{3}{2}\left(\frac{\kappa}{m_o\eta}\right)^2 + \frac{15}{4}\left(\frac{\kappa}{m_o\eta}\right)^4 + \ldots\ldots\right\}\left\{\frac{1}{2} - \frac{1}{12\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^2 + \frac{1}{240\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^4 + \ldots\right\}$$

$$+ \left\{\left(\frac{\kappa}{m_o\eta}\right)^2 + \frac{5}{2}\left(\frac{\kappa}{m_o\eta}\right)^4 + \ldots\right\}\left\{\frac{1}{4}\left(\frac{\epsilon\pi}{2}\right) - \frac{1}{16\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^3 + \frac{1}{288\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^5 + \ldots\right\}$$

$$+ \left(\frac{\kappa}{m_o\eta}\right)^4\left\{\frac{1}{6}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{1}{20\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^4 + \frac{1}{336\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^6 + \ldots\right\} + \ldots\Big] \tag{16}$$

$$J = \left[\frac{3}{2} - \frac{11}{16}\frac{1}{\epsilon^2} + \frac{133}{256}\frac{1}{\epsilon^4}\cdots\right]\Bigg\{ G\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{3}{4\epsilon^2}\left[\sqrt{\frac{\epsilon\pi}{2}}\, e^{\frac{\epsilon\pi}{2}} \frac{\sqrt{\pi}}{2} Er\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\left\{\frac{1}{3}\left(\frac{\epsilon\pi}{2}\right) - \frac{1}{2}\right\}\right.$$

$$\left. + \left(\frac{\epsilon\pi}{2}\right)\left\{\frac{1}{4} - \frac{1}{12}\left(\frac{\epsilon\pi}{2}\right)\right\} + \frac{1}{2}G\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\right] + \frac{7}{48\epsilon^4}\left[\sqrt{\frac{\epsilon\pi}{2}}\, e^{\frac{\epsilon\pi}{2}} \frac{\sqrt{\pi}}{2} Er\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\left\{\frac{1}{7}\left(\frac{\epsilon\pi}{2}\right)^3 - \frac{1}{2}\left(\frac{\epsilon\pi}{2}\right)^2 + \frac{5}{4}\left(\frac{\epsilon\pi}{2}\right) - \frac{15}{8}\right\}\right.$$

$$\left. + \left(\frac{\epsilon\pi}{2}\right)\left\{\frac{15}{16} - \frac{5}{16}\left(\frac{\epsilon\pi}{2}\right) + \frac{1}{12}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{1}{56}\left(\frac{\epsilon\pi}{2}\right)^3\right\} + \frac{15}{8}G\left(\sqrt{\frac{\epsilon\pi}{2}}\right)\right]\cdots\cdots\Bigg\}$$

$$- \left[\frac{1}{2} - \frac{11}{16}\frac{1}{\epsilon^2} + \frac{133}{256}\frac{1}{\epsilon^4}\cdots\right]\left[\frac{1}{2}\left(\frac{\epsilon\pi}{2}\right) - \frac{1}{12\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^3 + \frac{1}{240\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^5\cdots\right]$$

$$+ \left[\frac{11}{24}\frac{1}{\epsilon^2} - \frac{133}{384}\frac{1}{\epsilon^4}\cdots\right]\left[\frac{1}{4}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{1}{16\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^4 + \frac{1}{288\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^6 + \cdots\right]$$

$$+ \frac{1}{12}\frac{1}{\epsilon^2} - \frac{133}{960}\frac{1}{\epsilon^4}\cdots\Bigg]\left[\frac{1}{6}\left(\frac{\epsilon\pi}{2}\right)^3 - \frac{1}{20\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^5 + \frac{1}{336\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^7\cdots\right]$$

$$- \left[\frac{19}{480}\frac{1}{\epsilon^4}\cdots\right]\left[\frac{1}{8}\left(\frac{\epsilon\pi}{2}\right)^4 - \frac{1}{24\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^6 + \frac{1}{384\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^8\cdots\right]$$

$$- \left[\frac{1}{240}\frac{1}{\epsilon^4}\cdots\right]\left[\frac{1}{10}\left(\frac{\epsilon\pi}{2}\right)^5 - \frac{1}{28\epsilon^2}\left(\frac{\epsilon\pi}{2}\right)^7 + \frac{1}{432\epsilon^4}\left(\frac{\epsilon\pi}{2}\right)^9\cdots\right]\cdots\cdots \qquad (17)$$

$$K = F\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{1}{2\epsilon^2}\left[\frac{3}{4}F\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{1}{2}e^{\frac{\epsilon\pi}{2}}\sqrt{\frac{\epsilon\pi}{2}}\left\{\frac{3}{2} - \left(\frac{\epsilon\pi}{2}\right)\right\}\right]$$

$$+ \frac{1}{24\epsilon^4}\left[\frac{105}{16}F\left(\sqrt{\frac{\epsilon\pi}{2}}\right) - \frac{1}{2}e^{\frac{\epsilon\pi}{2}}\sqrt{\frac{\epsilon\pi}{2}}\left\{\frac{105}{8} - \frac{35}{4}\left(\frac{\epsilon\pi}{2}\right) + \frac{7}{2}\left(\frac{\epsilon\pi}{2}\right)^2 - \frac{9}{2}\left(\frac{\epsilon\pi}{2}\right)^3 + \left(\frac{\epsilon\pi}{2}\right)^4\right\}\right]\cdots \qquad (18)$$

At the end of the launching path, the trajectory is horizontal. Therefore, if T is the launching flight time, then $\omega T^2 = \frac{\pi}{2}$. Hence, $\epsilon\omega t^2 = \frac{\epsilon\pi}{2}$.

Then
$$\frac{1}{\eta} = \frac{1}{\sqrt{\epsilon\omega}} = \frac{T}{\sqrt{\frac{\epsilon\pi}{2}}} \qquad (19)$$

Now take $\epsilon = \frac{2}{\pi}$, then $\frac{\epsilon\pi}{2} = 1$. Therefore, in this special case $\frac{1}{\eta} = T$.

Then
$$\frac{\kappa}{m_o\eta} = \frac{\kappa T}{m_o} = \zeta = \frac{\text{Fuel Mass}}{\text{Initial Mass}}, \quad \epsilon = \frac{2}{\pi} \qquad (20)$$

For this case, ***the value of the end velocity*** v_e ***and the end altitude*** y_e ***are given by the following equations:***

$$v_e = \left(1 - \frac{D_o}{F}\right)C\left[2.02861\zeta + 0.85914\zeta^2 + 0.51431\zeta^3 + 0.35914\zeta^4 + 0.27146\zeta^5 + 0.21828\zeta^6 + 0.17864\zeta^7 + \cdots\right] - 2.16955gT + 2.71828v_o \qquad (21)$$

$$y_e = \left(1 - \frac{D_o}{F}\right) CT\Big[0.42504\zeta + 0.11987\zeta^2 + 0.15264\zeta^3 + 0.02979\zeta^4 + 0.02050\zeta^5 + 0.01836\zeta^6 + 0.02054\zeta^7 + \ldots\Big] - 0.46876gT^2 + 1.01226\nu_o T \qquad (22)$$

In these equations, C is the effective exhaust velocity of the rocket. It is related to the other variables by the expression

$$C = \frac{FT}{m_o\zeta} = \frac{FT}{\text{Total Propellant Mass}} \qquad (23)$$

Fig. 4 is computed by putting $v_o = 0$, $C = 6400$ ft/sec, $D_o/F = 0.1$.

PART IV

PROPERTIES OF LONG RANGE ROCKET TRAJECTORIES IN VACUO

By

G. B. SCHUBAUER

PART IV

PROPERTIES OF LONG RANGE ROCKET TRAJECTORIES IN VACUO

INTRODUCTION

With the increasing use of the rocket as a long-range missile, the question naturally arises, how far can a rocket be projected and yet carry a worth-while load of explosives? The answer to this question depends primarily on the answer to two more fundamental questions. First, how much energy is required to shoot a rocket to a distant point, hundreds or perhaps thousands of miles away? Second, what amounts of energy lie within the reach of known types of rocket motors and fuels? The object of this paper is to throw some light on energy requirements by showing what initial velocities are required to project a particle in vacuo for great distances over a spherical earth under the action of gravity. The general nature of the vacuum trajectory will be investigated to find how the range is related to the initial velocity and direction, the maximum height, and the time of flight. We do not consider how the initial velocity is acquired and therefore do not deal with the second fundamental question.

Since rocket flights with ranges up to 250 miles are now an accomplished fact, little attention need be devoted to ranges of lesser magnitude in a treatment of this sort. In order to overlap somewhat, the lower limit to be considered will arbitrarily be set at 100 miles. The upper limit will be set at 6000 miles, since there is scarcely a conceivable need for still greater ranges.

This treatment should be regarded as one devoted entirely to long ranges, since it is only for long ranges that the greater part of the trajectory is above the atmosphere where the assumption of flight in vacuo is valid. In our idealized problem we therefore find minimum energy requirements to which must be added the energy losses incurred in penetrating the atmosphere. It is assumed that such losses would be minimized in any practical case by launching the rocket vertically in order to take the shortest path through the atmosphere, and then setting the course of the rocket by some control device to the flight path calculated to reach the target. When atmospheric losses are taken into account it may possibly be found that the best vacuum trajectory above the atmosphere is not the best practical trajectory because of the long path through the atmosphere on descent. Other considerations may enter the problem, such as some necessary limitation on the velocity to avoid excessive heating by atmospheric compression and friction. It is not because such problems can be ignored, but because the vacuum trajectory forms the basis of departure for the practical problem, that we devote this treatment to the vacuum trajectory, assuming the natural gravitational trajectory to start from the ground.

Whereas, in the simple classical treatment of the vacuum trajectory, the earth is regarded as flat and the gravitational pull is regarded as constant, both the curvature of the earth and the variation of gravity with height are taken into account in the treatment to follow. In fact, the spherical earth and the inverse-square law of gravity are here the basic concepts. Fundamental gravitational trajectories are derived without taking the rotation of the earth into account. Having the characteristics of the trajectories on a nonrotating earth, the effects of the earth's rotation can be treated as a separate problem in relative motion when the latitude and longitude of the end points of the trajectory are specified. The manner in which this may be done and the nature and magnitude of the effects are illustrated by an example.

Numerical data are given for ranges from 100 to 6000 miles. These are presented in tables and curves for convenient reference. It should be emphasized that these results are not intended to serve as firing data. This problem was proposed to the author by Dr. Hugh L. Dryden and helpful suggestions were given by him with regard to the general character of the treatment.

THEORY

Since the gravitational attraction on all objects above the earth's surface varies inversely as the square of the distance from the center of the earth, the problem to be solved is simply that of the motion of a particle under a central force which varies inversely as the square of the distance from the center of attraction. This is one of the basic problems in celestial mechanics and is so well known that it is only necessary to state the concepts here and present the equations in a convenient form for our specific needs.

We assume, in effect, that the mass of the earth is concentrated at a point at its center and determine the various orbits of a particle of unit mass moving about this point with different amounts of energy. The only orbits that have any connection with a projectile or rocket problem are those that pass at some point or other within the earth's radius. The complete orbit is, therefore, purely hypothetical; but that part lying outside the sphere defined by the radius of the earth becomes a trajectory, if the particle has the velocity vector characteristic of the orbit at the surface of the sphere.

As far as the basic problem is concerned, it makes no difference whether the earth is rotating or not, since the particle has no connection with the earth outside of the attraction toward the center. The attraction is, of course, independent of the motion of the earth. Since the orbital velocity is a velocity with respect to the center, the velocity of the surface of the earth, due to rotation about the center, must be added vectorially to the orbital velocity in order to obtain the velocity with respect to the surface. This can be done for any location on the earth when the orbital velocity is known, and will be treated as a separate problem. Thus the fundamental problem deals with velocities with respect to the center of the earth or with respect to the surface of a nonrotating earth. It will be found quite convenient to think in terms of a nonrotating earth.

The orbit is a conic in all cases, with the center of attraction (center of the earth) at one focus. In polar coordinates with the origin at the center of attraction the equation of the conic is

$$r = \frac{p^2}{g\, r_0^2\, (1 - e \cos \Theta)} \tag{1}$$

where

- r = the radius vector,
- Θ = the polar angle measured from the axis of symmetry in the sense shown in Fig. 1,
- r_0 = the mean radius of the earth, the earth being regarded as a sphere,
- g = acceleration of gravity at the surface of a nonrotating earth,
- e = eccentricity of the conic,
- p = angular momentum per unit mass of the particle; p is a constant for a particular conic and is defined by $p = r^2\, d\theta/dt$, where t is time.

The orbit defined by the conic is a plane curve, and all motion takes place in the plane of the curve. The conic is an ellipse, parabola, or hyperbola according as e is less than, equal to, or greater than unity. Since we wish the particle to return to the earth, we are interested only in elliptical orbits, or values of e less than unity. Two elliptical orbits are shown in Fig. 1. The range R is defined as the arc distance formed by the intersection of the plane bounded by the ellipse and the sphere of radius r_0. On a nonrotating earth R is the great circle distance from the starting to the landing point, or the arc ABC in Fig. 1. If Θ_0 is the angle subtended by the arc AB, which is half the arc ABC, the range may be expressed by

$$R = \frac{4\pi \Theta_0 r_0}{360} \tag{2}$$

where Θ_0 is in degrees.

For any particular range, Θ_0 is a constant, and p may be found from equation (1) by substituting r_0 and Θ_0 for r and Θ. We find

$$p^2 = r_0^2 g\ (1 = e \cos\Theta_0) \tag{3}$$

Replacing p^2 in equation (1) by its equivalent in equation (3), we obtain

$$r = \frac{r_0 (1 - e \cos \Theta_0)}{1 - e \cos \Theta} \tag{4}$$

for the equation of the ellipse in terms of Θ_0, r_0, and e.

The semimajor axis, a, and the semiminor axis, b, are respectively,

$$a = \frac{r_0(1 - e \cos \Theta_0)}{1 - e^2},\ b = a\sqrt{1 - e^2}. \tag{5}$$

Equation (4) shows that e is the parameter of a family of ellipses all intersecting the circle of radius r_0 at the same points. This means that a particle can traverse the same range over a number of trajectories. In order to find what physical quantities determine any particular trajectory, we must find the relation of e to the following quantities:

V, the velocity at any point in the trajectory relative to the center of the earth or to the surface of a nonrotating earth.

V_o, the initial velocity at the surface of the earth relative to the center of the earth or to the surface of a nonrotating earth.

ψ, the angle between the radius vector and the tangent to the ellipse at any point.

ψ_o, the angle between the radius vector r_o and the tangent to the ellipse. ψ_o is the angle between the perpendicular and the trajectory at the surface of the earth (see Fig. 1).

H, the highest point of the trajectory above the surface of the earth.

T, the time of flight over the trajectory.

The velocity of the particle at any point is given by

$$V^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \tag{6}$$

where t is time. By differentiating equation (1) with respect to t, regarding e and p as constants, and using the relation $p = r^2 d\theta/dt$ we obtain, after replacing p by means of equation (3),

$$V^2 = \frac{gr_o^2}{r}\left(\frac{e^2 - 1}{1 - e\cos\theta} + 2\right). \tag{7}$$

When $r = r_o$ and $\theta = \theta_o$, equation (7) becomes

$$V_o^2 = gr_o\left(\frac{e^2 - 1}{1 - e\cos\theta_o} + 2\right). \tag{8}$$

The angle ψ is given by

$$\tan\psi = -\frac{r\,d\theta}{dr}. \tag{9}$$

By differentiating equation (4) with respect to θ, regarding e and θ_o as constants, then substituting in equation (9) and simplifying by equation (4), we obtain

$$\tan\psi = \frac{r_o(1 - e\cos\theta_o)}{r\,e\sin\theta}. \tag{10}$$

When $r = r_o$ and $\theta = \theta_o$, equation (10) becomes

$$\tan\psi_o = \frac{1 - e\cos\theta_o}{e\sin\theta_o}. \tag{11}$$

Since the distance from the focus to the center of the ellipse is ae, it follows that

$$H = a + ae - r_o \tag{12}$$

and by substituting for a from the first of equations (5) we find

$$H = r_o\left(\frac{1 - e\cos\theta_o}{1 - e} - 1\right) \tag{13}$$

From the relation $p = r^2 d\theta/dt$ we obtain

$$dt = \frac{r^2}{p}\,d\theta. \tag{14}$$

Substituting for r from equation (1) and for p by equation (3) we obtain

$$dt = \sqrt{\frac{r_o}{g}}\,(1 - e\cos\theta_o)^{3/2}\;\frac{d\theta}{(1 - e\cos\theta)^2}. \tag{15}$$

Integrating between the limits $\theta = o$ and $\theta = \theta_o$ and remembering that the result must be multiplied by two to obtain the total time because of the limits chosen, we obtain

$$T = 2\sqrt{\frac{r_o}{g}}\,\frac{(1 - e\cos\theta_o)^{3/2}}{1 - e^2}\left[\frac{e\sin\theta_o}{1 - e\cos\theta_o} + \frac{2}{\sqrt{1 - e^2}}\,\text{arc tan}\left(\sqrt{\frac{1 + e}{1 - e}}\tan\frac{\theta_o}{2}\right)\right]. \tag{16}$$

From the foregoing relations it is clear that we need only to choose some value of e between zero and one to define a possible trajectory and to determine the characteristics of the motion. By calculating trajectories for a number of values of e we can find the trajectory for which the initial velocity is a minimum. This is the best trajectory, for then the energy required for a given range is a minimum. However, the value of e for minimum initial velocity can be found directly by differentiating equation (8) with respect to e and setting the result equal to zero. Denoting this value of e by e_1 we find

$$e_1 = \frac{1 - \sin\theta_o}{\cos\theta_o}. \tag{17}$$

From equations (11) and (17) we find

$$\tan\psi_1 = \frac{\cos\theta_o}{1 - \sin\theta_o} = \frac{1}{e_1} \tag{18}$$

and finally from equations (8) and (17) we find

$$V_1^2 = 2gr_o\left(\frac{\sin\theta_o - 1}{\cos^2\theta_o} + 1\right) \tag{19}$$

where V_1 is the minimum initial velocity required for the range given by θ_o, and ψ_1 is the corresponding angle between the vector V_1 and the vertical. The associated values of H_1 and T_1 corresponding to the trajectory of minimum energy are conveniently obtained by substitution of e_1 in equations (13) and (16).

COMPUTATIONS AND RESULTS

The relations derived in the previous section were used to compute numerical values of V_o, V_1, ψ_o, ψ_1, H, H_1, and T_1 for ranges from 100 to 6000 statute miles. Since perturbing effects, such as might come from the influence of the moon and other heavenly bodies, are assumed to be negligible (although such effects were not investigated), the laws of orbital motion are considered to be exact, and the accuracy of the results depends only on the values selected for r_o and g and the approximations involved in regarding them as constants. The earth was assumed to be a sphere with constant radius r_o and constant acceleration of gravity, g, at the surface. While no extensive search was made for the best values of these constants, some thought was given to an appropriate choice for reasonable accuracy. For r_o the mean radius of the earth was used, the value selected being $r_o = 3956.3$ statute miles $= 20.889 \times 10^6$ ft. The statute mile equal to 5280 ft is used throughout. Wherever the term miles is used it is to be understood to mean statute miles.

Since the velocity V is the velocity relative to the center of the earth and since the gravitational pull on objects detached from the earth is unaffected by the earth's rotation, the value of g was taken as the average value found on the surface of a non-rotating earth, the value selected being $g = 32.227$ ft/sec². The accuracy obtainable with these values of r_o and g is not as great as the number of significant figures would indicate, due mainly to the fact that r_o and g are not strictly constant. The units of r_o and g determine the units of each computed quantity. Consistent units must be used in each equation unless dimensional constants are introduced.

The first step in the computation procedure was to select a range R and determine Θ_o from equation (2), which equation reduces to

$$\Theta_o \text{ (degrees)} = \frac{R \text{ (miles)}}{138.100} . \qquad (20)$$

A convenient order of procedure was then found to be as follows:

Calculate	From Equation
e_1	(17)
ψ_1	(18)
V_1	(19)
H_1	(13) with $e = e_1$
T_1	(16) with $e = e_1$
ψ_o	(11) for range of e greater than and less than e_1
V_o	(8) for range of e greater than and less than e_1
H	(13) for range of e greater than and less than e_1

Since the angle of the trajectory to the horizontal is more commonly used than the angle to the vertical, the results will be expressed in terms of

$$\varphi_{o,1} = 90^\circ - \psi_{o,1} \qquad (21)$$

where both φ and ψ are in degrees and

φ_o = initial angle of the trajectory to the horizontal, termed the angle of elevation.

φ_1 = angle of elevation for minimum initial velocity.

When the velocity is represented by a vector, φ_o or φ_1 is the acute angle between V_o or V_1 and the horizontal at the earth's surface.

The results are given in Tables I and II and shown graphically in Figs. 1, 2, 3, and 4. Fig. 1 is given mainly to illustrate the geometry of the problem and to assist in the definition of terms. The two elliptical orbits were calculated for a 6000-mile range and are shown in their true proportion to the size of the earth. The "best" trajectory is the minimum-energy trajectory corresponding to the symbols with the subscript "1." Table I gives the calculated values of V_1, φ_1, H_1 and T_1 for the minimum-energy trajectories corresponding to the ranges from 100 to 6000 miles. These quantities are represented graphically in Figs. 2 and 3. It may be of interest to note in Fig. 2 that φ_1 starts from 45°at zero range and decreases linearly with range. Thus φ_1 is the counterpart of the angle of maximum range, associated with a flat horizontal range and a constant acceleration of gravity. For any fixed initial velocity, φ_1 is the angle of maximum range, but for each initial velocity, φ_1 has a different value, always less than 45°. For ordinary ballistic ranges the departure from 45° is small.

While the range is used as the independent variable in Fig. 2, we may, on the other hand, regard each pair of values V_1, φ_1 as determining a given range. For ranges so determined the time of flight and the maximum height attained are given by Fig. 3. Actually the range, expressed in terms of Θ_o, serves in the computations as the independent variable on which V_1, φ_1, H_1, and T_1 depend.

Table II contains values of V_o, φ_o, and H for specific ranges from 100 to 6000 miles. In order to show how the initial velocity depends on the angle of elevation, V_o and φ_o were plotted for each range to obtain the family of curves shown in Fig. 4. At the minimum point of each curve is a pair of values V_o, φ_o, corresponding to V_1 φ_1, in Table I and Fig. 2. Since the curves are rather flat in the neighborhood of the minimum, the angle of elevation may vary by 5° above and below the value at the minimum without affecting the velocity by more than about 1%.

EFFECT OF ROTATION OF THE EARTH AS SHOWN BY EXAMPLE

The rotation of the earth will now be taken into account in an imaginary rocket flight, first from New York to Berlin, and second in the reverse direction, from Berlin to New York. On a nonrotating earth the range would be the great-circle distance of 3957 statute miles between the two cities, and in this case the minimum initial velocity and angle of elevation required to project the rocket over this range can be found at once from Fig. 2. The direction in which to launch the rocket could readily be found from the latitude and longitude of New York and Berlin. We now wish to determine how conditions are modified by the earth's rotation.

To get a picture of one aspect of the effect of rotation, imagine the rocket to be already in flight from New York toward Berlin with sufficient velocity to cover the distance of 3957 miles. As we have seen in the previous sections, the trajectory must lie in a plane containing the center of the earth. The intersection of the plane bounded by the elliptical orbit and the surface of the earth is an arc of a great circle which, in the case of a nonrotating earth, begins at New York and ends at Berlin, but which, in the case of the rotating earth, moves from east to west as seen by an observer on the ground. Therefore during the time of 1450 sec (found from Fig. 3) that the rocket is in transit, the Berlin end of the arc moves directly westward for a distance readily calculated to be about 250 miles. Thus the rocket would fall short of its mark unless it were launched for a greater range. This illustrates the first step in the calculation, namely, to determine the range for the rocket to go from where New York is at the beginning of the flight, to where Berlin will be at the end of the flight. The final step is to determine the initial velocity both in magnitude and direction relative to an observer on the ground at New York, in order to cover the range with the trajectory of minimum energy. The calculations can be carried out by a systematic procedure outlined as follows:

The first part of the problem concerns the solution of the spherical triangle in Fig. 5. The sides k and l and the included angle α are determined from the latitude and longitude given as follows:

	Latitude	Longitude referred to Greenwich
New York	40° 48′ 35″	73° 57′ 30″ W
Berlin	52° 31′ 31″	13° 21′ 51″ E

The meridian distances k and l in terms of the angle subtended at the center of the earth are 90° latitude. Thus

$k = 49° \; 11' \; 25''$ $\qquad l = 37° \; 28' \; 29''$

α = sum of E and W longitude = 87° 19′ 21″.

The great circle distance x in angular measure, is found from

$$\cos x = \cos k \cos l + \sin k \sin l \cos \alpha \qquad (22)$$

and the angle L and K are found from

$$\tan 1/2\,(K + L) = \frac{\cos 1/2\,(k-l) \cot 1/2\alpha}{\cos 1/2\,(k + l)} \qquad (23)$$

$$\tan 1/2\,(K - L) = \frac{\sin 1/2\,(k-l) \cot 1/2\alpha}{\sin 1/2\,(k + l)}$$

With the given values of k, l, and α we find

$x = 57° \; 18' \; 20'' = 57.306°$

L = 46° 14′, bearing of Berlin from New York N 46° 14′ E

K = 63° 57′, bearing of New York from Berlin N 63° 57′ W.

To reduce x to statute miles, we take the mean circumference of the earth to be 24,858 statute miles and use the relation

$$\frac{24{,}858\,x^{o}}{360} = x \text{ (miles)} \qquad (24)$$

from which we find x = 3957 miles. We now have the bearings and the great circle distance between the two cities.

FLIGHT FROM NEW YORK TO BERLIN

First approximation to range:

Regarding $x = 3957$ miles as the range, time of flight equals 1450 sec from Fig. 3. Since during this time the rocket is not rotating with the earth, the longitude of Berlin effectively increases, increasing α to

$$\alpha_1 = \alpha + \frac{1{,}450}{86{,}164} 360 = \alpha + \frac{1450}{239.3} = \alpha + (6^\circ\ 3'\ 32'') = 94^\circ\ 22'\ 53'' \quad (25)$$

where 86,164 is the number of seconds in a sidereal day.

From equations (22) and (24) with α_1, in place of α we find $x_1 = 4189$ miles.

Second approximation to range:

Regarding $x_1 = 4189$ miles as the range, time of flight equals 1502 sec from Fig. 3. The longitude of Berlin effectively increases, increasing α to

$$\alpha_2 = \alpha + \frac{1502}{239.3} = \alpha + (6^\circ\ 16'\ 37'') = 93^\circ\ 35'\ 58'' \quad (26)$$

and from equations (22) and (24) with α_2, in place of α we find $x_2 = 4190$ miles.

Since the first and second approximations agree to a mile, a third approximation is unnecessary. The correct range is therefore taken to be 4190 miles. This is not a great circle traced on the rotating earth by the course of the rocket, but is a great-circle arc with respect to the center of the earth. We now picture the elliptic orbit corresponding to a range of 4190 miles and imagine the earth's surface to be cutting the plane bounded by the orbit so that the range is given by the arc of intersection. At the instant the rocket leaves New York the far end of the arc has the latitude of Berlin but a longitude $6^\circ\ 16'\ 37''$ greater than Berlin. Therefore at the instant of launching, the plane of the orbit must have a bearing from New York found from the spherical triangle with sides l and k and an included angle α_2.

From equation (23) with α_2 in place of α, we find $L_2 = 44.145^\circ = 44^\circ\ 8'\ 42''$ where L_2 is the angle between the plane of the orbit and the meridian through New York at the instant of launching. If δ is the angle between this plane and the line of latitude, known as the parallel, $\delta = 90^\circ - L_2 = 45^\circ\ 51'\ 18''$.

We find from Fig. 2 that the minimum initial velocity required for a range of 4190 miles is $V_1 = 21{,}280$ ft/sec and that the proper angle of elevation is $\varphi_1, = 29.82^\circ$. We now have the length of the velocity vector V_1 with respect to the center of the earth, making the angle φ_1, to the horizontal plane and the angle δ to the vertical east-west plane. This is the condition existing at New York at the intsant of launching as seen by an observer fixed with respect to the center of the earth. It now remains to find the velocity vector relative to an observer on the ground at New York. This part of the problem is solved by the aid of Fig. 6. The vector W, representing the velocity of the earth at New York with respect to the center, points directly eastward, and its length is given by

$$W = \frac{\text{mean circumference of earth in ft}}{\text{sec in sidereal day}} \cos\ (\text{latitude N. Y.}) \quad (27)$$

$$W = \frac{131.250 \times 10^6}{86{,}164} \cos\ (40^\circ\ 48'\ 35'') = 1153 \text{ ft/sec}$$

The angle β between V_1 and W is given by

$$\sin \beta = \cos \varphi_1 \sqrt{\sin^2 \delta + \tan^2 \varphi_1} \qquad (28)$$

$$\beta = 52^\circ\ 49'\ 30''$$

The resultant of V_1 and W, denoted by V_2, is given by

$$V_2^2 = V_1^2 + W^2 - 2V_1W \cos \beta \qquad (29)$$

from which we find $V_2 = 20{,}605$ ft/sec where V_2 is the magnitude of the velocity vector with respect to an observer on the ground. This vector makes an angle φ_2, to the horizontal plane and δ_2 to the vertical east-west plane, where φ_2 and δ_2 are given by

$$\sin \varphi_2 = \frac{V_1}{V_2} \sin \varphi_1 \qquad (30)$$

$$\sin \delta_2 = \frac{V_1}{V_2} \frac{\cos \varphi_1 \sin \delta}{\cos \varphi_2}.$$

Using the values of V_1, V_2, φ_1, and δ we find

$$\varphi_2 = 30.90^\circ = 30^\circ\ 54'$$

$$\delta_2 = 48.53^\circ = 48^\circ\ 31'\ 48'' \qquad (31)$$

The bearing $L_3 = 90^\circ - \delta_2 = 41^\circ\ \ 28'\ \ 12''$.

The velocity V_2 is the actual minimum velocity required to reach Berlin, φ_2 is the proper angle of elevation to reach Berlin with the trajectory of minimum energy, and L_3 is the proper bearing on which to launch to hit Berlin.

The effect of rotation is illustrated by the following comparisons:

New York to Berlin

	Hypothetical Non-Rotating Earth	*Real Rotating Earth*
Range	3,957 miles	4,190 miles
Minimum initial velocity	20,880 ft/sec	20, 605 ft/sec
Angle of elevation	30.7°	30.9°
Bearing	N 46° 14′ E	N 41° 28′ E
Time of flight	1,450 sec	1,502 sec
Maximum height	705 miles (Fig. 3)	728 miles (Fig. 3)

Since the flight from Berlin to New York is calculated in a manner similar to that just given, the detailed steps will be omitted. The results are:

Berlin to New York

	Hypothetical Non-Rotating Earth	*Real Rotating Earth*
Range	3,957 miles	3,735 miles
Minimum initial velocity	20,880 ft/sec	21,240 ft/se
Angle of elevation	30.7°	30.3°
Bearing	N 63° 57′ W	N 68° 40′ W
Time of flight	1,450 sec	1,386 sec
Maximum height	705 miles	680 miles

It will be noted that no attempt has been made to derive the trajectory with respect to the surface of the rotating earth. There appears to be no need to do so, so long as the salient facts can be gathered by considering only the end points.

It is realized that calculations of the sort just made mean very little with regard to an actual rocket flight. The presence of an atmosphere would call for greatly modified launching angles, and the initial velocity would have to be sufficiently greater than that given here to make up for the losses from air resistance. The purpose of the example is to show, first, how the rotation of the earth can be taken into account for specific cases when the fundamental data for orbital trajectories are given, and, second, to show the nature of the various effects and their order of magnitude. In the example used, the effects on the whole are small. Considering the initial velocities, we find changes of from 1 to 2% introduced by the rotation of the earth. It is interesting to note that the initial velocity required to shoot from New York to Berlin is 3% less, and the associated kinetic energy 6% less, than that required to shoot from Berlin to New York. On the other hand, it takes about 2 min longer for the rocket to reach Berlin than to reach New York. While the effects will vary with the location of the range, it is safe to state that the rotation of the earth is not a significant factor in energy requirements.

Table I

Minimum-Energy Trajectories

Values of Initial Velocity V_1, Angle of Elevation φ_1,
Maximum Height H_1, Time of Flight T_1.

Range (*statute miles*)	V_1 (*ft/sec*)	φ_1 (*degrees*)	H_1 (*statute miles*)	T_1 (*seconds*)
100	4,099	44.64	24.85	182.58
300	7,012	43.91	73.56	321.36
500	8,942	43.19	121.20	421.05
1,000	12,276	41.38	233.69	629.63
2,000	16,411	37.76	431.84	934.77
3,000	19,071	34.14	591.4	1,208.8
4,000	20,959	30.52	710.4	1,458.3
5,000	22,360	26.90	786.5	1,685.7
6,000	23,422	23.28	818.4	1,890.3

Table II

Values of Angle of Elevation φ_0, Initial Velocity V_o, and Maximum Height H for Ranges from 100 to 6000 Statute Miles

φ_o (*degrees*)	V_o (*ft/sec*)	H (*statute miles*)	φ_o (*degrees*)	V_o (*ft/sec*)	H (*statute miles*)
	Range 100 Miles			Range 300 Miles	
26.69	4,578	12.33	30.42	7,412	44.54
31.67	4,330	15.49	35.39	7,165	54.02
36.03	4,198	18.27	38.39	7,076	60.34
39.54	4,132	20.76	41.81	7,022	68.24
44.64	4,100	24.85	45.70	7,019	78.38
48.45	4,117	28.41	50.14	7,093	91.02
51.15	4,153	31.29	55.18	7,285	110.87
57.20	4,306	39.20	60.85	7,668	139.30
64.18	4,645	52.37	67.18	8,385	186.68
72.04	5,377	78.71	74.07	9,783	281.45
	Range 500 Miles			Range 1000 Miles	
26.69	9,712	64.02	26.04	13,119	126.20
29.18	9,484	71.21	30.64	12,673	154.06
32.04	9,278	80.00	34.35	12,443	178.82
35.37	9,106	90.99	38.69	12,301	211.20
39.26	8,986	105.12	41.13	12,277	231.44
43.81	8,948	123.96	43.77	12,296	255.36
46.37	8,957	135.95	46.63	12,370	284.06
49.14	9,040	150.34	49.71	12,513	319.14
55.34	9,347	189.90	53.02	12,746	362.99
58.78	9,604	218.16	56.56	13,101	419.42
62.46	10,037	255.84	64.32	14,369	599.80
70.46	11,507	387.71			
	Range 2000 Miles			Range 3000 Miles	
23.68	17,248	233.42	19.06	20,087	280.82
28.51	16,762	293.29	22,60	19,653	343.22
31.91	16,550	339.80	26.64	19,311	421.24
34.41	16,456	377.11	30.29	19,133	499.26
38.55	16,413	445.62	34.34	19,070	596.8
41.59	16,470	502.8	36.53	19,094	655.3
43.19	16,530	535.8	38.82	19,163	722.2
44.86	16,616	572.6	41.23	19,285	799.3
48.37	16,876	660.0	45.03	19,587	940.3
54.07	17,484	841.2	49.07	20,065	1,123.4
60.26	18,753	1,131.1	53.30	20,773	1,371.2
			63.80	23,789	2,527.6

Table II (Continued)

φ_o (*degrees*)	V_o (*ft/sec*)	H (*statute miles*)	φ_o (*degrees*)	V_o (*ft/sec*)	H (*statute miles*)
Range 4000 Miles			Range 5000 Miles		
16.59	21,828	329.88	16.07	22,873	411.33
23.29	21,186	494.77	19.23	22,614	509.3
27.18	21,007	604.8	22.65	22,437	625.0
30.52	20,958	710.4	26.34	22,361	764.0
33.27	20,991	807.3	30.29	22,408	933.7
36.12	21,095	919.0	34.49	22,610	1,145.9
41.18	21,459	1,154.6	38.90	22,995	1,418.7
46.57	22,129	1,484.4	43.54	23,610	1,782.6

φ_o (*degrees*)	V_o (*ft/sec*)	H (*statute miles*)
Range 6000 Miles		
11.86	23,986	361.34
14.78	23,731	464.58
17.88	23,546	583.7
21.19	23,441	722.7
23.26	23,422	817.8
24.68	23,431	886.9
28.36	23,532	1,083.9
32.20	23,763	1,324.8
36.17	24,146	1,625.9
44.39	25,461	2,529.2

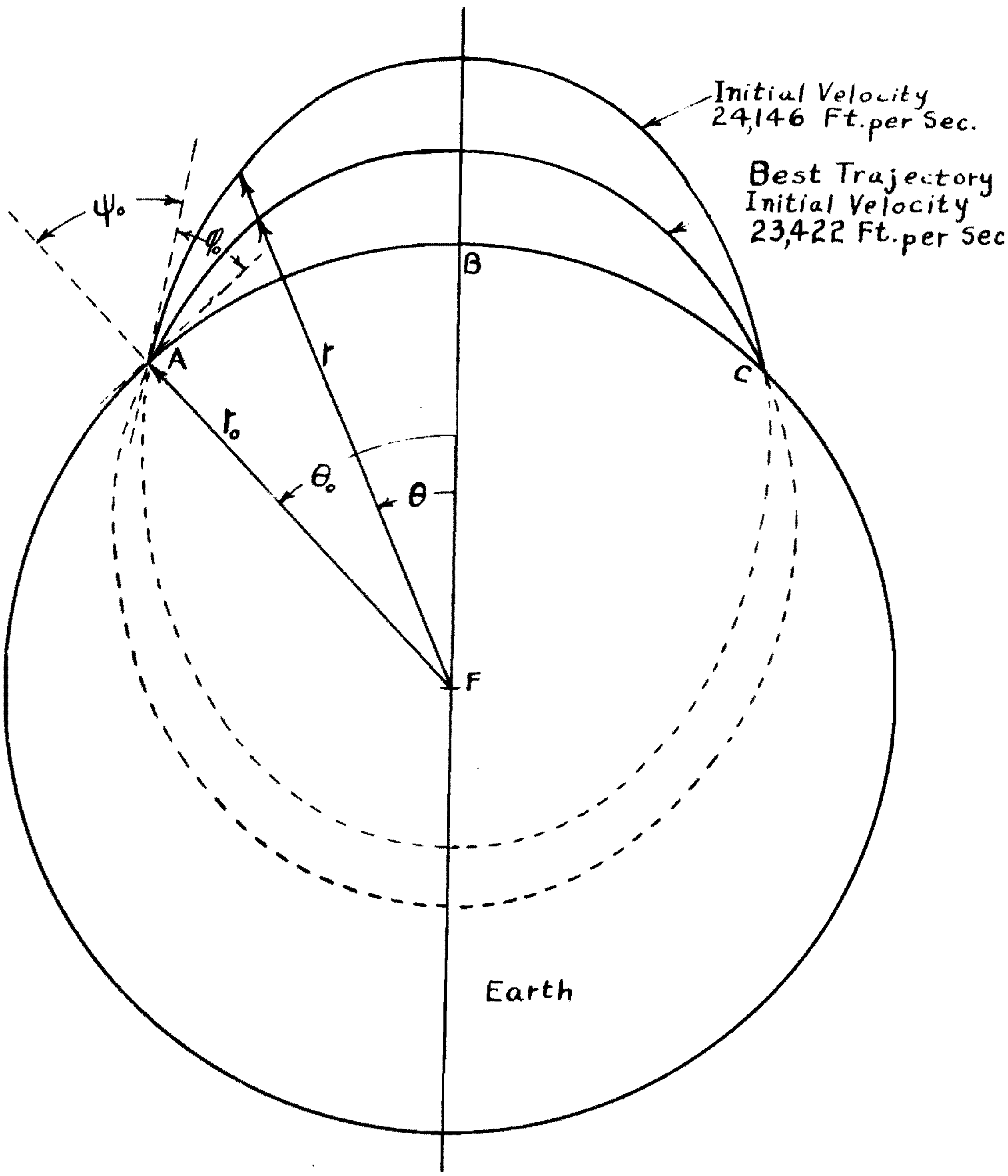
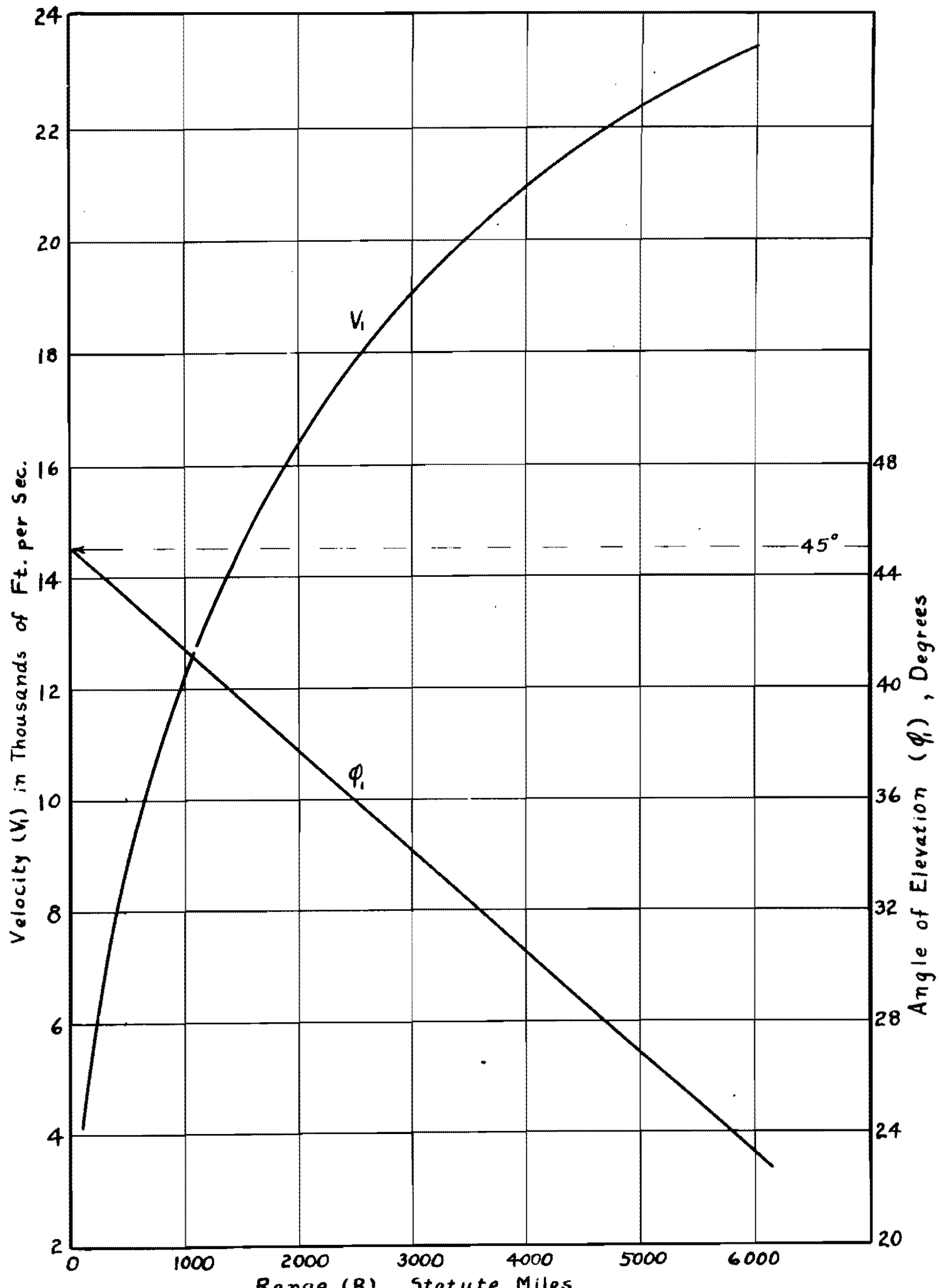


Figure 1 — Elliptical Trajectories (Range R = 6000 Miles)



24
22
20
18
16
14
12
10
8
6
4
2
0
1000
2000
3000
4000
5000
6000
48
45°
44
40
36
32
28
24
20
V_1
φ_1
Velocity (V_1) in Thousands of Ft. per Sec.
Angle of Elevation (φ_1), Degrees
Range (R), Statute Miles

Figure 2

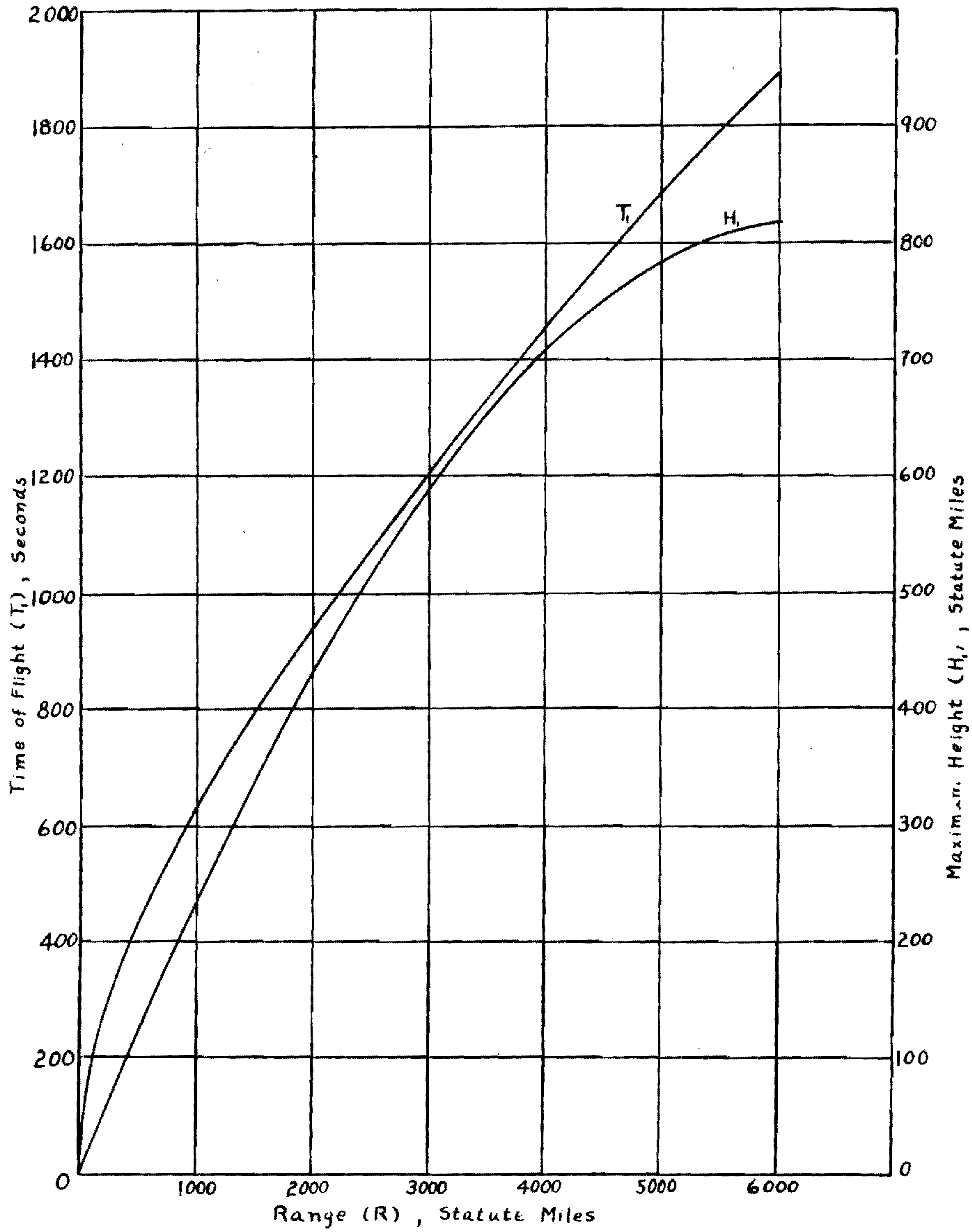

2000
1800
1600
1400
1200
1000
800
600
400
200
0
900
800
700
600
500
400
300
200
100
0
T₁
H₁
Time of Flight (T₁), Seconds
Maximum Height (H₁), Statute Miles
1000
2000
3000
4000
5000
6000
Range (R), Statute Miles

Figure 3

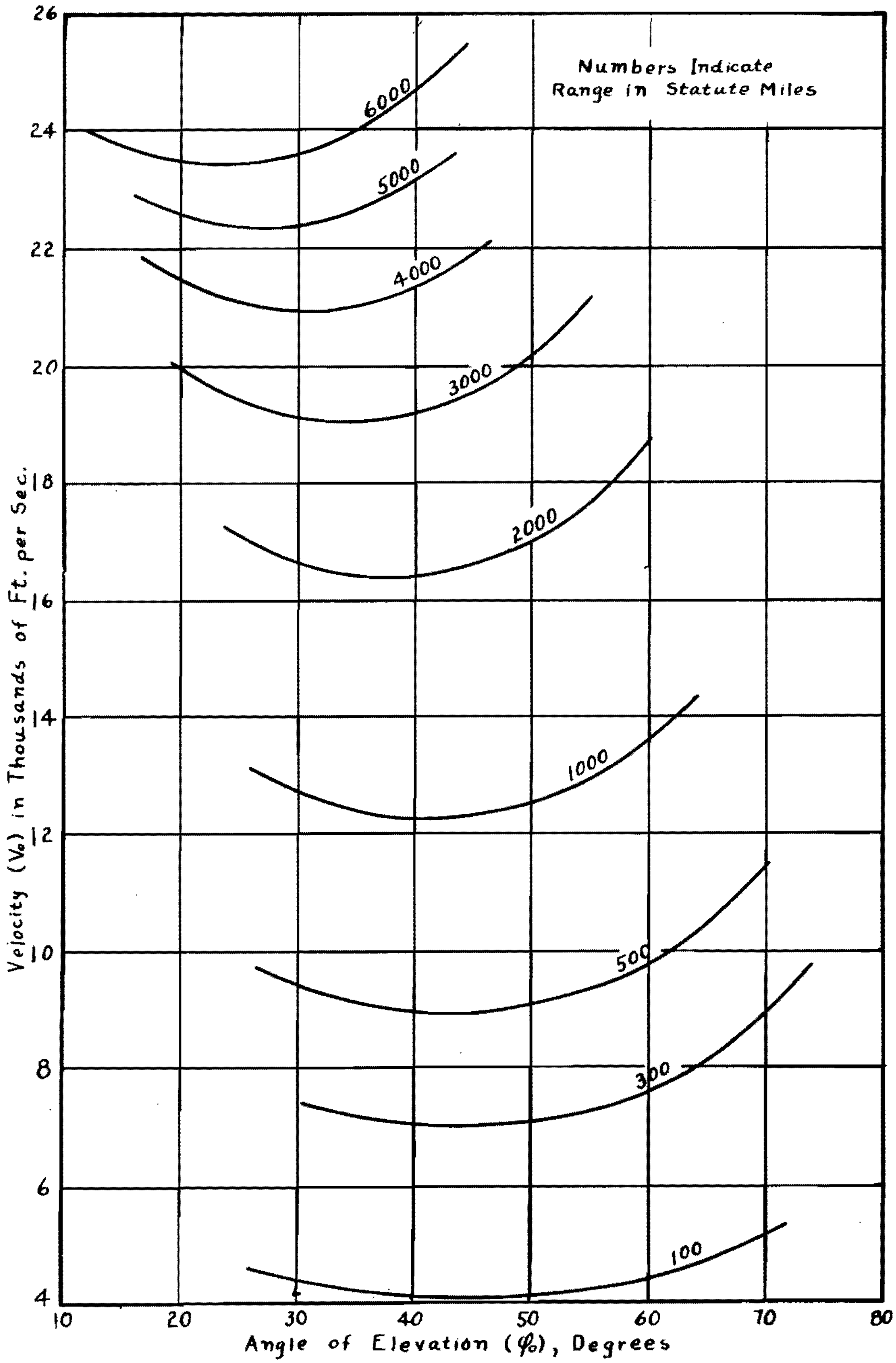

Numbers Indicate
Range in Statute Miles
6000
5000
4000
3000
2000
1000
500
300
100
26
24
22
20
18
16
14
12
10
8
6
4
10
20
30
40
50
60
70
80
Velocity (V_0) in Thousands of Ft. per Sec.
Angle of Elevation (ϕ_0), Degrees

Figure 4

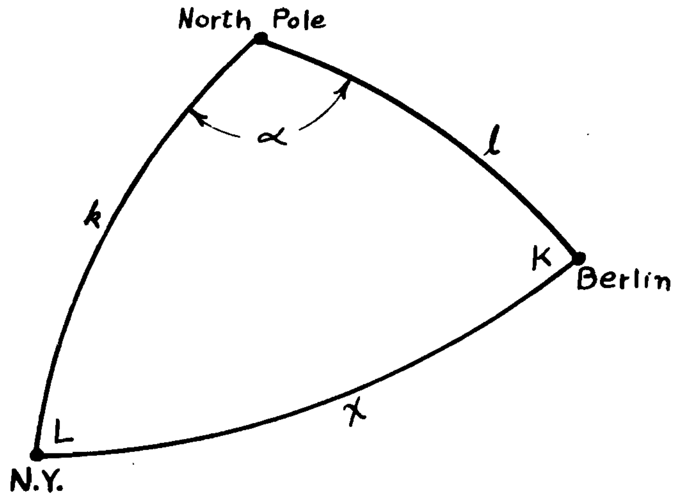


Figure 5

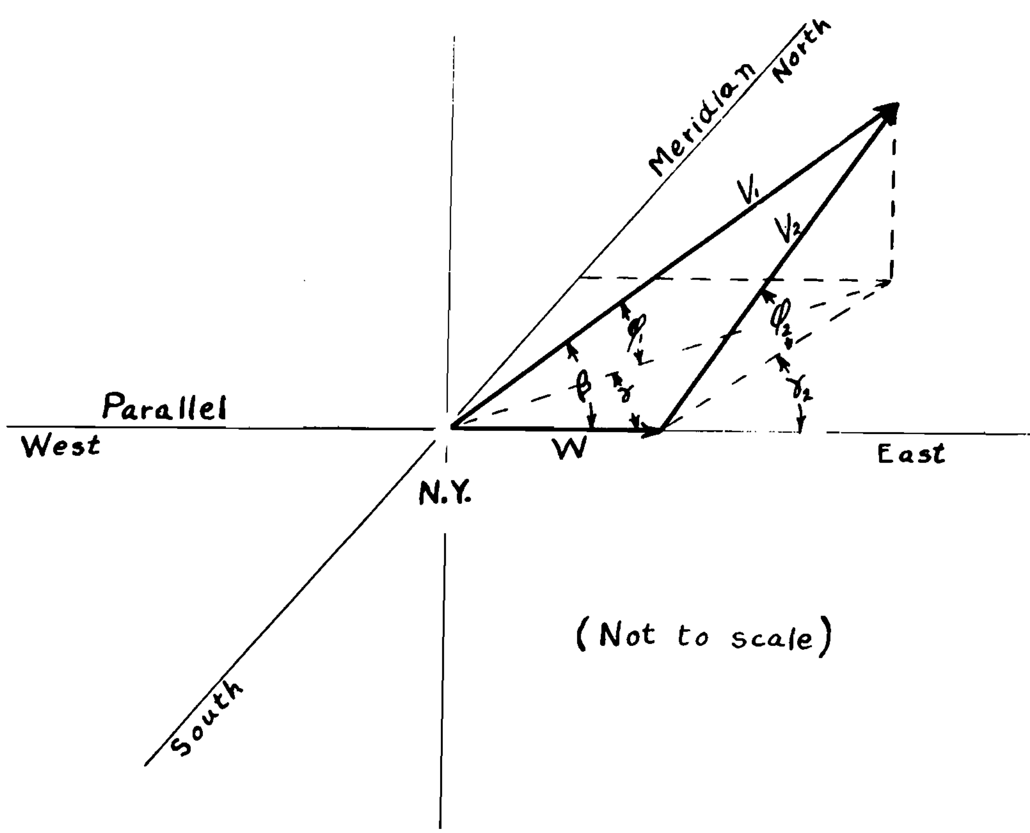


Figure 6

RESTRICTED

RESTRICTED