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NATIONAL SECURITY AGENCY
FORT GEORGE G. MEADE, MARYLAND 20755-6000

FOIA Case: 68177B
9 December 2020

This responds to your Freedom of Information Act (FOIA) request of 7 July 2012 for "a copy of Military Cryptanalytics, Part III, by Lambros D. Callamahos. Please review the sections marked as classified for possible declassification and release." A copy of your request is enclosed. Your request has been processed under the FOIA and the document you requested is on the enclosed CD. Certain information, however, has been deleted from the enclosure.

Some of the withheld information has been found to be currently and properly classified in accordance with Executive Order 13526. The information meets the criteria for classification as set forth in Subparagraph (c) of Section 1.4 and remains classified SECRET as provided in Section 1.2 of Executive Order 13526. The information is classified because its disclosure could reasonably be expected to cause serious damage to the national security. Because the information is currently and properly classified, it is exempt from disclosure pursuant to the first exemption of the FOIA (5 U.S.C. Section 552(b)(1)). The information is exempt from automatic declassification in accordance with Section 3.3(b)(1) of E.O. 13526.

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NATIONAL SECURITY AGENCY

MILITARY CRYPTANALYTICS

Part III

By

LAMBROS D. CALLIMAHOS

October 1977

Classified by DIRNSA/CHCSS (NSA/CSSM 123-2)
Exempt from GDS, EO 11652, Cat 2
Declassify Upon Notification by the Originator

National Security Agency
Fort George G. Meade, Maryland

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Give me an ounce of civet, good apothecary, to sweeten my imagination.

—Shakespeare.
(King Lear, Act IV, Sc. 6)

Preface

1. I wish to acknowledge my indebtedness to William F. Friedman in drawing upon portions of his early work, "Military Cryptanalysis, Part III," for much of the material treated in Chapters I–V. Chapters IV–XI are revisions of seven of my monographs in the *NSA Technical Literature Series, viz.:* Monograph No. 19, "The Cryptanalysis of Ciphertext and Plaintext Autokey Systems"; Monograph No. 20, "The Analysis of Systems Employing Long or Continuous Keys"; Monograph No. 21, "The Analysis of Cylindrical Cipher Devices and Strip Cipher Systems"; Monograph No. 22, "The Analysis of Systems Employing Geared Disk Cryptomechanisms"; Monograph No. 23, "Fundamentals of Key Analysis"; Monograph No. 15, "An Introduction to Teleprinter Key Analysis"; and Monograph No. 18, "Ars Conjectandi: The Fundamentals of Cryptodiagnosis."

2. I also wish to acknowledge my indebtedness to Francis T. Leahy for keeping me out of statistical mischief, and to Bruce W. Fletcher for his expert assistance in the final proofreading, and for checking the cryptograms and the various diagrams.

—L. D. C.

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CHAPTER I

INTRODUCTION

1. Preliminary remarks.—*a.* This text constitutes the third in the series of six basic texts on the science of cryptanalytics.[1] The first two texts together have covered most of the necessary fundamentals of cryptanalytics; this and the remaining three texts will be devoted to more specialized and more advanced aspects of the science.

b. It is assumed that the cryptanalyst reader has studied *Military Cryptanalytics, Parts I* and *II*, and is familiar with the cryptologic terminology, concepts, principles, and techniques of solution of the various cryptosystems treated in those texts. *This general background is a necessary prerequisite to the thorough understanding of the principles expounded in this and the succeeding volumes.* Where appropriate, however, reference will be made to particular portions of the first two volumes; the reader would be wise to have these volumes handy when undertaking the study of this present text.

c. The text immediately preceding this one dealt with various types of periodic polyalphabetic substitution, commonly called *repeating-key systems.* It was seen in these repeating-key systems how a regularity in the employment of a limited number of alphabets, or even the employment of a complete set of alphabets in succession as in a progressive alphabet system, results in the manifestation of periodicity or cyclic phenomena in the cryptogram, by means of which the latter may be solved. The difficulty of solution is directly correlated with the type and number of cipher alphabets employed in specific examples.

d. Two procedures might suggest themselves to an enemy cryptographer for consideration if he realizes the foregoing circumstances and he thinks of methods to eliminate the weaknesses inherent in repeating-key ciphers. First, noting that the difficulties in solution increase as the length of the key increases, he might consider employing much longer keys as a means of increasing the security of the messages. Upon second thought, however, if the enemy cyptographer recognizes that, as a general rule, the first step in the solution of these ciphers consists in ascertaining the number of alphabets employed, it might seem to him that the most logical thing to do would be to use a procedure which will avoid periodicity altogether, eliminating the cyclic phenomena that are normally manifested within cryptograms of periodic construction, and thus foil even a first step towards solution. In other words, the cryptographer might progress from the use of rather short repeating keys (of perhaps no more than a dozen letters or so) to the use of key phrases of, let us say, 25–40 letters or thereabouts; subsequently, he might embark upon the use of keying procedures which would have the effect of producing keys of a length approximately equal to that of the average message being enciphered; and finally, he might advance to a stage of keying sophistication wherein the key consists of hundreds or thousands of elements, or even of an infinite number of elements (as, for example, in autokey systems).

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[1] Before the echoes of the first sentence of this third volume have died down, the distinction between the *science* of cryptanalytics and the *art* of cryptanalysis should be re-emphasized. The cryptanalyst pursues studies along general and detailed lines, in order to equip himself technically for the duties of the moment or of the future. This parallels quite closely the technical studies of a violinist, who progresses from elementary exercises to the études of Kreutzer and Rode and finally to the Caprices of Paganini; in the meanwhile, the violinist has also studied various solo works and chamber music as a means of enhancing his comprehension and appreciation of music in general. All that a technical background does for the violinist is to give him the means of artistic expression or synthesis of musical thoughts from the coding of clefs, keys, and notes; all that a technical background does for the cryptanalyst is to give him the means for imaginative expression or synthesis of plaintext meanings from the coding of systems, keys, and characters. See also in this connection footnote 5 on p. 3 of *Military Cryptanalytics, Part I.*

e. At this point in our discussion it would be well to examine two terms defined in the previous volume:

(1) *periodic system.* A system in which the enciphering process is repetitive in character and which usually results in the production of cyclic phenomena in the cryptographic text.

(2) *aperiodic system.* A system in which the method of keying does not bring about cyclical phenomena in the cryptographic text.

The foregoing are *practical* definitions—nobody in his right mind (and that of course includes all of our readers) [2] would classify a Hagelin C–38 system [3] as periodic just because it really *is* periodic with a finite cycle of 26x25x23x21x19x17 or 101,405,850; nor would the same right-minded individual quibble with the classification of a system as aperiodic if the length of the key is only 1000 letters and messages very rarely exceed that length. In brief, what we are in effect saying is that, even if a system embraces in its principle a fixed cycle or period, unless the period is considerably shorter than the messages being enciphered (thus permitting the manifestation of cyclic phenomena), the system may nevertheless for all practical purposes be considered as aperiodic *since the solution of a message is not predicated on writing the cipher text on several superimposed cycles and then solving the cryptographic depth thus produced.*

f. In this text we shall first examine varieties of aperiodic (as just defined) polyalphabetic substitution systems; then we shall study methods of extending or lengthening short mnemonic keys, followed by systems using lengthy keys (to include digital and teleprinter systems). Subsequently, we shall study methods of solution of some typical cryptomechanisms and cipher machines, and aperiodic combination systems. The text proper will end with a discussion of principles of key analysis as applied in manual and machine cryptosystems, followed by an extensive treatment of cryptodiagnosis. The appendices include useful cryptologic and cryptomathematical reference material, concluding with a course of problems designed to insure comprehension of the principles expounded in this volume.

2. General remarks on cryptographic periodicity.—*a.* When we consider the nature of periodicity in polyalphabetic substitution systems, we note that it is composed of *two* fundamental factors, because there are in reality *two* elements involved in its production. We have appreciated the fact that periodicity necessitates the use of a keying element employed in a cyclic manner; now we begin to realize that there is also another element involved, *viz.*, that unless the key is applied to constant-length plaintext groupings, no periodicity will be manifested externally in the cipher text, despite the repetitive or cyclic use of a constant-length key. This realization is quickly followed by the idea that possibly all periodicity may be avoided or suppressed by either or both of two ways: (1) by using constant-length keying units to encipher variable-length plaintext groupings, or (2) by using variable-length keying units to encipher constant-length plaintext groupings.

b. In the usual types of polyalphabetic substitution systems, successive letters of the repeating key are applied to successive letters of the text. With respect to the employment of the key, the cryptographic process may be said to be *constant* or *fixed* in character. This is true even if a single keying unit serves to encipher two or more letters at a time, provided only the groupings of plaintext letters are constant in length. In all such cases of encipherment by constant-length groupings, the apparent length of the period (as found by applying the factoring process to the cryptograms) is a multiple of the real length and the multiple corresponds to the length of the groupings, i.e., the number of plaintext letters enciphered by the same key letter. It is to be noted, however, that all these cases are still periodic, because *both* the keying units and the plaintext groupings are constant in length.

3. Effects of varying the length of plaintext groupings.—*a.* Now let us consider the effects of making either one or the other of these two elements *variable* in length. Suppose that the plaintext groups are made variable in length and that the keying units are kept constant in length. Then, even though the key may be cyclic and may repeat itself many times in the course of encipherment, external periodicity is suppressed, *unless the law governing the variation in plaintext groupings is itself cyclic, and the length of the message is greater than that of the cycle applicable to this variable grouping.*

[2] To scholars of English who experience a quick intake of breath at this point, the author hastens to clarify that the parenthetical phrase is intended to modify only the four immediately preceding words.

[3] Cf. pp. 458–464 of *Military Cryptanalytics, Part II.*

b. As an example illustrating the italicized portion of the preceding sentence, let us suppose the correspondents agree to use reversed standard cipher alphabets with the key word SIGNAL, and that in the encryption the message is divided into groups as shown below:

S	I	G	N	A	L	S	I	G	N	A	L	S	I	G
1	12	123	1234	12345	1	12	123	1234	12345	1	12	123	1234	12345
C	OM	MAN	DING	GENER	A	LF	IRS	TARM	YHASI	S	SU	EDO	RDER	SEFFE
Q	UW	UGT	KFAH	UWNWJ	L	HN	ARQ	NGPU	PGNVF	I	TR	OPE	RFER	OCBBC

N	A	L	S	I	G	N	A	L	S	I	G	N	A	L
1	12	123	1234	12345	1	12	123	1234	12345	1	12	123	1234	12345
C	TI	VET	WENT	YFIRS	T	AT	NOO	NDIR	ECTIN	G	TH	ATT	ELEP	HONES
L	HS	QHS	WOFZ	KDARQ	N	NU	NMM	YIDU	OQZKF	C	NZ	NUU	WPWL	EXYHT

S	I	G	N	A	L	S	I
1	12	123	1234	12345	1	12	123
C	OM	MAS	WITC	HBOAR	D	SC	OMM...
Q	UW	UGO	RFUL	TZMAJ	I	AQ	UWW

Cryptogram

QUWUG TKFAH UWNWJ LHNAR QNGPU PGNVF ITROP ERFER OCBBC LHSQH
SWOFZ KDARQ NNUNM MYIDU OQZKF CNZNU UWPWL EXYHT QUWUG ORFUL
TZMAJ IAQUW W...

The cipher text in this example shows a tetragraphic and a pentagraphic repetition. The two occurrences of QUWUG$_c$ (=COMMA$_p$) are separated by an interval of 90 letters; the two occurrences of ARQN$_c$ (=IRST$_p$) by 39 letters. The first repetition (QUWUG$_c$), it will be noted, is a true periodic repetition, since the plaintext letters, their groupings, and the key letters are identical. The interval in this case, if counted in terms of letters, is the product of the keying cycle, 6, and the grouping cycle, 15. The second repetition (ARQN$_c$) is not a true periodic repetition in the sense that both cycles have been completed at the same point, as is the case in the first repetition. It is true that ARQN$_c$, representing IRST$_p$ both times, is a *causal* repetition produced by the action of the same combination of key letters, I and G, but the enciphering points in the grouping cycle are different in the two occurrences. Repetitions of this type may be termed *partially periodic* repetitions, to distinguish them from those of the *completely periodic* type.

c. When the intervals between the two repetitions noted above are more carefully studied, especially from the point of view of the interacting cycles which brought them about, it will be seen that, counting according to *groupings* and not according to single letters, the two pentagraphs QUWUG$_c$ are separated by an interval of 30 groupings. Or, if one prefers to look at the matter in the light of the keying cycle, the two occurrences of QUWUG$_c$ are separated by 30 key letters. Since the key is but 6 letters long, it has gone through 5 cycles. Thus, the number 30 is the product of the number of letters in the keying cycle (6) and the number of different-length groupings in the grouping cycle (5). The interaction of these two cycles is like that of two gears in mesh, one driven by the other. One of these gears has 6 teeth, the other 5, and the teeth are numbered. If the two gears are adjusted so that the teeth marked "1" are adjacent to each other and the gears are caused to revolve, these two teeth will not come together again until the larger gear has made 5 revolutions and the smaller one 6. During this time, a total of 30 meshings of individual teeth will have occurred. But since one revolution of the smaller gear (=the grouping cycle) represents the encipherment of 15 letters (when translated in terms of letters), the 6 complete revolutions of this gear mean the encipherment of 90 letters. This accounts for the period of 90, when stated in terms of letters.

d. The two occurrences of the other repetition, ARQN$_c$, are at an interval of 39 letters; but in terms of the number of intervening groupings, the interval is 12, which is obviously two times the length of the keying cycle. In other words, the key has in *this* case passed through two cycles.

e. In a long message enciphered according to such a scheme as the foregoing, there would be many repetitions of both types discussed above (the completely periodic and the partially periodic) so that the cryptanalyst might encounter some difficulty in his attempts to reach a solution, especially if he had no information as to the basic system. It is to be noted in this connection that if any one of the groupings exceeds, say, 5 letters or so in length, the scheme may give itself away rather easily, since it is clear that *within each grouping the encipherment is strictly monoalphabetic.* Therefore, in the event of groupings of more than 5 or 6 letters, the monoalphabetic equivalents of telltale words such as ATTACK, BATTALION, DIVISION, would stand out. This system is most efficacious, therefore, with short groupings.

f. It should also be noted that there is nothing about the scheme which requires a regularity in the grouping cycle such as that embodied in the example. A lengthy grouping cycle guided by a key of its own may just as easily be employed; for example, the *number* of dots and dashes contained in the International Morse signals [4] for the letters composing the 25-letter key phrase DECLARATION OF INDEPENDENCE might be used. Thus, A (. _) has 2, B (_ . . .) has 4, and so on. Hence:

D	E	C	L	A	R	A	T	I	O	N	O	F	I	N	D	E	P	E	N	D	E	N	C	E
3	1	4	4	2	3	2	1	2	3	2	3	4	2	2	3	1	4	1	2	3	1	2	4	1

The grouping cycle is 3+1+4+4+2+ . . . , or 60 letters in length, and if the same phrase is used as a repeating key the total period would of course be 60, since after the encipherment of 60 letters the first key letter would be used again to encipher 3 letters, and so on, repeating the cycle. Suppose, however, that the foregoing 60-element keying pattern were used in conjunction with a different literal sequence for the actual key letters, say the 38-letter phrase CONSTITUTION OF THE UNITED STATES OF AMERICA. The period would then be the least common multiple of 38 and 60, or 1140 letters. This system might appear at first glance to yield a fairly high degree of cryptographic security; but this is not the case, as we shall presently see.

4. Primary and secondary periods; resultant periods.—*a.* It has been noted that the length of the complete period in a system such as the foregoing is the least common multiple of the length of the two component or interacting periods. In a way, therefore, since the *component* periods constitute the basic elements of the scheme, they may be designated as the *basic* or *primary* periods. These are also *hidden* or *latent* periods. The *apparent* or *patent* period, that is, the complete period, may be designated as the *resultant* or *secondary* period. In certain types of cipher machines there may be more than two primary periods which interact to produce a resultant period; also, there are cases in which the latter may interact with another primary period to produce a tertiary period, and so on.[5] The *final,* or *resultant,* or *apparent* period is sometimes the one which is usually ascertained first as a result of the study of the intervals between repetitions. This may or may not be broken down into its component primary periods.

b. Although a solution may often be obtained without breaking down a resultant period into its component primary periods, the reading of many messages pertaining to a widespread system of secret communication is much facilitated when the analysis is pushed to its lowest level, that is, to the point where the final cryptographic scheme has been reduced to its simplest terms. This may involve the discovery of a multiplicity of simple elements which interact in successive cryptographic strata.

5. Cryptographic principles of aperiodic systems.—*a.* A discussion of the methods for avoiding periodicity was contained in the preceding text.[6] A brief résumé of these methods is given below:

(1) Elements of a fixed or invariable-length key are applied to variable or irregular-length groupings of the plain text.

(2) Elements of irregular-length (variable-length) key are applied to regular and fixed groupings of the plain text.

(3) The principles of (1) and (2) are combined into a single system.

[4] Cf. p. 23, *Military Cryptanalytics, Part I.*

[5] An example of a cipher machine with several interacting latent periods is the Hagelin C–38. This machine produces in effect at any given moment six simultaneous reversed-standard-alphabet monoalphabetic substitutions in all 26 combinations of their presence or absence. The activity of each contributing monoalphabetic substitution is strictly periodic, with cycles of 26, 25, 23, 21, 19, or 17, conforming to the six regularly stepping pinwheels of the stated sizes. The total cycle of the machine is the product of the six relatively prime numbers, but the presence of individual subcycles constitutes one of the serious weaknesses of the machine.

[6] Cf. par. 99, *Military Cryptanalytics, Part II.*

(4) The key does not repeat itself; this is brought about either by constructing a nonrepeating key, or by employing the key in a special manner (such as in plaintext- and ciphertext interruptor systems and plaintext- and ciphertext autokey systems).

b. From the standpoint of cryptographic mechanics, aperiodic systems may be divided into two main classes, *viz.:*

(1) Systems in which the key elements are not in any way determined or influenced by any elements of the plain or cipher text; and

(2) Systems in which the key elements are generated or governed by the plain text being enciphered or by the resultant cipher text.

[redacted]

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[7] Cf. par. 65 (on p. 157) of *Military Cryptanalytics, Part II.*

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

CHAPTER II

SYSTEMS USING CONSTANT-LENGTH KEYING UNITS TO ENCIPHER VARIABLE-LENGTH PLAINTEXT GROUPINGS

7. General remarks.—*a.* The system described in subpar. 3*b* is obviously not to be classified as aperiodic in nature, despite the incorporation into the cryptosystem of a variable which in that case consisted of irregularity in the length of one of the two elements (key text and plain text) involved in polyalphabetic substitution. The variable there was subject to a law which in itself was periodic in character.

b. To make such a system truly aperiodic (under the definition given in subpar. 1*e*), by elaborating upon the basic scheme for producing variable-length plaintext groupings, would be possible, but impractical. For example, using the Morse code method illustrated in subpar. 3*f* for determining the key and simultaneously the lengths of the groupings, one might employ the text of a book; and if the book is longer than the message to be enciphered, the cryptogram would certainly show no periodicity as regards the intervals between any repetitions which might occur. However, as already indicated, such a scheme would not be very practical for regular intercommunication between a large number of correspondents, for reasons which are quite apparent. Encipherment and decipherment would be slow, cumbersome, onerous, and very subject to error; the book would have to be safeguarded as would a code book; and, unless the same key text were used for all messages, methods or indicators would have to be adopted to show exactly where encipherment begins in each message. Therefore a simpler method is desirable for producing constantly changing, aperiodic plaintext groupings.

8. Aperiodic encipherment produced by plaintext sequences grouped according to word lengths.—*a.* The simplest method for producing aperiodic plaintext groupings is encipherment according to the actual word lengths of the message being encrypted. Although the *average* number of letters composing the words of any alphabetical language is fairly constant, *successive* words comprising plain text vary a great deal in this respect, and this variation is subject to no law.[1] In telegraphic English, for example, the mean length of words is 5.2 letters; the words may contain from 1 to 15 or more letters, but the successive words vary in length in an extremely irregular manner, no matter how long the text may be.

b. As a consequence, the use of word lengths for determining the number of letters to be enciphered by each key letter of a repetitive key suggests itself to a cryptographer as soon as he comes to understand the way in which repeating-key ciphers are solved. For, he asks, if there is no periodicity in the cryptograms, how can the letters of the cipher text written in 5-letter groups be distributed into their respective monoalphabets? And if this very first step is impossible, how can the cryptograms be solved? We shall see.

9. Solution when known cipher alphabets are involved.—*a.* Despite the foregoing rhetorical questions, the solution is really quite simple when the cipher alphabets involved are standard alphabets or are otherwise composed of known sequences. All that is involved is the completion of the plain-component sequence (preceded by, if the situation so demands, conversion into plain-component equivalents). In monoalphabetic substitution systems, all of the words of the entire message come out on a single gen-

[1] It is true, of course, that the differences between the vocabularies of two writers are often marked and can be measured. These differences may be subject to certain laws, but these laws are psychological rather than mathematical. See Rickert, E., *New Methods for the Study of Literature,* University of Chicago Press, Chicago, 1927.

eratrix in the completion diagram; in the case of the system discussed in subpar. 8*b*, since the individual, separate words of a message are enciphered by different key letters, *these words will reappear on different generatrices of the diagram.* All the cryptanalyst has to do is to pick them out; he can do this once he has found a good starting point, by using a little imagination and following clues afforded by the context.

b. As an example, let us consider the following intercepted message:

SUHPZ TCEPL GLQKC XHVKM VJLZA KXWHA

YTOWN HBAFE XAVEQ AUVZI EBPOB

In the course of routine study of the message, the plain-component sequence is completed for the first 15 letters of the cryptogram, on the assumptions of direct and reversed standard cipher alphabets, as shown in Figs. 2*a* and *b*, respectively, below:[2]

S	U	H	P	Z	T	C	E	P	L	G	L	Q	K	C
T	V	I	Q	A	U	D	F	Q	M	H	M	R	L	D
U	W	J	R	B	V	E	G	R	N	I	N	S	M	E
V	X	K	S	C	W	F	H	S	O	J	O	T	N	F
W	Y	L	T	D	X	G	I	T	P	K	P	U	O	G
X	Z	M	U	E	Y	H	J	U	Q	L	Q	V	P	H
Y	A	N	V	F	Z	I	K	V	R	M	R	W	Q	I
Z	B	O	W	G	A	J	L	W	S	N	S	X	R	J
A	C	P	X	H	B	K	M	X	T	O	T	Y	S	K
B	D	Q	Y	I	C	L	N	Y	U	P	U	Z	T	L
C	E	R	Z	J	D	M	O	Z	V	Q	V	A	U	M
D	F	S	A	K	E	N	P	A	W	R	W	B	V	N
E	G	T	B	L	F	O	Q	B	X	S	X	C	W	O
F	H	U	C	M	G	P	R	C	Y	T	Y	D	X	P
G	I	V	D	N	H	Q	S	D	Z	U	Z	E	Y	Q
H	J	W	E	O	I	R	T	E	A	V	A	F	Z	R
I	K	X	F	P	J	S	U	F	B	W	B	G	A	S
J	L	Y	G	Q	K	T	V	G	C	X	C	H	B	T
K	M	Z	H	R	L	U	W	H	D	Y	D	I	C	U
L	N	A	I	S	M	V	X	I	E	Z	E	J	D	V
M	O	B	J	T	N	W	Y	J	F	A	F	K	E	W
N	P	C	K	U	O	X	Z	K	G	B	G	L	F	X
O	Q	D	L	V	P	Y	A	L	H	C	H	M	G	Y
P	R	E	M	W	Q	Z	B	M	I	D	I	N	H	Z
Q	S	F	N	X	R	A	C	N	J	E	J	O	I	A
R	T	G	O	Y	S	B	D	O	K	F	K	P	J	B

FIGURE 2*a*

S	U	H	P	Z	T	C	E	P	L	G	L	Q	K	C
H	F	S	K	A	G	X	V	K	O	T	O	J	P	X
I	G	T	L	B	H	Y	W	L	P	U	P	K	Q	Y
J	H	U	M	C	I	Z	X	M	Q	V	Q	L	R	Z
K	I	V	N	D	J	A	Y	N	R	W	R	M	S	A
L	J	W	O	E	K	B	Z	O	S	X	S	N	T	B
M	K	X	P	F	L	C	A	P	T	Y	T	O	U	C
N	L	Y	Q	G	M	D	B	Q	U	Z	U	P	V	D
O	M	Z	R	H	N	E	C	R	V	A	V	Q	W	E
P	N	A	S	I	O	F	D	S	W	B	W	R	X	F
Q	O	B	T	J	P	G	E	T	X	C	X	S	Y	G
R	P	C	U	K	Q	H	F	U	Y	D	Y	T	Z	H
S	Q	D	V	L	R	I	G	V	Z	E	Z	U	A	I
T	R	E	W	M	S	J	H	W	A	F	A	V	B	J
U	S	F	X	N	T	K	I	X	B	G	B	W	C	K
V	T	G	Y	O	U	L	J	Y	C	H	C	X	D	L
W	U	H	Z	P	V	M	K	Z	D	I	D	Y	E	M
X	V	I	A	Q	W	N	L	A	E	J	E	Z	F	N
Y	W	J	B	R	X	O	M	B	F	K	F	A	G	O
Z	X	K	C	S	Y	P	N	C	G	L	G	B	H	P
A	Y	L	D	T	Z	Q	O	D	H	M	H	C	I	Q
B	Z	M	E	U	A	R	P	E	I	N	I	D	J	R
C	A	N	F	V	B	S	Q	F	J	O	J	E	K	S
D	B	O	G	W	C	T	R	G	K	P	K	F	L	T
E	C	P	H	X	D	U	S	H	L	Q	L	G	M	U
F	D	Q	I	Y	E	V	T	I	M	R	M	H	N	V
G	E	R	J	Z	F	W	U	J	N	S	N	I	O	W

FIGURE 2*b*

c. In the diagram in Fig. 2*b* we note the word CAN at the beginning of one generatrix, then in the very next six columns the words YOU and GET in two other generatrices. That we should get some three-letter words on various generatrices is not particularly remarkable; (note the short words produced purely by accident in the generatrices of Fig. 2*a*) but that these words should follow one another in direct sequence in succeeding columns, and that the three words in question should be in excellent *contextual relationship* to form a plausible and *convincing* sentence beginning such as "CAN YOU GET . . ."

[2] One of the first things, if not the very first, to be done to a cryptogram in an undiagnosed system is the completion of the plain-component sequence on the basis of standard alphabets. In certain cases a solution is sometimes achieved by this means that would be impossible by any other. The completion is painless if accomplished by sliding strips; its probability of success in an isolated case is small, but the ratio of the time expended to its potential value is very large. This is a typical illustration of the application of the maxim of the experienced cryptanalyst: *"Try the simplest thing first."*

is more than remarkable (=a probability of .01 of random occurrence)—it is astonishing (=random probability of .0001).[3]

d. From here on the rest of the solution follows easily. If the cryptanalyst comes to a temporary halt (as in the example in Fig. 2*b*) in recovering further words on the generatrices, he can search in subsequent positions of the generatrix diagram for more words to be disclosed, and then he can fill in the missing portions from context and take another look at the generatrices. Or, it might be simpler if the cryptanalyst recovers a fragment of the specific key for the message, and then expands this key by steps to assist in reading the rest of the plain text. For example, in the case under discussion the cryptanalyst would get U, N, and I as key letters[4] for the successive words of the plain text CAN YOU GET; these letters suggest the words UNION, UNITED, UNIVERSITY, etc. The complete solution is given below, with the recovered specific key being UNITED NATIONS.[5]

U	N	I	T	E	D	N	A	T	I	O	N	S
CAN	YOU	GET	IN	TOUCH	WITH	SECOND	DETACHMENT	STOP	LINES	OUT	OF	ORDER
SUH	PZT	CEP	LG	LQKCX	HVKW	VJLZAK	XWHAYTOWNH	BAFE	XAVEQ	AUV	ZI	EBPOB

The only minor difficulty of such a solution is that of making the first step and getting a good start on a word. If the words are short it is rather easy to overlook good possibilities and thus spend some time in fruitless searching. However, solution must come; if nothing good appears at the beginning of the message, search should be made in the interior of the cryptogram or at the end.

10. Solution when unknown cipher alphabets are involved.—*a.* It has been seen from the foregoing that solution of cryptograms involving word-length encipherment by standard alphabets is rather trivial, not because there is any magical quality about standard alphabets, but because the components are *known sequences.* If any other components had been used, say a plain component based upon a HYDRAULIC keyword-mixed sequence and a cipher component based on a QUESTIONABLY keyword-mixed sequence, and if these components were *known,* the problem would have been pursued in exactly the same way, *viz.,* conversion of the cipher letters of the cryptogram into their plain-component (HYDRAULIC . . . XZ) equivalents and completion of the plain-component (HYDRAULIC . . . XZ) sequence.

b. But what if one or both of the components are unknown mixed sequences? The simple procedure of completing the plain-component sequence obviously cannot be used. Since the messages are polyalphabetic, and since the process of factoring cannot be applied, it would seem that the solution of messages enciphered in different alphabets and according to word lengths would be a difficult matter. Nevertheless, as is about to be demonstrated, the solution, even of a single message, is not nearly so difficult as first impression might lead one to imagine; the *modus operandi* will be explained in pars. 11 and 12.

11. Solution by means of idiomorphs and the probable-word method.—*a.* The first case to be studied involving unknown alphabets will be one wherein the original word lengths are retained in the cryptogram; this case will be discussed not because it is often encountered in practical military cryptography, but because it affords a good introduction to the usual case in which the original word lengths are no longer in evidence in the cryptogram, the latter appearing in the customary 5-letter groups. If the words

[3] We must never forget that probabilities are influenced by the *amount* of material under examination; if we looked at *enough* material, it might not be at all astonishing if we obtained even a 10-letter word by accident. In all the probability considerations in this text, unless otherwise stated it is assumed that we are dealing with a limited amount of traffic, limited enough so that a probability of .01 is remarkable, and a probability of .001 exciting.

[4] The key letters are assumed to be under A_p as the index letter. Throughout this text, whenever encipherment processes are under discussion, the pair of enciphering equations commonly referred to as characterizing the so-called Vigenère method will be understood unless otherwise indicated. This method involves the pair of enciphering equations $\theta_{k/2}=\theta_{i/1}$; $\theta_{p/1}=\theta_{c/2}$. That is, the index letter, which is usually the initial letter of the plain component, is set opposite the key letter in the cipher component; the plaintext letter to be enciphered is sought in the plain component and its equivalent is the letter opposite it in the cipher component. See in this connection subpar. 13*f* of *Military Cryptanalytics Part II.*

[5] This is the specific key as recovered from this single message. It is quite possible that the complete key is UNITED NATIONS ORGANIZATION, UNITED NATIONS SECURITY COUNCIL, etc.; a longer message would prove whether the key is UNITED NATIONS used repetitively, or whether it is a phrase beginning with these two words.

of a message are enciphered monoalphabetically, the true and complete idiomorphs of word patterns will be patent, regardless of the identity of the particular alphabet used in the encryption of each word. These idiomorphs and word lengths can then be used as a basis for the probable-word method of attack.

b. Let us study the following low-echelon ground message in which the actual word lengths have been preserved in the cipher text:[6]

IUITD QHIWE LVCGWPCLZ RP NIV GYPYSYCV NC IXHCXWUJ ORS ZXH

GRPPRVQDOB SE OKYNMMHKV GUJLTN MYIN WZ IVURNI CLSWZVHS

We note some strong idiomorphic sequences, in particular the following:

(1) IUITD	(2) GYPYSYCV	(3) GRPPRVQDOB	(4) OKYNMMHKV
aba	abaca	abba	abcddea

Looking up these patterns in idiomorph lists,[7] and guided by the delimitations of the words, we arrive at the following assumptions:

(1) IUITD	(2) GYPYSYCV	(3) GRPPRVQDOB	(4) OKYNMMHKV
ENEMY	DIVISION	BATTALIONS	ARTILLERY

The cipher values of these plain-cipher equivalencies are entered into a sequence reconstruction matrix of four levels (representing the four word assumptions), as follows:

P:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
(1)					I								T	U											D	
(2)				G					Y					V	C				S			P				
(3)	R	G							Q			V		O	D				B	P						
(4)	O				H				N			M						K		Y					V	

FIGURE 3*a*

Noting in lines (2) and (3) that the intervals between the letters G, V, and P are the same in both cases, we can assume direct symmetry[8] of position. In a few moments our reconstruction matrix will look like this:

P:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
(1)	S	H	B	P	I	N			M		R	G	T	U	K		Y		Q			V	C	O	D	
(2)	M		R	G	T	U	K		Y		Q			V	C	O	D		S	H	B	P	I	N		
(3)	R	G	T	U	K		Y		Q			V	C	O	D		S	H	B	P	I	N			M	
(4)	O	D		S	H	B	P	I	N			M		R	G	T	U	K		Y		Q			V	C

FIGURE 3*b*

c. The rest of the plain text can be recovered either by (1) completion of the plain-component sequence, insofar as possible, in order to reveal further plaintext fragments which may be expanded and thus make possible the filling in of additional values in the cipher component, or by (2) recovery and expansion of the partial specific key for the message. An important additional step in solution is the recovery of the missing letters in the cipher component by analysis of the construction of the component in cases of systematic derivation. These points will be taken up in order in the subparagraphs below.

[6] Foolish as this may be, it has happened in operational practice.
[7] Cf. Appendix 3, *Military Cryptanalytics, Part I.*
[8] Cf. par. 28, *Military Cryptanalytics, Part II.*

(1) Let us complete the plain-component sequence on the second and third words of the message, after first converting the cipher letters into their plain-component equivalents (where known), using for this purpose the uppermost cipher alphabet given in Fig. 3*b*. This is shown in the illustration below:

IUITD	QHIWE	LVCGWPCLZ
ENEMY	SBE	VWL DW
	TCF	WXM EX
	UDG	XYN FY
	VEH	YZO GZ
	WFI	ZAP HA
	XGJ	ABQ IB
	YHK	BCR JC
	ZIL	CDS KD
	AJM	*DET LE
	BKN	EFU MF
	*CLO	FGV NG
	DMP	GHW OH
	ENQ	HIX PI
	*FOR	IJY QJ
	GPS	JKZ RK
	HQT	KLA SL
	IRU	LMB TM
	JSV	MNC UN
	KTW	NOD VO
	LUX	OPE WP
	MVY	PQF XQ
	NWZ	QRG YR
	OXA	RSH ZS
	PYB	*STI AT
	QZC	TUJ BU
	*RAD	UVK CV

FIGURE 4*a*

LVCGWPCLZ
GVWL DW
HWXM EX
IXYN FY
JYZO GZ
KZAP HA
LABQ IB
MBCR JC
NCDS KD
ODET LE
PEFU MF
QFGV NG
RGHW OH
SHIX PI
TIJY QJ
UJKZ RK
VKLA SL
WLMB TM
XMNC UN
YNOD VO
ZOPE WP
APQF XQ
BQRG YR
CRSH ZS
DSTI AT
ETUJ BU
FUVK CV

FIGURE 4*b*

LVCGWPCLZ
HVWL DW
IWXM EX
JXYN FY
KYZO GZ
LZAP HA
MABQ IB
NBCR JC
OCDS KD
PDET LE
QEFU MF
RFGV NG
SGHW OH
THIX PI
UIJY QJ
VJKZ RK
WKLA SL
XLMB TM
YMNC UN
ZNOD VO
AOPE WP
BPQF XQ
CQRG YR
DRSH ZS
*ESTI AT
FTUJ BU
GUVK CV

FIGURE 4*c*

The generatrices with the most plausible possibilities for the continuation of plain text are marked with an asterisk. If the context of the message cannot be gotten from this diagram, what we can do is to take the third word, LVCGWPCLZ, and assume that the letters for which we have no plain-component equivalents in the first cipher alphabet of Fig. 3*b* represent one of the eight missing plaintext letters, G, H, J, P, R, T, U, or Z. If we assume that the first letter (L_c) of this word represents G_p (on the first or conversion row of the generatrix diagram just beneath the ciphertext letters), we obtain the result shown in Fig. 4*b*; when we try $L_c=H_p$, as shown in Fig. 4*c*, we obtain an excellent plaintext tetragraph on the third generatrix from the bottom, and see that the word is ESTIMATED. The newly recovered values in the cipher alphabet will aid in establishing the remaining unknown letters in the generatrix diagrams for other words of the message.

(2) For the second method, let us refer again to Fig. 4*a*. The key letter used to encipher the first word, ENEMY, is S_k (assuming A_p to be the index letter in the usual Vigenère equation), since $I_c=E_p$. Now for the second word, if $Q_c=C_p$ (one of the asterisked good generatrices for this word), the key is Y_k; if $Q_c=F_p$, $\theta_k=U$; and if $Q_c=R_p$, $\theta_k=H$. The first key digraphs thus formed, SY, SU, and SH, are all compatible as English word beginnings. For the third word of the message, considering the two asterisked generatrices in Fig. 4*a*, if $V_c=D_p$, $\theta_k=Q$; if $V_c=S_p$, $\theta_k=P$. Therefore the first three key letters are now resolved as SYP or SUP; SYP is quickly discarded, and SUP should be followed by an E, I (less likely), P, or R, suggesting words such as SUPERIOR, SUPPORT, or SUPREME. A quick check on the message establishes that, with $\theta_k=R$, the fourth word deciphers to AT_p. Proceeding in this fashion, we are able to recover the key and simultaneously the plain text in record time.

(3) In cases wherein the cipher component has been constructed in some systematic manner, analysis of its derivation will make possible recovery of the component in its entirety after a sufficient number of values has already been placed correctly.[9] What constitutes "a sufficient number of values" depends upon the type of construction of the component, as well as the vagaries of the particular situation at hand. Taking for example the cipher component as established in Fig. 3*b*,

S H B P I N . . M . R G T U K . Y . Q . . V C O D .,

we observe the digraphic fragments BP and GT. If these are a part of a transposition-mixed sequence, the mechanics of the system would indicate that the fragments are part of the diagram

B . . . G,
P Q R S T

which means that three of the letters CDEF lie in order between B and G, and that directly above them are the letters composing the key word for the transposition matrix. However, since R immediately precedes the G in the sequence, it appears that R is part of the key word and not part of the remaining alphabetic portion. Thus the fragmentary matrix

H Y D R
B . . G
P Q S T

can be reconstructed, from which, with but little imagination, the key word HYDRAULIC may be seen emerging, so that the entire component is derivable from the following diagram:

4	9	3	7	1	8	6	5	2
H	Y	D	R	A	U	L	I	C
B	E	F	G	J	K	M	N	O
P	Q	S	T	V	W	X	Z	

d. By means of the foregoing methods, we can establish that the primary components are the following:

P:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
C:	A	J	V	C	O	D	F	S	H	B	P	I	N	Z	L	M	X	R	G	T	U	K	W	Y	E	Q

The complete message and the specific key are given below:

S	U	P	R	E	M	E	C	O	U
ENEMY	FORCE	ESTIMATED	AT	ONE	DIVISION	OF	INFANTRY	AND	TWO
IUITD	QHIWE	LVCGWPCLZ	RP	NIV	GYPYSYCV	NC	IXHCXWUJ	ORS	ZXH

R	T	O	F	T	H	E	U	(NITED STATES)
BATTALIONS	OF	ARTILLERY	MOVING	WEST	OF	NEWTON	JUNCTION	
GRPPRVQDOB	SE	OKYNMMHKV	GUJLTN	MYIN	WZ	IVURNI	CLSWZVHS	

Now that the components have become known sequences, the solution of subsequent messages enciphered with these components but with different specific keys is a simple matter, involving only a conversion of the cipher letters into their plain-component equivalents and a completion of the plain-component sequence. This point required re-emphasizing because in actual operational problems it is frequently forgotten.

e. The example in subpar. *b* involved a case of direct symmetry of position. If both the plain and the cipher components had involved mixed sequences, indirect symmetry of position would have

[9] For a treatment of the cryptographic mechanics of systematically mixed sequences and their cryptanalytic recovery, see par. 51 (on pp. 86–90) of *Military Cryptanalytics, Part I.*

applied.[10] As an example of such a case, let us suppose that the cipher text of the message in question had been different, and that the sequence reconstruction matrix in Fig. 3*a* had been the following:

Ø	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1					H								U	L											O	
2				J					P					Y	D				U			I				
3	R	C							L			U		M	N				Q	S						
4	O				W				S			Q						N		C					K	

FIGURE 5

(1) We observe the proportion AR (Ø–1, 3–1) = ON (Ø–15, 3–15) which is duplicated in AR (Ø–1, Ø–18) = ON (4–1, 4–18);[11] this is indicative that symmetry extends to the Ø line, and therefore that the plain and cipher components are identical sequences. Consequently, we are able to chain to the Ø line, deriving the following sets of partial chains:

```
Ø–1  EH   MU   NL   YO
Ø–2  ODJ   VIP   NY   SU
Ø–3  AR   BC   ILU   ONM   TSQ
Ø–4  AO   EW   IS   LQ   RN   TC   YK
```

FIGURE 6

(2) We note that the fragmentary chains ONM and TSQ of the Ø–3 set appear to be parts of a keyword-mixed sequence in reverse; so, proceeding with the graphical method [12] of indirect symmetry, we assign to these chains the notation →, and then we arbitrarily assign the notation ↓ to the Ø–2 chains. The four sets of fragmentary chains will then be amalgamated into the diagram shown in Fig. 7*a*, below.

```
    C B
    Q S T V
      U L I
  K M N O P
      Y D R A
        J
```

FIGURE 7*a*

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
J K M N O P Q S T V . . . . . . . . . . . . . . . .
      Y D R A U L I C B
        J K M N O P Q S T V
```

FIGURE 7*b*

This diagram may then be expanded into that shown in Fig. 7*b*, consisting of the integration of two major chains tied together by the vertical VIP relationship.

(3) Now noting in Fig. 7*b* the sequence VCS on a diagonal and the letters S.V in the top row, we realize that the distance V to C when measured on the primary component should be 12, i.e., one-half of the distance (24) between V and S on the top row. Consequently, we may place the C at a position 12 spaces to the right of the V, which permits us to expand our diagram into the following:

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
J K M N O P Q (S) T (V) . . . . Y D R A U L I (C) B . . .
      Y D R A U L I C B
        J K M N O P Q (S) T V
```

FIGURE 7*c*

(4) The fragmentary chains EH and EW in Fig. 6 could have been placed in their proper positions earlier in Figs. 7*a*–*c*; however, in order to illustrate a point, we have delayed their amalgamation until now. We note that the Ø–1 chains in Fig. 6 are at a decimation of −10 in the sequence in Fig. 7*c*; there is only one possible placement of the letters E and H at this interval, which then fixes the position of the

[10] Cf. Chapter VI, *Military Cryptanalytics, Part II.*

[11] This notation has been discussed in footnote 2 on p. 92, *Military Cryptanalytics, Part II.*

[12] Cf. par. 46, *Military Cryptanalytics, Part II.*

last unused letter, W, the placement of which heretofore could have been ambiguous. These letters fit into the reconstructed sequence as follows:

J K M N O P Q S T V W . . H Y D R A U L I C B E . .

It is a pleasure to use, without encountering a risk of cavilation the word "obvious" [13] as regards the positions of the missing letters:

J K M N O P Q S T V W X Z H Y D R A U L I C B E F G

f. The immediately preceding example treated a case of identical sequences proceeding in the same direction for the plain and cipher components. If the cipher component had run in the reverse direction, or if the components had been two different [unknown] mixed sequences, indirect symmetry would still have applied, with the exception (and a very important exception indeed) that chains to the Ø line would have been excluded, all chaining being done within the matrix. This prohibition would result in the situation that not only would a single short message encrypted in such a system be well-nigh unsolvable, but that even if we had a long message or a small volume of traffic, it would probably be necessary to make a fairly large number of assumptions, all correct, before there would be enough data available to permit their manipulation and exploitation by indirect symmetry.

g. Now that we have understood the details of solution of cases wherein the true word lengths have been preserved, we will take up the situation wherein the cipher text has been transmitted in its usual form of 5-letter groups.

(1) Let us suppose that we have a number of messages, all of which are known to have been enciphered monoalphabetically by word lengths with the same pair of unknown primary mixed components and (although this is not a vital consideration) in the same message key.[14] Five messages have been selected from the aggregate because of the presence of polygraphic repetitions between them; the beginnings of these messages are shown in Fig. 8*a*, below:

1. G K B S A M K U H Q P J C G K K L J H K C F V T Y . . .
2. A L E J Q A K G L Y L W H R H C D H K U V B V P V . . .
3. S T T J U M A M K U Z I U V S V N R L Z O K L Z P . . .
4. L K Q A M G I J E U M G P J C G K K L J H B E K V . . .
5. B K J U A I E S A A S B R H S L Y L W H H Q Y E P . . .

FIGURE 8*a*

[13] The reader is reminded of the pithy anecdote on the word "obvious" quoted in footnote 11 on p. 6 of *Military Cryptanalytics, Part I*.

[14] If this latter fact had not been known, it could have been conjectured, from an examination of the I.C.'s of groups of columns, that the same message key was used for all the messages. In the particular example in Fig. 8*a*, the I.C. of the first 5 columns (taken collectively) is 1.56, while that of the first 10 is 1.76, and thereafter the I.C. drops off very rapidly even though we are adding more data to our distribution for evaluation. The grouped I.C.'s for the first N columns are summarized in the diagram below:

N	I.C.	N	I.C.
5	1. 56	12	1. 53
6	1. 55	15	1. 33
8	1. 73	20	1. 37
10	1. 76	25	1. 24

The reason for the low I.C.'s of the first 5 and the first 6 columns is that the sample was insufficient to portray what we expect of English plain text; on the other hand, the reason for the high I.C.'s of the first 8 and the first 10 columns is that the beginning words of these messages probably exceed the average length (5.2 letters) of all English words.

(2) The 5-letter and 9-letter repetitions have the length and idiomorphic patterns of ENEMY and ARTILLERY, respectively. Taking into account that the average word length in telegraphic English plain text is 5.2 letters, it appears that both of these words were probably enciphered by the third letter of the message key,[15] although the relative numerical identity of the particular alphabet is really of no concern to us at the moment. On the basis of the idiomorphic beginning, Message No. 3 could start with the word AMMUNITION, making the 4-letter repetition TION which is cryptolinguistically titillating; the first word of Message No. 1, LOCATION, comes immediately thereafter, which is followed by COUNTERATTACK at the beginning of Message No. 5, HOSTILE at the beginning of Message No. 4, and THRUST at the beginning of Message No. 2. From the solution of the first three words of these five messages, and with the concurrent exploitation of the direct symmetry manifested, the primary cipher component is established as

S H B P I . Z L M . R G T U K W Y E Q A J V C . D F

(3) The key letters (under A_p) of the first three alphabets are S, U, and P. The rest of the solution proceeds either by the generatrix method as outlined in subpar. 11*c*(1), or by analysis of the key as illustrated in subpar. 11*c*(2). The complete texts of the message beginnings are shown in Fig. 8*b*, below:

```
1.  G K B S A  M K U|H Q| P J C G K  K L J H|K  C F V R T . . .
    L O C A T  I O N|O F| A R T I L  L E R Y|E  M P L A C

2.  A L E J Q  A|K G|L Y  L W H|R H  C D H K U| V B V P V . . .
    T H R U S  T|B Y|E N  E M Y|A R  M O R E D| E L E M E

3.  S T T J U  M A M K U| Z I U V S| V N R L Z  O K L Z|P . . .
    A M M U N  I T I O N| T R A I N| S C H E D  U L E D|T

4.  L K Q A M  G I|J E U  M G|P J C  G K K L J  H|B E K V . . .
    H O S T I  L E|H E A  V Y|A R T  I L L E R  Y|S H E L

5.  B K J U A  I E S A A  S B R|H S| L Y L W H| H Q Y E Z| . . .
    C O U N T  E R A T T  A C K|O N| E N E M Y| R I G H T|
```

FIGURE 8*b*

(4) It may be seen from the foregoing example that the general theory of idiomorphic attack and the probable-word method remains the same for 5-letter texts as it is for text divided into bona fide word lengths; only the details of the execution differ. Where a small volume of homogeneous traffic is at hand, and something is known about the correspondents and the nature of the messages, solution should pose no problems (other than usual cryptanalytic headaches concomitant with operational situations of minor systems in which only a few messages are available).

12. Solution by means of isomorphs.—*a*. The phenomenon of isomorphism and an illustration of the exploitation of isomorphs in cipher text were covered in the previous volume.[16] In practical cryptanalysis the phenomena of isomorphism afford a constantly astonishing source of clues and aids in solution. The alert cryptanalyst is always on the lookout for situations in which he can take advantage of these phenomena, for they are among the most interesting and most important in cryptanalytics.

[15] We have already noted that the common plain-cipher equivalencies $E_p=L_c$ and $Y_p=H_c$ in the two words establish the fact that these words were enciphered by the same alphabet.

[16] Cf. par. 71, *Military Cryptanalytics, Part II.*

b. Let us consider the case of word-length encipherment involving an unknown pair of primary components, the cipher text being transmitted in the customary 5-letter groups. The following cryptogram is available for study:

```
LHJJT  YZLDX  ZHYPH  ZFOCX  LIMDF  GOOBD
PFQXX  QGYJP  RXGJG  LTSRM  KSPGZ  ZIJFP
KEFGJ  IMKHX  WIYDC  CTAUE  EDTFK  HUNFZ
HSGRG  EGJKL  IBWXW  DVBBO  WTDXS  TVWMT
FBDJZ  IYZBE  XXXXX
```

c. There are no long polygraphic repetitions in evidence. An isomorphic search,[17] however, uncovers several isomorphic sequences indicated by the dotted lines above; these are grouped into the following two sets of isomorphs:

Set "A"		Set "B"	
(α)	L H J J T Y Z L D X Z H Y	(δ)	D F G O O B D
(β)	P G Z Z I J F P K E F G J	(ε)	T A U E E D T
(γ)	D V B B O W T D X S T V W		

If these isomorphs are causal isomorphs, i.e., isomorphs produced by the different encryptions of identical plaintext sequences, then the relationships between corresponding letters of the isomorphs reflect the relationships between different juxtapositions or slides of a cipher component against a plain component; these relationships, latent in the isomorphs, may be made patent through the mechanics of indirect symmetry.

d. The partial chains derivable from these isomorphs are given below:

```
α–β: LP   HG   YJZF   TI   DK   XE
α–γ: LDXS   HV   JB   ZTO   YW
β–γ: PD   GV   ZB   IO   JW   FT   KX   ES
δ–ε: BDT   FA   GU   OE
```

Using the graphical method of indirect symmetry, these partial chains may be amalgamated into the diagrams shown in Figs. 9*a* and *b*, below. We note in Fig. 9*a* the $\underset{2}{\llcorner 1}$ relationship of the letters **XZ**, **ST**, **OP**, and **EF**, and conclude that the cipher component must be a keyword-mixed sequence. We now expand the diagram of Fig. 9*a* by placing the **W** in position diagonally ahead of the **XZ**, and we duplicate the remaining letters in their proper position with respect to the **W** just placed a moment before; this

```
Y W
.
J B
L D X S
Z T O
P K E
F I
A
```

FIGURE 9a

```
H V
.
G
U
```

FIGURE 9b

```
Y   W
.
J   B       Y  (W)
L   D  (X)  S
(Z) T   O   J   B
P   K   E   L   D   X   S
F   I       Z   T   O
A           P   K   E
            F   I
            A
```

FIGURE 9c

```
Y   W
                    H  (V)
.
J   B       Y  (W)  .
L   D  (X)  S       G
(Z) T   O   J   B   U
P   K   E   L   D   X   S
F   I       Z   T   O
A           P   K   E
            F   I
            A
```

FIGURE 9d

[17] For a systematic method of searching for isomorphs, see footnote 7 on p. 174 of *Military Cryptanalytics, Part II*.

is shown in Fig. 9*c*. This facilitates placing the diagram of Fig. 9*b* into the array (on the basis of the VWXZ diagonal), resulting in the final diagram shown in Fig. 9*d*. From this latter figure, the original cipher component, minus 5 letters, may be chained out:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
B	E	F	G	J	K	.	.	O	P	.	S	T	V	W	X	Z	H	Y	D	.	A	U	L	I	.

Our old friend HYDRAULIC.

e. We have the sequence for the cipher component, but now what? We could assume the plain component to be the normal sequence, direct and then reversed, and we could convert the first few cipher letters into their plain-component equivalents on these hypotheses and then complete the plain-component sequence; if we are correct in our assumption, a plaintext word would be revealed on one generatrix, another word on a different generatrix, etc. We could also assume the plain component to be the same as the cipher, in the same or in the reverse direction, and we could complete the plain-component sequence accordingly. All of these attempts fail, so it means that we are faced with a plain component of unknown composition. We have the cipher component at hand, it is true, but *unless we know or can deduce the motion of the cipher component,*[18] it will be impossible for us to convert the cipher text into monoalphabetic terms; in other words, the original cipher is already reduced as far as it will go. Plaintext assumptions are now an absolute necessity.

f. It can be seen by referring to the two sets of isomorphs in sub-par. *c* that Set "A" has the complete idiomorphic pattern for COMMUNICATION, and that Set "B" has the idiomorphic pattern contained in ARTILLERY. If the now known cipher component is set down and the plaintext equivalents for the first occurrences of the assumed COMMUNICATION and ARTILLERY are recorded in the rows labelled P_1 and P_2 of the diagram below, direct symmetry of position will of course apply, provided that there

C:	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z
P_1	O	N	A				C							M								U			T	I
P_2			R							E		T	I			A		L	Y							

is a tie-in letter between the sequences; there happen to be three such letters, so that the plain component may be expanded as follows:

O N A . L Y C . . . . . . M . R . . . . . U E . T I

Since there are manifested the phenomena of a keyword-mixed sequence in the plain component, we may further expand the sequence into the following:

O N A . L Y C D F G H J K M . R . . . . . U E . T I

If the key word for the sequence cannot be guessed from this partial sequence, we might finish the solution by a modification of the method indicated in subpar. 11*c*(1), with the difference that, in this case, not only will some of the cipher letters not have plain-component equivalents, but also that the plain component itself will have gaps in its sequence in the plain-component completion diagram. After the key word (QUESTIONABLY) for the plain component has been recovered, the solution can be completed by the generatrix method, keeping in mind the reconstruction of the message key as a means of quick

[18] See in this connection subpar. 71*d* on p. 175 of *Military Cryptanalytics, Part II*.

analysis after a few letters of the key have been derived. The complete decipherment of the message is shown below:

```
S                      T         R         I
L H J J T  Y Z L D X  Z H Y|P H  Z F|O C X  L I|M D F  G O O B D
C O M M U  N I C A T  I O N|W I  T H|F I R  S T|A R T  I L L E R

  K        E   W           H           I     L
P|F Q X X| Q G|Y J P  R X G J|G  L T S R|M  K S|P G Z  Z I J F P
Y|W I L L| B E|T H R  O U G H|C  O R P S|A  N D|C O M  M U N I C

           E       T           H                  E
K E F G J| I M K H|X  W I Y D C| C T A U E  E D T F|K  H U N F Z
A T I O N| W I T H|S  E C O N D| A R T I L  L E R Y|T  H R O U G

  I                R       O     N                            I
H|S G R G  E G J K|L  I B W|X W| D V B B O  W T D X S  T V W|M T
H|D I V I  S I O N|S  T O P|N O| C O M M U  N I C A T  I O N|A F

      S
F B D|J Z  I Y Z B E  X|X X X X
T E R|M I  D N I G H  T|
```

FIGURE 10

The key for the message, under Q_p as the index letter, is "STRIKE WHILE THE IRON IS . . . (HOT?)"[19]

g. In connection with the solution of the problem in this paragraph, let us take a closer look at the isomorphs listed in subpar. *c.* These are given below, together with their plaintext equivalents:

```
            Set "A"                            Set "B"

      C O M M U N I C A T I O N          (A)R T I L L E R(Y)
(α)   L H J J T Y Z L D X Z H Y      (δ)    D F G O O B D
(β)   P G Z Z I J F P K E F G J      (ε)    T A U E E D T
(γ)   D V B B O W T D X S T V W
```

In case it had escaped attention before, note the ciphertext fragments XZHY, EFGJ, and STVW at the ends of the isomorphs of Set "A". These three tetragraphs, transparent in the cipher text, are actually fragments of the keyword-mixed sequence constituting the cipher component. The reason for their presence is not hard to find: the plaintext equivalent of the isomorphs ended with TION and the letters TION happened to be a fragment of the keyword-mixed sequence constituting the *plain* component. (Note also, from Set "B", that AU must also be in sequence in the cipher component.) This information would have been of assistance to us in the chaining process pursued in subpar. *d*; for pedagogical reasons, however, we delayed drawing attention to this situation until now. Needless to say, this situation or a recognizable variation of it could be of considerable assistance in the solution of a difficult problem of only a few messages in actual operations.

h. One more very important facet of isomorphism should be discussed at this point. Let us suppose that we have recovered the cipher component of the message under study through the exploitation of isomorphs as just demonstrated; but let us suppose that the two plaintext assumptions (COMMUNICATION and ARTILLERY) were insufficient to disclose enough of the sequence for the plain component to permit its facile recovery.[20] Additional plaintext assumptions are necessary, but we seem to have milked the

[19] Hot.

[20] This would be the case if the plain component were not a keyword-mixed sequence but were, let us say, a transposition-mixed sequence.

cipher text dry with the two cribs we have already placed. The problem confronting us is how to make further "educated guesses" that might display a trace (or more, we hope) of erudition.

i. Since the cipher component has become a known sequence, let us set it down, numbering its elements serially from 1 to 26, as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z

Let us first replace the letters of the cipher text by their numerical equivalents according to the HYDRAULIC sequence. We will then take a delta or lateral difference stream [21] from these numerical values, by subtracting each number from the following one;[22] however, instead of recording the numerical difference, we will record the literal equivalent of this numerical difference according to the HYDRAULIC sequence above. The result of this process is shown in Fig. 11, below:

```
L  H  J  J  T    Y  Z  L  D  X    Z  H  Y  P  H    Z  F  O  C  X    L  I  M  D  F    G  O  O  B  D
7  1  14 14 22   2  26 7  3  25   26 1  2  19 1    26 12 18 9  25   7  8  16 3  12   13 18 18 10 3
  Q  G  Z  I   U  W  L  T  T    H  H  H  N  I    X  F  U  N  M    I  H  I  G  C    H  A  Z  O  P

P  F  Q  X  X    Q  G  Y  J  P    R  X  G  J  G    L  T  S  R  M    K  S  P  G  Z    Z  I  J  F  P
19 12 20 25 25   20 13 2  14 19   4  25 13 14 13   7  22 21 4  16   15 21 19 13 26   26 8  14 12 19
M  P  I  A  Z    S  P  K  F  A    E  S  J  H  X    Q  K  X  C  F    X  U  W  Q  G    Z  I  U  W  L

K  E  F  G  J    I  M  K  H  X    W  I  Y  D  C    C  T  A  U  E    E  D  T  F  K    H  U  N  F  Z
15 11 12 13 14   8  16 15 1  25   24 8  2  3  9    9  22 5  6  11   11 3  22 12 15   1  6  17 12 26
T  T  H  H  H    Q  I  X  F  W    X  B  Q  H  U    Z  G  C  H  A    Z  O  P  M  D    F  A  E  S  J

H  S  G  R  G    E  G  J  K  L    I  B  W  X  W    D  V  B  B  O    W  T  D  X  S    T  V  W  M  T
1  21 13 4  13   11 13 14 15 7    8  10 24 25 24   3  23 10 10 18   24 22 3  25 21   22 23 24 16 22
H  Q  O  N  C    W  Y  H  H  O    H  Y  J  H  X    A  Q  G  Z  I    U  W  L  T  T    H  H  H  O  U

F  B  D  J  Z    I  Y  Z  B  E    X  X  X  X  X
12 10 3  14 26   8  2  26 10 11   25 25 25 25 25
M  W  P  E  F    I  Q  W  B  H    J  Z  Z  Z  Z
```

FIGURE 11

j. We can see, by comparing Fig. 11 with the original plain text as given in Fig. 10, that the delta stream has revealed *all* of the polygraphic repetitions of trigraphs or better in the underlying plain text.[23] Note the IXF repetition in the delta stream, which means that the ciphertext sequences PHZF and IMKH must represent the same plain text (probably the word WITH, since it is a four-letter repetition following COMMUNICATION); the PHZF and IMKH sequences are actually isomorphic, but we were unable to recognize them as such until now. Also note the delta repetition FAESJH, which means that the ciphertext sequences YJPRXGJ and KHUNFZH (an isomorphic pair whose isomorphism we were unable to trust before, because of a lack of sufficient corroborative values in the isomorphic repetition pattern) must represent the same plain text (in this case, the assumption of the word THROUGH would be permitted). Note further the HH digraphic fragment in the delta stream, which means that GJK_c must represent ION_p (from COMMUNICATION); since this is not preceded by T_p, the assumption of S_p and therefore DIVISION is encouraged. With these plaintext values and those which follow as a direct result of our analysis thus far, it would be a simple matter to reconstruct the plain component almost *in toto* and the plain text of the message in its entirety.

13. Additional remarks.—*a*. One of the practical difficulties in employing systems in which the keying process shifts according to word lengths is that in handling such a message the deciphering clerk is often not exactly certain when the termination of a word has been reached, which results in the loss of time and effort. For instance, in deciphering a word such as INFORM, the clerk would not know whether he now has the complete word and should shift to the next key letter or not; the word might be INFORMS,

[21] The application of delta stream techniques to the solution of digital cipher systems has been illustrated in Chapters XI and XII of *Military Cryptanalytics, Part II*.

[22] In this process, subtraction is performed *mod* 26: i.e., if we are to subtract a large number from a smaller, we add 26 to the smaller before subtraction. For example, $1-7=(1+26)-7=20=Q$ in the scale above. 26 is equivalent to Ø in this modulus, so that $14-14=Ø=Z$.

[23] The plaintext repetitions are foreshortened by one letter in the delta stream; e.g., COMMUNICATION, a 13-letter word, appears as a 12-letter sequence in the delta stream.

INFORMED, INFORMING, INFORMAL, INFORMATION, etc. The past tense of verbs, the plural of nouns, and terminations of various sorts capable of being added to word roots would give rise to difficulties, and the latter would be especially troublesome if the messages contained a few telegraphic errors to boot. Consequently, *word separators* are often adopted to circumvent this source of trouble.[24] These separators usually consist of an infrequent letter, such as X_p or Q_p, which is placed after every word of the plain text and is encrypted along with the rest of the message.[25]

(1) When word separators are employed and this fact is once suspected or discovered, their presence is of as much aid to the cryptanalyst in his solution as it is to the clerks who are to decipher the messages. As an example, let us study the following cryptogram:

IWJIR	NPTXS	FIWCM	SDFEW	SBLXQ	LBHFL	TYIFD	UVLUL	JRLYG	HRZYI
FMZXD	GRMCR	SWPTX	SFIWC	KAMWZ	XLXWQ	BAARN	FLTVQ	AMQDZ	LVUQK
GQZZO	IHMIR	OLOMI	DXZFG	PLKIS	CAHQZ	MGNWX	BTIYQ	BDLTP	NPQUD
LYLGU	FINSX	LOHZA	SXAFD	XTFIZ	PJXMM	QDCPE	WYIBZ	QGHBH	RXDTX
IOOLU	IKVGC	MGITZ	HWDRG	GIWMY	RZWNP	FDCEM	YFASY	PJWHX	JZGWW
XFQXO	TMCNA	UUEJJ	IKVGH	RZYIP	MWIDL	RDCWI	PGAQC	SACWP	

Collateral information indicates that the cryptosystem involves monoalphabetic encipherment by word lengths, a word separator being used to signal the change to a new key letter; the key letters themselves form a plaintext word as a mnemonic key.

(2) If the encipherment is by word lengths and a word separator is used, the average length of words should be 6.2 letters. Since a key word is used to control the selection of alphabets, if a polygraphic repetition of significant length is present in the cipher text, the interval between the first and second occurrences should give a fair indication of the length of the key, unless there are repeated letters in the key and these polygraphic repetitions happen to be produced by identical key letters in *different* positions in the key word. We note the 8-letter repetition PTXSFIWC at an interval of 56 letters; this would seem to indicate that the key word is $\frac{56}{6.2}$ or 9 letters long, give or take a letter. Since there is another polygraphic repetition present, GHRZYI at an interval of 224 letters, the division $\frac{224}{6.2}=36=4\times 9$ furnishes corroboration of the length of the key word, and dispels fears that these repetitions may have been produced by identical key letters in different positions in the key word.

(3) When word separators have been used, the first and last letters of long polygraphic repetitions are most likely to be word separators;[26] consequently, in the case of the first repeated sequence, PTXSFIWC (representing either the second or third word of the message), P_c and C_c should represent the word separators. Now if the cipher text of the message is written out in lines of 50–60 letters or so using the repeated sequence PTXSFIWC as a sort of base, we might be able to pick out the successive word separators; this is shown in the diagram below:

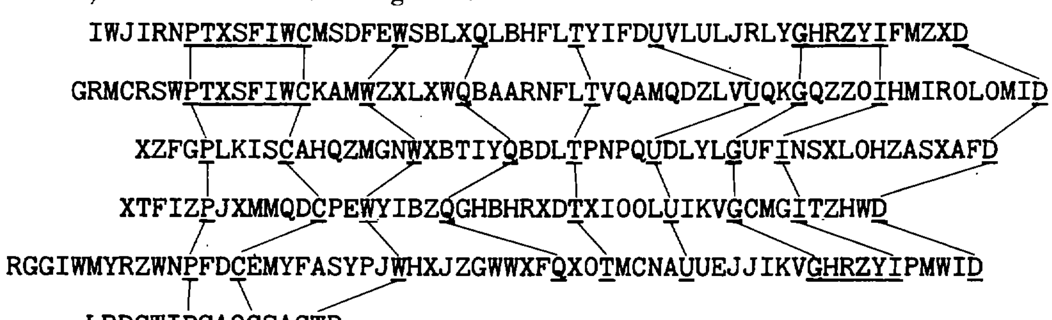

FIGURE 12

[24] See the discussion on word separators in subpar. 100*d* of *Military Cryptanalytics, Part II.*

[25] Occasionally, *unenciphered* word separators are encountered, there being employed for this purpose a character not otherwise used in the cryptographic scheme.

[26] The occasional exceptions would be cases of the partial repetitions arising from pairs of words such as INFORMS and INFORMATION, wherein the initial letters of the ciphertext repetitions would represent a word separator, but the final letters would represent M_p, the last letter of the root word.

Note that the next-to-last separator, I_c, was preferred to a preceding Z_c (also a possibility as a word separator) because of the final letter of the GHRZYI repetition. With the word divisions now known, it is an easy matter to make plaintext assumptions based on word lengths and idiomorphs; the rest of the solution is left to the reader as an exercise.

(4) The foregoing example involved but a single message and a relatively short message key so that during the encryption five complete cycles of the message key were employed; it was this re-use of the keying sequence that permitted a solution. It is obvious that, regardless of the length of the key, if we had five or six messages or so in the same key, we could have written them out over one another and by careful scrutiny we could have determined the word separators much in the same fashion as was illustrated by the diagram of Fig. 12, above.

b. The systems thus far discussed are all based upon word-length encipherment using different cipher alphabets. Words are markedly irregular in regard to this feature of their construction and thus aperiodicity is imparted to such cryptograms. But variations in the method, aimed at making the latter somewhat more secure, are possible. Instead of enciphering according to natural word lengths, the irregular groupings of the text may be regulated by other agreements. For example, suppose that the numerical value (in the normal sequence) of each key letter be used to control the number of letters enciphered by the successive cipher alphabets. Depending then upon the composition of the key word or key phrase, there would be a varying number of letters enciphered in each alphabet. If the key word were PREPARE, for instance, then the first cipher alphabet would be used for 16 (=P) letters, the second cipher alphabet, for 18 (=R) letters, and so on. Monoalphabetic encipherment would therefore allow plenty of opportunity for telltale word patterns to manifest themselves in the cipher text. Once an entering wedge is found in this manner, solution would be achieved rather rapidly.

c. If the key of the system described in the foregoing subparagraph is short, and the message is long, periodicity will be manifested in the cryptograms, so that it would be possible to ascertain the length of the basic cycle (in this case, the length of the key) from a single message, despite the irregular groupings in encipherment. The determination of the length of the cycle might, however, present difficulties in some cases, since the basic or fundamental period would not be clearly evident because of the presence of repetitions which are not periodic in their origin. For example, suppose the word PREPARE were used as a key, each key letter being employed to encipher a number of letters corresponding to its numerical value in the normal sequence. It is clear that the length of the basic period, in terms of letters, would here be the sum of the numerical values of P(=16)+R(=18)+E(=5)+ . . ., totalling 79 letters. But because the key itself contains repeated letters and because encipherment by each key letter is monoalphabetic, there would be plenty of cases in which the first letter P would encipher the same or part of the same word as the second letter P, producing repetitions in the cipher text. The same would be true as regards encipherments by the two R's and the two E's in this key word. Consequently, the basic period of 79 would be distorted or masked by aperiodic repetitions, the intervals between which would not be a function of, nor bear any relation to, the length of the key. Cases are frequently encountered in which a fundamental periodicity is masked or obscured by the presence of ciphertext repetitions not attributable to a fundamental cycle; the experienced cryptanalyst is on the lookout for phenomena of this type, when he finds in a polyalphabetic cipher plenty of repetitions but with no factorable constancy which leads to the disclosure of a short period. He may conclude, then, either that the cryptosystem involves several primary periods which interact to produce a long resultant period, or that it involves a fairly long fundamental cycle within which repetitions of a nonperiodic origin are present and obscure the phenomena manifested by repetitions of periodic origin.[27]

d. A logical extension of the principle of polyalphabetic encipherment of variable-length plaintext groupings is the case in which these plaintext groupings rarely exceed 4 letters, so that a given cipher alphabet is in play for only a very short time, thus breaking up what might otherwise appear as fairly long repetitions in the cipher text.

[27] See also in this connection footnote 8 on p. 26 of *Military Cryptanalytics, Part II.*

(1) For example, suppose that the letters of the alphabet to be used as key letters are arranged in the order of their relative frequencies in English plain text, and are set off into four groups of 5, 6, 7, and 8 letters, respectively, as follows:

E T N R O	A I S D L H	C F P U M Y G	W V B X Q K J Z
Group 1	Group 2	Group 3	Group 4

Suppose that a key letter in Group 1 means that one letter will be enciphered; a letter in Group 2, that two letters will be enciphered; and so on. Suppose, next, that a rather lengthy phrase were used as a key; for example, I KNOW NOT WHAT COURSE OTHERS MAY TAKE BUT AS FOR ME GIVE ME LIBERTY OR GIVE ME DEATH. Suppose, finally, that each letter of the key were used to control the number of letters to be enciphered by the selected alphabet, according to the scheme outlined above. Such an enciphering scheme, using the HYDRAULIC . . . XZ primary cipher component sliding against a normal plain component, would yield the following groupings:

Grouping:	2	4	1	1	4	1	1	1	4	2	2	1	3	1	3	1	2	1	1	1	2
Key:	I	K	N	O	W	N	O	T	W	H	A	T	C	O	U	R	S	E	O	T	H
Plain:	TW	ENTI	E	T	HREG	I	M	E	NTHE	AD	QU	A	RTE	R	SNO	W	LO	C	A	T	ED
Cipher:	HR	PYIV	S	E	AKYR	X	R	Z	ENAY	HR	SX	T	ZYG	C	WPQ	Z	UC	G	O	K	AR

Grouping:	1	1	2	3	2	3	1	2	4	1	4	3	1	2	2	3	1	1	
Key:	E	R	S	M	A	Y	T	A	K	E	B	U	T	A	S	F	O	R	
Plain:	N	E	AR	HEA	DQ	UAR	T	ER	SOFF	O	RTYS	ECO	N	DR	EG	IME	N	T	. . .
Cipher:	W	I	SF	VQM	IS	TYP	K	CT	LDQQ	X	HDIY	BIQ	C	IT	XH	QWM	A	V	

(2) Here it will be seen that any tendency for the formation of lengthy repetitions would be counteracted by the short groupings and quick shifting of alphabets. Before a long plaintext passage can be enciphered by *exactly* the same sequence of key letters, an interval of exactly 135 letters (the sum of the values of the letters in the key phrase) or a multiple thereof must intervene between the two occurrences of the plaintext passage.[28] When, however, a repeated plaintext passage is at an interval of only one or two letters off from 135 or a multiple of 135, there can occur in the system under discussion a phenomenon of intermittent coincidences; i.e., coincidences not among all the ciphertext letters representing the repeated plaintext passage, but among only a few of these ciphertext letters. As an example, let us consider the following message beginnings of two messages in flush depth:

Grouping:	2	4	1	1	4	1	1	1	4	2	2	1	3	1	3	1	2	1	1	1	2	
Key:	I	K	N	O	W	N	O	T	W	H	A	T	C	O	U	R	S	E	O	T	H	
Msg "A":	TW	ENTI	E	T	HREG	I	M	E	NTHE	AD	QU	A	RTE	R	SNO	W	LO	C	A	T	ED	. . .
Cipher "A":	HR	PYIV	S	E	AKYR	X	R	Z	ENAY	HR	SX	T	ZYG	C	WPQ	Z	UC	G	O	K	AR	
Msg "B":	FI	FTYF	I	R	STDI	V	I	S	IONH	EA	DQ	U	ART	E	RSI	S	MO	V	I	N	G	. . .
Cipher "A":	GM	QIGQ	X	C	MNHU	F	Z	J	UFEA	AH	IS	M	CZY	T	VWJ	T	LC	U	Z	C	L	

The word HEADQUARTERS is offset one position to the right in Message "B" with respect to its position in Message "A". (This same situation would arise if the second occurrence of HEADQUARTERS in Message "A" were at an interval of 136 letters from the first occurrence, or 271 letters, or any other multiple of 135 plus one more letter.) If we set down the cipher equivalents of the two occurrences of HEADQUARTERS under the plain text, we have the following:

H	E	A	D	Q	U	A	R	T	E	R	S
A	Y	H	R	S	X	T	Z	Y	G	C	W
A	A	H	I	S	M	C	Z	Y	T	V	W

[28] Note that, in the case of this particular key, two occurrences of a 14-letter plaintext passage could receive identical encipherments at two different positions of the key at which there are identical fragments (GIVE ME) in the key phrase; the intervals between these repeated ciphertext sequences would have nothing to do with the length of the period.

We notice that the cipher equivalents agree only in the first, third, fifth, eighth, ninth, and twelfth letters. The repetitions here extend only to one or two letters; longer repetitions can occur only exceptionally. The two encipherments yield only occasional coincidences, i.e., places where the cipher letters are identical; moreover, the *distribution* of the coincidences is quite irregular and of an intermittent character. This phenomenon of intermittent coincidences, involving coincidences of single letters, pairs of letters, or short sequences (rarely ever exceeding pentagraphs) is one of the characteristics of this general class of polyalphabetic substitution, wherein the cryptograms commonly manifest what appears to be disturbed or distorted periodicity.

e. As has already been noted, in aperiodic systems wherein the key is determined or generated *apart* from the plain text being enciphered (as is the case of the example in the foregoing subparagraph), cryptographic depth is possible; therefore the analyst may be able, if keying conditions permit, to superimpose messages and solve the resultant superimposition. On the other hand, in systems wherein the plain or cipher text influences or governs in any way the selection of keys, cryptographic depth is usually impossible of establishment, except in very special circumstances.

f. The essence of the systems described in this chapter really comprises *monoalphabetic* substitution in irregular, and usually small, segments; nevertheless, these segments were large enough to permit of their isolation and exploitation. As the size of these segments decreases, ultimately to units of single letters, so does the difficulty of solution increase—but not beyond the potentials of cryptanalysis, as will shortly be demonstrated.

CHAPTER III

SYSTEMS USING VARIABLE-LENGTH KEYING UNITS TO ENCIPHER CONSTANT-LENGTH PLAINTEXT GROUPINGS

14. General.—*a.* The systems treated in the preceding chapter incorporated simple methods of eliminating or avoiding periodicity by enciphering variable-length groupings of the plain text, using constant-length keying units; the essence of those systems was really *monoalphabetic* encipherment by sections,[1] the sections comprising irregular-length plaintext groupings. In subpar. 2*a*, however, it was pointed out that periodicity can also be suppressed by applying variable-length keying units to constant-length plaintext groupings; the essence of such systems is polyalphabetic substitution applied to the plaintext units (usually single letters). One such method consists in *irregularly interrupting* the keying sequence, if the latter is of a limited or fixed length, and recommencing it (from its initial point) after such interruption, so that the keying sequence becomes equivalent to a series of keys of different lengths. Thus, the key phrase BUSINESS MACHINES might be expanded, by a particular keying convention, into a series of irregular-length keying sequences, such as BUSI/BUSINE/BU/BUSINESSM/BUSINESSMAC, etc. Various schemes or prearrangements for determining the type or character of the interruptions may be adopted. Several typical methods will now be described.

b. There are many methods of interrupting a keying sequence which is basically cyclic and which therefore would give rise to periodicity if not interferred with in some way. These methods may, however, be classified into six general cases as regards what happens after the interruption occurs.[2]

Case I: The keying sequence merely stops and begins again at the initial point of the cycle.

Case II: Certain elements of the keying sequence may "stutter" or be repeated a fixed or a variable number of times.

Case III: One or more of the elements in the keying sequence may be omitted from time to time irregularly.

Case IV: The keying sequence irregularly alternates in its direction of progression.[3]

Case V: The keying sequence irregularly alternates in its direction of progression, and, in addition, certain elements of the keying sequence may be repeated one or more times.

Case VI: The keying sequence irregularly alternates in its direction of progression, and, in addition, one or more of the elements in the keying sequence are omitted from time to time irregularly.

c. The foregoing methods may, for clarity, be represented graphically as follows. Suppose the key consists of a cyclic sequence of 10 elements represented symbolically by the series of numbers 1, 2, 3, . . . 0. Indicating an interruption by a vertical line,[4] we show in Fig. 13, below, the relationship between

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[1] See in this connection subpar. 84*b*(1) on p. 220 of *Military Cryptanalytics, Part I.*

[2] In addition to these cases, a "septimum quid" could be listed, as a catchall for "everything else," which includes progressions so irregular as to defy classification.

[3] It is to be noted that this fourth case could be treated as though it were a special form (with irregularly occurring small or large skips) of the third case.

[4] What *specifically* brings about the interruption is here not stated, nor for that matter does it concern us at the moment. Suffice it to say that, whatever the cause of the interruption, it is not a function of the plain or of the cipher text, but is in this case predetermined by an external convention with steps 3, 1, 6, 7, 5, 8, 2, 4, 9 . . .

the letter at each position of the message and the identity of the element of the keying sequence in the six general cases discussed above.

Letter No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Key elements, Case I:	1	2	3	1	1	2	3	4	5	6	1	2	3	4	5	6	7	1	2	3	4	5	1	2	3
Key elements, Case II:	1	2	3	3	3	4	5	6	7	8	8	9	0	1	2	3	4	4	5	6	7	8	8	9	0
Key elements, Case III:	1	2	3	5	7	8	9	0	1	2	4	5	6	7	8	9	0	2	3	4	5	6	8	9	0
Key elements, Case IV:	1	2	3	2	3	4	5	6	7	8	7	6	5	4	3	2	1	2	3	4	5	6	5	4	3
Key elements, Case V:	1	2	3	3	3	4	5	6	7	8	8	7	6	5	4	3	2	2	3	4	5	6	6	5	4
Key elements, Case VI:	1	2	3	5	7	8	9	0	1	2	4	3	2	1	0	9	8	0	1	2	3	4	6	5	4

Letter No.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	
Key elements, Case I:	4	5	6	7	8	1	2	1	2	3	4	1	2	3	4	5	6	7	8	9	1	2	3	4	5	. . .
Key elements, Case II:	1	2	3	4	5	5	6	6	7	8	9	9	0	1	2	3	4	5	6	7	7	8	9	0	1	. . .
Key elements, Case III:	1	2	3	4	5	7	8	0	1	2	3	5	6	7	8	9	0	1	2	3	5	6	7	8	9	. . .
Key elements, Case IV:	2	1	0	9	8	9	0	9	8	7	6	7	8	9	0	1	2	3	4	5	4	3	2	1	0	. . .
Key elements, Case V:	3	2	1	0	9	9	0	0	9	8	7	7	8	9	0	1	2	3	4	5	5	4	3	2	1	. . .
Key elements, Case VI:	3	2	1	0	9	1	2	4	3	2	1	3	4	5	6	7	8	9	0	1	3	2	1	0	9	. . .

FIGURE 13

Note that in Cases III and VI the amount of skip is here portrayed as being constant. This is not a necessary condition in these keying methods; the amount of skip could have consisted of irregular jumps as established by the keying convention employed.

d. If we *knew* just when the interruptions take place, and if we also knew the exact nature of the effect of each interruption,[5] then the successive ciphertext sections of encrypted messages in the foregoing six cases could be properly superimposed so as to be in true cryptographic depth. In the diagrams below, the digits in the top line represent the ten keying elements, while the numbers 1–50 underneath this line represent the positional identities of the first 50 ciphertext letters.

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)							
	(4)									
	(5	6	7	8	9	10)				
	(11	12	13	14	15	16	17)			
	(18	19	20	21	22)					
Case I	(23	24	25	26	27	28	29	30)		
	(31	32)								
	(33	34	35	36)						
	(37	38	39	40	41	42	43	44	45)	
	(46	47	48	50 . . .						

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)							
			(4)							
			(5	6	7	8	9	10)		
	→14	15	16	17)				(11	12	13→
				(18	19	20	21	22)		
Case II	→26	27	28	29	30)			(23	24	25→
					(31	32)				
						(33	34	35	36)	
	→39	40	41	42	43	44	45)		(37	38→
	→50 . . .						(46	47	48	49→

[5] These two points could have been determined either through physical compromise of a cipher machine incorporating such principles, or through the *analytical* compromise of a cryptosystem.

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)		(4)		(5	6	7	8→
	→ 9	10)		(11	12	13	14	15	16	17)
Case III		(18	19	20	21	22)		(23	24	25→
	→26	27	28	29	30)		(31	32)		(33→
	→34	35	36)		(37	38	39	40	41	42→
	→43	44	45)		(46	47	48	49	50	. . .

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)							
		(4)	(5	6	7	8	9	10)		
	(17	16	15	14	13	12	11)			
		(18	19	20	21	22)				
Case IV	←27	26	25	24	23)			(30	29	28←
									(31	32)
						(36	35	34	33)	
	→41	42	43	44	45)		(37	38	49	40→
	←49	48	47	46)					. . .	50←

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)							
			(4)							
			(5	6	7	8	9	10)		
		(17	16	15	14	13	12	11)		
		(18	19	20	21	22)				
Case V	←28	27	26	25	24	23)			(30	29←
									(31	32)
							(36	35	34	33)
	→41	42	43	44	45)		(37	38	39	40→
	. . . 50	49	48	47	46)					

	1	2	3	4	5	6	7	8	9	Ø
	(1	2	3)		(4)					
	→ 9	10)					(5	6	7	8→
	←14	13	12	11)				(17	16	15←
	→19	20	21	22)						(18→
Case VI	←28	27	26	25	24	23)			(30	29←
	(31	32)								
	(36	35	34	33)						
	→45)		(37	38	39	40	41	42	43	44→
	←48	47	46)					. . .	50	49←

Obviously, if we did not know when or how the interruptions take place, then the successive sections of keying elements cannot be superimposed as indicated above.

e. The interruption of the fundamental cyclic keying sequence usually takes place according to some prearranged plan or convention. The *identity* of the plaintext letters being enciphered might be involved in the determination of the interruption (as in plaintext interruptor systems); [6] or the identity of the ciphertext letters might be a factor (as in ciphertext interruptor systems); or, finally, the interruption of the fundamental cyclic keying sequence might be predicated upon a separate convention,

[6] In the Wheatstone cipher device the interruption of the keying sequence of the 26 cipher alphabets used in sequential progression is predicated upon the relative *position in the plain component* of a plaintext letter with respect to the *position* (in the plain component) occupied by the next plaintext letter to be encrypted. (See Chapter VIII in this connection.)

mechanism, or prearrangement, without regard to the plain text or the cipher text. Some basic methods of interruption will now be taken up, using a short mnemonic key as an example.

15. Plaintext interruptor systems.—*a*. Suppose the correspondents agree that the interruption in the key will take place after the occurrence of a specified letter in the plain text, after which the key begins anew at its initial position.[7] Since there is nothing fixed about the time the interruption will occur—it will take place at no fixed intervals—not only does the interruption become quite irregular, following no pattern, but also the method never reverts to one having periodicity. Let us assume that the correspondents have agreed upon R_p as the interruptor letter, and that they are using the normal sequence for the plain component and the HYDRAULIC . . . XZ sequence for the cipher component. If the mnemonic key phrase is BUSINESS MACHINES, this key would be interrupted by the occurrences of R_p as in the following example:

Key:	B U S I N E S S M A C H I B U S B U S I B U S I N E
Plain:	A M M U N I T I O N F O R F I R S T A R T I L L E R
Cipher:	B O L Y R P J D R O J K X K J F Y X S X D J U P S Y

Key:	B U S I N E S S M A C H I N E S B U B U S I N E S S M A C H I
Plain:	Y W I L L B E L O A D E D A F T E R A M M U N I T I O N F O R
Cipher:	I Y D P Y F X U R A F A E N M J J V B O L Y R P J D R O J K X

Key:	B U S I B U S B U S I N E B U S I N
Plain:	T H I R D A R T I L L E R Y S T O P . . .
Cipher:	D G D X G U F D J U P S Y I W J T U

The final cipher text, in groups of five letters, would be the following:

B O L Y R P J D R O J K X K J F Y X S X D J U P S Y I Y D P

Y F X U R A F A E N M J J V B O L Y R P J D R O J K X D G D

X G U F D J U P S Y I W J T U . . .

It will be noted that the two long polygraphic repetitions are at intervals of 44 and 34, respectively, which intervals have nothing in common with 16, the length of the basic, uninterrupted period.

b. Instead of employing an ordinary plaintext letter as the interruptor letter, one might use a 25-letter plain component, combining I with J, and then use the 26th character (J) as a null plaintext letter which is inserted at random by the encipherer to serve as the interruptor letter. Note the following example:

Key:	B U S I N E S S M A B U S I N E S S M A C H I N E S B U S B U S I N
Plain:	P R O C E E D T O J R O A D I U N C T I O N S I X T W O J F I V E J . . .

c. It is obvious that repetitions would be plentiful in cryptograms of this construction, regardless of whether a letter of high-, medium-, or low-frequency is selected as the signal for key interruption. If a letter of high frequency is chosen, repetitions will occur quite often, not only because that letter will certainly be a part of many common words, but also because it will be followed by words that are frequently repeated; and since the key starts again with each such interruption, these frequently repeated words will be enciphered by the same sequence of cipher alphabets. This is the case in the first of the two foregoing examples. It is clear, for instance, that every time the word ARTILLERY appears in the cryptogram the cipher equivalents of TILLERY must be the same. If the interruptor letters were A_p instead of R_p, the repetition would include the cipher equivalents of RTILLERY; if it were T_p, ILLERY, and so on. On the other hand, if a letter of very low frequency were selected as the interruptor letter, then the encipherment would tend to approximate that of normal periodic substitution, and repetitions would be plentiful on that basis alone. Of course, the intervals between the repetitions in any of the

[7] This is Case I of subpar. 14*b*.

foregoing cases (except perhaps that in which the plaintext interruptor is a letter of very low frequency) would be markedly irregular, so that periodicity would not be manifested.

16. Ciphertext interruptor systems.—*a.* In the systems of the preceding paragraph, a plaintext letter serves as the interruptor letter. But now suppose the correspondents agree that the interruption in the key will take place immediately after a previously agreed-upon letter, say Q_c, occurs in the cipher text. The key would then be interrupted as shown in the following example:

```
Key:    B U S I N E S S M A C H I N E S B U S I N E S S M
Plain:  A M M U N I T I O N F O R F I R S T A R T I L L E
Cipher: B O L Y R P J D R O J K X T P F Y X S X B P U U Q

Key:    B U S I N E S S M A C H I N B U S I N E S S M A C H B U
Plain:  R Y W I L L B E L O A D E D A F T E R A M M U N I T I O
Cipher: H R N M Y T T X H P C R F Q B E J F I E L L B O N Q O Q

Key:    B U S I N E S S M A C H B U S I N E S S M A
Plain:  N F O R T H I R D A R T I L L E R Y S T O P . . .
Cipher: V E C X B O D F P A Z Q O N U F I C G J R Q
```

The cipher text in 5-letter groups is as follows:

```
BOLYR  PJDRO  JKXTP  FYXSX  BPUUQ  HRNMY
TTXHP  CRFQB  EJFIE  LLBON  QOQVE  CXBOD
FPAZQ  ONUFI  CGJRQ
```

b. In the foregoing example, there are no significant repetitions; such as do occur comprise only digraphs, several of which are purely accidental. But the absence of significant, long repetitions is itself purely accidental, for had the interruptor letter been a letter other than Q_c, then the phrase AMMUNITION FOR (which occurs twice) might have been enciphered identically both times. If a short key is employed, repetitions may be plentiful. For example, note the following, in which S_c is the interruptor letter: [8]

```
Key:    B A N D S B A N D S B A N D S B A N D S B A N B A N D S B B A
Plain:  F R O M F O U R F I V E T O F O U R F I F T E E N W I L L B E . . .
Cipher: K T A K Z W X I I D A C B N Z W X I I D K W S J O G E U S E C
```

c. This last example gives a clue to one method of attacking this type of system. There will be repetitions within short sections, and the interval between them will sometimes permit ascertaining the length of the basic key. In such short sections, the letters which intervene between the repeated sequences may be eliminated as possible interruptor letters. Thus, in the foregoing example, we can deduce that the length of the basic key is 5 letters, and that the cipher letters A, C, B, and N may be eliminated as interruptor letters. By extension of this principle to the letters intervening between other repetitions, one may more-or-less quickly ascertain what ciphertext letter serves as the interruptor.[9]

d. The ciphertext interruptor might be a letter which is not otherwise used in the cryptographic scheme; for example, the plain component might be a 25-letter sequence (combining I and J) and the cipher component a 25-letter sequence excluding, let us say, Z. This letter Z may then be inserted in appropriate places in the cipher text to signal the interruptions in the keying cycle. In some cases such a special interruptor letter may be used in *addition* to a ciphertext interruptor which arises from the bona fide encryption of a plaintext letter, as a means of insuring that interruption of the keying cycle will take place frequently enough to suit the cryptographer or his procedures-prescribing superiors.

[8] Note that the periodic repetitive phenomena manifested would also have arisen in a *plaintext* interruptor system, if the interruptor had been, let us say, A_p—or, for that matter, *any* other letter not present in the fragment FOURFIVETOFOURFI.

[9] The method described in this subparagraph may also be applied in the case of plaintext interruptor systems, with certain modifications.

(For that matter, there is nothing to bar the use of two or more letters as interruptors in the usual manner, in either plaintext interruptor or ciphertext interruptor systems.)

17. Systems employing externally generated or determined keys.—*a*. In subpars. 3*b* and *f* we have seen two examples of keying procedures which do not depend upon conventions affiliated with identities of plaintext or ciphertext letters, but which are established by an independent, external keying convention. The keying methods of subpar. 3*b*, if modified to incorporate variable-length *polyalphabetic* keying units (as contrasted with the variable-length monoalphabetic keying units illustrated in that example), could take on a form such as the following:

```
S  IG  GNA  NALS  ALSIG  L  SI  IGN  GNAL  NALSI  A  LS  SIG  IGNA  GNALS
1  12  123  1234  12345  1  12  123  1234  12345  1  12  123  1234  12345

N  AL  LSI  SIGN  IGNAL  G  NA  ALS  LSIG  SIGNA  I  GN  NAL  ALSI  LSIGN
1  12  123  1234  12345  1  12  123  1234  12345  1  12  123  1234  12345

S  IG  GNA  NALS  ALSIG  L  SI  IGN etc.
1  12  123  1234  12345  1  12  123
```

Similarly, the keying method of subpar. 3*f*, modified to embrace the aspect of variable-length polyalphabetic keying units, could be transformed into one of the following, among other possibilities:

(1) D E F/E/C D E F/L M N O/A B/R S T/A B/T/I J/O P Q/N O/O P Q/F G H I/. . .

(2) D E C/E/C L A R/L A R A/A R/R A T/A T/T/I O/O N O/N O/O F I/F I N D/. . .

b. The foregoing methods have as their purpose the establishment of keys of fair length from a short mnemonic key. There are other simple methods for accomplishing this, as illustrated in the examples which follow. Let us consider the mnemonic key HYDRAULIC, and derive from it a numerical key:

```
H Y D R A U L I C
4 9 3 7 1 8 6 5 2
```

We may now take the key letters in numerical-key order, and in groupings as determined by the numerical key, so that the original key of only 9 letters is expanded to one of 45 letters. Thus:

(1) A/C H/D R A/H Y D R/I C H Y D/L I C H Y D/R A U L I C H/
U L I C H Y D R/Y D R A U L I C H/

Two other methods of deriving 45-element key sequences from the basic 9-letter key word are shown below:

(2) H Y D R/Y D R A U L I C H/D R A/R A U L I C H/A/
U L I C H Y D R/L I C H Y D/I C H Y D/C H/
(3) H Y D R A/H Y D R A U L I C/H Y D/H/H Y D R A U L I/
H Y D R A U L/H Y D R /H Y D R A U/H Y/

Method (2) is essentially the same as (1), except that the key fragments are taken in the order in which they appear in the key word. Method (3) involves taking the successive sections of the numerical key, these sections terminating with the successive numbers 1, 2, 3, . . . of the numerical key.[10]

c. Many other methods exist for the establishment of keys consisting of variable-length keying units. Furthermore, some of these methods merge into the domain of methods of lengthening or extending keys in general, apart from any considerations of variable-length keying units. Several of the most important of these methods will be discussed in subsequent chapters of this text.

18. Solution when known cipher alphabets are employed.—*a*. (1) Let us suppose that a particular cryptosystem has been in use for some time, and that the general nature of the system and the cipher

[10] This method is equivalent to an interrupted-key columnar transposition system. See in this connection subpar. 51*h* on p. 89 of *Military Cryptanalytics, Part I*.

alphabets have become known, either through successful cryptanalysis or through light-fingered techniques coming under the formal term of "physical compromise," which includes among its manifold tachydactylurgic aspects that which has been referred to colloquially as "wastebasket cryptanalysis." [11] Only the specific key to messages remains unknown. The cipher text is examined for repetitions, and an attack is made on the basis of searching for a probable word. Thus, taking the cryptogram in subpar. 15*a* as an example (quoted here below for convenience), suppose the presence of the word ARTILLERY is suspected.

```
BOLYR  PJDRO  JKXKJ  FYXSX  DJUPS  YIYDP
XFXUR  AFAEN  MJJVB  OLYRP  JDROJ  KXDGD
XGUFD  JUPSY  IWJTU . . .
```

Attempts are made to locate this word, basing the search upon the recognition of an intelligible key; we will assume in this case that the cipher component is the HYDRAULIC . . . XZ sequence sliding against the normal sequence for the plain component.

(2) Beginning with the very first letter of the message, we juxtapose the word ARTILLERY against the cipher text and ascertain the key letters. Thus:

```
Key:    B H J Q P I B F U
Cipher: B O L Y R P J D R
Plain:  A R T I L L E R Y
```

Since this "key" is certainly not intelligible text, the assumed word is moved one letter to the right and the test repeated, and so on until the 19th position in the text is reached.[12]

```
Key:    S I B U S I N E B
Cipher: S X D J U P S Y I
Plain:  A R T I L L E R Y
```

(3) The sequence BUSINE suggests BUSINESS; moreover, it is noted that the key appears to be interrupted both times by the letter R_p. The key may now be applied to the beginning of the message, to see whether the whole key or only a portion of it has been recovered. Thus:

```
Key:    B U S I N E S S B U S I N E S
Cipher: B O L Y R P J D R O J K X K J . . .
Plain:  A M M U N I T I U M T H I E T
```

(4) It is obvious that BUSINESS is only a part of the key. But the first word of the message is plainly AMMUNITION. When this is tried, the key is extended to BUSINESS MA . . . This key crib is now slid through the rest of the cipher text and the remainder of the message is quickly deciphered and the entire key recovered.

---

[11] This is really not stealing. For the pure in heart, this should be thought of as conversion of raw data, and that the parties so generously supplying these raw data are, unknowingly, cooperating in government work.

[12] In actual practice, the search for the placement of the probable word would have been accomplished by means of the following diagram (see in this connection subpar. 22*d* on pp. 41–42 of *Military Cryptanalytics, Part II*):

```
    B O L Y R P J D R O J K X K J F Y X S X D J U P S Y I Y D P . . .
A | B O L Y R P J D R O J K X K J F Y X S X D J U P S Y I Y D P
R |   H M E G Y V F G H V W I W V S E I R I F V K Y R E N E F Y
T |     J C E Z S B E X S T U T S P C U Y U B S G Z Y C K C B Z
I |       Q T E U S T B U L N L U R Q N G N S U W E G Q Z Q S E
L |         P I D O P L D R J R D H N J B J O D S I B N V N O I
L |           I D O P L D R J R D H N J B J O D S I B N V N O I
E |                                                 N
R |                                                   E
Y |                                                     B
```

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

d. (1) Another technique, if we know or can assume the method of key interruption (e.g., a skip over one element of the key after the occurrence of a previously designated ciphertext letter, in this case W_c), involves writing out the modified cipher text of a single message on trial widths in order to see if any cyclic properties are present in the basic, uninterrupted key. We can then determine statistically when the correct cyclic write-out is reached by the application of a technique discussed in the preceding text.[16] As an example, let us assume the following message is at hand:

```
GSWWT  RHZDW  GLNUJ  WXRWR  HNQLS  YXTEV
GCVBW  CWZUV  IAVFG  XXFNP  HGPHA  MIKDR
VCTEA  VCAWG  JICGG  CISNS  IVCJB  SZSRW
VLKZR  JBHCC  CAYQV  WJMRL  WTLRS  DJXFN
ZZIAF  MQJCX
```

(2) If we know the method of interruption and also the identity of the ciphertext interruptor,[17] we would write out the appropriately modified cipher text on various widths, testing each hypothesis in turn, until a satisfactory I.C. is reached for an entire columnar array. For example, if we know that the enemy is using key words and phrases from 11 to 40 letters in length as the basic key sequence, we would begin by writing out the modified cipher text (on the assumption of W_c as the interruptor letter) on a width of 11 as shown below, together with the appropriate ϕ values for the computations:

	1	2	3	4	5	6	7	8	9	10	11	
	G	S	W	.	W	.	T	R	H	Z	D	
	W	.	G	L	N	U	J	W	.	X	R	
	W	.	R	H	N	Q	L	S	Y	X	T	
	E	V	G	C	V	B	W	.	C	W	.	
	Z	U	V	I	A	V	F	G	X	X	F	
	N	P	H	G	P	H	A	M	I	K	D	
	R	V	C	T	E	A	V	C	A	W	.	
	G	J	I	C	G	G	C	I	S	N	S	
	I	V	C	J	B	S	Z	S	R	W	.	
	V	L	K	Z	R	J	B	H	C	C	C	
	A	Y	Q	V	W	.	J	M	R	L	W	
	.	T	L	R	S	D	J	X	F	N	Z	
	Z	I	A	F	M	Q	J	C	X			
ϕ:	6	6	4	2	4	2	12	6	6	14	2	$\Sigma\phi=64$
N:	12	11	13	12	13	11	13	12	12	12	9	

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(3) The row of numbers immediately beneath the write-out represents the ϕ values of the columns; the row beneath the "ϕ" row, labelled "N", represents the number of letters in the columns. Since there

[16] Cf. Subpar. 18*e* on pp. 28–31 of *Military Cryptanalytics, Part II.*

[17] If worst came to worst, we could test each of the 26 letters in turn as the ciphertext interruptor. This testing, coupled with the writing out of the cipher text on the various widths, would be quite laborious and time-consuming by manual methods; data processing machine techniques would here be very useful.

are 3 columns of 13 letters each, 5 columns of 12 letters each, 2 columns of 11 letters, and 1 column of 9 letters, the expected value of ϕ random (ϕ_r) is given by the formula

$$\phi_r=\frac{3(13\cdot12)+5(12\cdot11)+2(11\cdot10)+1(9\cdot8)}{26}=\frac{1420}{26}=54.6$$

The δI.C. is defined as the ratio of the observed value of ϕ to the expected value of ϕ random, or $\frac{\phi_o}{\phi_r}$. Now since the ϕ_o (which is the sum of all the ϕ values for the columns) is 64, our I.C. formula becomes, by simple algebraic transformation,

$$\delta\text{I.C.}=\frac{26\cdot64}{3(13\cdot12)+5(12\cdot11)+2(11\cdot10)+1(9\cdot8)}=\frac{1664}{1420}=1.17$$

(4) This I.C. of 1.17 is not satisfactory, so we continue testing successively greater widths, until the width of 32 is reached:

```
            5         10             15             20             25             30  32
    G S W . W . T R H Z D W . G L N U J W . X R W . R H N Q L S Y X
    T E V G C V B W . C W . Z U V I A V F G X X F N P H G P H A M I
    K D R V C T E A V C A W . G J I C G G C I S N S I V C J B S Z S
    R W . V L K Z R J B H C C C A Y Q V W . J M R L W . T L R S D J
    X F N Z Z I A F M Q J C X
φ:  — — — 2 2 — — 2 — 2 — 4 — 2 — 2 — 2 2 — 2 — — — — 2 — — — 6 — —  Σφ=30
N:  5 5 4 4 5 4 5 5 4 5 5 4 3 4 4 4 4 4 4 2 4 4 4 3 4 3 4 4 4 4 4 4
```

At this width, the I.C. calculation becomes

$$\delta\text{I.C.}=\frac{26\cdot30}{7(5\cdot4)+21(4\cdot3)+3(3\cdot2)+1(2\cdot1)}=\frac{780}{412}=1.89,$$

giving statistical credence to the assumption of 32 as the correct width, since we were looking for an I.C. in the vicinity of 1.73 for the correct case.[18] Knowing the components involved, we may complete the plain-component sequence on the letters of the columns to effect a speedy solution.

19. Solution when unknown cipher alphabets are employed.—*a*. In the first text in this series, it was pointed out that "in the final analysis, the solution of every cryptogram involving a form of substitution depends upon its reduction to monoalphabetic terms, if it is not originally in those terms."[19] In the preceding volume, it was observed that when in the course of solution of an ordinary repeating-key cipher the text is written out in period-lengths, "another way of looking at the matter is to conceive of the text as having thus been transcribed into *superimposed periods;* in such a case the letters in each

[18] The reader might be interested in the I.C.'s for all the widths from 11 to 40; these are shown in the following table:

w	δ	w	δ	w	δ	w	δ	w	δ	w	δ
11	1. 17	16	1. 11	21	0. 91	26	1. 06	31	1. 08	36	1. 33
12	1. 09	17	0. 95	22	1. 35	27	1. 24	32	1. 89	37	1. 69
13	1. 06	18	1. 03	23	1. 18	28	0. 98	33	0. 79	38	1. 28
14	0. 91	19	1. 02	24	1. 17	29	1. 67	34	0. 68	39	0. 98
15	1. 29	20	0. 86	25	1. 20	30	1. 29	35	1. 14	40	1. 01

Note the I.C. of 1.67 for the width of 29, and the I.C. of 1.69 for the width of 37. These I.C.'s are certainly satisfactory; however, the widths from which they were derived are incorrect, so that they represent the vagaries of the touch not of a Mephistophelian finger, but rather that of a Bernoullian digit when an insufficient number of trials is involved.

[19] *Military Cryptanalytics, Part I*, subpar. 17*b*.

column have undergone the same kind of treatment by the same elements (plain and cipher components of the cipher alphabet)." [20] It follows that, even if the repeating key is very long, if there are many short cryptograms all enciphered by exactly the same key and each message begins at the same point in the key, the distributions applicable to the successive columns of text can be solved.[21] Even in aperiodic systems, if there is available a number of messages starting out in the same key which then diverges in the course of encipherment according to the nature of the cryptosystem, this *solution by superimposition* may be applicable in particular cases, so long as the key divergence is not too radical for cryptanalytic comfort.

b. Let us study the following beginnings of 30 messages, passed between correspondents known to have used various types of aperiodic keying:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1.	Y	F	W	F	M	R	I	Q	M	X	X	E	L	M	J	. . .
2.	H	W	W	T	T	E	C	T	D	O	Z	F	D	O	V	. . .
3.	T	P	Y	F	K	S	O	V	W	I	H	F	N	C	J	. . .
4.	Y	P	E	P	S	N	L	S	K	Z	N	V	T	J	B	. . .
5.	E	A	Q	U	Z	D	V	E	S	K	C	I	U	P	A	. . .
6.	Z	C	G	M	W	T	N	B	I	M	K	U	S	N	L	. . .
7.	E	P	D	O	Z	F	D	O	V	B	I	L	V	L	W	. . .
8.	E	P	T	L	E	S	A	C	R	B	M	P	T	P	J	. . .
9.	W	M	L	S	O	T	O	Z	E	E	J	Z	G	V	K	. . .
10.	Z	C	F	F	D	C	F	R	J	W	H	L	P	D	T	. . .
11.	H	C	Q	E	D	T	P	Y	I	L	N	R	E	M	V	. . .
12.	C	L	C	T	Z	I	K	S	O	E	O	Z	C	T	T	. . .
13.	H	C	Q	E	F	D	K	I	F	Q	W	O	C	L	M	. . .
14.	E	P	T	W	K	S	U	Z	N	V	V	A	U	C	S	. . .
15.	Z	A	B	M	Z	H	G	O	F	X	Q	I	G	M	M	. . .
16.	G	O	E	Q	B	Q	O	T	L	E	S	A	C	R	B	. . .
17.	W	T	S	R	G	X	M	Z	T	V	S	J	Q	L	X	. . .
18.	W	T	E	V	F	C	I	B	T	S	P	R	C	A	T	. . .
19.	Z	C	V	Y	M	B	V	N	Y	W	Q	U	Z	G	U	. . .
20.	Z	C	T	T	Z	W	C	T	T	I	K	H	Q	U	T	. . .
21.	Z	C	C	T	S	N	E	S	K	O	U	B	M	P	T	. . .
22.	A	F	E	S	J	O	N	K	T	D	V	E	S	K	C	. . .
23.	Z	C	F	F	D	T	N	P	F	D	H	D	T	P	F	. . .
24.	V	Z	I	E	X	R	X	R	F	F	U	N	T	Q	E	. . .
25.	E	P	S	N	L	S	K	L	O	H	W	P	T	R	G	. . .
26.	Y	T	S	V	W	L	S	T	L	E	S	A	C	R	B	. . .
27.	W	A	Z	X	Z	Q	A	C	H	Q	U	T	L	S	T	. . .
28.	N	O	F	T	Z	N	L	H	Q	U	T	J	H	Z	A	. . .
29.	V	R	C	W	K	M	O	L	N	X	W	S	D	O	L	. . .
30.	S	P	R	C	P	F	X	E	O	J	C	Q	F	W	M	. . .

[20] *Military Cryptanalytics, Part II*, subpar. 65*a*.
[21] Cf. subpar. 65*b*, *Military Cryptanalytics, Part II*.

The presence of digraphic and polygraphic repetitions in the initial columns could mean that the messages start out in flush depth, and the presence of offset repetitions could be an indication of shifts in the keying sequence. Frequency distributions for the columns are made and are shown in Fig. 16, below, accompanied by their I.C.'s:

1. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 2.63

2. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 3.41

3. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.31

4. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.37

5. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.85

6. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.20

7. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.08

8. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.08

9. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.13

10. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 0.95

11. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.02

12. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 0.72

13. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.43

14. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.02

15. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1.61

FIGURE 16

c. The first two columns are certainly monoalphabetic; after that, there is a rapid falling off in monoalphabeticity, with the exceptions of cols. 5, 13 and 15 which could be due to chance. We note the digraph ZC_c which occurs 6 times in cols. 1 and 2; this could well be the equivalent of RE_p, and HC_c in cols. 1 and 2 could stand for SE_p. On this basis, ZCFFD in Messages 10 and 23 could represent REFER, and HCQE in Messages 11 and 13 could be SEND. We then note

```
                 5         10        15
23.  Z C F F D T N P F D H D T P F . . .
     R E F E R
                 
 6.  Z C G M W T N B I M K U S N L . . .
     R E
```

(In both lines, "T N" is underlined.)

which could represent REFERENCE and REGIMENT. We now turn our attention to the following four message beginnings:

```
                5           10            15
28. N O F T Z N L H Q U T J H Z A . . .
          F   R
20. Z C T T Z W C T T I K H Q U T . . .
    R E       R
21. Z C C T S N E S K O U B M P T . . .
    R E
12. C L C T Z I K S O E O Z C T T . . .
              R
```

If we assume that T_c in col. 4 represents O_p, then in No. 28—FOR . . . becomes INFORM(ATION), in No. 20 RE–OR– becomes REPORT, in No. 21 RE–O . . . becomes RECOMMEND, and in No. 12—COR––N– becomes ACCORDING.

d. The plain-cipher equivalencies from the foregoing assumptions are entered into a sequence reconstruction matrix, as shown below:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	C								N									Z	H							
2			L		C									O												
3			C			F	G							Q		T										
4				(E)	F				(M)																	
5													$^{W}_{B}$					$^{D}_{Z}$								
6				I	T								N							W						
7	L				E				K					N												
8			P											S						$^{B}_{H}$						
9				(K)	F		O		(Q)																	
10															U											
11														T												

Conflicts are noted in lines 5 and 8, and between lines 4 and 9; however, possibility of direct symmetry is noted in the top four lines, which indicates that the recoveries in these lines could well be homogeneous, not having been affected by vagaries of the keying. Transferring values among these four lines, we will develop the reconstruction matrix into the following, in which the cipher components are slides of what is patently a keyword-mixed sequence (derived values in lower case):

P:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	C		e	f	g			m	N	o		q		t				Z	H						l	
2			L		C		e	f	g			m	n	O		q		t				z	h			
3	l		C		e	F	G			m	n	o		Q		T				z	h					
4		c		E	F	g			M	n	o		q		T				z	h						l

Our work sheet will now look as illustrated in Fig. 17. which includes the values in the first four columns obtained by direct symmetry shown in lower case:

```
            5         10        15                   5         10        15
 1. Y F W F M R I Q M X X E L M J . . . 16. G O E Q B Q V O C F X E S N H . . .
      h   E                                 e N e m

 2. H W W T T E C T D O Z F D O V . . . 17. W T S R G X M Z T V S J Q L X . . .
    S     O                                   r

 3. T P Y F K S O V W I H F N C J . . . 18. W T E V F C I B T S P R C A T . . .
    n     E                                   r e         T

 4. Y P E P S N L S K Z N V T J B . . . 19. Z C V Y M B V N Y W Q U Z G U . . .
        e   M M A N D                       R E

 5. E A Q U Z D V E S K C I U P A . . . 20. Z C T T Z W C T T I K H Q U T . . .
    c   N   R                               R E P O R T

 6. Z C G M W T N B I M K U S N L . . . 21. Z C C T S N E S K O U B M P T . . .
    R E G I M E N T                         R E C O M M E N D

 7. E P D O Z F D O V B I L V L W . . . 22. A F E S J O N K T D V E S K C . . .
    c     k R                                 h e       N

 8. E P T L E S A C R B M P T P F . . . 23. Z C F F D T N P F D H D T P F . . .
    c   P z                                 R E F E R E N C E

 9. W M L S O T O Z E E J Z G V K . . . 24. V Z I E X R X R F F U N T Q E . . .
      l a     E                               v   D         E

10. Z C F F D C F R J W H L P H X . . . 25. E P S N L S K L O H W P T R G . . .
    R E F E R                               c     j     I   G

11. H C Q E D T P Y I L N R E M V . . . 26. Y T S V W L S T L E S A C R B . . .
    S E N D R E                               r     M

12. C L C T Z I K S O E O Z C T T . . . 27. W A Z X Z Q A C H Q U T L S T . . .
    A C C O R D I N G                           t   R

13. H C Q E F D K I F Q W O C L M . . . 28. N O F T Z N L H Q U T J H Z A . . .
    S E N D     I   E                       I N F O R M A T I O N

14. E P T W K S U Z N V V A U C S . . . 29. V R C W K M O L N X W S D O L . . .
    c   P                                       C

15. Z A B M Z H G O F X Q I G M M . . . 30. S P R C P F X E O J C Q F W M . . .
    R     I R       E                             b         G
```

FIGURE 17

e. At this point more plain text could be assumed in the messages from the fragments already present; the cipher component would be recovered in its entirety, the basic key determined, and the cause of the key interruptions (as manifested by the apparent garbles) ascertained. Or, as another approach, we might take an introspective look at the first 5 letters of matched plain and cipher of the following three message beginnings:

```
10. Z C F F D . . .
    R E F E R
20. Z C T T Z . . .
    R E P O R.
28. N O F T Z . . .
    I N F O R
```

We note the $D_c=R_p$ and $Z_c=R_p$ in position 5 of Messages 10 and 20, and observe that, if the system employs a ciphertext interruptor, it may be either F_c or T_c; but if $TZ_c=OR_p$ in Messages 20 and 28 is causal, F_c and T_c are eliminated and therefore there is *no* ciphertext interruptor in the cryptosystem. We then note the common $F_c=F_p$ between Messages 10 and 28 and the fact that in position 5 of Message 28 $Z_c=R_p$, and we may conclude that, if a plaintext interruptor is present, it must be O_p. We find this to be true, and when we finish the solution of the problem we find the cipher component to be our perennial friend, the HYDRAULIC . . . XZ sequence, and the basic key to be CALIFORNIAGOLDR(USH).

20. Additional remarks.—*a.* We have seen in the preceding paragraph a demonstration of solution of only one irregularly keyed system involving unknown cipher alphabets. The solution involved a set of very fortunate circumstances indeed, all of which were happily present awaiting rapid exploitation by the cheerful cryptanalyst. Modern cryptanalysis is quite often contingent upon miracles—minor miracles for minor systems, and healthy miracles for some of the complex systems encountered in present-day operations. When we come right down to it, all cryptanalysis is astonishing; it certainly is so to a layman, and it is so even to an expert—if he pauses long enough from his breaking of one system after another to marvel at the phenomenal luck he has had, shuddering at the thought of what would have happened *if* (i.e., *if* the enemy had done this instead of that, if he had used this instead of that, and *if* . . .). On the other hand, all cryptanalysis is quite commonplace:[22] after all, messages have been encrypted with certain invariant mathematico-philosophical-procedural elements, and all the cryptanalyst does is to discover and exploit these elements. And, in retrospect, after a problem has been solved, we often shrug our shoulders and say "Well, how else would one have done it?" Many systems of the types treated in this volume could be virtually unsolvable, or might appear to be so, if only a small amount of traffic is available for study, and if little or nothing is known about the nature of the cryptosystem. However, as happens time and again in actual operations, Fortuna smiles and the incredible is shorn of its prefix.

b. Operationally, cryptodilemmas are resolved by the exploitation of contingencies which are by now well-known to the reader: (1) messages in the same or nearly the same keys; (2) depths and partial depths; (3) polygraphic repetitions; (4) cribs; (5) various kinds of cryptographic errors; (6) isologs; (7) matched plain and cipher; (8) isomorphs; (9) indicators. Each problem presents a very special case, and therefore demands its own special requirements for solution.

c. Most of the types of aperiodic substitution discussed in this chapter are rather unsuitable for practical military usage. Encipherment is slow and subject to error. In some cases encipherment can be accomplished only by a single-letter operation. For, in interruptor systems, if the interruptor is a cipher letter the key is interrupted by a letter which cannot be known in advance; if the interruptor is a plaintext letter, while the interruptions can be indicated before encipherment is begun, the irregularities occasioned by the interruptions in keying cause confusion and quite materially retard the enciphering process. In deciphering, the rate of speed would be just as slow in either method. It is obvious that one of the principal disadvantages in all these methods is that if an error in transmission is made, if some letters are omitted, or if anything happens to the interruptor letter, the message becomes difficult or impossible to decipher by the ordinary cipher clerk. In spite of all these objections, plus the fact that the degree of cryptosecurity attainable by most of these methods is not sufficient for military purposes, these systems have been and are still occasionally encountered—which is what makes the cryptologic world go round.

[22] It must have been a deep thinker who first uttered the statement that "all problems in cryptanalysis, like mathematics, are either trivial or impossible."

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

Chapter IV

CIPHERTEXT AUTOKEY SYSTEMS

21. The cryptography of autokey encipherment.—*a.* The mechanics of autokey encipherment were treated briefly in the preceding volume.[1] In autokey systems there are two possible sources for successive key letters: the cipher text or the plain text of the message itself. In either case, the *initial* key letter or key letters are supplied by prearrangement between the correspondents, or are designated by means of an indicator; after that, the text letters that are to serve as the key are displaced 1, 2, 3 . . . intervals to the right, depending upon the length of the prearranged key.

b. An example of ciphertext autokey encipherment is shown below, wherein the cipher alphabets are direct standard alphabets, and the single letter **X** is the prearranged initial key:

```
K: X|Q X F W Z Q U A I U Y L E G U G S S F I X L D W I W R Z M
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Q X F W Z Q U A I U Y L E G U G S S F I X L D W I W R Z M S
```

Figure 18*a*

Instead of having a single letter serve as the initial key, a word or even a long phrase may be used, as in the example below wherein the word **FORTUNE** is used as the initial key:

```
K: F O R T U N E|Y V Z K X E I E D L O K X K S P X O X A Z G H
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Y V Z K X E I E D L O K X K S P X O X A Z G H Q A L V H T N
```

Figure 18*b*

Sometimes only the last cipher letter resulting from the use of the prearranged key word is used as the key letter for enciphering the autokeyed portion of the text. Thus, in the preceding example, the plain text beginning **GIMENT** . . . would be enciphered differently as follows:

```
K: F O R T U N E|I O W I M Z S U I U G G T W L Z R K W K F N A
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Y V Z K X E I O W I M Z S U I U G G T W L Z R K W K F N A G
```

Figure 18*c*

c. In plaintext autokey encipherment the procedure is quite similar, as is shown in the following example wherein the prearranged initial key is the letter **X**:

```
K: X|T H I R D R E G I M E N T C O M M A N D P O S T M O V I N
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Q A P Z U U V K O U Q R G V Q A Y M N Q S D G L F A J D V T
```

Figure 19*a*

[1] Cf. subpar. 99*e* on pp. 310–311 of *Military Cryptanalytics, Part II.*

If the word FORTUNE were used as the initial key, the plain text would be enciphered as follows:

```
K: F O R T U N E|T H I R D R E G I M E N T C O M M A N D P O S
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Y V Z K X E I Z P U V Q K G U U Y E A W R C E F M B Y X B Y
```

FIGURE 19*b*

d. In the foregoing examples, direct standard alphabets were used; however, mixed alphabets, either interrelated or independent,[2] may be used just as readily. Furthermore, instead of the ordinary type of cipher alphabets, the cryptographic process may employ a mathematical process of addition, but the difference between the latter process and the ordinary one using sliding alphabets is more apparent than real. For example, let us consider the following numerical sequence for the 26 letters

H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	0

and let the plaintext message be the same as before. Let us assume that the cryptographic rules prescribe that the first plaintext letter will be self-enciphered,[3] and that each cipher letter from that point on is produced in turn by finding the sum (mod 26) of the numerical equivalents of the preceding cipher letter and the plaintext letter to be enciphered; in other words, a type of numerical ciphertext autokey system. This is shown in the diagram below, wherein P' denotes the numerical equivalents of the plain text, C' the sum of the key and the numerical (i.e., intermediate) plain text, and C the conversion into letters of the intermediate cipher text C'.

```
K : 0  22 23 5  9  12 16 1  14 22 12 23 14 10 19 11 1  17 22 13 16 9  1  22 18 8  0  23 5  22
P : T  H  I  R  D  R  E  G  I  M  E  N  T  C  O  M  M  A  N  D  P  O  S  T  M  O  V  I  N  G . . .
P': 22 1  8  4  3  4  11 13 8  16 11 17 22 9  18 16 16 5  17 3  19 18 21 22 16 18 23 8  17 13
C': 22 23 5  9  12 16 1  14 22 12 23 14 10 19 11 1  17 22 13 16 9  1  22 18 8  0  23 5  22 9
C : T  V  A  C  F  M  H  J  T  F  V  J  B  P  E  H  N  T  G  M  C  H  T  O  I  Z  V  A  T  C
```

FIGURE 20*a*

e. That the difference between the types of encipherment in the preceding subparagraph and the ordinary method of ciphertext autokey encipherment is illusory is demonstrated by the example in Fig. 20*b*, below:

```
K: Z T V A C F M H J T F V J B P E H N T G M C H T O I Z V A T
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: T V A C F M H J T F V J B P E H N T G M C H T O I Z V A T C
```

FIGURE 20*b*

In this example, the plain and cipher components are keyword-mixed sequences based upon HYDRAULIC, and Z_p is the index letter against which the key letters in the cipher component are set; the cryptographic results are identical to those obtained in Fig. 20*a*, above.

f. Since the analysis of ciphertext autokey systems is usually easier than the analysis of plaintext autokey systems, the former will be the first to be discussed.

22. Solution of ciphertext autokeyed cryptograms when known cipher alphabets are employed.—*a.* First of all, it is to be noted that if the cryptanalyst knows the cipher alphabets which were employed in encipherment, the solution hardly presents any problem. It is only necessary to decipher the message beyond the key-letter or key-word portion and the initial part of the plain text enciphered by this key letter or key word can be filled in from the context.

---

[2] For instance, an autokey system might incorporate independent, random alphabets such as those illustrated in Fig. 33 on p. 70.

[3] This, on a numerical scale, is tantamount to the effect of a key of Ø.

(1) For example, let us consider the following beginning of an intercepted message:

```
Q X F W Z   Q U A I U   Y L E G U   G S S F I . . .
```

On the assumption of ciphertext autokey involving direct standard alphabets, if we write the cipher text as key letters, displaced one interval to the right, we obtain the following decipherment:

```
K:   Q X F W Z Q U A I U Y L E G U G S S F
C: Q X F W Z Q U A I U Y L E G U G S S F I . . .
P:   H I R D R E G I M E N T C O M M A N D
```

The introductory key letter required to make $Q_c = T_p$ is found to be X_k.

(2) As a second example, let us consider the following beginning of a cryptogram suspected to have been enciphered by ciphertext autokey with direct standard alphabets:

```
B P A U V   N L F J A   L Y M L Q   N A E L R . . .
```

Assuming an introductory key of one letter, we obtain the following decipherment:

```
K:   B P A U V N L F J A L Y M L Q N A E L
C: B P A U V N L F J A L Y M L Q N A E L R . . .
P:   O L U B S Y U E R L N O Z F X N E H G
```

Nothing. We now assume that the introductory key consisted of two letters, and we get the following results:

```
K:     B P A U V N L F J A L Y M L Q N A E
C: B P A U V N L F J A L Y M L Q N A E L R . . .
P:     Z J V T Q S Y V C Y B N E C K R L N
```

Still nothing. We make several more trials, and finally, on the assumption of an introductory key of 8 letters, the following is obtained.

```
K:                 B P A U V N L F J A L Y
C: B P A U V N L F J A L Y M L Q N A E L R . . .
P:                 I L L E R Y F I R E A T
```

It is clear that the introductory key is 8 letters in length. Doing what comes naturally,

```
K:           N U M B P A U V N L F J A L Y
C: B P A U V N L F J A L Y M L Q N A E L R . . .
P:           A R T I L L E R Y F I R E A T
```

shows that the introductory key ends with NUM; now with but little experimentation, either with the letters of the key or with the beginning of the message plain text, we obtain the complete solution:

```
K: A L U M I N U M B P A U V N L F J A L Y
C: B P A U V N L F J A L Y M L Q N A E L R . . .
P: B E G I N A R T I L L E R Y F I R E A T
```

(3) In a third case, we will assume that the following is the beginning of an intercepted message:

```
D I T G C   M G T Z B   P C V D Q   K Y S K P . . .
```

Again assuming direct standard alphabets, writing the cipher text as key letters displaced one interval to the right and deciphering, we obtain the following:

```
K:   D I T G C M G T Z B P C V D Q K Y S K
C: D I T G C M G T Z B P C V D Q K Y S K P . . .
P:   F L N W K U N G C O N T I N U O U S F
```

We note the plain text "CONTINUOUS F . . ." emerging, preceded by the NG which is probably ING; this indicates that ciphertext autokey is involved with an initial key of 7 letters, and that the last letter of the initial key is used to start the autokeyed portion. After a little experimentation with the initial portion of the message text and the key,[4] we recover the key word and the first word of the message, as follows:[5]

```
K: M E R C U R Y|G T Z B P C V D Q K Y S K
C: D I T G C M G T Z B P C V D Q K Y S K P . . .
P: R E C E I V I N G C O N T I N U O U S F
```

b. A mechanical method of solution for ciphertext autokeyed cryptograms when the components are known sequences may be of interest. The method involves the use of sliding alphabet strips aligned in such a manner that, as one progresses from left to right across the strips, each key letter is set opposite the letter k on the preceding strip:[6] the plain text will appear to the left of the pertinent cipher letter on each strip. In other words, what we have is a mechanical method of correlating the letters of the key, cipher, and plain text; the method is best understood by examples.

(1) In Fig. 21 is illustrated the arrangement of standard-alphabet strips for the first 10 letters of putative key, QXFWZQUAIU, for the message beginning given in subpar. *a*(1), above. If we assume that a one-letter introductory key has been used, the key letters just named were used to key the 2d through 11th cipher letters, XFWZQUAIUY; therefore we search for these cipher letters consecutively across the strips and we note the letters to their immediate left. In this case the plain text HIRDREGIME is manifested and the problem is solved.

---

[4] The I of ING gives a key of Y, which should be preceded by a T, A, L, or very few other letters.

[5] The key word and the first word of the message may be recovered by working backwards from ING_p, or by assuming various initial digraphs for the plain text or the key; a trial of RE_p for the message beginning would yield ME_k, and we could go on from here to read this "depth of one."

[6] This is under the assumption that A_p is the index letter in the cryptographic equations.

	Figure 21	Figure 22a	Figure 22b
(K):	(Q X F W Z Q U A I U)	(B P A U V N L F J A)	(B P A U V N L F J A)
(C):	(X F W Z Q U A I U Y)	(P A U V N L F J A L)	(J A L Y M L Q N A E)
	A Q N S O N D X X F Z	A B Q Q K F S D I R R	A B Q Q K F S D I R R
	B R O T P O E Y Y G A	B C R R L G T E J S S	B C R R L G T E J S S
	C S P U Q P F Z Z H B	C D S S M H U F K T T	C D S S M H U F K T T
	D T Q V R Q G A A I C	D E T T N I V G L U U	D E T T N I V G L U U
	E U R W S R H B B J D	E F U U O J W H M V V	E F U U O J W H M V V
	F V S X T S I C C K E	F G V V P K X I N W W	F G V V P K X I N W W
	G W T Y U T J D D L F	G H W W Q L Y J O X X	G H W W Q L Y J O X X
	H X U Z V U K E E M G	H I X X R M Z K P Y Y	H I X X R M Z K P Y Y
	I Y V A W V L F F N H	I J Y Y S N A L Q Z Z	I J Y Y S N A L Q Z Z
	J Z W B X W M G G O I	J K Z Z T O B M R A A	J K Z Z T O B M R A A
	K A X C Y X N H H P J	K L A A U P C N S B B	K L A A U P C N S B B
	L B Y D Z Y O I I Q K	L M B B V Q D O T C C	L M B B V Q D O T C C
	M C Z E A Z P J J R L	M N C C W R E P U D D	M N C C W R E P U D D
	N D A F B A Q K K S M	N O D D X S F Q V E E	N O D D X S F Q V E E
	O E B G C B R L L T N	O P E E Y T G R W F F	O P E E Y T G R W F F
	P F C H D C S M M U O	P Q F F Z U H S X G G	P Q F F Z U H S X G G
	Q G D I E D T N N V P	Q R G G A V I T Y H H	Q R G G A V I T Y H H
	R H E J F E U O O W Q	R S H H B W J U Z I I	R S H H B W J U Z I I
	S I F K G F V P P X R	S T I I C X K V A J J	S T I I C X K V A J J
	T J G L H G W Q Q Y S	T U J J D Y L W B K K	T U J J D Y L W B K K
	U K H M I H X R R Z T	U V K K E Z M X C L L	U V K K E Z M X C L L
	V L I N J I Y S S A U	V W L L F A N Y D M M	V W L L F A N Y D M M
	W M J O K J Z T T B V	W X M M G B O Z E N N	W X M M G B O Z E N N
	X N K P L K A U U C W	X Y N N H C P A F O O	X Y N N H C P A F O O
	Y O L Q M L B V V D X	Y Z O O I D Q B G P P	Y Z O O I D Q B G P P
	Z P M R N M C W W E Y	Z A P P J E R C H Q Q	Z A P P J E R C H Q Q
(P):	(H I R D R E G I M E)	(O L U B S Y U E R L)	(I L L E R Y F I R E)

FIGURE 21 FIGURE 22*a* FIGURE 22*b*

(2) The next example in Fig. 22*a* illustrates the strip arrangement for the first 10 letters of key, BPAUVNLFJA, for the message beginning given in subpar. *a*(2). If a one-letter introductory key has been used, these key letters apply to the 2d through 11th cipher letters, PAUVNLFJAL; the decipherment of these letters is found to their immediate left, which is OLUBSYUERL, obviously not plain text. On the same diagram we then search for the decipherment of the 3d through 12th letters, assuming that a two-letter introductory key was employed; again this yields no valid plain text. Finally, on the 8th trial, on the assumption that an 8-letter introductory key is involved, we obtain the plain text ILLERY FIRE; this is shown in Fig. 22*b*.

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(3) For the third and final example, there is illustrated in Fig. 23*a* the strip arrangement for the first 10 letters of assumed key, DITGCMGTZB, for the message beginning given in subpar. *a*(3).

```
(K):  (D I T G C M G T Z B)
(C):  (I T G C M G T Z B P)

       A D L E K M Y E X W X
       B E M F L N Z F Y X Y
       C F N G M O A G Z Y Z
       D G O H N P B H A Z A
       E H P I O Q C I B A B
       F I Q J P R D J C B C
       G J R K Q S E K D C D
       H K S L R T F L E D E
       I L T M S U G M F E F
       J M U N T V H N G F G
       K N V O U W I O H G H
       L O W P V X J P I H I
       M P X Q W Y K Q J I J
       N Q Y R X Z L R K J K
       O R Z S Y A M S L K L
       P S A T Z B N T M L M
       Q T B U A C O U N M N
       R U C V B D P V O N O
       S V D W C E Q W P O P
       T W E X D F R X Q P Q
       U X F Y E G S Y R Q R
       V Y G Z F H T Z S R S
       W Z H A G I U A T S T
       X A I B H J V B U T U
       Y B J C I K W C V U V
       Z C K D J L X D W V W

(P):(F L N W K U N G C O)
```

FIGURE 23*a*

```
(Z B P C V D Q K Y S)
(B P C V D Q K Y S K)

 A Z A P R M P F P N F
 B A B Q S N Q G Q O G
 C B C R T O R H R P H
 D C D S U P S I S Q I
 E D E T V Q T J T R J
 F E F U W R U K U S K
 G F G V X S V L V T L
 H G H W Y T W M W U M
 I H I X Z U X N X V N
 J I J Y A V Y O Y W O
 K J K Z B W Z P Z X P
 L K L A C X A Q A Y Q
 M L M B D Y B R B Z R
 N M N C E Z C S C A S
 O N O D F A D T D B T
 P O P E G B E U E C U
 Q P Q F H C F V F D V
 R Q R G I D G W G E W
 S R S H J E H X H F X
 T S T I K F I Y I G Y
 U T U J L G J Z J H Z
 V U V K M H K A K I A
 W V W L N I L B L J B
 X W X M O J M C M K C
 Y X Y N P K N D N L D
 Z Y Z O Q L O E O M E

(C O N T I N U O U S)
```

FIGURE 23*b*

Nothing is seen here, so a number of additional trials is made, sliding the assumed key over successive 10-letter segments of the cipher text, all without success. We could now assume that an introductory key word was used, and that the autokeyed portion began with the last letter of cipher text after the end of the introductory key. With this in mind, we take as hypothetical key some text after the beginning of the cryptogram, say the 9th through 18th letters, ZBPCVDQKYS; trying this as key for the 10th through 19th letters, BPCVDQKYSK, we are successful on the first trial as shown in Fig. 23*b* with the emergence of the plain text CONTINUOUS.[7]

c. The foregoing mechanical method serves in helping to understand the mechanics of solution of ciphertext autokey encipherment involving known components. A simpler approach, however, is the use of the method of searching for the location of a probable word, as illustrated in the previous volume.[8]

[7] Had we been a little more observant, we could have noticed what appears to be a good plaintext fragment NGCO in the very first trial in Fig. 23*a*; this was a contrived lapse of observation, the better to illustrate a pedagogical point in Fig. 23*b*.

[8] Cf. Subpar. 22*d* on pp. 41–42 of *Military Cryptanalytics, Part II*.

(1) For example, if we were to test the message beginning given in subpar. *a*(2), above, for the possibility of ciphertext autokey involving direct standard alphabets and an introductory key of unknown length, we would construct the following diagram:

```
           5         10        15        20
   B P A U V N L F J A L Y M L Q N A E L R . . .
B|   O Z T U M K E I Z K X L K P M Z D K Q
P|     L F G Y W Q U L W J X W B Y L P W C
A|       U V N L F J A L Y M L Q N A E L R
U|         B T R L P G R E S R W T G K R X
V|           S Q K O F Q D R Q V S F J Q W
```

FIGURE 24

In this diagram, the top row contains the cipher letters, and at the left are the first five cipher letters (the putative key); the row just below the line consists of the decipherments of the cipher letters with the first key letter; the second row below the line consists of the decipherments with the second key letter; and so forth. On a diagonal under the 9th cipher letter may be seen the plaintext fragment ILLER, proving that the introductory key was 8 letters in length.

(2) Taking as another example the message beginning given in subpar. *a*(3), above, we construct the following diagram:

```
           5         10        15        20
   D I T G C M G T Z B P C V D Q K Y S K P
D|   F Q D Z J D Q W Y M Z S A N H V P H M
I|     L Y U E Y L R T H U N V I C Q K C H
T|       N J T N A G I W J C K X R F Z R W
G|         W G A N T V J W P X K E S M E J
C|           K E R X Z N A T B O I W Q I N
```

FIGURE 25

Nothing of significance is seen, so, testing the possibility of autokeying from the last letter of a long introductory key, we construct the diagram shown below, in which we have arbitrarily taken as tentative key elements the cipher letters starting at position 11:

```
           5         10        15        20
   D I T G C M G T Z B P C V D Q K Y S K P
P|                       N G O B V J D V A
C|                         T B O I W Q I N
V|                           I V P D X P U
D|                             N H V P H M
Q|                               U I C U Z
```

FIGURE 25*b*

On the very first diagonal, the plain text fragment NTINU manifests itself, showing that the single-letter offset keying begins at least by the 12th cipher letter, if not before (it actually begins at the 8th position, after a 7-letter introductory key, as can be quickly determined).

d. The index letter was A_p in the foregoing examples; if some other letter were used as the index letter, only a slight modification of the general procedure is necessary. Let us study the following example enciphered with direct standard alphabets, with Q_p as the index letter:[9]

```
K: X|A R J K X Y M C U Q E B E Q O K G Q N A Z X Z C Y W B T Q
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: A R J K X Y M C U Q E B E Q O K G Q N A Z X Z C Y W B T Q G
```

[9] Note that, although the plain text and introductory keys are identical with those of the example in Fig. 18*a*, nevertheless the two cipher texts are *not* isomorphic, since the change of index letter eliminates any causal isomorphism between the two versions.

If we had only the cipher text, and assumed that it was the result of ciphertext autokey encipherment with direct standard alphabets and a one-letter introductory key, we would perform the following decipherment on the basis of A_p as the index letter:

```
K :     A R J K X Y M C U Q E B E Q O K G Q N
C :   A R J K X Y M C U Q E B E Q O K G Q N A . . .
"P":    R S B N B O Q S W O X D M Y W W K X N
```

The "decipherment" does not yield plain text; but if we *complete the plain-component sequence on the result of this decipherment*, we will obtain the true plain text on one of the generatrices, as shown in the diagram below:

```
 R S B N B O Q S W O
 S T C O C P R T X P
 T U D P D Q S U Y Q
 U V E Q E R T V Z R
 V W F R F S U W A S
 W X G S G T V X B T
 X Y H T H U W Y C U
 Y Z I U I V X Z D V
 Z A J V J W Y A E W
 A B K W K X Z B F X
 B C L X L Y A C G Y
 C D M Y M Z B D H Z
 D E N Z N A C E I A
 E F O A O B D F J B
 F G P B P C E G K C
 G H Q C Q D F H L D
*H I R D R E G I M E
 I J S E S F H J N F
 J K T F T G I K O G
 K L U G U H J L P H
 L M V H V I K M Q I
 M N W I W J L N R J
 N O X J X K M O S K
 O P Y K Y L N P T L
 P Q Z L Z M O Q U M
 Q R A M A N P R V N
```

23. Principles of solution by frequency analysis.—*a.* It is apparent that repetitions in ciphertext autokey systems will not be nearly as plentiful in the cipher text as they are in the plain text, because in these systems before a repetition can appear two things must happen simultaneously. First, of course, the plaintext sequence must be repeated, and second, one or more ciphertext letters (depending upon the length of the introductory key) immediately before the second appearance of the plaintext sequence must be identical with one or more ciphertext letters immediately before the first appearance of the plaintext sequence. This can happen only as the result of chance. In the following example the introductory key is the single letter **X**, and the components are direct standard alphabets employed in the usual Vigenère manner:

```
K: X|C K B T M D H N V H L Y . . . . K D K S J M D H N V H L Y
P: F I R S T R E G I M E N T . . . . T H I R D R E G I M E N T
C: C K B T M D H N V H L Y R . . . K D K S J M D H N V H L Y R
```

The repeated plaintext word, REGIMENT, has only 8 letters but the repeated ciphertext sequence contains 9, of which only the last 8 letters actually represent the plaintext repetition. In order that the word REGIMENT be enciphered by DHNVHLYR the second time this word appeared in the text, it was necessary that the key letter for its first plaintext letter, R, be M *both* times; no other key letter will produce the same cipher sequence for the word REGIMENT in this case. Each different key letter for enciphering the first letter of REGIMENT will produce a different encipherment for the word, so that the chance for a repetition in this case is 1 in 26. This is the principal cause for the reduction in repetitions in this system. If an introductory key of two letters were used, it would be necessary that the two cipher letters immediately before the second appearance of the repeated word REGIMENT be identical with the two cipher letters immediately before the first appearance of the word; therefore the chance for a repetition in this case is 1 in 26^2. In general, then, an n-letter repetition in the cipher text, represents an $(n-k)$ letter repetition in the plain text, where n is the length of the ciphertext repetition and k is the length of the introductory key.

b. There is a second phenomenon of interest in connection with ciphertext autokey systems. Let the letter opposite which the key letter is placed (when using sliding components for encipherment) be termed, for convenience in reference, the "base letter." Normally the base letter is the initial letter of the plain component, but it has been pointed out in the preceding volume that this is only a convention. Now when the introductory key is a single letter, if the base letter occurs as a plaintext letter its cipher equivalent is identical with the immediately preceding cipher letter; that is, there is produced a double letter in the cipher text, no matter what the cipher component is and no matter what the key letter happens to be for that encipherment. For example, using the HYDRAULIC . . . XZ sequence for both primary components, with H (the initial letter of the plain component) as the base letter, and using the introductory key letter X, the following encipherment is produced:

```
K: X|U N F F T T V K U H H M B N
P: I F T H E H Y P O T H E S I S . . .
C: U N F F T T V K U H H M B N E
```

Note the doublets FF, TT, and HH. Each time such a doublet occurs it means that the second letter represents H_p, which is the base letter in this case (the initial letter of the plain component). Now if the base letter happens to be a high-frequency letter in normal plain text, for example the letter E or T, then the cipher text will show a large number of doublets; if it happens to be a low-frequency letter then the cipher text will show very few doublets. In fact, the number of doublets will be directly proportional to the frequency of the base letter in normal plain text. Thus, if the cryptogram contains 1,000 letters there should be about 72 occurrences of doublets if the base letter is A, since in 1,000 letters of plain text there should be about 72 A's. Conversely, if a cryptogram of 1,000 letters shows about 72 doublets, the base letter is likely to be A; if it shows about 90, it is likely to be T, and so on. Furthermore, when a clue to the identity of the base letter has been obtained in this manner, it is possible immediately to insert the corresponding plaintext letter throughout the text of the message. The distribution of this letter may not only serve as a check (if no inconsistencies develop) but may also lead to the assumption of values for other cipher letters.

c. When the introductory key is two letters, then this same phenomenon will produce groups of the formula ABA, where A and B may be any letters, but the first and third must be identical. The occurrence of patterns of this type in this case indicates the encipherment of the base letter.

d. The phenomenon noted above can be used to considerable advantage in the solution of ciphertext autokey cryptograms. For instance, if it is known that the ordinary Vigenère method of encipherment ($\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$) is used, then the initial letter of the plain component is the base letter. If, further, it is known that the plain component is the normal A–Z sequence, then the base letter is A and a word such as BATTALION will be enciphered by a group having the pattern AABCCDEFG. If the plain component is a mixed sequence and happens to start with the letter E, then a word such as ENEMY would be enciphered by a sequence having the pattern AABBCD.[10] Sequences such as these are, of course, idiomorphic and if words yielding such idiomorphisms are frequent in the text there will be produced in the latter several

[10] Six letters are shown because the idiomorphism in this case extends over that many letters.

or many cases of isomorphism. When these are analyzed by the principles of indirect symmetry of position, a quick solution may follow.

e. A final principle underlying the solution of ciphertext autokeyed cryptograms remains to be discussed; it concerns the nature of the frequency distribution required for the analysis of such cryptograms. Consider the message beginning illustrated in Fig. 18*a* in subpar. 21*b*. It happens that the letter W_c occurs three times in this short message beginning and, because of the nature of the ciphertext autokeying method, this letter must also appear three times in the key. Now it is obvious that all plaintext letters enciphered by key letter W_k will be in the same cipher alphabet; in other words, if the key text is cipher text offset one letter to the right of the cipher text, *then every cipher letter which immediately follows a* W_c *in the cryptogram will belong to the same cipher alphabet*, and this alphabet may be designated conveniently as the W cipher alphabet. Now if there were sufficient text, so that there were, say, 30 to 40 W_c's in it, then a frequency distribution of the letters immediately following the W_c's will exhibit monoalphabeticity. What has been said of the letters following the W_c's applies equally well to the letters following all the other letters of the cipher text, the A_c's, B_c's, C_c's, and so on. In short, if 26 distributions are made, one for each letter of the alphabet, showing the cipher letter immediately succeeding each different letter of the cipher text, the text of the cryptograms can be allocated into 26 uniliteral, monoalphabetic frequency distributions which can be solved by frequency analysis, provided that there are sufficient data for this purpose.

(1) The foregoing principle has been described as pertaining to the case when the introductory key is a single letter; that is, when the key text is offset or displaced but one interval to the right of the cipher text. But it applies equally to cases wherein the key text is offset more than one interval, provided that the frequency distributions are based upon the proper interval, as determined by the displacement due to the length of the introductory key. For instance, suppose the introductory key consists of two letters, as in the following example.

```
K: X Z|M R H F H G F N Q R X O M R M V W E E
P: R E L I A B L E I N F O R M A T I O N . . .
C: M R H F H G F N Q R X O M R M V W E E . . .
```

The key text in this case is offset two intervals to the right of the cipher text; therefore if we made frequency distributions by taking the cipher letters one interval to the right of a given cipher letter (each time that letter occurs), these distributions will not be monoalphabetic because some letter not related at all to the given cipher letter is the key letter for enciphering the letter one interval to the right of the letter. For example, note the three R_c's in the foregoing illustration. The first R_c is followed by H_c, representing the encipherment of L_p by M_k; the second R_c is followed by X_c, representing the encipherment of F_p by Q_k; the third R_c is followed by M_c, representing the encipherment of A_p by M_k. The three cipher letters H, X, and M are here entirely unrelated and do not belong to the same cipher alphabet because they represent encipherments by three different key letters. On the other hand, the cipher letters *two* intervals to the right of the R_c's, *viz.*, F, O, and V, are in the same cipher alphabet because these cipher letters are the results of enciphering plaintext letters I, O, and T, respectively, by the *same* key letter, R. It is obvious, then, that when the introductory key consists of two letters and the key text is displaced two intervals to the right of the cipher text, the proper frequency distributions for monoalphabeticity will be based upon the letter at the second interval to the right of each cipher letter. Likewise, if the introductory key consists of three letters and the key text is displaced three intervals to the right of the cipher text, the distributions must be based upon the third interval, and so on, in each case the interval used corresponding to the amount of displacement between key text and cipher text.

(2) Conversely, in solving a problem of this type, when the length of the introductory key and therefore the amount of displacement are not known, the appearance of the frequency distributions based upon various intervals after each different cipher letter will disclose this unknown factor, since only one set of distributions will exhibit monoalphabeticity and the interval corresponding to that set will be the correct interval.

24. Example of solution by frequency analysis.—*a*. It will be assumed that previous studies have disclosed that the enemy is using ciphertext autokey systems; it will be further assumed that these studies have also disclosed that (1) the introductory key is usually a single letter, (2) the usual Vigenère method of employing sliding primary components is used, and (3) the plain component is usually the normal A–Z sequence, the cipher component a mixed sequence which changes daily. The following cryptograms, all of the same date, have been intercepted:

Message No. 1

I J X W X   E E C D A   C N Q E T   U K N M V   D I W P P   Q Z S X D
H I F E L   N N J J I   D I V E Y   G T C Z M   E H H L M   R V C U R
G D I E Q   S G T A R   J J Q Q Y   C A R P H   M G L D Y   F Y T C D
G Y F K R   F K S E T   T D I Q K   K M L T U   R Q G G N   K M K I X
J X W K A   O K N T B   T Z J O Q   Y S C D I   D G E T X   G

Message No. 2

G R V R M   Z W K X G   W P C K K   R M X A N   J C C X U   R T N J U
A K O B L   N L M W K   Y Y Z J U   C S U H F   F H I J A   Q B M L T
P U R R S   U E Q E V   Z E Y G C   F F N F I   B W N Y S   T C E T P
D G T T Z   R R Q H Q   A O O X D   B U Y N K   L B W C D   G G K

Message No. 3

R W K A O   L T C J M   Z D K V U   J C D D Y   B Z E L M   M W T Q O
H Q V G X   C H O L M   W V G R K   I B R X D   L A Q Y U   K I R O Z
T Q Y U

Message No. 4

X J J P M   L T Z K X   E C A Q Z   N T T O C   O N D U C   T U T C V
G R J P F   F D I P P   D I X C E   S E T W W   S U M U J   C S L G X
H X M O Z   E K A Q I   S U A O

Message No. 5

G I S U H   W Z H S T   T Z O I D   D H O O V   N B T J G   X C T B S
F K I R H   M M V Y M   I I V U U   C Z M J E   H A G I E   W M E H H
L M W K Y   P P D Q Z   G B O I W   P S F A J   U Q Z H Z   M T F H Z
M L A C Z   R O V D I   W P V I B   O B C C X   N N D G I   E S J O C
K B J H Q   M U Z E L   Y O O V U   J W K I E   I B B O Z   A J I E F
F O R S A   J L N Q M   B Q

Message No. 6

T B J P A   A R Y Y P   V H I D I   T U X N J   M X G S S   B D A Q Y
M M T T F   U U N M G   Q P U X M   O V U Y E   C E C Z M   M W O H C
F O B H V   N K A Z C   K M

Message No. 7

T B J P A   Q A A Z T   R X A L X   F K K M E   I A A B D   S F T Q T
C J J G J   O V M R G   L V W T T   J U A W L   X U K T X   G G B O X
M X D I D   S P B S F   L Y Z K C   F

b. A distribution table is now compiled, the results of which are shown in Fig. 26, below; in originally making the distribution, tallies had been recorded in the appropriate cell in the pertinent horizontal line of the table to indicate the cipher letter which immediately followed each occurrence of the letter to which that line applies. Obviously, the best method of compiling the data is to treat the text biliterally, taking the first and second letters, the second and third letters, and so on, distributing the digraphs as tallies in a digraphic distribution.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	N	φ	I.C.
A	3	1	2				1			3	1	1		1	4		6	3					1			2	29	64	2.05
B		1	1	2				1		3		1	1		4		1	1	2	2	1		2	2		1	24	26	1.22
C	2		2	5	3	3		1		2	3			1	1				2	2	1	1		2		4	35	62	1.35
D	2	1		2			5	2	10		1	1					1		2		1				2		30	120	3.59
E			4		1	1		3	2		1	3					2		2	5		1	1		2		28	52	1.79
F	1			1	1	4		2	1		4	1		1	2					1	1				1		21	28	1.73
G		2	1	1	1		3		3	1	1	2		1			1	3	1	3			1	3	1		29	32	1.02
H	1		1			1		2	3			2	2		2		3		1			1	1	1		2	23	22	1.13
I	1	4		5	5	1			1	2						1	1	2	1	1		2	3	3			33	70	1.72
J	1		3		1		2	1	2	4		1	2		3	4	1				4		1	2			32	56	1.47
K	4	1	1						5		3	1	4	2	1			2	1	1		1		2	2		31	58	1.62
L	2	1		1			1						5	3						4		1		2	2		22	44	2.48
M		1			3		2		1	1	1	4	4		2			2		2	2	2	5	3		2	37	70	1.37
N		1	2		1					4	3	1	2	2			2			2					1		21	28	1.73
O		3	2					2	2		1	2		1	3		1	1				5		2		3	28	48	1.66
P	2	1	1	3		1		1					1			3	1		1		2	2					19	18	1.37
Q	2	1			2		1	1	1		1		2		1	1	1		1	1		1			5	4	26	38	1.52
R						1	2	1		2	1		2		2	1	2	2	2	1		2	1	2	1		25	18	0.78
S	1	1	1		2	4	1			1		1				1			1	2	5			1			22	36	2.03
T	1	4	6	1		2				2				1	1	2	3	1		6	4		1	2		4	41	110	1.74
U	3		3		1			2		3	3		1	1			1	4		1	2			2	2	1	30	44	1.31
V			1	2	1		3	1	1				1	2				1			4		1		1	1	20	22	1.51
W			1								6	1	1	1	1	4			1	2		1	1	1		1	22	44	2.48
X	2		3	4	2	1	4	1		2			3	2							2		2				28	50	1.72
Y		1	1		1	2	2						2	1	1	2			2	1	2				2	2	22	16	0.90
Z	1		1	1	4		1	2		2	2		5	1	1			2	1	2			1				27	42	1.56
																											705	1218	42.85

FIGURE 26

c. The individual frequency distributions give every appearance of being monoalphabetic, which confirms the assumption that the enemy is using ciphertext autokey with a *single-letter* introductory key. The average I.C. of the rows of the matrix is $\frac{42.85}{26}=1.65$, which is fine;[11] or, as a better approach, we could calculate the *digraphic* I.C. of the matrix by considering the sum (1218) of the ϕ values of Fig. 26 as the observed value of ϕ and substituting in the formula $\delta=\frac{676\Sigma f(f-1)}{N(N-1)}=\frac{676(1218)}{705\times 704}=1.66$, again substantiating the same assumption.[12] (This discrepancy between the two figures lies in the round-off errors introduced in obtaining an average I.C.)

[11] The arithmetic mean here suffices because the values of N involved are fairly close to one another; since, as has been previously stated, in ciphertext autokey systems the cipher letters are equiprobable (the over-all I.C. of the cipher text in this example is 1.005), a weighted mean is unnecessary.

[12] Ciphertext autokey systems may therefore be identified statistically from the appropriate digraphic distribution (i.e., on the assumption of the correct length of the introductory key) by the fact that the digraphic I.C. will reflect the *monographic* I.C. of the language.

d. The total number of letters of text is 712, comprising 705 digraphs. If the base letter is A, then there should be approximately 705×7.4%=52 cases of doubled letters in the text. There are actually 53 doublets, which checks very well with the expectancy. The letter A is substituted throughout the text for the second letter of each doublet.

e. The following sequence is noted at the beginning of Message No. 5:

```
G I S U H   W Z H S T  T Z O I D  D H O  O V   N B T J G   X C T B S
                       A          A      A
```

Assume that the sequence DDHOOVNBT represents the word BATTALION, in which case we will have the following key-cipher-plain relationships:

```
K: I D D H O O V N B T
C: D D H O O V N B T
P: B A T T A L I O N
```

If this assumption is correct, the frequency of H_c in the D alphabet should be high, since $H_c=T_p$; the H_c has only two occurrences. Likewise, the frequency of O_c ($=T_p$) in the H alphabet should be high; it is also only two. The frequency of V_c in the O alphabet should be medium or low, since it would equal L_p; it is five, which is too high. The rest of the letters of the assumed word are similarly checked against the appropriate frequency distributions, with the result that, on the whole, the assumption that the DDHOOVNBT sequence represents BATTALION does not appear to be warranted. Similar attempts are made at other points in the text, with the same or other probable words. Some of these attempts may have to be carried to the point where the placement of values in the tentative cipher component leads to serious inconsistencies. Finally, attention is fixed upon the following sequence in the second line of Message No. 6:

```
M M T T F U U N M G
  A   A     A
```

If we assume that this skeleton represents the word AVAILABLE, the following fragment of key, cipher, and plain should be true:

```
K: M M T T F U U N M G
C: M T T F U U N M G
P: A V A I L A B L E
```

Reference is now made to the appropriate frequency distributions to see how well the actual individual frequencies correspond to the expected ones; these data are tabulated in the diagram below:

Alphabet	Assumed		Frequency		Approximation
	θ_c	θ_p	Expected	Actual	
M	T	V	Low	2	Fair
T	F	I	High	2	Fair
F	U	L	Medium	1	Good
U	N	B	Low	1	Good
N	M	L	Medium	2	Fair
M	G	E	High	2	Poor

This assumption of AVAILABLE cannot be discarded just yet. Let the values derivable from the assumption be inserted in their proper places in a cipher component, and, using the latter in conjunction with a normal A–Z sequence as the plain component, let an attempt be made to find corroboration for these values. The following placements may be made: [13]

```
P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C: M     F G                 U N           T
```

The letter M_c appears twice in the cipher sequence and when this partially reconstructed cipher component is tested it is found that the value $L_p(N_k)=M_c$ is corroborated. Having the letters M, F, G, U, N, and T tentatively placed in the cipher component, it is possible to insert certain plaintext values in the text. For example, in the M alphabet, $F_c=D_p$, $G_c=E_p$, $U_c=O_p$, $N_c=P_p$, and $T_c=V_p$. In the F alphabet, $G_c=B_p$, $U_c=L_p$, $N_c=M_p$, $T_c=S_p$, and $M_c=X_p$. The other letters yield additional values in the appropriate alphabets. The plaintext values thus obtainable are inserted in the cipher text. No inconsistencies appear and, moreover, certain good digraphs are brought to light. For instance, note what is manifested at the end of the third line of Message No. 5:

```
K:   U Q Z H  Z M T F H
C: U Q Z H Z  M T F H Z
P:              V I
```

Now if the letter H can be placed in the cipher component, several values might be added to this partial decipherment. We note that F and G are sequent in the cipher component; now let us suppose that H follows G therein, and we obtain the following:

```
K:   U Q Z H  Z M T F H
C: U Q Z H Z  M T F H Z
P:              V I C
```

Suppose the VIC is the beginning of VICINITY. This assumption permits the placement of A, C, L, and Z in the cipher component, as follows:

```
P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C; M A     F G H       L       Z U N           T       C
```

These additional values check in very nicely and presently the entire cipher component is reconstructed. It is found to be as follows:

```
P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C: M A B F G H J K L Q S V X Z U N D E R W O T Y P I C
```

The key phrase is clearly based upon UNDERWOOD TYPEWRITER COMPANY. All the messages may now be deciphered with ease. The following gives a letter-for-letter decipherment of the first three groups of each message: [14]

```
                K: K|I J X W  X E E C D  A C N Q E
Message No. 1   C: I J X W X  E E C D A  C N Q E T . . .
                P: R I G H T  F A I R L  Y Q U I E

                K: E|G R V R  M Z W K X  G W P C K
Message No. 2   C: G R V R M  Z W K X G  W P C K K . . .
                P: N O T H I  N G O F S  P E C I A
```

[13] Note that, had we not known (or assumed) the plain-component sequence, we would first have entered these values in a 26×26 square rather than in a single strip for the cipher component, and then we would exploit any manifestations of direct or indirect symmetry present.

[14] The introductory keys for these messages are presumed to have been specified by prearrangement, or indicated by the message number, file time, or some other element of the message externals.

```
                  K: R|R W K A   O L T C J   M Z D K V
Message No. 3     C: R W K A O   L T C J M   Z D K V U . . .
                  P: A B O U T   O N E H U   N D R E D

                  K: J|X J J P   M L T Z K   X E C A Q
Message No. 4     C: X J J P M   L T Z K X   E C A Q Z . . .
                  P: G U A R D   I N S U F   F I C I E

                  K: E|G I S U   H W Z H S   T T Z O I
Message No. 5     C: G I S U H   W Z H S T   T Z O I D . . .
                  P: N U M E R   O U S F L   A S H E S

                  K: B|T B J P   A A R Y Y   P V H I D
Message No. 6     C: T B J P A   A R Y Y P   V H I D I . . .
                  P: T H E R E   A R E A B   O U T S I

                  K: B|T B J P   A Q A A Z   T R X A L
Message No. 7     C: T B J P A   Q A A Z T   R X A L X . . .
                  P: T H E R E   I S A M I   X U P H E
```

f. In the foregoing example the plain component was the normal sequence, so that with the Vigenère method of encipherment the base letter is A. If the plain component is a mixed sequence, the base letter may no longer be A, but in accordance with the principle set forth in subpar. 23*b*, the frequency of doublets in the cipher text will correspond with the frequency of the base letter as a letter of normal plain text.[15] If a good clue is afforded by the frequency of doublets in the cipher text, the insertion of the corresponding base letter in the plain text will lead to further clues. The solution from there on can be handled along the lines indicated above.

25. Solution by means of isomorphs.—*a*. It was stated in subpar. 23*d* that in ciphertext autokey systems the production of isomorphs is a frequent phenomenon and that analysis of these isomorphs may yield a quick solution. An example of this sort will now be studied, using as an illustration the following three messages which are suspected of being in a ciphertext autokey system:

Message No. 1

```
U S Y P W   T R X D I   M L E X R   K V D B D   D Q G S U   N S F B O
B E K V B   M A M M O   T X X B W   E N A X M   Q L Z I X   D I X G Z
P M Y U C   N E V V J   L K Z E K   U R C N I   F Q F N N   Y G S I J
T C V N I   X D D Q Q   E K K L R   V R F R F   X R O C S   S J T B V
E F A A G   Z R L F D   N D S C D   M P B B V   D E W R R   N Q I C H
A T N N B   O U P I T   J L X T C   V A O V E   Y J J L K   D M L E G
N X Q W H   U V E V Y   P L Q G W   U P V K U   B M M L B   O A E O T
T N K K U   X L O D L   W T H C Z   R
```

Message No. 2

```
B I I B F   G R X L G   H O U Z O   L L Z N A   M H C T Y   S C A A T
X R S C T   K V B W K   O T G U Q   Q F J O C   Y Y B V K   I X D M T
K T T C F   K V K R O   B O E P L   Q I G N R   I Q O V J   Y K I P H
J O E Y M   R P E E W   H O T J O   C R I I X   O Z E T Z   N K
```

Message No. 3

```
H A L O Z   J R R V M   M H C V B   Y U H A O   E O V A C   Q V V J L
K Z E K U   R F R F X   Y B H A L   Z O F H M   R S J Y L   A P G R S
X A G X D   M C U N X   X L X G Z   J P W U I   F D B B Y   P V F Z N
B J N N B   I T M L J   O O S E A   A T K P B   Y
```

[15] If the plain and cipher components had been identical sequences, this fact together with the identity of the base letter could have been determined from the digraphic distribution: one of the rows of the distribution (the row corresponding to the base letter) would reflect an approximation of the normal frequency distribution, i.e., peaks for the letters **AEINORST**, and blanks or near-blanks for **JKQXZ**. Furthermore, the reconstruction matrix would have displayed symmetry about the main diagonal (from upper left to lower right); see in this connection subpar. 33*e* in the next chapter.

b. Frequency distributions are made, based upon the second letters of pairs, as in the preceding example. These distributions are shown in the table in Fig. 27, below. The digraphic I.C. is $\frac{676(500)}{451 \times 450}=$ 1.67, confirming ciphertext autokey with a single-letter introductory key. Nevertheless, the data in each distribution are relatively scanty and it would appear that the solution is going to be a rather difficult matter.

2d letter (columns); 1st letter (rows)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	N	φ
A	3		1		1		2					2	2		2	1				3				1			18	20
B		2		1	1	1		1	2	1			2		4							3	2		3		23	32
C	1			1		1		1						2			1	1	1	2	1	3			1	1	17	8
D		2		2	1				2			1	4	1			2		1								16	20
E	1				1	1	1				4			1	2	1				1		2	2	1	2		20	20
F	1	1		2			1	1		1	1			1			1	2						2		1	15	6
G								1						2				1	3		1		1	1		3	13	14
H	4		3							1			1		2						1						12	20
I		1	1			2	1		2	1			1			1	1			2				5			18	26
J										1		4		1	4	1		1		2					2		16	28
K				1					2		2	1			1	1		1		1	4	4				2	20	30
L	1	1			2	1	1			1	3	1			2		2	1					1	2		3	22	20
M	1		1					2				4	3		1	1	1	2		1					1		18	22
N	2	3		1	1				2		2			3			1	1	1					2	1		20	20
O	1	2	3	1	3	1		1				1			1				1	4	2	3				2	26	36
P		2			1		1		1			2	1									2	2				12	8
Q					1	2	2		2			1			1		2					1	1				13	8
R			1			4			2		1	1		1	2	1		2	3			2		2			22	28
S			3		1	1			1	2									1		1			1	1		12	8
T		1	3				1	1		2	3		1	2				1		2				2	1	1	21	20
U		1	1					1	1					2		2	1	2	1			1		1		1	15	6
V	2	3		2	3	1				3	3		1	1				1				2			1		23	30
W					1			2			1							1		2	2						9	6
X	1	1		5			2					3	1		1		1	3	1	1				2	1		23	36
Y		2					1			1	1	1	1			3					2				1		13	10
Z					3				1	2				3	2	1		2									14	18
																											451	500

FIGURE 27

c. Before becoming discouraged too quickly, however, we make a search throughout the text to see if any isomorphs are present. Fortunately, there appear to be several of them. Note the following:

	(1)	. . . D B D D Q G S U N S F B O B E K . . .
Message No. 1	(2)	. . . N E V V J L K Z E K U R C N I F . . .
	(3)	. . . T N K K U X L O D L W T H C Z R]
Message No. 2	(4)	. . . C R I I X O Z E T Z N K]
Message No. 3	(5)	. . . C Q V V J L K Z E K U R F R F X . . .

First, it is necessary to delimit the length of the isomorphs. Isomorph (2) shows that the isomorphism begins with the doubled letters; for there is an E before VV in that case and also an E within the isomorph. If the phenomenon included the E, then the letter immediately before the DD in the case of isomorph (1) would have to be an N, to match its homolog, E, in isomorph (2), which it is not. Corroborating data are given by isomorphs (3), (4), and (5) in this respect. Hence, we may take it as established that the isomorphism begins with the doubled letters. As for the end of the isomorphism, the fact that isomorphs (2) and (5) consist of the same set of 10 letters seems to indicate that this number defines the length of the isomorphism. The fact that Message No. 2 ends 2 letters after the last tie-in letter, Z, corroborates this assumption. It is at least certain that the isomorphism does not extend beyond 11 letters because the recurrence of R in isomorph (5) is not matched by the recurrence of R in isomorph (2), nor by the recurrence of T in isomorph (3). Hence it may be assumed that the isomorphic sequence is probably 10 letters in length, possibly 11. But to be on safe ground it is best to proceed on the 10-letter basis.

d. By applying the principles of indirect symmetry to the superimposed isomorphs, partial chains may be constructed, as shown below:

```
(1-2) D V   Q J   G L   S K   F U Z   N E   B R
(1-3) N D K   Q U O   G X   S L   F W   B T
(1-4) D I   Q X   G O   S Z   U E   F N T   B K
(2-3) V K L X   J U W   Z O   E D   R T
(2-4) V I   J X   L O   R K Z E T   U N
(3-4) D T K I   U X O E   L Z   W N
```

These partial chains may be amalgamated into the following sequence:

L O D J X B S U N . G W . Q . . . F V I . R K Z E T

Noting the J K at an interval of –7, and also W X Z at the same interval, we conclude that a keyword-mixed sequence is involved, and we derive the original sequence as

W X Z . . D R . U L I . B E F G J K . N O . Q S T V,

whereupon we recognize our perennial friend HYDRAULIC and fill in the missing six letters.

e. We now have the cipher component, and the plain component remains to be reconstructed. The simplest and most foolproof solution ordinarily is a reduction to monoalphabetic terms, using the recovered cipher component and the known offset of the cipher text against itself as key.[16] However, the probable word method, if the probable words are at all probable, may be used to good advantage. A good crib to assume for the 10-letter repetition found in Message Nos. 1 and 3 is ARTILLERY (especially since the doublet rate of the distribution in Fig. 27 is $\frac{33}{451}=7.3\%$, which is just right for a base letter of A_p to represent the doublet in the repetition). This single assumption is sufficient to place 7 letters in the plain component, thus:

```
K:   V V J L K Z E K U R
C: V V J L K Z E K U R
P:   A R T I L L E R Y
```

A . . . E . . . I . . L . . . . . R . T . . . . Y .

These few letters (few, but how beautifully spaced!) are sufficient to suggest that the plain component is in all probability the normal sequence. A few moments' testing proves this to be true. The two components are therefore:

P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z

[16] Note that if the LODJX . . . sequence in subpar. *d,* above, had not been of systematic construction to enable us to analyze its derivation and thus fill in the missing 6 letters, we still could have converted most of the cipher text to monoalphabetic terms, solved the text, and recovered both components.

f. With the two components at hand, the decipherment of the message is a low-order triviality. Since a single-letter introductory key is known to have been used,[17] we decipher the first five groups of Message No. 1 as follows:

```
K: ? U S Y P  W T R X D  I M L E X  R K V D B  D D Q G S
C: U S Y P W  T R X D I  M L E X R  K V D B D  D Q G S U . . .
P: ? P H R F  Y I V E F  I R E O F  L I G H T  A R T I L
```

The mangled beginning is the result either of garbles, or of specialized keying procedure wherein the *last* letter of an introductory key was used as the introductory key letter for enciphering the subsequent autokeyed portion of the text (see Fig. 18*c* in subpar. 21*b*). If we assume that the IVE before the word FIRE is the ending of the first word of the plain text, and that this word is INTENSIVE, the introductory key word is found to be WICKER. Thus:

```
K: W I C K E R|T R X D I M L E X R K V D B D D Q G S
C: U S Y P W T R X D I M L E X R K V D B D D Q G S U . . .
P: I N T E N S I V E F I R E O F L I G H T A R T I L
```

The beginnings of the other two messages are recoverable in the same way and are found to be as follows:

```
K: P R O M I S E|R X L G H O U Z
C: B I I B F G R X L G H O U Z O . . .
P: R E Q U E S T V I G O R O U S

K: C H A R G E D|R R V M M H C V
C: H A L O Z J R R V M M H C V B . . .
P: S E C O N D B A T T A L I O N
```

g. The example solved in the foregoing subparagraphs offers an important lesson to the student, insofar as it teaches him that *he should not immediately feel discouraged when confronted with a problem presenting only a small quantity of text and therefore affording what seems at first glance to be an insufficient quantity of data for solution.* For in this example, while it is true that there are insufficient data for analysis by simple principles of frequency, it turned out that solution was achieved *without any recourse to the principles of frequency of occurrence.* Here, then, is one of those interesting cases of substitution ciphers of rather complex construction which are solvable without any study whatsoever of frequency distributions. Indeed, it will be found to be true that in more than a few instances the solution of quite complicated cipher systems may be accomplished not by the application of the principles of frequency, but by recourse to inductive and deductive reasoning based upon other considerations, even though the latter may often appear to be very tenuous and to rest upon quite flimsy supports.

26. Solution of isologs involving the same pair of unknown primary components.—*a.* Two messages containing identical plain text encrypted in a ciphertext autokey system with two different single-letter introductory keys may be solved in a manner identical to that described in the last paragraph, since what we really have is a pair of long isomorphs one letter shorter than the length of the messages. Even if the introductory keys are words of different lengths and compositions, if the key usage is similar to that illustrated in Fig. 18*c* in subpar. 21*b* the message can be solved very rapidly by reconstructing the primary components, since the cryptographic texts of such messages will be isomorphic after the initial keyword portions.

[17] We know this from (a) statistical evidence of the digraphic distribution at an offset of 1, (b) the indications of the correct plain component emerging from a tentative decipherment of the 10-letter repetition, and (c) the unlikelihood that with three rather short messages a long ciphertext repetition would have manifested itself if the offset were more than 1 letter. We knew that the three messages were autokeyed at the same offset, otherwise the isomorphs would have not appeared among all three messages.

(1) Note the two following superimposed messages, in which isomorphism between the two cryptograms is both obvious and consistent after their 6th letters:

Msg "A":	T S B J S	K B N L O	C F H A Z	L W J A M	B N F N S
Msg "B":	B K K M J	X Y C X B	H R P V O	X M U V I	Y C R C G
Msg "A":	M V J R E	H F P R X	C P C R R	E H F M U	H R A X C
Msg "B":	I K U T D	P R E T N	H E H T T	D P R I W	P T V N H
Msg "A":	N F D U B	A T F Q R			
Msg "B":	C R S W Y	V J R F T			

Starting with any pair of superimposed letters (after the 6th pair), the following chains are derived:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
(1)	Z	O	B	Y	.	.	.	.						
(2)	L	X	N	C	H	P	E	D	S	G	.	.	.	.
(3)	Q	F	R	T	J	U	W	M	I	.	.	.	.	
(4)	A	V	K	.	.	.	.							

The foregoing fragments either are part of two 13-letter chains, or they are parts of a complete 26-letter sequence. If the former is the case, then the two 13-letter chains must be (Z O B Y Q F R T J U W M I) and (L X N C H P E D S G A V K); and, a few moments later, noting phenomena associated with keyword-mixed sequences in the two chains, we superimpose them in the diagram [18]

```
Z O B Y Q F R T J U W M I
H P E D S G A V K L X N C
Y Q F R T J U W M I Z O B
```

from which we speedily obtain the HYDRAULIC . . . XZ sequence.[19]

(2) Only the cipher component has been recovered thus far. If we assume that the plain component is the same as the cipher, the initial key words and the message plain texts are at once deciphered; it will be found that the initial key word for Message "A" is PENCE, and that for Message "B" is LATERAL.[20] If the plain component had not been guessed in this case, we could have "deciphered" the message text using an arbitrary plain component (say, the A–Z sequence), resulting in a conversion of the complex cipher text into monoalphabetic terms which can then speedily be solved.

(3) The foregoing solution affords a clue to the solution of cases in which the texts of two or more messages are not completely identical but are in part identical because they happen to have similar beginnings or endings, or contain nearly similar information or instructions. The progress in such cases is not so rapid as in the case of messages with wholly identical texts because much care must be exercised in blocking out the isomorphic sequences upon which the reconstruction of the primary components will be based.

b. In the preceding example the autokeyed portions of the texts started with the last letters of the introductory keys. If full autokeying (i.e., the method shown in Fig. 18*b*), had been employed the solution would hardly be more difficult.

---

[18] See subpar. 44*i* on pp. 89–90 of *Military Cryptanalytics, Part II*.

[19] If the four fragments (1), (2), (3), and (4) had been parts of a complete 26-letter sequence, there would have been only 6 ways to permute them, *viz.*, 1-2-3-4, 1-2-4-3, 1-3-2-4, 1-3-4-2, 1-4-2-3, and 1-4-3-2; therefore the problem would still be solvable without too much effort, even if the cipher component has been a random sequence.

[20] The reason that the cryptographic texts are isomorphic after the initial keyword portions is, of course, that since the text beyond the key word is enciphered autokey fashion by the preceding cipher letter, the letters before the last letter of the key have no effect upon the encipherment at all. Hence two messages having identical plain text cannot be other than isomorphic after the initial keyword portions.

(1) In order to illustrate such a case, let the same plain texts used in the preceding example be enciphered by introductory key words of the same lengths but different compositions: PENCE and LATER. Thus:

Message "A"

```
K: PENCE| TSBJS  MMNRU  LPUIH  JBTXF  INNRM
P: REQUE  STINF  ORMAT  IONOF  SITUA  TIONI
C: TSBJS  MMNRU  LPUIH  JBTXF  INNRM  DWIQV

K: DWIQV  PCKAO  DPAZO  BCMRI  AFNWO  GLIHT
P: NFIFT  EENTH  INFAN  TRYSE  CTORA  TONCE
C: PCKAO  DPAZO  BCMRI  AFNWO  GLIHT  IWWCU
```

Message "B"

```
K: LATER| BKKMJ  RBTUX  SGEBQ  YRHHA  TETUC
P: REQUE  STINF  ORMAT  IONOF  SITUA  TIONI
C: BKKMJ  RBTUX  SGEBQ  YRHHA  TETUC  NOGTM

K: NOGTM  LDQLE  NGBYE  WDSUH  PUTZE  HHGDK
P: NFIFT  EENTH  INFAN  TRYSE  CTORA  TONCE
C: LDQLE  NGBYE  WDSUH  PUTZE  HHGDK  TODEX
```

(2) Now let the cipher texts be superimposed and isomorphisms be sought. They are shown underlined below:

```
Msg A: TSBJS  MMNRU  LPUIH  JBTXF  INNRM  DWIQV
Msg B: BKKMJ  RBTUX  SGEBQ  YRHHA  TETUC  NOGTM

Msg A: PCKAO  DPAZO  BCMRI  AFNWO  GLIHT  IWWCU
Msg B: LDQLE  NGBYE  WDSUH  PUTZE  HHGDK  TODEX
```

It will be noted that the intervals between identical vertical pairs show a constant factor of 5, indicating that the messages have been enciphered with 5-letter introductory key words.

(3) The vertical pairs beyond the first five letters of the messages are now distributed in a reconstruction matrix according to their position based upon this interval of 5, similar to the treatment of vertical pairs in periodic-cipher isologs arising from the use of repeating keys of the same lengths.[21] This is shown below:

Ø	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	P	W		N			H		T	Y		S	R			L										
2		R	D			U						H	B	E		G							O			
3	B								G		Q		S	T						H	E		D			
4	L		E					D	B								T	U					Z	H		Y
5						A		Q	H				C		E					K	X	M				

From the values in this matrix the original cipher component, the HYDRAULIC . . . XZ sequence, may quickly be recovered, because the Ø line may be included in the chaining.

[21] Cf. subpar. 60f, *Military Cryptanalytics, Part II.*

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

28. Further remarks on ciphertext autokey systems.—*a.* All of the discussion on ciphertext autokey systems thus far has been limited to alphabetical systems employing sliding primary components (or the equivalent form of a square table). There is no reason, of course, why a set of 26 unrelated random sequences in a table such as that in Fig. 33, below, could not be used for the cipher alphabets. In such

Plain

Key	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	T	O	K	Z	B	L	R	X	S	P	W	N	A	Q	C	E	I	G	D	J	F	V	U	Y	M	H
B	S	B	A	C	D	E	H	F	I	J	K	T	L	M	O	U	V	Y	G	Z	N	P	Q	X	R	W
C	Y	Q	R	T	V	W	L	A	D	K	O	M	J	U	B	G	E	P	H	S	C	Z	I	N	X	F
D	Z	S	A	E	D	C	B	I	F	G	J	H	L	K	M	R	U	O	Q	V	P	T	N	W	Y	X
E	S	L	W	E	M	Z	V	X	G	A	F	N	Q	U	K	D	O	P	I	T	J	B	R	H	C	Y
F	G	P	O	C	I	X	L	U	R	N	D	Y	Z	H	W	B	J	S	Q	F	K	V	M	E	T	A
G	W	A	H	X	J	E	Z	B	N	I	K	P	V	R	O	G	S	Y	D	U	L	C	F	M	Q	T
H	G	T	D	X	A	I	H	P	J	O	B	W	K	C	V	F	Z	L	Q	E	R	Y	N	S	U	M
I	A	J	D	S	K	Q	O	I	V	T	Z	E	F	H	G	Y	U	N	L	P	M	B	X	W	C	R
J	J	G	H	O	N	M	T	P	R	Q	S	V	Z	U	X	Y	W	I	C	A	K	E	L	B	D	F
K	V	Q	P	N	O	H	U	W	D	I	Z	Y	C	G	K	R	F	B	E	J	A	L	T	M	S	X
L	E	W	O	A	M	N	F	L	H	Q	G	C	U	J	T	B	Y	P	Z	K	X	I	S	R	D	V
M	D	H	B	M	K	G	X	U	Z	T	S	W	Q	Y	V	O	R	P	F	E	A	N	C	J	I	L
N	D	W	P	K	J	V	I	U	Q	H	Z	C	T	X	B	L	E	G	N	Y	R	S	M	F	A	O
O	S	G	U	E	N	T	C	X	O	W	F	Q	D	R	L	J	Z	M	A	P	B	V	H	I	Y	K
P	X	C	S	H	D	E	O	K	F	P	Y	A	Q	J	N	U	B	T	G	I	M	W	Z	R	V	L
Q	N	V	A	R	M	Y	O	F	T	H	E	U	S	Z	J	X	D	P	C	W	G	Q	I	B	K	L
R	O	Z	P	L	G	V	J	R	K	Y	T	F	U	I	W	X	H	A	S	D	M	C	N	E	Q	B
S	T	O	J	Y	L	F	X	N	G	W	H	V	C	M	I	R	B	S	E	K	U	P	D	Z	Q	A
T	Z	X	Q	L	Y	I	O	V	B	P	E	S	N	H	J	W	M	D	G	F	C	K	A	U	T	R
U	E	Y	B	F	S	J	M	U	D	Q	C	L	Z	W	T	I	P	A	V	N	K	H	R	G	O	X
V	X	P	U	C	O	T	Y	A	W	V	S	F	D	L	I	E	B	H	K	N	R	J	Q	Z	G	M
W	E	V	D	T	U	F	O	Y	H	M	L	S	I	Q	N	J	C	P	G	B	Z	A	X	K	W	R
X	M	V	K	B	Q	W	U	G	L	O	S	T	E	C	H	N	Z	F	R	I	D	A	Y	J	P	X
Y	W	J	L	V	G	R	C	Q	M	P	S	O	E	X	T	K	I	A	Z	D	N	B	U	H	Y	F
Z	T	B	R	E	M	X	Z	P	V	Q	Y	U	O	G	A	I	K	L	F	S	W	H	D	C	N	J

FIGURE 33

cases, the general methods treated in par. 22 still apply, with necessary modifications, as also do the methods in pars. 23 and 24, except that it is obvious that (a) there will be no determinable base letter, and (b) there will be no causal isomorphs. For that matter, even with a matrix such as that of Fig. 34 below, in which the key letters are designated by *arbitrary* letters to the left of the square (instead of

Key \ Plain	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z
A	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z
B	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H
C	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y
D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D
E	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R
F	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A
G	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U
H	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L
I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I
J	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C
K	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B
L	F	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E
M	G	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F
N	J	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G
O	K	M	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J
P	M	N	O	P	Q	S	T	W	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K
Q	N	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M
R	O	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N
S	P	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O
T	Q	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P
U	S	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q
V	T	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S
W	V	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T
X	W	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V
Y	X	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W
Z	Z	H	Y	D	R	A	U	L	I	C	B	E	F	G	J	K	M	N	O	P	Q	S	T	V	W	X

FIGURE 34

the letters in a column under a particular plaintext letter—the base letter), there is likewise no determinable base letter and no causal isomorphs can be produced, as can be shown by the following isologous message beginnings:

Message "A"

```
K: X| P M Q Q P P Z F G T R I R Z N D P Q L J Y M L L H X Q W G
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: P M Q Q P P Z F G T R I R Z N D P Q L J Y M L L H X Q W G P
```

Message "B"

```
K: Y| Q N S T T V U L P A E S J O U B N O A D T E X P A O E F T
P: T H I R D R E G I M E N T C O M M A N D P O S T M O V I N G . . .
C: Q N S T T V U L P A E S J O U B N O A D T E X P A O E F T U
```

The ciphertext doublets are the result of chance. Note in this example that, since the plain and cipher components are identical sequences, one of the distributions of the letters immediately following a particular cipher letter (in this case, A_c) will fit the normal, since the plaintext letters enciphered by this key letter will be self-enciphered; the distributions of the letters following the other cipher letters will of course be monoalphabetic.

b. The general principles of ciphertext autokeying apply equally well to digital systems, and for that matter to Baudot systems. The modulus in digital systems is the usual mod-10 arithmetic,[26] so that in effect the cipher component is a *known* sequence—thus a reduction to monoalphabetic terms suggests itself at once, if not sooner.

c. As an example, let us study the following message, suspected of having been enciphered in a ciphertext autokey system with the additive method of encipherment (P+K=C).[27]

```
15635 92001 13756 94093 05151 40094
22123 30476 69720 39498 79929 58112
76111 49369 28072 48301 90074 45548
40544 31589 12943 50529 52814 74516
71932 40506 11682 05793 10649 25695
19689 99661
```

If a single-digit introductory key has been used, the cipher text is offset against itself at an interval of 1 and a decipherment obtained, the beginning of which is shown below:[28]

```
K:   1 5 6 3  5 9 2 0 0  1 1 3 7 5  6 9 4 0 9  3 0 5 1 5
C: 1 5 6 3 5  9 2 0 0 1  1 3 7 5 6  9 4 0 9 3  0 5 1 5 1 . . .
P:   4 1 7 2  4 3 8 0 1  0 2 4 8 1  3 5 6 9 4  7 5 6 4 6
```

The I.C. of the entire deciphered message is 0.99, so the length of the introductory key was probably not 1. The cipher text is then offset against itself at intervals of 2, 3, 4 . . ., up to an interval of 9: the I.C.'s obtained from the resulting decipherments are all unsatisfactory, as may be seen from the following table:

Offset	I.C.	Offset	I.C.	Offset	I.C.
1	0.99	4	0.97	7	0.99
2	1.02	5	0.96	8	0.98
3	1.02	6	0.96	9	0.97

[26] If other than mod-10 arithmetic is used, say an arbitrary conversion-square encipherment with a square such as the following,

Plain

Key	0	1	2	3	4	5	6	7	8	9
0	3	1	6	7	5	8	2	4	9	0
1	1	2	3	5	4	6	8	9	0	7
2	6	4	0	2	8	1	5	3	7	9
3	7	3	2	9	0	5	6	1	8	4
4	5	8	4	0	9	2	3	7	1	6
5	8	5	1	6	7	9	0	2	4	3
6	2	9	8	1	3	7	4	0	6	5
7	4	6	9	8	2	0	7	5	3	1
8	9	0	7	3	6	4	1	8	5	2
9	0	7	5	4	1	3	9	6	2	8

the problem must be solved as a general case of ciphertext autokey as treated earlier in this chapter. (The conversion square shown here is a *Latin square*, with ten unique digits in each of the rows and in each of the columns; other conversion squares may have columns containing repeated digits.)

[27] As regards the effect of the use of different encipherment conventions, see subpars. 35*g* and *h* (on pp. 117–120).

[28] We have assumed that the enciphering procedure was the *additive* method (i.e., wherein P+K=C); see also the remarks in subpar. 35*h* in the next chapter if subtractive or minuend methods had been involved.

d. When an offset of 10 is tried, the I.C. of the deciphered text jumps to 1.52 and a long repetition is in evidence, revealing that 10 is the length of the introductory key. The decipherment and its appertaining frequency distribution are shown below:

```
K:                         1 5 6 3 5   9 2 0 0 1   1 3 7 5 6   9 4 0 9 3
C: 1 5 6 3 5   9 2 0 0 1   1 3 7 5 6   9 4 0 9 3   0 5 1 5 1   4 0 0 9 4
P:                         0 8 1 2 1   0 2 0 9 2   9 2 4 0 5   5 6 0 0 1

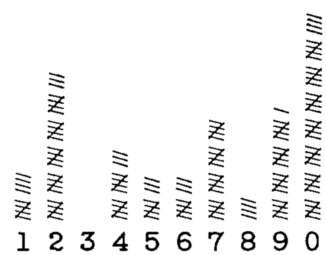
K: 0 5 1 5 1   4 0 0 9 4   2 2 1 2 3   3 0 4 7 6   6 9 7 2 0   3 9 4 9 8
C: 2 2 1 2 3   3 0 4 7 6   6 9 7 2 0   3 9 4 9 8   7 9 9 2 9   5 8 1 1 2
P: 2 7 0 7 2   9 0 4 8 2   4 7 6 0 7   0 9 0 2 2   1 0 2 0 9   2 9 7 2 4

K: 7 9 9 2 9   5 8 1 1 2   7 6 1 1 1   4 9 3 6 9   2 8 0 7 2   4 8 3 0 1
C: 7 6 1 1 1   4 9 3 6 9   2 8 0 7 2   4 8 3 0 1   9 0 0 7 4   4 5 5 4 8
P: 0 7 2 9 2   9 1 2 5 7   5 2 9 6 1   0 9 0 4 2   7 2 0 0 2   0 7 2 4 7

K: 9 0 0 7 4   4 5 5 4 8   4 0 5 4 4   3 1 5 2 9   1 2 9 4 3   5 0 5 6 9
C: 4 0 5 4 4   3 1 5 2 9   1 2 9 4 3   5 0 5 6 9   5 2 8 1 4   7 4 5 5 6
P: 5 0 5 7 0   9 6 0 8 1   7 2 4 0 9   2 9 0 4 0   4 0 9 7 1   2 4 0 9 7

K: 5 2 8 1 4   7 4 5 5 6   7 1 9 3 2   4 0 5 4 6   1 1 6 8 2   0 5 7 3 3
C: 7 1 9 3 2   4 0 5 4 6   1 1 6 8 2   0 5 7 3 3   1 0 6 4 9   2 5 6 3 5
P: 2 9 1 2 8   7 6 0 9 0   4 0 7 5 0   6 5 2 9 7   0 9 0 6 7   2 0 9 0 2

K: 1 0 6 4 9   2 5 6 3 5
C: 1 9 6 8 9   9 9 6 0 1
P: 0 9 0 4 0   7 4 0 7 6
```



From here on the solution of the intermediate text is a simple matter; monome-dinome characteristics are observed in the preliminary examination, and recovery of the plain text and of the enciphering matrix quickly follow.[29]

[29] The analysis of the intermediate text is given in par. 77 of *Military Cryptanalytics, Part I.*

CHAPTER V

PLAINTEXT AUTOKEY SYSTEMS

29. Preliminary remarks on plaintext autokeying.—*a*. If the cipher alphabets are unknown mixed sequences, plaintext autokeying gives rise to cryptograms of more intricate character than does ciphertext autokeying, as has already been intimated in the preceding chapter. As a cryptographic principle, it is very commonly encountered as a new and remarkable "invention" of tyros in the cryptographic art. It apparently gives rise to the type of reasoning to which attention has been directed once before, and which was then shown to be a popular delusion of the uninitiated. The novice to whom the plaintext autokey principle comes as a brilliant flash of the imagination sees only the apparent impossibility of penetrating a secret which enfolds another secret. His reasoning runs about as follows: "In order to read the cryptogram, the would-be solver must, of course, first know the key; but the key does not become known to the would-be solver until he has read the cryptogram and has thus found the plain text. Since this is reasoning around a circle, the system is indecipherable." How unwarranted such reasoning really is in this case, and how readily the problem is solved, will soon be demonstrated.

b. A consideration of the mechanics of the plaintext autokey method discloses that a repetition of n letters in the plain text will produce a repetition of $(n-k)$ letters in the cipher text, where n represents the length of the repetition and k the length of the introductory key. Therefore, when the introductory key consists of a single letter, there will be as many polygraphic repetitions in the cipher text as there are in the plain text, except for repetitions of digraphs only, which of course disappear. But on the other hand some accidental (i.e., noncausal) digraphic repetitions are to be expected, since it can happen that two different plaintext pairs, enciphered by different key letters, will produce identical cipher equivalents. Such accidental repetitions will happen less frequently, of course, in the case of longer polygraphs, so that when repetitions of four or more letters are found in the cipher text they may be taken to be true or causal repetitions. It is obvious that in studying repetitions in a cryptogram of this type, when the introductory key is a single letter, a 5-letter repetition in the cipher text, for example, represents a 6-letter word or sequence repeated in the plain text. When the introductory key is k letters in length then an n-letter repetition represents a repetition in the plain text of length $(n+k)$ letters.

c. The discussion in this chapter will, as usual, be divided into two principal cases: (1) those in which the cipher alphabets are known, and (2) those in which they are unknown. Under each case the introductory key may consist of a single letter, a word, or a short phrase. Furthermore, in the solution of plaintext autokey systems there are important differences whether the components are identical sequences progressing in the same direction, or running against each other in opposite directions,or whether the components are two different sequences. In addition, complications in solution are introduced when the wrong base letter is assumed, as will presently be demonstrated.

30. Solution of plaintext autokey systems when known cipher alphabets are employed and the introductory key consists of a single letter.—*a*. Note the following plaintext autokeyed encipherment, wherein the introductory key is a single letter, of such commonly encountered words as COMMANDING, BATTALION, DIVISION, and CAPTAIN, using two identical primary components (in this case direct standard alphabets), with base letter A:

```
    K: . B A T T A L I O N              K: . D I V I S I O N
(1) P: B A T T A L I O N .          (2) P: D I V I S I O N .
    C: . B T M T L T W B .              C: . L D D A A W B .

    K: . C O M M A N D I N G            K: . C A P T A I N
(3) P: C O M M A N D I N G .        (4) P: C A P T A I N .
    C: . Q A Y M N Q L V T .            C: . C P I T I V .
```

The following characteristics may be noted:[1]

(1) The cipher equivalent of A_p is the plaintext letter which immediately precedes A_p (see the two A's in BATTALION, in example 1 above). When the key is A, a plaintext letter is self-enciphered (see the first T_c and the L_c in example 1).

(2) A plaintext sequence of the pattern ABA yields a doublet as the cipher equivalent of the final two letters (see IVI and ISI in DIVISION, in the second example).

(3) Every plaintext trigraph having A_p as its central letter yields a cipher equivalent the last two letters of which are identical with the initial and final letters of the plaintext trigraph (see MAN in COMMANDING, in the third example).

(4) Every plaintext tetragraph having A_p as the initial and the final letter yields a cipher equivalent the second and fourth letters of which are identical with the second and third letters of the plaintext tetragraph (see APTA in CAPTAIN, in the fourth example; also ATTA in BATTALION, in the first example).

b. (1) From the foregoing characteristics and the fact that a repetition of a sequence of n plaintext letters will yield, in the case of a one-letter introductory key, a repetition of a sequence of $n-1$ cipher letters, it is obvious that an easy method of solving this type of cipher is that of the probable word. Indeed, if the system were used for regular traffic it would not be long before the solution would consist merely in referring to lists of cipher equivalents of commonly used words (as found from previous messages) and searching through the traffic for these sequences, aided by their idiomorphic patterns.

(2) Note how easily the following message can be solved:

```
BECJI BTMTL TWBPQ AYMNQ HVNET WAALC . . .
```

We take note of the sequence BTMTLTWB, which is in the list of equivalents in subpar. *a*, above, and we insert the word BATTALION in the proper position. Thus:

```
K:           B A T T A  L I O N
C: B E C J I B T M T L  T W B P Q . . .
P:         B A T T A L  I O N
```

With this as a start, the decipherment may proceed forward or backward with ease, as shown below:

```
             5          10          15          20          25          30
K:   E A C H  B A T T A  L I O N C  O M M A N  D E R W I  L L P L A
C: B E C J I  B T M T L  T W B P Q  A Y M N Q  H V N E T  W A A L C . . .
P: E A C H B  A T T A L  I O N C O  M M A N D  E R W I L  L P L A C
```

c. The foregoing example is based upon the normal Vigenère method of encipherment ($\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$). If in encipherment the plaintext letter is sought in the second (i.e., lower) component, and its equivalent taken in the first (i.e., upper) component ($\theta_{k/2}=\theta_{1/1}$; $\theta_{p/2}=\theta_{c/1}$), the steps in solution are identical, except that the list of cipher equivalents of probable words must be modified accordingly.

[1] The reader is cautioned that the characteristics noted apply only to the case where two *identical* components are used, with the base letter A.

For instance if the components are direct standard sequences the word BATTALION will now be enciphered as BATTALION.
ZTAHLXGZ

d. If reversed standard alphabets are used, the word BATTALION will be enciphered as BATTALION
BHATPDUB
which also presents idiomorphic characteristics leading to the easy recognition of the word in the cipher texts of messages.

e. All of the foregoing phenomena are based upon standard alphabets, but when mixed cipher alphabets are used and these have been reconstructed, similar observations may be recorded and the results employed in the solution of additional messages enciphered by the same components.

f. (1) Let us again consider the case of known components wherein two identical sequences progress in the same direction; for the sake of illustration, let both of these sequences be normal sequences. Let it also be known that plaintext autokey with a single-letter introductory key is involved, and that the enciphering equations are those of the normal Vigenère method.

(2) A message beginning QVGLB TPJTF . . . is intercepted; the only unknown factor is the initial key letter. Of course, one could try to decipher the message using each key letter in turn, beginning with A and continuing until the correct key letter is tried, whereupon plain text will be obtained. But it seems logical to think that all 26 possible "decipherments" might be derived from the first one, so that the process might be much simplified; this is true, as will now be shown. If we take the two cipher groups under consideration and decipher them with initial key letter A, we obtain the following:

```
K: A Q F B K R C N W X
C: Q V G L B T P J T F . . .
P: Q F B K R C N W X I
```

The deciphered text is certainly not plain text. But if one completes the sequences initiated by these letters, *using the direct standard sequence for the even columns, the reversed standard for the odd columns*, the plain text HOSTILE FOR(CE) will appear in one generatrix. From this it is clear that instead of going through the labor of making 26 successive trials, which would consume considerable time, all that is necessary is to have a set of strips bearing the normal direct sequence and another set bearing the reversed normal sequence, and to align the strips, alternatively direct and reversed, to the first "decipherment." The plain text will now reappear on one generatrix of the completion diagram, as shown in Fig. 35, below:

Initial key ltr.	1	2	3	4	5	6	7	8	9	10
	Q	V	G	L	B	T	P	J	T	F
A	Q	F	B	K	R	C	N	W	X	I
B	P	G	A	L	Q	D	M	X	W	J
C	O	H	Z	M	P	E	L	Y	V	K
D	N	I	Y	N	O	F	K	Z	U	L
E	M	J	X	O	N	G	J	A	T	M
F	L	K	W	P	M	H	I	B	S	N
G	K	L	V	Q	L	I	H	C	R	O
H	J	M	U	R	K	J	G	D	Q	P
I	I	N	T	S	J	K	F	E	P	Q
J	*H	O	S	T	I	L	E	F	O	R
K	G	P	R	U	H	M	D	G	N	S
L	F	Q	Q	V	G	N	C	H	M	T
M	E	R	P	W	F	O	B	I	L	U
N	D	S	O	X	E	P	A	J	K	V
O	C	T	N	Y	D	Q	Z	K	J	W
P	B	U	M	Z	C	R	Y	L	I	X
Q	A	V	L	A	B	S	X	M	H	Y
R	Z	W	K	B	A	T	W	N	G	Z
S	Y	X	J	C	Z	U	V	O	F	A
T	X	Y	I	D	Y	V	U	P	E	B
U	W	Z	H	E	X	W	T	Q	D	C
V	V	A	G	F	W	X	S	R	C	D
W	U	B	F	G	V	Y	R	S	B	E
X	T	C	E	H	U	Z	Q	T	A	F
Y	S	D	D	I	T	A	P	U	Z	G
Z	R	E	C	J	S	B	O	V	Y	H

FIGURE 35

g. The peculiar nature of the phenomenon just observed, *viz.*, a completion diagram with the vertical sequences in adjacent columns progressing in opposite directions, those in alternate columns in the same direction, calls for an explanation. Although the matter might seem a bit mysterious, it is not hard to understand. First, it is clear why the letters in column 1 of Fig. 35 form the descending sequence QPO . . . these letters are merely the ones resulting from the successive "decipherment" of Q_c by the successive key letters A, B, C . . ., as shown below:

Initial key A:	Initial key B:	Initial key C:
K: A Q F B K R C N W X	K: B P G A L Q D M X W	K: C O H Z M P L E Y V
C: Q V G L B T P J T F	C: Q V G L B T P J T F	C: Q V G L B T P J T F
P: Q F B K R C N W X I	P: P G A L Q D M X W J	P: O H Z M P E L Y V K

Now since the decipherment obtained from the 1st cipher letter in any row in Fig. 35 becomes the key letter for deciphering the 2d cipher letter in the same row, it is apparent that as the letters in the 1st column progress in a reversed normal (i.e., descending) order, the letters in the 2d column *must* progress in a direct normal (i.e., ascending) order. The matter may perhaps become more clear if encipherment is regarded as a process of addition, and decipherment as a process of subtraction. Instead of primary components or a Vigenère square, we may use mod-26 arithmetic, assigning numerical values to the letters of the alphabet, beginning with A=Ø and ending with Z=25. For example, if we consider the usual Vigenère enciphering equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$, the letter H_p enciphered by key letter M_k with direct standard alphabets yields T_c; or, by using the following numerical values,

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

the same result may be obtained thus: $H_p(M_k)=7+12=19=T_c$. (Every time the number 25 is exceeded in addition, we subtract 26 from it and find the equivalent for the remainder.) In decipherment, the process is one of subtraction.[2] For example, $T_c(M_k)=19-12=7=H_p$; $D_c(R_k)=3-17=(26+3)-17=29-17=12=M_p$. Using this arithmetical equivalent of normal sliding-strip encipherment, the phenomenon just noted can be set down in the form of a diagram (Fig. 36) which perhaps will make the matter clear.

```
                                     1 2 3 4 5 6 7 8 9 10
                                     Q V G L B T P J T F
Qc (Ak) =16- 0=16=Q               →  Q F B K R . . .
Vc (Qk) =21-16= 5=F
Gc (Fk) = 6- 5= 1=B
Lc (Bk) =11- 1=10=K
Bc (Kk) = 1-10=17=R
        * * * * * * * * * * * * * * * * * * * * * *
Qc (Bk) =16- 1=15=P               →  P G A L Q . . .
Vc (Pk) =21-15= 6=G
Gc (Gk) = 6- 6= 0=A
Lc (Ak) =11- 0=11=L
Bc (Lk) = 1-11=16=Q
        * * * * * * * * * * * * * * * * * * * * * *
Qc (Ck) =16- 2=14=O               →  O H Z M P . . .
Vc (Ok) =21-14= 7=H
Gc (Hk) = 6- 7=25=Z
Lc (Zk) =11-25=12=M
Bc (Mk) = 1-12=15=P
```

FIGURE 36

Note how homologous letters of the three rows (joined by vertical dotted lines) form alternately descending and ascending normal sequences.

h. But now let us consider what happens when the base letter is incorrectly assumed. Note the following direct standard alphabet encipherments:

```
   Base letter B:             Base letter C:             Base letter D:
K: J H O S T I L E F O     K: J H O S T I L E F O     K: J H O S T I L E F O
P: H O S T I L E F O R     P: H O S T I L E F O R     P: H O S T I L E F O R
C: P U F K A S O I S E     C: O T E J Z R N H R D     C: N S D I Y Q M G Q C
```

FIGURE 36*a* FIGURE 36*b* FIGURE 36*c*

---

[2] It will be noted that if the letters of the alphabet are numbered from 1 to 26, in the usual manner, the arithmetical method must be modified in a minor particular in order to obtain the same results as are given by employing the normal Vigenère square. This modification consists merely in subtracting 1 from the numerical value of the key letter. Thus:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

$$Hp(Mk) = 8 + (13 - 1) = 8 + 12 = 20 = Tc$$
$$Tc(Mk) = 20 - (13 - 1) = 20 - 12 = 8 = Hp$$

When we make our initial "decipherments" of the foregoing cipher texts, assuming erroneously that the base letter is A_p, we have the following:

K:	A	P	F	A	K	Q	C	M	W	W
C:	P	U	F	K	A	S	O	I	S	E
P:	P	F	A	K	Q	C	M	W	W	I

FIGURE 37*a*

K:	A	O	F	Z	K	P	C	L	W	V
C:	O	T	E	J	Z	R	N	H	R	D
P:	O	F	Z	K	P	C	L	W	V	I

FIGURE 37*b*

K:	A	N	F	Y	K	O	C	K	W	U
C:	N	S	D	I	Y	Q	M	G	Q	C
P:	N	F	Y	K	O	C	K	W	U	I

FIGURE 37*c*

Note that the even letters FKCWI of these three decipherments match. When we make generatrix diagrams from these decipherments, it may be seen in Figs. 38*a–c*, below, that the true plain text appears on *two* generatrices, and that the interval between these generatrices reflects the identity of the particular base letter involved. This phenomenon holds true only for those cases in which the plain and cipher components are identical sequences running in the same direction.

P	U	F	K	A	S	O	I	S	E
P	F	A	K	Q	C	M	W	W	I
Q	E	B	J	R	B	N	V	X	H
R	D	C	I	S	A	O	U	Y	G
S	C	D	H	T	Z	P	T	Z	F
T	B	E	G	U	Y	Q	S	A	E
U	A	F	F	V	X	R	R	B	D
V	Z	G	E	W	W	S	Q	C	C
W	Y	H	D	X	V	T	P	D	B
X	X	I	C	Y	U	U	O	E	A
Y	W	J	B	Z	T	V	N	F	Z
Z	V	K	A	A	S	W	M	G	Y
A	U	L	Z	B	R	X	L	H	X
B	T	M	Y	C	Q	Y	K	I	W
C	S	N	X	D	P	Z	J	J	V
D	R	O	W	E	O	A	I	K	U
E	Q	P	V	F	N	B	H	L	T
F	P	Q	U	G	M	C	G	M	S
G	Ⓞ	R	Ⓣ	H	Ⓛ	D	Ⓕ	N	Ⓡ
Ⓗ	N	Ⓢ	S	Ⓘ	K	Ⓔ	E	Ⓞ	Q
I	M	T	R	J	J	F	D	P	P
J	L	U	Q	K	I	G	C	Q	O
K	K	V	P	L	H	H	B	R	N
L	J	W	O	M	G	I	A	S	M
M	I	X	N	N	F	J	Z	T	L
N	H	Y	M	O	E	K	Y	U	K
O	G	Z	L	P	D	L	X	V	J

FIGURE 38*a*

O	T	E	J	Z	R	N	H	R	D
O	F	Z	K	P	C	L	W	V	I
P	E	A	J	Q	B	M	V	W	H
Q	D	B	I	R	A	N	U	X	G
R	C	C	H	S	Z	O	T	Y	F
S	B	D	G	T	Y	P	S	Z	E
T	A	E	F	U	X	Q	R	A	D
U	Z	F	E	V	W	R	Q	B	C
V	Y	G	D	W	V	S	P	C	B
W	X	H	C	X	U	T	O	D	A
X	W	I	B	Y	T	U	N	E	Z
Y	V	J	A	Z	S	V	M	F	Y
Z	U	K	Z	A	R	W	L	G	X
A	T	L	Y	B	Q	X	K	H	W
B	S	M	X	C	P	Y	J	I	V
C	R	N	W	D	O	Z	I	J	U
D	Q	O	V	E	N	A	H	K	T
E	P	P	U	F	M	B	G	L	S
F	Ⓞ	Q	Ⓣ	G	Ⓛ	C	Ⓕ	M	Ⓡ
G	N	R	S	H	K	D	E	N	Q
Ⓗ	M	Ⓢ	R	Ⓘ	J	Ⓔ	D	Ⓞ	P
I	L	T	Q	J	I	F	C	P	O
J	K	U	P	K	H	G	B	Q	N
K	J	V	O	L	G	H	A	R	M
L	I	W	N	M	F	I	Z	S	L
M	H	X	M	N	E	J	Y	T	K
N	G	Y	L	O	D	K	X	U	J

FIGURE 38*b*

N	S	D	I	Y	Q	M	G	Q	C
N	F	Y	K	O	C	K	W	U	I
O	E	Z	J	P	B	L	V	V	H
P	D	A	I	Q	A	M	U	W	G
Q	C	B	H	R	Z	N	T	X	F
R	B	C	G	S	Y	O	S	Y	E
S	A	D	F	T	X	P	R	Z	D
T	Z	E	E	U	W	Q	Q	A	C
U	Y	F	D	V	V	R	P	B	B
V	X	G	C	W	U	S	O	C	A
W	W	H	B	X	T	T	N	D	Z
X	V	I	A	Y	S	U	M	E	Y
Y	U	J	Z	Z	R	V	L	F	X
Z	T	K	Y	A	Q	W	K	G	W
A	S	L	X	B	P	X	J	H	V
B	R	M	W	C	O	Y	I	I	U
C	Q	N	V	D	N	Z	H	J	T
D	P	O	U	E	M	A	G	K	S
E	Ⓞ	P	Ⓣ	F	Ⓛ	B	Ⓕ	L	Ⓡ
F	N	Q	S	G	K	C	E	M	Q
G	M	R	R	H	J	D	D	N	P
Ⓗ	L	Ⓢ	Q	Ⓘ	I	Ⓔ	C	Ⓞ	O
I	K	T	P	J	H	F	B	P	N
J	J	U	O	K	G	G	A	Q	M
K	I	V	N	L	F	H	Z	R	L
L	H	W	M	M	E	I	Y	S	K
M	G	X	L	N	D	J	X	T	J

FIGURE 38*c*

i. When the method of encipherment based upon enciphering equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/2}=\theta_{c/1}$ is used instead of that based upon the usual Vigenère equations, the process indicated above is simplified by the fact that no alteration in the direction of the sequences in the completion diagram is required. For example, if the first two groups of a cipher message were YHEBP DTBJD the "decipherments" with key letters A and B, respectively, are shown below:

K:	A	Y	F	J	K	Z	C	V	W	F	
C:	Y	H	E	B	P	D	T	B	J	D	. . .
P:	Y	F	J	K	Z	C	V	W	F	I	

K:	B	Z	G	K	L	A	D	W	X	G	
C:	Y	H	E	B	P	D	T	B	J	D	. . .
P:	Z	G	K	L	A	D	W	X	G	J	

The entire completion diagram would therefore be as follows:

Initial	1	2	3	4	5	6	7	8	9	10
key ltr.	Y	H	E	B	P	D	T	B	J	D
A	Y	F	J	K	Z	C	V	W	F	I
B	Z	G	K	L	A	D	W	X	G	J
C	A	H	L	M	B	E	X	Y	H	K
D	B	I	M	N	C	F	Y	Z	I	L
E	C	J	N	O	D	G	Z	A	J	M
F	D	K	O	P	E	H	A	B	K	N
G	E	L	P	Q	F	I	B	C	L	O
H	F	M	Q	R	G	J	C	D	M	P
I	G	N	R	S	H	K	D	E	N	Q
J	*H	O	S	T	I	L	E	F	O	R
K	I	P	T	U	J	M	F	G	P	S
L	J	Q	U	V	K	N	G	H	Q	T
M	K	R	V	W	L	O	H	I	R	U
N	L	S	W	X	M	P	I	J	S	V
O	M	T	X	Y	N	Q	J	K	T	W
P	N	U	Y	Z	O	R	K	L	U	X
Q	O	V	Z	A	P	S	L	M	V	Y
R	P	W	A	B	Q	T	M	N	W	Z
S	Q	X	B	C	R	U	N	O	X	A
T	R	Y	C	D	S	V	O	P	Y	B
U	S	Z	D	E	T	W	P	Q	Z	C
V	T	A	E	F	U	X	Q	R	A	D
W	U	B	F	G	V	Y	R	S	B	E
X	V	C	G	H	W	Z	S	T	C	F
Y	W	D	H	I	X	A	T	U	D	G
Z	X	E	I	J	Y	B	U	V	E	H

FIGURE 39

j. (1) In the foregoing examples the primary components were normal sequences, but the case of identical mixed components may be handled in a similar manner. Note the following example, based upon

H Y D R A U L I C B E F G J K M N O P Q S T V W X Z

the primary component which we have reconstructed from previous work. Let the first two groups of an intercepted message be XOFMD JNTDS . . .

(2) First, the message is "deciphered" with the initial key letter **A** and base letter H_p, and then a completion diagram is made, using sliding strips bearing the mixed primary component, alternate strips bearing the same sequence in reverse. Note Fig. 40, in which the plain text HOSTILE FOR(CE) reappears on a single generatrix.

```
K: A S W K Y Y G A O F
C: X O F M D J N T D S
P: S W K Y Y G A O F B
   T V M H D F U N G C
   V T N Z R E L M J I
   W S O X A B I K K L
   X Q P W U C C J M U
   Z P Q V L I B G N A
  *H O S T I L E F O R
   Y N T S C U F E P D
   D M V Q B A G B Q Y
   R K W P E R J C S H
   A J X O F D K I T Z
   U G Z N G Y M L V X
   L F H M J H N U W W
   I E Y K K Z O A X V
   C B D J M X P R Z T
   B C R G N W Q D H S
   E I A F O V S Y Y Q
   F L U E P T T H D P
   G U L B Q S V Z R O
   J A I C S Q W X A N
   K R C I T P X W U M
   M D B L V O Z V L K
   N Y E U W N H T I J
   O H F A X M Y S C G
   P Z G R Z K D Q B F
   Q X J D H J R P E E
```

FIGURE 40

```
K: A D K O P E H A B K
C: X T W Z L X H Z R X
P: D K O P E H A B K N
   E L P Q F I B C L O
   F M Q R G J C D M P
   G N R S H K D E N Q
  *H O S T I L E F O R
   I P T U J M F G P S
   J Q U V K N G H Q T
   K R V W L O H I R U
   L S W X M P I J S V
   M T X Y N Q J K T W
   N U Y Z O R K L U X
   O V Z A P S L M V Y
   P W A B Q T M N W Z
   Q X B C R U N O X A
   R Y C D S V O P Y B
   S Z D E T W P Q Z C
   T A E F U X Q R A D
   U B F G V Y R S B E
   V C G H W Z S T C F
   W D H I X A T U D G
   X E I J Y B U V E H
   Y F J K Z C V W F I
   Z G K L A D W X G J
   A H L M B E X Y H K
   B I M N C F Y Z I L
   C J N O D G Z A J M
```

FIGURE 41

```
K: A L W H Y J G N O W
C: X B W Z K Y V Z S K
P: L W H Y J G N O W B
   I X Y D K J O P X E
   C Z D R M K P Q Z F
   B H R A N M Q S H G
   E Y A U O N S T Y J
   F D U L P O T V D K
   G R L I Q P V W R M
   J A I C S Q W X A N
   K U C B T S X Z U O
   M L B E V T Z H L P
   N I E F W V H Y I Q
   O C F G X W Y D C S
   P B G J Z X D R B T
   Q E J K H Z R A E V
   S F K M Y H A U F W
   T G M N D Y U L G X
   V J N O R D L I J Z
   W K O P A R I C K H
   X M P Q U A C B M Y
   Z N Q S L U B E N D
  *H O S T I L E F O R
   Y P T V C I F G P A
   D Q V W B C G J Q U
   R S W X E B J K S L
   A T X Z F E K M T I
   U V Z H G F M N V C
```

FIGURE 42

k. (1) Next to be considered is the case in which the two primary components progress in opposite directions. Let us assume that XTWZL XHZRX . . . are the first two groups of a message known to have been enciphered by plaintext autokeying with a single letter introductory key and reversed standard alphabets. The procedure in this case is exactly the same as before, except that it is not necessary to have any alternation in direction of the completion sequences; note the solution in Fig. 41. Let the student ascertain why the alternation of the completion sequences is not necessary in this case.

(2) In the foregoing case the alphabets were reversed standard, produced by the sliding of the normal sequence against its reverse. But the underlying principle of solution is the same even if a mixed sequence were used instead of the normal; so long as the sequence is known, the procedure to be followed is exactly the same as demonstrated in subpar. (1), above. For example, let the reconstructed primary components be

```
P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
C: Z X W V T S Q P O N M K J G F E B C I L U A R D Y H
```

and let the first two groups of an intercepted message be XBWZK YVZSK . . . . Referring to Fig. 42, we may note that the primary mixed sequence is used for the completion sequence and that the plain text, HOSTILE FOR(CE), comes out on one generatrix. It is immaterial whether the direct or reversed mixed component is used for the completion sequence, so long as *all* the sequences in the diagram progress in the same direction.

l. When the enciphering alphabets are identical sequences but running against each other in opposite directions, an incorrectly assumed base letter has an effect on the same manner of reading the generatrix diagrams. In Fig. 41, above, we assumed correctly that the base letter was A_p, and the plain text reappeared on a single generatrix. If, however, we had assumed the base letter to be B_p, our generatrix diagram would have been that of Fig. 43*a*, below; and if we had assumed a base letter of C_p, the generatrix diagram would have been that of Fig. 43*b*. Note that the plain text in Fig. 43*a* progresses upward on a simple diagonal, while that in Fig. 43*b* progresses upward on a steeper diagonal, two rows apart. The diagonal for an assumed base letter of D_p would have been three rows apart, and that for an assumed base letter of E_p would have been four rows apart, and so on: this shows that the angle of reading the diagonal, then, depends upon the distance between the true base letter from the assumed base letter, as measured on the component.

```
K: A E M R T J N H J T
C: X T W Z L X H Z R X
P: E M R T J N H J T X
   F N S U K O I K U Y
   G O T V L P J L V Z
   H P U W M Q K M W A
   I Q V X N R L N X B
   J R W Y O S M O Y C
   K S X Z P T N P Z D
   L T Y A Q U O Q A E
   M U Z B R V P R B F
   N V A C S W Q S C G
   O W B D T X R T D H
   P X C E U Y S U E I
   Q Y D F V Z T V F J
   R Z E G W A U W G K
   S A F H X B V X H L
   T B G I Y C W Y I M
   U C H J Z D X Z J N
   V D I K A E Y A K O
   W E J L B F Z B L P
   X F K M C G A C M Q
   Y G L N D H B D N R
   Z H M O E I C E O S
   A I N P F J D F P T
   B J O Q G K E G Q U
   C K P R H L F H R V
   D L Q S I M G I S W
```

FIGURE 43*a*

```
K: A F O U X O T O R C
C: X T W Z L X H Z R X
P: F O U X O T O R C H
   G P V Y P U P S D I
   H Q W Z Q V Q T E J
   I R X A R W R U F K
   J S Y B S X S V G L
   K T Z C T Y T W H M
   L U A D U Z U X I N
   M V B E V A V Y J O
   N W C F W B W Z K P
   O X D G X C X A L Q
   P Y E H Y D Y B M R
   Q Z F I Z E Z C N S
   R A G J A F A D O T
   S B H K B G B E P U
   T C I L C H C F Q V
   U D J M D I D G R W
   V E K N E J E H S X
   W F L O F K F I T Y
   X G M P G L G J U Z
   Y H N Q H M H K V A
   Z I O R I N I L W B
   A J P S J O J M X C
   B K Q T K P K N Y D
   C L R U L Q L O Z E
   D M S V M R M P A F
   E N T W N S N Q B G
```

FIGURE 43*b*

m. (1) There remains now to be considered only the case in which the two components are different mixed sequences. Let the two components be

```
P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
C: Q U E S T I O N A B L Y C D F G H J K M P R V W X Z
```

and let us examine the cipher text resulting from the following encipherment:

```
K: X H O S T I L E F O
P: H O S T I L E F O R
C: X N Q X Y Y P D I B
```

(2) First "decipher" the message in plaintext autokey fashion with any arbitrarily selected initial key letter, say A, assuming the base letter to be H_p, the initial letter of the plain component. Now in the first column, under the first decipherment, complete any arbitrary 26-letter sequence (in this example, the normal sequence), as shown in Fig. 44*a*, below.

K:	A	N	H	E	V	M	P	H	W	C
C:	X	N	Q	X	Y	Y	P	D	I	B
P:	N	H	E	V	M	P	H	W	C	W
	O									
	P									
	Q									
	R									
	S									
	T									
	U									
	V									
	W									
	X									
	Y									
	Z									
	A									
	B									
	C									
	D									
	E									
	F									
	G									
	H									
	I									
	J									
	K									
	L									
	M									

FIGURE 44*a*

K:	A	N	H	E	V	M	P	H	W	C
C:	X	N	Q	X	Y	Y	P	D	I	B
P:	N	H	E	V	M	P	H	W	C	W
	O	Y								
	P	J								
	Q	I								
	R	G								
	S	A								
	T	R								
	U	L								
	V	F								
	W	E								
	X	B								
	Y	V								
	Z	C								
	A	Z								
	B	X								
	C	T								
	D	S								
	E	U								
	F	Q								
	G	P								
	H	O								
	I	D								
	J	N								
	K	M								
	L	W								
	M	K								

FIGURE 44*b*

K:	A	N	H	E	V	M	P	H	W	C
C:	X	N	Q	X	Y	Y	P	D	I	B
P:	N	H	E	V	M	P	H	W	C	W
	O	Y	M	U						
	P	J	B	M						
	Q	I	T	S						
	R	G	F	E						
	S	A	P	A						
	T	R	U	W						
	U	L	N	O						
	V	F	G	B						
	W	E	X	H						
	X	B	O	P						
	Y	V	A	N						
	Z	C	K	L						
	A	Z	Y	J						
	B	X	D	F						
	C	T	V	D						
	D	S	W	Y						
	E	U	Z	Z						
	F	Q	H	C						
	G	P	L	K						
	*H	O	S	T						
	I	D	J	I						
	J	N	Q	X						
	K	M	I	Q						
	L	W	R	R						
	M	K	C	G						

FIGURE 44*c*

Then prepare a strip bearing the cipher component *reversed*, and set it below the plain component so that $H_p = N_c$, a setting given by the first two letters of NHEVMPHWCW, the spurious plain text obtained by the arbitrary decipherment with the key letter A. Thus:

P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
C: N O I T S E U Q Z X W V R P M K J H G F D C Y L B A

(3) Now opposite each letter of the completion sequence in column 1, write in column 2 its plain-component equivalent, as given by the juxtaposed sequences above. This gives what is shown in Fig. 44*b*. Then reset the two sequences so that $E_p = H_c$ (to correspond with the 2d and 3d letters of the spurious

P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
C: Q Z X W V R P M K J H G F D C Y L B A N O I T S E U

plain text); write down the plain-component equivalents of the letters in column 2, forming column 3. Continue this process, scanning the generatrices from time to time, resetting the two components and finding equivalents from column to column, until it becomes evident on what generatrix the plain text is reappearing. In Fig. 44*c* it is seen that the plaintext generatrix is the one beginning HOST; from this point on, the solution may be obtained directly, by using the two primary components.

n. When two different components are involved, if the base letter is incorrectly assumed there is nothing much that can be done about it except tedious trial and error. There are no shortcut procedures such as those exemplified in subpars. *h* or *l*: successive base letters with their corresponding generatrix diagrams must be tried in turn, until a solution is forthcoming.

o. Another "mechanical" solution for the foregoing cases will now be described because it presents rather interesting cryptanalytic sidelights. Let us take the message "REFERENCE HIS PREFERENCE IN REFERENCE BOOKS AND REFERENCE CHARTS . . ." and encipher it by plaintext autokey, with direct standard alphabets, A_p as the base letter, and the single-letter initial key $A_p=G_c$. Then note the underscored repetitions:

```
           5         10        15        20        25        30
K: G R E F E R E N C E H I S P R E F E R E N C E I N R E F E R E N C
P: R E F E R E N C E H I S P R E F E R E N C E I N R E F E R E N C E
C: X V J J V V R P G L P A H G V J J V V R P G M V E V J J V V R P G

     35        40        45        50        55
K: E B O O K S A N D R E F E R E N C E C H A R T
P: B O O K S A N D R E F E R E N C E C H A R T S . . .
C: F P C Y C S N Q U V J J V V R P G G J H R K L
```

Now suppose the message has been intercepted and is to be solved, assuming that the only unknown factor is the initial key letter. Let the message be "deciphered" by means of any initial key letter,[3] say A_k, and then note the underscored repetitions in the spurious plain text:

```
           5         10        15        20        25        30
K: A X Y L Y X Y T W K B O M V L K Z K L K H I Y O H X Y L Y X Y T W
C: X V J J V V R P G L P A H G V J J V V R P G M V E V J J V V R P G
P: X Y L Y X Y T W K B O M V L K Z K L K H I Y O H X Y L Y X Y T W K

     35        40        45        50        55
K: K V U I Q M G H J L K Z K L K H I Y I B G L Z
C: F P C Y C S N Q U V J J V V R P G G J H R K L
P: V U I Q M G H J L K Z K L K H I Y I B G L Z M
```

The original four 8-letter repetitions now turn out to be two different sets of 9-letter repetitions. This calls for an explanation. Let the spurious plain text, together with its real plain text, be transcribed as though we were dealing with a periodic cipher involving two alphabets, as shown in Fig. 45, below:

1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2
XY	LY	XY	TW	KB	OM	VL	KZ	KL	KH	IY	OH	XY	LY
RE	FE	RE	NC	EH	IS	PR	EF	ER	EN	CE	IN	RE	FE
XY	TW	KV	UI	QM	GH	JL	KZ	KL	KH	IY	IB	GL	ZM . . .
RE	NC	EB	OO	KS	AN	DR	EF	ER	EN	CE	CH	AR	TS

FIGURE 45

It will here be seen that the letters in column 1 have been monoalphabetically enciphered, as have been those in column 2. In other words, an autokey cipher, which is an aperiodic polyalphabetic substitution, has been converted into a 2-alphabet periodic polyalphabetic substitution. The two repetitions of XYLYXYTWK represent encipherments of the word REFERENCE, in alphabets 1–2–1–2–1–2–1–2–1; the two repetitions of LKZKLKHIY likewise represent encipherments of the same word but in alphabets 2–1–2–1–2–1–2–1–2. Later on it will be seen how this method of converting an autokey cipher into a periodic cipher may be applied to the case where an introductory key word is used as the initial keying element instead of a single letter, as in the present case.

[3] Except the actual key letter or a letter 13 intervals from it. If the actual key letter is used, we will of course obtain plain text; if we use the letter 13 intervals away from the actual key letter, the cipher text will be converted to monoalphabetic terms. (See subpar. *r*, below.)

p. The student has probably already noted that the phenomena observed in the preceding subparagraph are the same as those observed in subpar. *g*, in which it was seen that the direction of the sequences in alternate columns had to be reversed in order to bring out the plain text on one generatrix. If this reversal is not done, then obviously the plain text would appear on *two* generatrices, which is equivalent to having the plain text reduced to two monoalphabets.

q. When reciprocal components are employed, the spurious plain text obtained by "decipherment" with a key setting other than the actual one will be monoalphabetic throughout. Note the following encipherment (with initial key $A_p=G_c$, using reversed standard alphabets) and its "decipherment" by setting the two components at $A_p=A_c$.

```
 P  : R E F E R E N C E H I S P R E F E R E N C E . . .
 C  : P N Z B N N R L Y X Z Q D Y N Z B N N R L Y . . .
"P" : L Y Z Y L Y H W Y B C M J L Y Z Y L Y H W Y . . .
```

Here the spurious plain text is wholly monoalphabetic.

r. The reason for the exception noted in footnote 3 in subpar. *o* now becomes clear. For if the actual initial key letter (G) were used, of course the decipherment yields the correct plain text; if a letter 13 intervals removed from G is used as the key letter, the cipher alphabet selected for the first "decipherment" is the reciprocal of the actual initial cipher alphabet and thereafter all alternate cipher alphabets are reciprocal. Hence the spurious plain text obtained from such a "decipherment" must be monoalphabetic.

s. In the example illustrated in subpar. *o*, the primary components were identical normal sequences progressing in the same direction. If they were mixed sequences, the phenomena observed above would still hold true, and, so long as the sequences are known, the indicated method of solution may be applied. When the two primary components are known but differently mixed sequences, this method of solution is too involved to be practical. It is more expedient to try successive initial key letters, noting the plain text each time and resetting the strips until the correct setting has been ascertained, as will be evidenced by obtaining intelligible plain text.

31. Solution of plaintext autokey systems involving known cipher alphabets when the introductory key consists of several letters.—*a*. In the foregoing discussion of plaintext autokeying, the introductory key was assumed to consist of a single letter, so that the subsequent key letters are displaced one letter to the right with respect to the text of the message itself. But sometimes a word or phrase may serve this function, in which case the subsequent key is displaced as many letters to the right of the initial plaintext letter of the message as there are letters in the introductory key. This will not, as a rule, interfere in any way with the application of the general principles of solution set forth to that part of the cryptogram subsequent to the introductory key, and a solution by the probable-word method and the study of repetitions can be reached. However, it may happen that trial of this method is not successful in certain cryptograms because of the paucity of repetitions, or because of failure to find a probable word in the text. When the cipher alphabets are known, there is another point of attack which is useful and interesting. The method consists of finding the length of the introductory key and then solving by frequency principles; this method will be described in the subparagraphs below.

b. Suppose that the introductory key word is the 10-letter word PARLIAMENT, that the plaintext message is as below, and that identical primary components progressing in the same direction are used to encipher the message, with the normal Vigenère enciphering convention. Let the components be the normal sequence. The encipherment, shown here written out on a width of 20 (i.e., twice the length of the introductory key) is as follows:

```
    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
K:  P A R L I A M E N T  R  E  C  E  I  V  I  N  G  H
P:  R E C E I V I N G H  E  A  V  Y  A  R  T  I  L  L
C:  G E T P Q V U R T A  V  E  X  C  I  M  B  V  R  S

K:  E A V Y A R T I L L  E  R  Y  F  I  R  E  O  N  M
P:  E R Y F I R E O N M  Y  L  E  F  T  F  L  A  N  K
C:  I R T D I I X W Y X  C  C  C  K  B  W  P  O  A  W

K:  Y L E F T F L A N K  S  T  O  P  E  N  E  M  Y  I
P:  S T O P E N E M Y I  S  M  A  S  S  I  N  G  H  I
C:  Q E S U X S P M L S  K  F  O  H  W  V  R  S  F  Q

K:  S M A S S I N G H I  S  T  R  O  O  P  S  T  O  L
P:  S T R O O P S T O L  E  F  T  F  R  O  N  T  A  N
C:  K F R G G X F Z V T  W  Y  K  T  F  D  F  M  O  Y

K:  E F T F R O N T A N  D  C  O  N  C  E  N  T  R  A
P:  D C O N C E N T R A  T  I  N  G  A  R  T  I  L  L
C:  H H H S T S A M R N  W  K  B  T  C  V  G  B  C  L

K:  T I N G A R T I L L  E  R  Y  T  H  E  R  E  S  T
P:  E R Y T H E R E S T  O  P  R  E  I  N  F  O  R  C
C:  X Z L Z H V K M D E  S  G  P  X  P  R  W  S  J  V

K:  O P R E I N F O R C  E  M  E  N  T  S  I  M  P  E
P:  E M E N T S I M P E  R  A  T  I  V  E  T  O  H  O
C:  S B V R B F N A G G  V  M  X  V  O  W  B  A  W  S

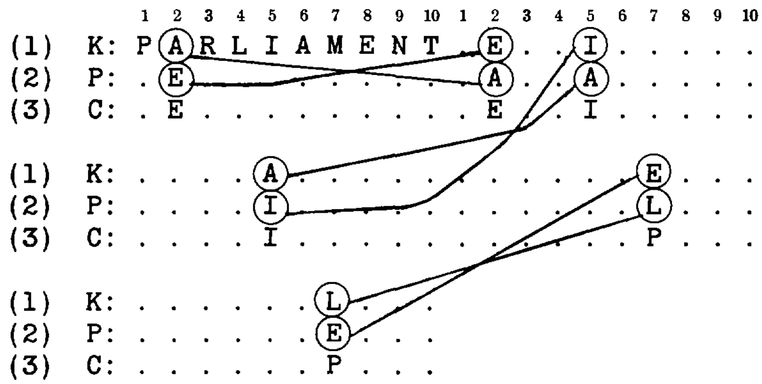
K:  R A T I V E T O H O
P:  L D P O S I T I O N
C:  C D I W N M M W V B
```

It will now be noted that, since the introductory key consists of 10 letters, the 11th letter of the message is enciphered by the 1st letter of the plain text, the 12th by the 2d, and so on. Likewise, the 21st letter is enciphered by the 11th, the 22d by the 12th, and so on. An important step in the solution of a message of this kind would therefore involve ascertaining the length of the introductory key. This step will now be explained.

c. Since the plain text itself constitutes the key letters in this system (after the introductory key), these key letters will occur with their normal frequencies, and this means that there will be many occurrences of E, T, N, R, O, A, I, S, enciphered by E_k; there will be many occurrences of these same high-frequency letters enciphered by T_k, by N_k, by R_k, and so on. In fact, the number of times each of these combinations will occur may be calculated statistically. With the enciphering conditions set forth under subpar. *b*, above, if E_p is enciphered by T_k, for example, it will yield the same cipher equivalent as T_p enciphered by E_k; in other words, two encipherments of any pair of letters of which either may serve as the key for enciphering the other must yield the same cipher resultant.[4] It is the cryptographic effect

[4] It is important to note that this observation applies only to cases in which the two components are *identical sequences progressing in the same direction*; if this is not the case, the entire reasoning is inapplicable.

of these two phenomena working together which permits ascertaining the length of the introductory key in such a case. For every time a given letter, θ_p, occurs in the plain text, it will occur n letters later as a key letter, θ_k, where n equals the length of the introductory key. Note the following illustration:



It may be seen that the key-plain combination $A_k(E_p)=E_c$, yields the same cipher resultant 10 positions later as the combination $E_k(A_p)=E_c$; likewise, $I_k(A_p)=I_c$, and 10 positions away, $A_k(I_p)=I_c$; also, $E_k(L_p)=P_c$, and 10 positions later we have the same cipher resultant from $L_k(E_p)=P_c$. Two identical cipher letters at an interval equal to the length of the introductory key is a function of the key-plain reversibility just demonstrated. But not every pair of identical letters in the cipher text represents a case of this type. For in this system, identity in two cipher letters may be the result of the following three conditions each having a statistically ascertainable probability of occurrence:

(1) A given plaintext letter is enciphered by the same key letter two different times, at an interval which is purely accidental; the cipher equivalents are identical, but could not be used to give any information about the length of the introductory key.

(2) Two different plaintext letters are enciphered by two different key letters; the cipher equivalents are fortuitously identical.

(3) A given plaintext letter is enciphered by a given key letter, and later on the same plaintext letter serves to encipher another plaintext letter which is identical with the first key letter; the cipher equivalents are *causally* identical.

d. It can be proved that the probability for identities of the third type, *for that interval which corresponds with the length of the introductory key,* is greater than that for identities of either or both 1st and 2d types; that is, if a tabulation is made of the intervals between identical letters in such a system as the one being studied, the interval which occurs most frequently should coincide with the length of the introductory key. In a random case, i.e., cases (1) and (2), above, the probability of a ciphertext identity is for all intents and purposes 1/26 or .0385, whereas in the causal case (case 3, above) the probability is the kappa plain of the language, which for English is .0667 or 1/15. As a practical demonstration of this point, let us transcribe the cipher text of the message in subpar. *b* on trial widths; those for widths of 7, 8, 9, and 10 are shown in Fig. 46, below:

```
1 2 3 4 5 6 7
G E T P Q V U
R T A V E X C
I M B V R S I
R T D I I X W
Y X C C C K B
W P O A W Q E
S U X S P M L
S K F O H W V
R S F Q K F R
G G X F Z V T
W Y K T F D F
M O Y H H H S
T S A M R N W
K B T C V G B
C L X Z L Z H
V K M D E S G
P X P R W S J
V S B V R B F
N A G G V M X
V O W B A W S
C D I W N M M
W V B
```

```
1 2 3 4 5 6 7 8
G E T P Q V U R
T A V E X C I M
B V R S I R T D
I I X W Y X C C
C K B W P O A W
Q E S U X S P M
L S K F O H W V
R S F Q K F R G
G X F Z V T W Y
K T F D F M O Y
H H H S T S A M
R N W K B T C V
G B C L X Z L Z
H V K M D E S G
P X P R W S J V
S B V R B F N A
G G V M X V O W
B A W S C D I W
N M M W V B
```

```
1 2 3 4 5 6 7 8 9
G E T P Q V U R T
A V E X C I M B V
R S I R T D I I X
W Y X C C C K B W
P O A W Q E S U X
S P M L S K F O H
W V R S F Q K F R
G G X F Z V T W Y
K T F D F M O Y H
H H S T S A M R N
W K B T C V G B C
L X Z L Z H V K M
D E S G P X P R W
S J V S B V R B F
N A G G V M X V O
W B A W S C D I W
N M M W V B
```

```
1 2 3 4 5 6 7 8 9 10
G E T P Q V U R T A
V E X C I M B V R S
I R T D I I X W Y X
C C C K B W P O A W
Q E S U X S P M L S
K F O H W V R S F Q
K F R G G X F Z V T
W Y K T F D F M O Y
H H H S T S A M R N
W K B T C V G B C L
X Z L Z H V K M D E
S G P X P R W S J V
S B V R B F N A G G
V M X V O W B A W S
C D I W N M M W V B
```

FIGURE 46

In each transcription, every pair of superimposed letters is noted and the number of identities is indicated by circling the letters involved, as shown above. In the diagram below, *w* is the trial width, C the number of comparisons, *r* the random expectation (obtained by dividing C by 26), *p* the expectation if the width is correct (obtained by dividing C by 15), and *i* is the observed number of identities. It is obvious that the width of 10 is probably the correct width.

w	C	*r*	*p*	*i*
7	143	5.5	9.5	4
8	142	5.5	9.5	8
9	141	5.4	9.4	2
10	140	5.4	9.3	10

e. Once we have found the length of the introductory key, two lines of attack are possible if the enciphering alphabets are known: (1) completion of the plain-component sequence on the columns of the correct write-out, or (2) conversion of the aperiodic cipher text to periodic terms and its solution as a repeating-key cipher. The first line of attack will be discussed first, using the cipher message of subpar. *b* as an example.

f. The cipher letters of column 1 and column 2 (of the write-out on a width of 10) are written in a row, and the key of A is arbitrarily chosen to start an autokey decipherment of the letters. Then, from these decipherments, the plain-component sequence is completed on the assumption of direct standard alphabets, running alternate columns in the reverse direction, as shown in Figs. 47*a* and *b*, below:[5]

	Column 1
K:	A G P T J H D H P S E T Z T C
C:	G V I C Q K K W H W X S S V C
P:	G P T J H D H P S E T Z T C A
6	G P T J H D H P S E T Z T C A
6	H O U I I C I O T D U Y U B B
	~~I N V H J B J N U C V X V A C~~
	~~J M W G K A K M V B W W W Z D~~
	~~K L X F L Z L L W A X V X Y E~~
	~~L K Y E M Y M K X Z Y U Y X F~~
	~~M J Z D N X N J Y Y Z T Z W G~~
9	N I A C O W O I Z X A S A V H
4	O H B B P V P H A W B R B U I
	~~P G C A Q U Q G B V C Q C T J~~
	~~Q F D Z R T R F C U D P D S K~~
12	R E E Y S S S E D T E O E R L
7	S D F X T R T D E S F N F Q M
3	T C G W U Q U C F R G M G P N
2	U B H V V P V B G Q H L H O O
7	V A I U W O W A H P I K I N P
	~~W Z J T X N X Z I O J J J M Q~~
	~~X Y K S Y M Y Y J N K I K L R~~
	~~Y X L R Z L Z X K M L H L K S~~
	~~Z W M Q A K A W L L M G M J T~~
5	A V N P B J B V M K N F N I U
7	B U O O C I C U N J O E O H V
5	C T P N D H D T O I P D P G W
	~~D S Q M E G E S P H Q C Q F X~~
7	E R R L F F F R Q G R B R E Y
	~~F Q S K G E G Q R F S A S D Z~~

FIGURE 47*a*

	Column 2
K:	A E A R L T M T F C I R P M A
C:	E E R C E F F Y H K Z G B M D
P:	E A R L T M T F C I R P M A D
8	E A R L T M T F C I R P M A D
	~~F Z S K U L U E D H S O N Z E~~
5	G Y T J V K V D E G T N O Y F
	~~H X U I W J W C F F U M P X G~~
	~~I W V H X I X B G E V L Q W H~~
3	J V W G Y H Y A H D W K R V I
	~~K U X F Z G Z Z I C X J S U J~~
7	L T Y E A F A Y J B Y I T T K
	~~M S Z D B E B X K A Z H U S L~~
5	N R A C C D C W L Z A G V R M
2	O Q B B D C D V M Y B F W Q N
6	P P C A E B E U N X C E X P O
5	Q O D Z F A F T O W D D Y O P
	~~R N E Y G Z G S P V E C Z N Q~~
4	S M F X H Y H R Q U F B A M R
7	T L G W I X I Q R T G A B L S
	~~U K H V J W J P S S H Z C K T~~
	~~V J I U K V K O T R I Y D J U~~
	~~W I J T L U L N U Q J X E I V~~
	~~X H K S M T M M V P K W F H W~~
5	Y G L R N S N L W O L V G G X
	~~Z F M Q O R O K X N M U H F Y~~
	~~A E N P P Q P J Y M N T I E Z~~
	~~B D O O Q P Q I Z L O S J D A~~
6	C C P N R O R H A K P R K C B
	~~D B Q M S N S G B J Q Q L B C~~

FIGURE 47*b*

[5] Generatrices with three or more of the letters **JKQXZ** have been eliminated.

The R generatrix of the letters of column 1 (with a two-category score of 12), and the E generatrix of column 2 (with a score of 8) are clearly the correct ones, and the solution is off to a flying start.

g. In the decipherment step of the preceding subparagraph, the base letter was correctly assumed to be A_p. Had we assumed an incorrect base letter, for example D_p, our steps would have been those shown below:

Column 1

K: A J P W J K D K P V E W Z W C
C: G V I C Q K K W H W X S S V C
P: J P W J K D K P V E W Z W C D

~~J P W J K D K P V E W Z W C D~~
~~K O X I L C L O W D X Y X B E~~
3 L N Y H M B M N X C Y X Y A F Ø 3
~~M M Z G N A N M Y B Z W Z Z G~~
7 N L A F O Z O L Z A A V A Y H 6 1
~~O K B E P Y P K A Z B U B X I~~
~~P J C D Q X Q J B Y C T C W J~~
~~Q I D C R W R I C X D S D V K~~
7 R H E B S V S H D W E R E U L 6 1
6 S G F A T U T G E V F Q F T M 4 2
4 T F G Z U T U F F U G P G S N 2 2
7 U E H Y V S V E G T H O H R O 1 6
6 V D I X W R W D H S I N I Q P 3 3
~~W C J W X Q X C I R J M J P Q~~
~~X B K V Y P Y B J Q K L K O R~~
~~Y A L U Z O Z A K P L K L N S~~
~~Z Z M T A N A Z L O M J M M T~~
7 A Y N S B M B Y M N N I N L U 4 3
~~B X O R C L C X N M O H O K V~~
~~C W P Q D K D W O L P C P J W~~
~~D V Q P E J E V P K Q F Q I X~~
~~E U R O F I F U Q J R E R H Y~~
8 F T S N G H G T R I S D S G Z 4 4
7 G S T M H G H S S H T C T F A 5 2
6 H R U L I F I R T G U B U E B 3 3
~~I Q V K J E J Q U F V A V D G~~

FIGURE 48*a*

Column 2

K: A H A U L W M W F F I U P P A
C: E E R C E F F Y H K Z G B M D
P: H A U L W M W F F I U P P A G

3 H A U L W M W F F I U P P A G Ø 3
~~I Z V K X L X E G H V O Q Z H~~
~~J Y W J Y K Y D H G W N R Y I~~
~~K X X I Z J Z C I F X M S X J~~
5 L W Y H A I A B J E Y L T W K 3 2
~~M V Z G B H B A K D Z K U V L~~
3 N U A F C G C Z L C A J V U M 3 Ø
6 O T B E D F D Y M B B I W T N 2 4
8 P S C D E E E X N A C H X S O 4 4
3 Q R D C F D F W O Z D G Y R P 1 2
~~R Q E B G G G V P Y E F Z Q Q~~
5 S P F A H B H U Q X F E A P R 3 2
9 T O G Z I A I T R W G D B O S 5 4
~~U N H Y J Z J S S V H C C N T~~
~~V M I X K Y K R T U I B D M U~~
~~W L J W L X L Q U T J A E L V~~
~~X K K V M W M P V S K Z F K W~~
~~Y J L U N V N O W R L Y G J X~~
~~Z I M T O U O N X Q M X H I Y~~
6 A H N S P T P M Y P N W I H Z 4 2
~~B G O R Q S Q L Z O O V J G A~~
~~C F P Q R R R K A N P U K F B~~
~~D E Q P S Q S J B M Q T L E G~~
8 E D R O T P T I C L R S M D D 5 3
7 F C S N U O U H D K S R N C E 4 3
5 G B T M V N V G E J T Q O B F 4 1

FIGURE 48*b*

The numbers to the left of the generatrices give the two-category score for the generatrix as a whole, while the numbers to the right give the two-category scores for the odd and even letters, respectively. It may be seen that the correct generatrices are those with the ringed letters, and that from the intervals between them the base letter must have been not D_p, but three letters earlier, or A_p.

h. If the message in subpar. *b* had been enciphered with the same introductory key, PARLIAMENT, but with reciprocal alphabets (e.g., reversed standard), the only way we could have arrived at the length of the introductory key is by trial and error, since the technique demonstrated in subpar. *d* does not apply. We would have had to make generatrix diagrams for the first several columns of trial widths, until a solution was reached. In Fig. 49*a*, below, is the autokey decipherment of the letters of column 1 obtained from a write-out of the cipher text on a width of 10, together with the appertaining generatrix diagram, on the assumption of reversed standard alphabets; the correct plain text comes out on a generatrix which has been underlined. In Fig. 49*b* we have the decipherment of the letters of column 1, under the incorrect assumption of the base letter as D_p. Note that the correct letters are those ringed, in an ascending diagonal pattern. This demonstrates the messiness of the case of plaintext autokey encipherment with reversed sequences, an unknown length of introductory key, and an unknown base letter.

```
K: A C P P J D D D P O E P Z P C
C: Y N A G G A A O B K P Q K N G
P: C P P J D D D P O E P Z P C W

   C P P J D D D P O E P Z P C W
   D Q Q K E E E Q P F Q A Q D X
   E R R L F F F R Q G R B R E Y
   F S S M G G G S R H S C S F Z
   G T T N H H H T S I T D T G A
   H U U O I I I U T J U E U H B
   I V V P J J J V U K V F V I C
   J W W Q K K K W V L W G W J D
   K X X R L L L X W M X H X K E
   L Y Y S M M M Y X N Y I Y L F
   M Z Z T N N N Z Y O Z J Z M G
   N A A U O O O A Z P A K A N H
   O B B V P P P B A Q B L B O I
   P C C W Q Q Q C B R C M C P J
   Q D D X R R R D C S D N D Q K
   R E E Y S S S E D T E O E R L
   S F F Z T T T F E U F P F S M
   T G G A U U U G F V G Q G T N
   U H H B V V V H G W H R H U O
   V I I C W W W I H X I S I V P
   W J J D X X X J I Y J T J W Q
   X K K E Y Y Y K J Z K U K X R
   Y L L F Z Z Z L K A L V L Y S
   Z M M G A A A M L B M W M Z T
   A N N H B B B N M C N X N A U
   B O O I C C C O N D O Y O B V
```

FIGURE 49*a*

```
K: A F V Y V S V Y N P I W J C S
C: Y N A G G A A O B K P Q K N G
P: F V Y V S V Y N P I W J C S P

   F V Y V (S) V Y N P I W J C S P
   G W Z W T W Z O Q J X K D T Q
   H X A X U X A P R K Y L (E) U R
   I Y B (Y) V Y B Q S L Z M F V S
   J Z C Z W Z C R T M A N G W T
   K A D A X A D S U N B (O) H X U
   L B (E) B Y B E T V O C P I Y V
   M C F C Z C F U W P D Q J Z W
   N D G D A D G V X Q (E) R K A X
   O (E) H E B E H W Y R F S L B Y
   P F I F C F I X Z S G T M C Z
   Q G J G D G J Y A (T) H U N D A
   (R) H K H E H K Z B U I V O E B
   S I L I F I L A C V J W P F C
   T J M J G J M B (D) W K X Q G D
   U K N K H K N C E X L Y R H E
   V L O L I L O D F Y M Z S I F
   W M P M J M P (E) G Z N A T J G
   X N Q N K N Q F H A O B U K H
   Y O R O L O R G I B P C V L I
   Z P S P M P (S) H J C Q D W M J
   A Q T Q N Q T I K D R E X N K
   B R U R O R U J L E S F Y O (L)
   C S V S P (S) V K M F T G Z P M
   D T W T Q T W L N G U H A Q N
   E U X U R U X M O H V I B (R) O
```

FIGURE 49*b*

i. When the plain and cipher components are two different sequences, not only must we arrive at the correct length of the introductory key by trial and error, but each base letter must be laboriously assumed in turn for the specialized generatrix diagrams (cf. those in subpar. 30*m*) pertaining to each assumption of introductory key length, since the correct plaintext letters cannot be determined from any sort of systematic pattern from the generatrix diagram predicated on an incorrect assumption of the base letter.

j. In subpar. *e* it was stated that the aperiodic substitution resulting from plaintext autokey encipherment can be converted to periodic terms and solved as though it were a repeating-key cipher, provided that the primary components are known sequences. As a demonstration, we shall use as an example the cipher text of the message in subpar. *b* which was discovered to have been enciphered with an introductory key of 10 letters. This cipher text, written on a width of 10, is reproduced in Fig. 50, below:

	1	2	3	4	5	6	7	8	9	10
(1)	G	E	T	P	Q	V	U	R	T	A
(2)	V	E	X	C	I	M	B	V	R	S
(3)	I	R	T	D	I	I	X	W	Y	X
(4)	C	C	C	K	B	W	P	O	A	W
(5)	Q	E	S	U	X	S	P	M	L	S
(6)	K	F	O	H	W	V	R	S	F	Q
(7)	K	F	R	G	G	X	F	Z	V	T
(8)	W	Y	K	T	F	D	F	M	O	Y
(9)	H	H	H	S	T	S	A	M	R	N
(10)	W	K	B	T	C	V	G	B	C	L
(11)	X	Z	L	Z	H	V	K	M	D	E
(12)	S	G	P	X	P	R	W	S	J	V
(13)	S	B	V	R	B	F	N	A	G	G
(14)	V	M	X	V	O	W	B	A	W	S
(15)	C	D	I	W	N	M	M	W	V	B

FIGURE 50

```
(1) G E T P Q V U R T A          p f c u j o b p n u
(2) V E X C I M B V R S     (9)  H H H S T S A M R N
    p a e n s r h e y s          s c f y k e z x e t
(3) I R T D I I X W Y X     (10) W K B T C V G B C L
    t r p q q r q s a f          e i w v s r h e y s
(4) C C C K B W P O A W     (11) X Z L Z H V K M D E
    j l n u l f z w a r          t r p e p e d i f m
(5) Q E S U X S P M L S     (12) S G P X P R W S J V
    h t f a m n q q l b          z p a t a n t k e j
(6) K F O H W V R S F Q     (13) S B V R B F N A G G
    d m j h k i b c u p          t m v y b s u q c x
(7) K F R G G X F Z V T     (14) V M X V O W B A W S
    h t i z w p e x b e          c a c x n e h k u v
(8) W Y K T F D F M O Y     (15) C D I W N M M W V B
    p f c u j o b p n u          a d g z a i f m b g
```

FIGURE 51

Now using the assumed components of direct standard alphabets,[6] we will treat the first row of cipher text as key elements with which to decipher the second row, and this resultant "plain text" will then be used as key with which to decipher the third row, and so on through the 15th row. This step is shown in Fig. 51, above. We now write the first 10 letters of the cipher text, and all the subsequent "decipherments" of Fig. 51, into the rows of a rectangle of a width *twice* the length of the introductory key; this is shown in Fig. 52*a*, below. The original aperiodic cipher text has now been converted into periodic polyalphabetic substitution, as can be proved by a comparison with the original plain text written on a width of 20, as shown in Fig. 52*b*. The monoalphabetic columns of Fig. 52*a* can now be solved by the generatrix

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
G	E	T	P	Q	V	U	R	T	A	P	A	E	N	S	R	H	E	Y	S
T	R	P	Q	Q	R	Q	S	A	F	J	L	N	U	L	F	Z	W	A	R
H	T	F	A	M	N	Q	Q	L	B	D	M	J	H	K	I	B	C	U	P
H	T	I	Z	W	P	E	X	B	E	P	F	C	U	J	O	B	P	N	U
S	C	F	Y	K	E	Z	X	E	T	E	I	W	V	S	R	H	E	Y	S
T	R	P	E	P	E	D	I	F	M	Z	P	A	T	A	N	T	K	E	J
T	M	V	Y	B	S	U	Q	C	X	C	A	C	X	N	E	H	K	U	V
A	D	G	Z	A	I	F	M	B	G										

FIGURE 52*a*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
R	E	C	E	I	V	I	N	G	H	E	A	V	Y	A	R	T	I	L	L
E	R	Y	F	I	R	E	O	N	M	Y	L	E	F	T	F	L	A	N	K
S	T	O	P	E	N	E	M	Y	I	S	M	A	S	S	I	N	G	H	I
S	T	R	O	O	P	S	T	O	L	E	F	T	F	R	O	N	T	A	N
D	C	O	N	C	E	N	T	R	A	T	I	N	G	A	R	T	I	L	L
E	R	Y	T	H	E	R	E	S	T	O	P	R	E	I	N	F	O	R	C
E	M	E	N	T	S	I	M	P	E	R	A	T	I	V	E	T	O	H	O
L	D	P	O	S	I	T	I	O	N										

FIGURE 52*b*

---

[6] We know that the plain and cipher components must be identical sequences running in the same direction, otherwise the length of the introductory key could not have been discovered by the method employed in subpar. *d*.

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method, using direct standard alphabets which we have just proved to have been employed by the enciphering cryptographer.

k. The mystery of the conversion of a plaintext autokey encipherment to periodic terms will now be explained. First, let the key word PARLIAMENT be enciphered by the following alphabet:

	Ø	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
P:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
C:	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B

P:	P	A	R	L	I	A	M	E	N	T
C:	L	A	J	P	S	A	O	W	N	H

Then let the message RECEIVING HEAVY ARTILLERY FIRE . . . be enciphered by direct standard alphabets as before, but for the key add the monoalphabetic equivalents of PARLIAMENT (i.e., LAJPSAOWNH) to the key itself; that is, use the 20-letter key sequence PARLIAMENTLAJPSAOWNH in a repeating-key manner, as shown in the figure below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
K:	P	A	R	L	I	A	M	E	N	T	L	A	J	P	S	A	O	W	N	H	
P:	R	E	C	E	I	V	I	N	G	H	E	A	V	Y	A	R	T	I	L	L	
C:	G	E	T	P	Q	V	U	R	T	A	P	A	E	N	S	R	H	E	Y	S	
P:	E	R	Y	F	I	R	E	O	N	M	Y	L	E	F	T	F	L	A	N	K	. . .
C:	T	R	P	Q	Q	R	Q	S	A	F	J	L	N	U	L	F	Z	W	A	R	

FIGURE 53

The cipher resultants of this process of *enciphering* a message coincide exactly with those obtained from the *deciphering* operation that gave rise to Fig. 52*a*. How does this happen?

(1) First, let it be noted that the sequence LAJPSAOWNH which forms the second half of the key for enciphering the text in Fig. 53 may be described as the standard-alphabet *complement* of the sequence PARLIAMENT which forms the first half of the key. Arithmetically, the sum of a letter of the first half and its corresponding letter in the second half is Ø, mod 26. Thus:

$$P + L = 15 + 11 = 26 = \emptyset$$
$$A + A = \emptyset + \emptyset = \emptyset$$
$$R + J = 17 + 9 = 26 = \emptyset$$
$$L + P = 11 + 15 = 26 = \emptyset$$
$$I + S = 8 + 18 = 26 = \emptyset$$

In other words, the LAJPSAOWNH sequence is, by cryptographic arithmetic, equivalent to "minus PARLIAMENT." Therefore, in Fig. 53, *enciphering* the second half of each line by the key letters LAJPSAOWNH (i.e., adding 11, Ø, 9, 15, 18 . . .) is the same as *deciphering* by the key letters PARLIAMENT (i.e., subtracting 15, Ø, 17, 11, 8 . . .). For example, at position 11 of Fig. 53, $E_p(L_k)=4+11=15=P_c$, but if we use the key letter at position 1 and subtract it from E_p, we have $E_p(-P_k)=4-15=4-15+(26)=15=P_c$, the same cipher resultant.

(2) Refer now to Fig. 52*a*. The letters in the first half of line 1, beginning GETPQ . . . ., are identical with those in the first half of line 1 of Fig. 53. They must be identical because they are produced from identical elements. The letters in the second half of this same line in Fig. 52*a*, beginning PAENS . . . ., were produced by *deciphering* the letters in the second line of Fig. 50 (VEXCI . . .). Thus (taking for illustrative purposes only the first five letters in each case):

PAENS = VEXCI − GETPQ.
But VEXCI = EAVYA + RECEI
and GETPQ = RECEI + PARLI.
Hence, PAENS = (EAVYA + RECEI) − (RECEI + PARLI),
or PAENS = EAVYA − PARLI. [I]

As for the letters in the second half of line 1 of Fig. 53, also beginning PAENS . . ., these letters were the result of *enciphering* EAVYA by LAJPS. Thus:

PAENS = EAVYA + LAJPS.

But it has been shown in subpar. (1), above, that

LAJPS = – PARLI.
Hence, PAENS = EAVYA + (–PARLI),
or PAENS = EAVYA – PARLI. [II]

Thus, equations [I] and [II] turn out to be identical but arise from what appear to be quite diverse sources.

(3) What has been demonstrated in connection with the letters in line 1 of Figs. 52*a* and 53 also holds true for the letters in the other lines of those two figures, and it is not necessary to repeat the explanation. The steps show that the originally aperiodic, autokey cipher has been converted, through a knowledge of the primary components, into a repeating-key cipher with a period *twice the length of the introductory key*. The message may now be solved as an ordinary repeating-key cipher.

l. The procedure just described (in subpars. *j* and *k*) has the advantage over the generatrix methods of subpars. *g* and *h* in that it is not necessary to assume the correct base letter for the procedure to work.

m. The foregoing case is based upon encipherment by the normal Vigenère equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$. When encipherment has been accomplished by the equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/2}=\theta_{c/1}$, the conversion of a plaintext autokeyed cipher yields a repeating-key cipher with a period *equal to the length of the introductory key*. In this conversion the equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$ are used in finding equivalents. As an example, note the plaintext autokey encipherment of the following message by equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/2}=\theta_{c/1}$:

```
K: T U E S D A Y I N F O R M A T I O N F R O M R E L I A B L E S O U R C
P: I N F O R M A T I O N F R O M R E L I A B L E S O U R C E S I N D I C . . .
C: P T B W O M C L V J Z O F O T J Q Y D J N Z N O D M R B T O Q Z J R A
```

If the cipher text is written out in lines corresponding to the length of the introductory key, and each line is *enciphered* by the one directly above it, using the normal Vigenère equations in finding equivalents, the results are shown in Fig. 54*b*. But if the same is enciphered by equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/2}=\theta_{c/1}$, using the word TUESDAY as a repeating key, the cipher text (Fig. 54*c*) is identical with that obtained in Fig. 54*b* by enciphering each successive line with the line above.

```
Original cipher          Original cipher          Repeating key
     text                and converted            encipherment
                              text
                                                  T U E S D A Y
                                                  I N F O R M A
P T B W O M C <------> P T B W O M C <------> P T B W O M C
L V J Z O F O <------> L V J Z O F O
                                                  T I O N F R O
                       a o k v c r q <------> A O K V C R Q
T J Q Y D J N <------> T J Q Y D J N
                                                  M R E L I A B
                       t x a t f a d <------> T X A T F A D
Z N O D M R B <------> Z N O D M R B
                                                  L E S O U R C
                       s k o w r r e <------> S K O W R R E
T O Q Z J R A <------> T O Q Z J R A
                                                  E S I N D I C
                       l y e v a i e <------> L Y E V A I E
     (a)                    (b)                      (c)
```

FIGURE 54

Now note that the sequences joined by the arrows in Figs. 54*b* and *c* are identical; since it is certain that Fig. 54*c* is periodic in form because it was enciphered by the repeating-key method, it follows that Fig. 54*b* is now also in periodic form, and in that form the message could be solved as though it were a repeating-key cipher.

n. In the case of primary components consisting of reversed standard alphabets, the process of converting the plaintext autokeyed text to periodic terms is accomplished by using a *direct standard* alphabet and "deciphering" each line of the text (as transcribed in period-lengths) by the line above it. For example, here is a message enciphered by reversed standard alphabets, with the initial key word TUESDAY:

```
K: T U E S D A Y I N F O R M A T I O N F R O M R E L I A B L E S O U R C
P: I N F O R M A T I O N F R O M R E L I A B L E S O U R C E S I N D I C . . .
C: L H Z E M O Y P F R B M V M H R K C X R N B N M X O J Z H M K B R J A
```

The cipher text is transcribed in periods equal to the length of the initial key word (7 letters) and the 2d line is "deciphered" with key letters of the 1st line, using the normal Vigenère equations $\theta_{k/2}=\theta_{1/1}$; $\theta_{p/1}=\theta_{c/2}$. The resultant letters are then used as key letters to "decipher" the 3d line of text, and so on. The results are as seen in Fig. 55*b* below. Now let the original message be enciphered *as a repeating-key cipher* by reversed standard alphabets with the key word TUESDAY; the result is shown in Fig. 55*c*. Note that the odd or alternate lines of Fig. 55*b* and *c* are identical, showing that the autokeyed text has been converted into repeating-key cipher text.

Original cipher text		Original cipher and converted text		Repeating key encipherment
				T U E S D A Y
				I N F O R M A
L H Z E M O Y	←→	L H Z E M O Y	←→	L H Z E M O Y
P F R B M V M	←→	P F R B M V M		
				T I O N F R O
		a m q f y j k	←→	A M Q F Y J K
H R K C X R N	←→	H R K C X R N		
				M R E L I A B
		h d a h v a x	←→	H D A H V A X
B N M X O J Z	←→	B N M X O J Z		
				L E S O U R C
		i q m e j j w	←→	I Q M E J J W
H M K B R J A	←→	H M K B R J A		
				E S I N D I C
		p c w f a s w	←→	P C W F A S W
(a)		(b)		(c)

FIGURE 55

o. The foregoing procedures indicate a simple method of solving plaintext autokey ciphers when the primary components or the secondary cipher alphabets are known. It consists in assuming introductory keys of various lengths, converting the cipher text into repeating-key form, and then examining the resulting diagrams for repetitions. When a correct key length is assumed, repetitions will be as numerous as should be expected in ciphers of the repeating-key class; incorrect assumptions for key length will not show so many repetitions.[7] All of the foregoing presupposes a knowledge of the cipher alphabets involved. When these are unknown, recourse must be had to first principles, and the messages

[7] These repetitions need not be polygraphic repetitions in order to prove the key length. In the write-outs on the correct width, the average I.C. of the entire array of columns should be near 1.73, with acceptable deviations. See in this connection subpars. 18*e* *et seq.* in *Military Cryptanalytics, Part II.*

must be solved purely upon the bases of probable words and repetitions; the general approaches will be demonstrated.

32. Analysis of a case involving unknown components.—*a.* When the primary components in a plaintext autokey system are unknown, the observations noted under the preceding paragraphs are, of course, not applicable; nevertheless, solution is not difficult. Let us assume that we have available the following three cryptograms, all intercepted on the same day, and therefore suspected of being related.

Message No. 1

```
H U F I I   O C Q J J   I V Z O Z   V P D G O   V V V K W   U E W H U
U Q H U M   R Z V Q R   U A K V D   N N E Z V   G J P G H   A Y J D R
U W N G R   Y S K B L   Q V U X N   P H D P R   S V K Z P   P P K G S
L L P R V   R B H A K   W U A V W   Y U E Z Q   X A P Q Y   G P S V S
F N R A K   C I F G Z   U V C C P   D K C W V   X T W F M   R F K B V
R O Q O J   D R U W N   G R Y S K   B L
```

Message No. 2

```
J U F I I   O C Q J J   I V Z O Z   I B F E J   S U B R J   S P K T S
R Z V X T   W F M R F   Q H H F O   R F J P D   G O V V V   K W U H E
N D B D D   R H W U N   K C M P D   G O V Z S   E N D B D   D R H W U
N P P K P   E Q O Y
```

Message No. 3

```
F J U H F   F K D E N   A L U P Z   K Q M V B   J W V P K   E U B D D
R H W U M   R H V G P   D N C U J   C D Z C Y   R H U J U   F Z P Q P
Y Q C Y H   O E Q Z V   X K C Q F   T V H N S   V C C E J   P E A M P
A P O E P   B H M V J   U N M H H   W K C V G   D S W J U   E Q Z B O
F F Y U E   Z Q X A P   Q Y G P A   R P Z V X   C F N R A   K C I F G
Z U V C C   P D K C O   G J W Z H   A P U F Z   F V H A V   X M H F F
K M Y H S   T B S K C   V R Q I J   Y C P Z H   U H C B M   T H O F H
```

b. There are many repetitions within and between the messages, attesting to their cryptographic homogeneity. The intervals between repetitions show no common factor, and the δ I.C. of the composite uniliteral frequency distribution is 1.12; these manifestations admit of the possibility of a plaintext autokey system. Furthermore, the appearance of the repetitions in the first line of Message No. 1, with the isolated 1st and 16th letters, suggests a 1-letter introductory key.

c. The simplest assumption to make is that Vigenère encipherment with standard alphabets is involved; however, the trials both for direct standard and reversed standard alphabets are unsuccessful.[8] We next assume that the plain component is the normal sequence, the cipher component a mixed sequence. The various repetitions are studied intensively, in particular the 13-letter repetition JDRUWNGRYSKBL and the 10-letter repetition PDGOVVVKWU, which, on the hypothesis of a 1-letter introductory key, must represent 14-letter and 11-letter plaintext sequences or words.

d. If the normal Vigenère method of encipherment is in effect, then the base letter is probably A_p, in which case a good word to fit the 13-letter repetition is

```
K: . R E C (O) N N A I S S A N C E
P: R E C O |N| N A I S S A N C E
C:   J D R (U) W N G R Y S K B L
```

[8] The theoretical I.C. for plaintext autokey systems involving either direct or reversed standard alphabets has been computed and found to be 1.035; this then would tend to disprove the assumption of standard alphabets in the case under examination.

and a good word to fit the 10-letter repetition (note the corroborative vertical trigraph ONU with the preceding crib) is

```
K: . O B S E R V A T I O N
P: O B S E R V A T I O N
C:   P D G O V V V K W U
```

e. The values from these two assumptions are inserted in a reconstruction matrix, yielding the following:

Plain

Key	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A									G					K						V						
B																			D							
C					L										R											
D																										
E			D															O								
F																										
G																										
H																										
I															W				R							
J																										
K																										
L																										
M																										
N	N		B											W												
O		P												U												
P																										
Q																										
R					J																	V				
S	S				G														Y							
T									K																	
U																										
V	V																									
W																										
X																										
Y																										
Z																										

FIGURE 56*a*

On the hypothesis of direct symmetry of position, we are able to amalgamate the values in the square into a nearly complete sequence, as shown by the fragmentary matrix below (the derived letters are in lower case):

Plain

		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	A	a	d	j	u	s	t	i	n	G	b	c			K	l		o	p		V	w		y		r	e
	B	b	c				l		o	p		v	w				r	e		D	j	u			i	n	
	C	c				L		o	p		v	w				R	e		d	j	u			i	n		b
	D	d																									
	E	e		D															O								
	F	f																									
	G	g																									
	H	h														W				R							
	I	i																									
	J	j																									
	K	k																									
	L	l																									
Key	M	m																									
	N	N		B											W												
	O	o	P												U												
	P	p																									
	Q	q																									
	R	r				J																	V				
	S	S				G														Y							
	T	t								K																	
	U	u																									
	V	V																									
	W	w																									
	X	x																									
	Y	y																									
	Z	z																									

FIGURE 56*b*

The keyword-mixed sequence is apparent, and the original components must be the following:

P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C: R E A D J U S T I N G B C F H K L M O P Q V W X Y Z

With the primary components at hand, solution of the messages is now a trivial matter. The messages are found to have initial keys of R, L, and W, respectively.

f. The foregoing example used an unknown mixed cipher component sliding against what was first assumed (and later proved) to be the normal sequence. When both primary components are unknown mixed sequences but are identical, solution is more difficult, naturally, because the results of assuming values for repetitions cannot be proved and established so quickly as in the foregoing example; if the primary components are two different unknown mixed sequences, the problem becomes even more difficult. Nevertheless, the general method indicated, and the application of the principles of indirect symmetry of position will lead to solution, if there is an adequate amount of text available for study. When the introductory key consists of several letters, repetitions are much reduced and the difficulty of solution is considerable but by no means insurmountable. Under operational conditions, the inevitable cribs, collateral information, isologs, and compromised plain text, make analysis even of difficult plain-text autokey systems practicable.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

34. Analysis of digital plaintext autokey systems.—*a.* In digital plaintext autokey systems wherein the nature of the intermediate text is known, the methods of solution are identical with those of literal plaintext autokey systems involving known components. As an illustration, let us assume that it is known that the enemy is using plaintext autokey systems with one-digit introductory keys, in conjunction with the following variant matrix for producing the intermediate text:

		6	7	8	9	0
		1	2	3	4	5
6	1	A	B	C	D	E
7	2	F	G	H	I	K
8	3	L	M	N	O	P
9	4	Q	R	S	T	U
0	5	V	W	X	Y	Z

The encipherment of the word REGIMENTAL, for example, could then take on the following form:

R E G I M E N T A L
47 15 22 79 82 60 33 94 16 81

This intermediate text, when subjected to additive encipherment by the plaintext autokey method with the introductory key 4, would yield the following cipher text:

K: 4 4 7 1 5 2 2 7 9 8 2 6 0 3 3 9 4 1 6 8
P: 4 7 1 5 2 2 7 9 8 2 6 0 3 3 9 4 1 6 8 1
C: 8 1 8 6 7 4 9 6 7 0 8 6 3 6 2 3 5 7 4 9

Now if a message beginning with the groups 81867 49670 86362 35749 . . . were intercepted, the simplest procedure would be to make a trial "decipherment," and then complete the plain-component sequence down the columns in alternate directions. First, the "decipherment" with an arbitrary key of Ø:

K: 0 8 3 5 1 6 8 1 5 2 8 0 6 7 9 3 0 5 2 2
C: 8 1 8 6 7 4 9 6 7 0 8 6 3 6 2 3 5 7 4 9 . . .
8 3 5 1 6 8 1 5 2 8 0 6 7 9 3 0 5 2 2 7

Next, in Fig. 59, below, we show this decipherment, together with the generatrices formed from this first decipherment by running down the normal numerical sequence in the odd columns, and down the reversed normal sequence in the even columns;[12] the generatrix marked with an asterisk is the original plain text, which will decipher as "REGIMENTAL" with the known matrix.

K: 0 8 3 5 1 6 8 1 5 2 8 0 6 7 9 3 0 5 2 2
C: 8 1 8 6 7 4 9 6 7 0 8 6 3 6 2 3 5 7 4 9 . . .
P: 8 3 5 1 6 8 1 5 2 8 0 6 7 9 3 0 5 2 2 7
9 2 6 0 7 7 2 4 3 7 1 5 8 8 4 9 6 1 3 6
0 1 7 9 8 6 3 3 4 6 2 4 9 7 5 8 7 0 4 5
1 0 8 8 9 5 4 2 5 5 3 3 0 6 6 7 8 9 5 4
2 9 9 7 0 4 5 1 6 4 4 2 1 5 7 6 9 8 6 3
3 8 0 6 1 3 6 0 7 3 5 1 2 4 8 5 0 7 7 2
*4 7 1 5 2 2 7 9 8 2 6 0 3 3 9 4 1 6 8 1
5 6 2 4 3 1 8 8 9 1 7 9 4 2 0 3 2 5 9 0
6 5 3 3 4 0 9 7 0 0 8 8 5 1 1 2 3 4 0 9
7 4 4 2 5 9 0 6 1 9 9 7 6 0 2 1 4 3 1 8

FIGURE 59

[12] Since additive encipherment is really a digital Vigenère system with direct standard (numerical) alphabets, the cryptanalytic treatment must be identical to that of its literal counterpart. (See also subpar. 35*g* on subtractive and minuend methods.)

b. If the general nature of the intermediate text is known but the introductory key consists of an unknown number of digits, generatrix diagrams (similar to those employed in the analysis of literal plaintext autokey systems) for the various possibilities of key lengths would enable the cryptanalyst

	9	4	8	1	2	7	6	5	0	3
–	R	E	P	U	B	L	I	C		
0	A	D	F	G	H	J	K	M	N	O
3	Q	S	T	V	W	X	Y	Z		

to effect a speedy solution. As an example, let us suppose we know that the enemy is using the monome-dinome matrix shown above for his intermediate text, in conjunction with additive-enciphered plaintext autokey and introductory keys of varying lengths, and that the following cryptogram is at hand:

```
04166 94503 96343 36243 94770 44813
35073 48890 04265 64839 23734 94333
48934 08301 25077 74005 46288 92134
84311 55035
```

Assumptions of introductory key-lengths of 1, 2, 3, and 4 are tried, without success. On assuming a trial length of 5, we take every fifth digit of the cipher text, starting with the first, and subject these digits to a plaintext autokey decipherment with, say, an arbitrary initial key digit of Ø. From these "decipherments" we complete the plain-component sequence down the columns in alternate directions, as shown in Fig. 60*a*, below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
K:	0	0	9	0	3	6	8	5	9	1	5	7	2	2	8	4	3	1	8	0
C:	0	9	9	3	9	4	3	4	0	6	2	9	4	0	2	7	4	9	8	5
P:	0	9	0	3	6	8	5	9	1	5	7	2	2	8	4	3	1	8	0	5
	1	8	1	2	7	7	6	8	2	4	8	1	3	7	5	2	2	7	1	4
	2	7	2	1	8	6	7	7	3	3	9	0	4	6	6	1	3	6	2	3
	3	6	3	0	9	5	8	6	4	2	0	9	5	5	7	0	4	5	3	2
	4	5	4	9	0	4	9	5	5	1	1	8	6	4	8	9	5	4	4	1
	5	4	5	8	1	3	0	4	6	0	2	7	7	3	9	8	6	3	5	0
	6	3	6	7	2	2	1	3	7	9	3	6	8	2	0	7	7	2	6	9
	7	2	7	6	3	1	2	2	8	8	4	5	9	1	1	6	8	1	7	8
	8	1	8	5	4	0	3	1	9	7	5	4	0	0	2	5	9	0	8	7
	9	0	9	4	5	9	4	0	0	6	6	3	1	9	3	4	0	9	9	6

FIGURE 60*a*

Since it is difficult to recognize "plain text" when we have it, we shall have to resort to statistical recognition and weight the generatrices, using the following log weights computed for the intermediate text produced by this particular monome-dinome matrix:[13]

Plaintext digit:	0	1	2	3	4	5	6	7	8	9
Log weight:	9	1	1	8	7	1	3	0	5	5

[13] The computation of these log weights is shown in subpars. 87*b* and *c* (on pp. 246–248) of *Military Cryptanalytics, Part II*.

The generatrices of the diagram of Fig. 60*a*, when scored by the foregoing weights, become the following:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Score
9	5	9	8	3	5	1	5	1	1	0	1	1	5	7	8	1	5	9	1	85
1	5	1	1	0	0	3	5	1	7	5	1	8	0	1	1	1	0	1	7	49
1	0	1	1	5	3	0	0	8	8	5	9	7	3	3	1	8	3	1	8	75
8	3	8	9	5	1	5	3	7	1	9	5	1	1	0	9	7	1	8	1	92
7	1	7	5	9	7	5	1	1	1	1	5	3	7	5	5	1	7	7	1	86
1	7	1	5	1	8	9	7	3	9	1	0	0	8	5	5	3	8	1	9	91
3	8	3	0	1	1	1	8	0	5	8	3	5	1	9	0	0	1	3	5	65
0	1	0	3	8	1	1	1	5	5	7	1	5	1	1	3	5	1	0	5	54
5	1	5	1	7	9	8	1	5	0	1	7	9	9	1	1	5	9	5	0	89
5	9	5	7	1	5	7	9	9	3	3	8	1	5	8	7	9	5	5	3	114

FIGURE 60*b*

The last generatrix, with a high score of 114, shows us that we have arrived at the correct length of introductory key, and that this generatrix is the correct one. Similarly, the correct generatrices for the remaining four positions yield high scores of 124, 135, 135, and 130, and we have discovered thereby that the first five plaintext digits are 94503, making the introductory key 10663. The decipherment of the beginning of the message is shown below:

```
            5              10              15              20              25              30
K: 1 0 6 6 3 9 4 5 0 3 0 0 0 0 0 9 6 3 4 3 4 0 9 0 0 5 4 8 7 0
C: 0 4 1 6 6 9 4 5 0 3 9 6 3 4 3 3 6 2 4 3 9 4 7 7 0 4 4 8 1 3 . . .
P: 9 4 5 0 3 0 0 0 0 0 9 6 3 4 3 4 0 9 0 0 5 4 8 7 0 9 0 0 4 3
   R E C  O   N   N   A  I  S   S   A   N  C E P L   A   N   E   S
```

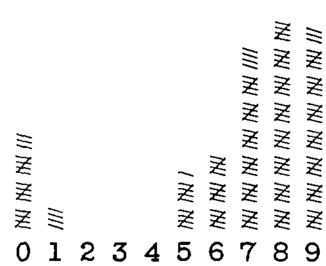
c. For the next example we shall consider the case of a digital plaintext autokey system involving a 1-digit introductory key, the intermediate text being of unknown composition. The message is as follows:

```
47466  46776  45863  67446  75368  67897
46744  44557  96269  87764  66366  68686
36744  42662  46456  67766  76534  78634
46978  74688  65534  78996  68657  63674
46753  68678  87787  77863  69965  67744
67743  67653
```

An arbitrary plain text of 0 is assumed for the first cipher digit and the text is deciphered on this basis; the first 30 digits are shown deciphered below:

```
K:  0779  77989  77806  79868  89679  97081
C: 47466  46776  45863  67446  75368  67897 . . .
P: 07797  79897  78067  98688  96799  70816
```

The distribution of the digits of the entire decipherment is given below:

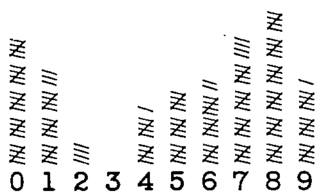


The δ I.C. is $\frac{10(4726)}{160\times 159}=1.89$, a high figure, certainly,[14] but one which must be compared with the I.C.'s of the other possible decipherments in order to be able to evaluate its significance.

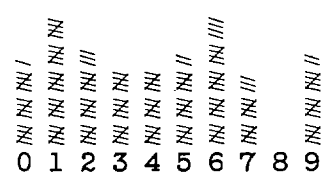
d. The first 30 digits of the next four decipherments, on the basis of plaintext 1, 2, 3, and 4 for the first cipher digit, are shown in Fig. 61, below, [15] followed by their appertaining distributions:

```
K:   1 6 8 8   8 6 0 7 0   6 8 7 1 5   8 8 9 5 9   7 0 5 8 8   0 6 1 7 2
C: 4 7 4 6 6   4 6 7 7 6   4 5 8 6 3   6 7 4 4 6   7 5 3 6 8   6 7 8 9 7 . . .
P:(1)6 8 8 8   6 0 7 0 6   8 7 1 5 8   8 9 5 9 7   0 5 8 8 0   6 1 7 2 5
                                 * * * * *
K:   2 5 9 7   9 5 1 6 1   5 9 6 2 4   9 7 0 4 0   6 1 4 9 7   1 5 2 6 3
C: 4 7 4 6 6   4 6 7 7 6   4 5 8 6 3   6 7 4 4 6   7 5 3 6 8   6 7 8 9 7 . . .
P:(2)5 9 7 9   5 1 6 1 5   9 6 2 4 9   7 0 4 0 6   1 4 9 7 1   5 2 6 3 4
                                 * * * * *
K:   3 4 0 6   0 4 2 5 2   4 0 5 3 3   0 6 1 3 1   5 2 3 0 6   2 4 3 5 4
C: 4 7 4 6 6   4 6 7 7 6   4 5 8 6 3   6 7 4 4 6   7 5 3 6 8   6 7 8 9 7 . . .
P:(3)4 0 6 0   4 2 5 2 4   0 5 3 3 0   6 1 3 1 5   2 3 0 6 2   4 3 5 4 3
                                 * * * * *
K:   4 3 1 5   1 3 3 4 3   3 1 4 4 2   1 5 2 2 2   4 3 2 1 5   3 3 4 4 5
C: 4 7 4 6 6   4 6 7 7 6   4 5 8 6 3   6 7 4 4 6   7 5 3 6 8   6 7 8 9 7 . . .
P:(4)3 1 5 1   3 3 4 3 3   1 4 4 2 1   5 2 2 2 4   3 2 1 5 3   3 4 4 5 2
```

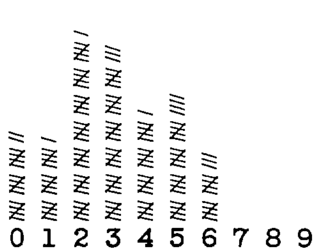
FIGURE 61



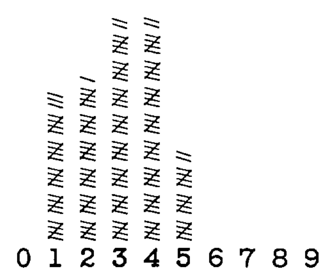
Base 1 $\delta=1.25$



Base 2 $\delta=1.11$



Base 3 $\delta=1.56$



Base 4 $\delta=2.12$

[14] If we had bothered to take an I.C. of the over-all cipher text, we would have found it to be 1.81; therefore the figure of 1.89 hardly reflects anything rougher than the original ciphertext population.

[15] Note that it would be simpler to generate the next four decipherments by means of a generatrix diagram such as that shown in Fig. 59; the example here is for the purpose of pedagogical clarity.

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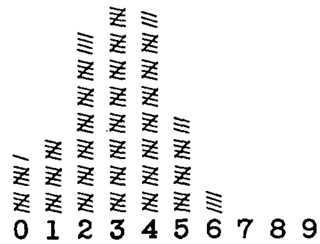
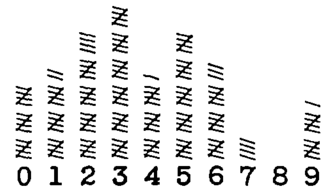
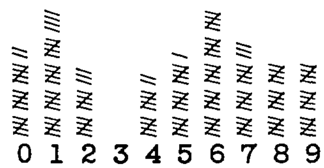
e. If we had made five more decipherments on the basis of plaintext digits 5, 6, 7, 8, and 9 as the equivalents for the first cipher digit, we would have obtained the following distributions:



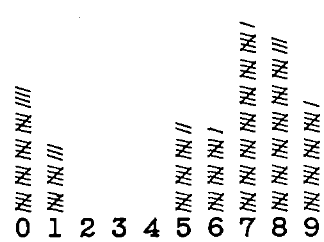
Base 5 $\delta=1.89$



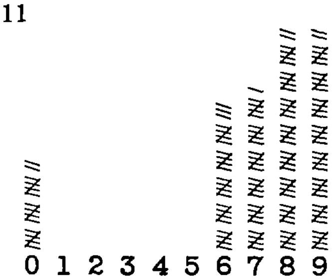
Base 6 $\delta=1.25$



Base 7 $\delta=1.11$



Base 8 $\delta=1.56$



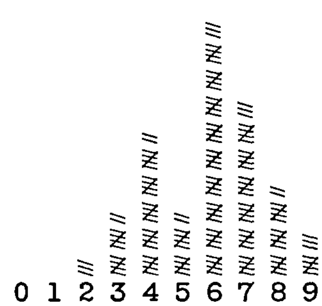
Base 9 $\delta=2.12$

Note that the distribution for base 5 is identical with that of base 0, except for a slide of 5; the distribution for base 6 is identical with that of base 1 (except for the slide of 5), and so on to the distribution for base 9 which is identical with that of base 4, with a slide of 5.[16] This means that we may dispense with the last 5 decipherments, since we can derive their distributions artificially from the first 5 distributions.

f. The greatest roughness is displayed by the distributions based on 4 or 9 as the initial plaintext digit; the problem may now be solved on either base, but the initial plaintext 4 seems, from the limitation of the span of digits 1–5, the more likely base. A dinomic distribution is made of the intermediate text, and it is quickly discovered that it is produced by a 5 x 5 square containing the normal sequence (I=J), with the digits 1 through 5 in normal order as the coordinates;[17] the first two words are found to read "SECOND REGIMENT."

[16] This phenomenon is a result of the mod-10 arithmetic involved; the explanation can be seen more clearly upon examination of rows five apart in Fig. 59.

[17] This is the square employed in the classic Nihilist system (cf. par. 81 of *Military Cryptanalytics, Part II*); the fact that we were faced with a Nihilist system could have been surmised from the distributional appearance of the original cipher text:

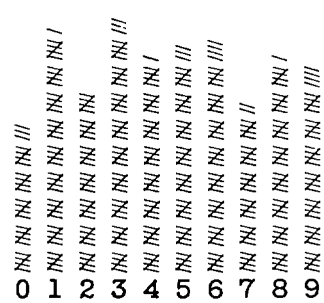


g. For the last example we shall consider the case of an unknown type of intermediate text coupled with an unknown length of introductory key.[18] The message under study is given below:

```
18445 73449 97016 44801 98866 70882
90848 83615 22430 40017 43067 78060
36407 83995 41461 17145 24311 13605
03396 24801 98883 83114 21462 08416
20217 78070 57684 38721 89376 31003
03278 02323 60397 48040 01746 32446
72176 28459 24780 23734 02370 31837
35198 36546 18937 63100 29468 44037
48509 50369 04424 80677 12650 87466
79765 43636 88700 96677 08889 14230
91374 03874 99548 57957 88832 95042
15301 39891 15937 80400 17463 20545
86318 66007 83363 83364 37144 74390
48766 06696 501
```

As in the previous example, we begin by assuming an arbitrary plaintext ∅ for the first cipher digit, at an offset of 1; the decipherment of the first 30 digits and the distribution for the entire decipherment are shown below:

```
K:  0868 70313 63465 13173 81715 16444
C: 18445 73449 97016 44801 98866 70882 . . .
P: 08687 03136 34651 31738 17151 64448
```



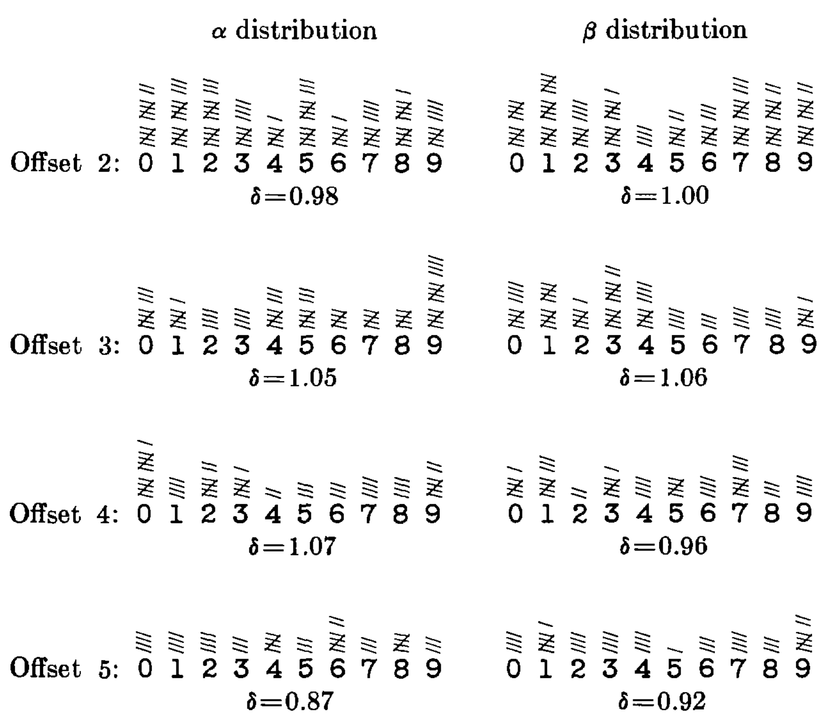
[18] The analysis of digital plaintext autokey systems when the length of the introductory key and the properties of the underlying plain text are unknown, previously considered impossible, was first published in an article ("The Analysis of Digital Plaintext Autokey Systems") by the author in the *NSA Technical Journal*, Vol. XIV, No. 2 (Spring 1969).

h. The δ I.C. is $\frac{10(16{,}330)}{403\times402}=1.01$ when Ø is taken as the equivalent of the first cipher digit; assumptions of 1, 2, 3, and 4 also give I.C.'s hovering near the random expectancy of 1.00. The conjecture is made that, at an incorrect offset, the deciphered text will display the I.C. of random, regardless of the base of the decipherment. Decipherments on a base of Ø for offsets of 2, 3, 4, and 5 are now made; Fig. 62, below, shows the "1st alphabet" of these decipherments:

```
K:     |0  |4  |1  |2  |2  |7  |3  |3  |1  |9  |0  |8  |8  |2
C: 1 8 |4 4|5 7|3 4|4 9|9 7|0 1|6 4|4 8|0 1|9 8|8 6|6 7|0 8|8 2 . . .
P: 0   |4  |1  |2  |2  |7  |3  |3  |1  |9  |0  |8  |8  |2  |6
                         * * * * *
K:       |0    |4    |9    |0    |0    |4    |6    |2    |4
C: 1 8 4 |4 5 7|3 4 4|9 9 7|0 1 6|4 4 8|0 1 9|8 8 6|6 7 0|8 8 2 . . .
P: 0     |4    |9    |0    |0    |4    |6    |2    |4
                         * * * * *
K:         |0      |5      |9      |1      |3      |6      |0
C: 1 8 4 4 |5 7 3 4|4 9 9 7|0 1 6 4|4 8 0 1|9 8 8 6|6 7 0 8|8 2 . . .
P: 0       |5      |9      |1      |3      |6      |0      |8
                         * * * * *
K:           |0        |7        |2        |2        |7
C: 1 8 4 4 5 |7 3 4 4 9|9 7 0 1 6|4 4 8 0 1|9 8 8 6 6|7 0 8 8 2 . . .
P: 0         |7        |2        |2        |7        |0
```

FIGURE 62

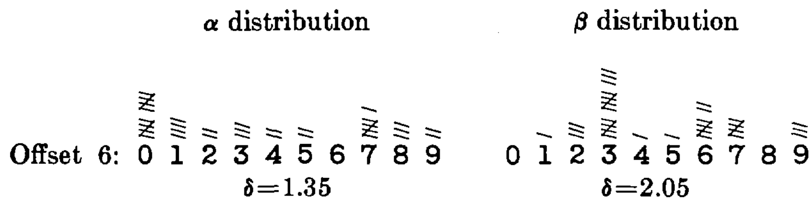
The digits of these decipherments are allocated into two distributions, α for the digits in the odd positions, and β for the digits in the even positions; thus, for offset 2, above, the digits 01231 . . . are tallied into the α distribution, while the digits 42739 . . . are tallied into the β distribution.[19] These distributions, together with their I.C.'s are shown below:



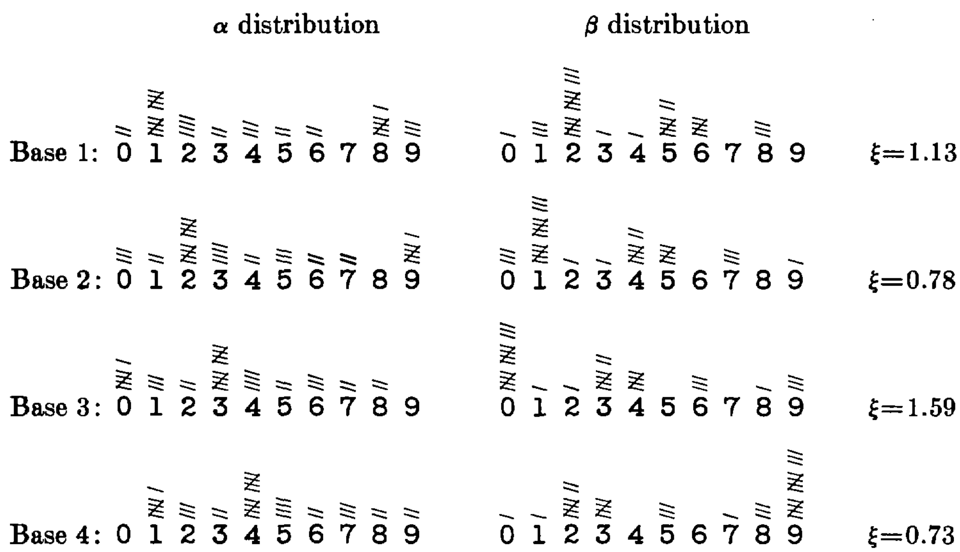
[19] Two distributions, rather than amalgamation into one, are necessary to compensate for the mechanics of the system, and to facilitate the establishment of the correct base.

None of these is significant. But at the very next offset, 6, we have a significant bulge in the I.C.'s for the decipherment (to the base Ø) of the "1st alphabet":

```
K:              |0            |3            |7            |3
C: 1 8 4 4 5 7 |3 4 4 9 9 7 |0 1 6 4 4 8 |0 1 9 8 8 6 |6 7 0 8 8 2 . . .
P: 0            |3            |7            |3            |3
```



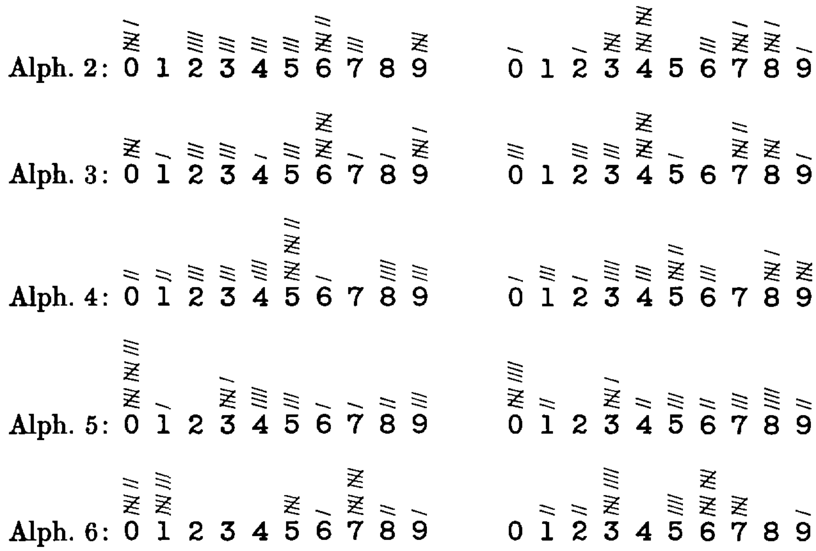
i. Now we're getting somewhere. If instead of base Ø we had made our decipherments on bases of 1, 2, 3, and 4, we merely would have displaced the α distribution successively one position to the right, and the β distribution one position to the left, as is shown below, together with their ξ I.C.'s:



It is evident that base 3 (or 8, which is congruent to 3, mod 5), with a high ξ I.C. of 1.59, is unquestionably the correct base for "alphabet 1"; it is immaterial whether we choose the base 3 or 8 as the norm—solution may be obtained either way.

j. Since we know the correct offset, 6, we now make the α and β distributions for the remaining "alphabets" on a base of ∅, as are shown below:

α distribution β distribution



The ξ I.C.'s of these distributions on the various bases are shown in the diagram below:

	Base 0	Base 1	Base 2	Base 3	Base 4
Alph. 2:	0.88	1.05	1.27	0.62	1.41
Alph. 3:	0.57	1.12	1.17	0.69	1.69
Alph. 4:	1.43	0.63	1.16	1.01	0.90
Alph. 5:	1.70	0.63	0.87	0.91	0.91
Alph. 6:	0.86	1.10	0.62	1.91	1.05

From these data it may be inferred that the correct bases for alphabets 2–6 are 4, 4, 0, 0, and 3, respectively. We can thereupon combine the α and β distributions for the correct offsets for each alphabet, as displayed below:

	19	4	3	17	9	2	6	2	3	3
Alph. 1, base 3:	0	1	2	3	4	5	6	7	8	9
	17	3	3	11	12	1	5	3	4	8
Alph. 2, base 4:	0	1	2	3	4	5	6	7	8	9
	20	2	1	13	10	2	6	3	4	6
Alph. 3, base 4:	0	1	2	3	4	5	6	7	8	9
	3	5	4	7	7	19	4		10	8
Alph. 4, base 0:	0	1	2	3	4	5	6	7	8	9
	22	3		12	6	6	3	4	6	5
Alph. 5, base 0:	0	1	2	3	4	5	6	7	8	9
	19	2	5	17	13		1		7	3
Alph. 6, base 3:	0	1	2	3	4	5	6	7	8	9

It is now also clear that the distribution for alph. 4 should be shifted 5 positions to be on base 5 (which is congruent to 0, mod 5), in order to make it match with the other distributions. When this is done, the cumulative total distribution is as follows:

116	18	12	80	58	14	26	16	31	32
0	1	2	3	4	5	6	7	8	9

The I.C. of the combined distributions is $\frac{10(26{,}398)}{403 \cdot 402}=1.63$, which, for this size of sample, would reflect fairly accurately the theoretical I.C. of the underlying intermediate text.[20]

k. The first six plaintext digits are 344503 (which we have determined from the bases for the six alphabets), so we can reduce the cipher text to monoalphabetic terms, as shown in the beginning fragment below:

```
            5         10        15        20        25        30
K: 8 4 0 9 5 4 3 4 4 5 0 3 0 0 0 4 9 4 0 1 6 0 5 4 0 0 3 8 3 2
C: 1 8 4 4 5 7 3 4 4 9 9 7 0 1 6 4 4 8 0 1 9 8 8 6 6 7 0 8 8 2 . . .
P: 3 4 4 5 0 3 0 0 0 4 9 4 0 1 6 0 5 4 0 0 3 8 3 2 6 7 7 0 5 0
```

When the entire text has been reduced to monoalphabetic terms, it is easily diagnosed as a monome-dinome cipher, and solution proceeds with alacrity and celerity. The opening words are "SECOND REGIMENT" and the monome-dinome matrix is reconstructed as the following:

	9	4	8	1	2	7	6	5	0	3
–	R	E	P	U	B	L	I	C		
0	A	D	F	G	H	J	K	M	N	O
3	Q	S	T	V	W	X	Y	Z		

As the final step in the solution, the initial key digits 840954 are subjected to scrutiny, and it is discovered that the introductory key is based upon the key word PEACE enciphered through the matrix.

l. The foregoing problem involved what at first blush appeared to be a very complex situation: a six-digit introductory key happened to have been used, and it could have been reasoned that, since there are 10^6 different 6-digit numbers, a maximum of 1,000,000 trials[21] with the aid of a computer might be necessary in order to establish the correct introductory key (if the key length were *known*). Instead of these million trials, however, we had to make only 6 trials to arrive at the correct key-length, and 5 more trials to establish the bases for alphabets 2–6.

35. Concluding remarks on autokey systems.—*a*. The plaintext autokey systems treated in this chapter involved those employing sliding primary components, and therefore having related secondary alphabets. As with ciphertext autokey systems, there is no reason why a table of 26 random, unrelated alphabets could not be used, such as that illustrated in Fig. 33 on p. 70. The problem of cryptanalysis in such a case becomes considerably more difficult, naturally, since we do not have the advantage of the exploitation of symmetry of position; nevertheless, as indicated in subpar. 32*f*, under operational conditions the inevitable cribs, collateral information, isologs, and compromised plain text make analysis even of difficult plaintext autokey systems practicable.

b. Ciphertext and plaintext autokey systems display characteristics which permit their diagnosis. Both cases will show repetitions in the cipher text. In ciphertext autokey systems there will be far fewer repetitions than in the original plain text, especially when introductory keys of more than one letter in length are employed; in plaintext autokey systems there will be nearly as many repetitions in the cipher text as in the original plain text unless long introductory keys are used. In either system the repetitions will show no constancy as regards intervals between them. Ciphertext autokey systems may be distinguished from plaintext autokey by the appearance of the frequency distribution of the second member of sets of two letters separated by the length of the introductory key (see subpar. 23*e*)—in the case of ciphertext autokey these frequency distributions will be monoalphabetic; in plaintext autokey systems such frequency distributions will not show monoalphabetic characteristics. Ciphertext autokey traffic should be replete with isomorphs; on the other hand, causal isomorphs cannot occur in plaintext autokey systems.

[20] The theoretical γ I.C. is in fact 1.63. (See also subpar. 89*b* on pp. 261–262 of *Military Cryptanalytics, Part II*.)

[21] Actually, only 999,990 trials, since we exclude the ten trivial cases wherein all the key digits are identical. This results in cutting down the work by a factor of one one-hundred-thousandth.

c. Whereas the expected I.C. of ciphertext autokey traffic is 1.00 (i.e., random), the I.C. of plaintext autokey will show a measurable departure from random. The "bulge" (i.e., the excess over 1.00) of the over-all I.C. may be calculated by the formula $\beta_c = \frac{\beta_k \times \beta_p}{c-1}$, where β_c is the bulge of the cipher text, β_p the bulge of the plain text (=.73), β_k the bulge of the key (which, since the key text is plain text, is also .73), and c is the number of categories (in this case, 26). Thus the expected bulge of plaintext autokey traffic, without regard to the identity of the components involved, is $\frac{.73 \times .73}{25} = .0213$, so the I.C. is 1.02 in the general case.

d. Where the components are known sequences, it is possible to establish the theoretical distribution for the ciphertext letters as well as the I.C. of the specific case. For example, in the case of direct standard alphabets, we would first construct a *deciphering* square (shown in fragmentary form in Fig. 63*b*, below) from the normal enciphering form (Fig. 63*a*) of the Vigenère square; we then replace the key letters and the plaintext letters within the deciphering square by their relative frequencies (per thousand letters) in

Plain

Key	A	B	C	D	E
A	A	B	C	D	E
B	B	C	D	E	F
C	C	D	E	F	G
D	D	E	F	G	H
E	E	F	G	H	I
X	X	Y	Z	A	B
Y	Y	Z	A	B	C
Z	Z	A	B	C	D

FIGURE 63*a*

Cipher

Key	A	B	C	D	E
A	A	B	C	D	E
B	Z	A	B	C	D
C	Y	Z	A	B	C
D	X	Y	Z	A	B
E	W	X	Y	Z	A
X	D	E	F	G	H
Y	C	D	E	F	G
Z	B	C	D	E	F

FIGURE 63*b*

Cipher

Key		A	B	C	D	E
A	74	74	10	31	42	130
B	10	1	74	10	31	42
C	31	19	1	74	10	31
D	42	5	19	1	74	10
E	130	16	5	19	1	74
X	5	42	130	28	16	34
Y	19	31	42	130	28	16
Z	1	10	31	42	130	28

FIGURE 63*c*

English plain text [22] (Fig. 63*c*), and we multiply the entries in each row by the key-letter value for that row.[23] Since the square is a deciphering square, the sum of the entries in each column will represent the relative frequencies of the *cipher* letters associated with the columns. For example, it will be found that the sum of the cross-multiplied values in the "A" column is 40,467, and that of the "B" column, 35,314; these of course represent frequencies per 1000^2 or 1,000,000 letters. The relative frequencies, reduced to a base of 1000 letters, are shown below:

A	40.5	G	45.9	L	41.4	Q	30.9	V	51.7
B	35.3	H	41.8	M	43.5	R	47.2	W	47.3
C	31.6	I	50.5	N	30.1	S	42.2	X	40.8
D	25.0	J	30.1	O	28.0	T	41.4	Y	31.5
E	47.5	K	35.8	P	34.3	U	30.4	Z	35.4
F	39.2								999.3

[22] Since the relative frequencies shown are frequencies per thousand, what we have in effect are the probabilities of the letters to three decimals. For reference, the probabilities of English plaintext letters to four decimals are as follows:

A	.0737	G	.0164	L	.0364	Q	.0035	V	.0153
B	.0097	H	.0339	M	.0247	R	.0758	W	.0156
C	.0307	I	.0735	N	.0795	S	.0612	X	.0046
D	.0424	J	.0016	O	.0753	T	.0919	Y	.0193
E	.1300	K	.0030	P	.0267	U	.0260	Z	.0010
F	.0283								1.0000

[23] In the actual computation, a pair of strips bearing the letter frequencies was used, as an equivalent of the bulkier table.

The sum of the squares of the probabilities of these cipher letters is found to be .0398; therefore the theoretical I.C. of plaintext autokey cipher with direct standard alphabets is 26(.0398) = 1.0348.

e. By a similar process, the theoretical ciphertext distribution and the I.C. of plaintext autokey using reversed standard alphabets may be determined. The distribution, expressed in frequencies per 1000, is found to be as follows:[24]

A 66.9	G 34.8	L 45.4	Q 37.7	V 34.4
B 39.9	H 33.9	M 37.3	R 38.1	W 44.1
C 34.1	I 31.3	N 46.7	S 31.3	X 32.1
D 32.1	J 38.1	O 37.3	T 33.9	Y 34.1
E 44.1	K 37.7	P 45.4	U 34.8	Z 39.9
F 34.4				999.8

The sum of the squares of the probabilities here too is .0398, so the I.C. of reversed standard alphabets is the same as that for direct standard alphabets, namely, 1.0348.

f. The foregoing theoretical ciphertext distributions may be used in matching operations for testing samples of cipher text for possibility of encipherment by plaintext autokey with direct or reversed standard alphabets. For a quick test to determine whether a plaintext autokey system involves direct standard alphabets, we note (in subpar. *d*, above) that the cumulative frequencies of the highest four cipher letters, VIER, sum to 197, and that the cumulative frequencies of the lowest four letters, JNOD, sum to 113; the ratio of 197 to 113, or 1.7, compared with the random expectancy of 1.0, is indicative that direct standard alphabets may have been used. Similarly, for a quick test to determine whether a plaintext autokey system involves reversed standard alphabets, we observe (in subpar. *e*) that the frequencies of the four highest letters, ANLP, sum to 204, and that the frequencies of the four lowest, DXIS, sum to 127; the ratio of 204 to 127, or 1.6, compared with the expectancy for random of 1.0, may be employed to establish the use of reversed standard alphabets. (Note that the indices 1.7 and 1.6 will obtain in these situations, regardless of the lengths of the introductory keys, or whether the traffic consists of messages in the same introductory key or in many different keys.)

g. The digital plaintext autokey systems illustrated in par. 36 all involved the *additive* method in their encipherment, wherein P+K=C. It was stated in the preceding volume[25] that an additive system may also be solved as a *subtractive* system (i.e., wherein P−K=C) or as a *minuend* system (i.e., wherein K−P=C), and that any one of these may be solved as either of the other two. Although this is true in the general cases of numerical polyalphabetic substitution, it is not true in the case of plaintext autokey systems: here an additive system can only be solved as an additive system, but a subtractive system may be solved as a minuend system, and a minuend system solved as a subtractive system, as is about to be demonstrated.

(1) Let us consider three different encipherments of the same intermediate plain text by plaintext autokey and with the same 1-digit introductory key, first by the additive method (Fig. 64*a*), then by the subtractive method (Fig. 64*b*), and finally by the minuend method (Fig. 64*c*):

```
    Additive encipherment       Subtractive encipherment      Minuend encipherment
K: 0 4 3 1 5 1 3 3 4 3      K: 0 4 3 1 5 1 3 3 4 3      K: 0 4 3 1 5 1 3 3 4 3
P: 4 3 1 5 1 3 3 4 3 3      P: 4 3 1 5 1 3 3 4 3 3      P: 4 3 1 5 1 3 3 4 3 3
C: 4 7 4 6 6 4 6 7 7 6      C: 4 9 8 4 6 2 0 1 9 0      C: 6 1 2 6 4 8 0 9 1 0
```

FIGURE 64*a* FIGURE 64*b* FIGURE 64*c*

It will be noted that the subtractive and minuend encipherments are complementary. Now let us see what happens when the cipher text of each type of encipherment is assumed to have been produced by the additive, subtractive, and minuend methods in turn, from the standpoint of their particular generatrix diagrams.

[24] Note the symmetry of the distribution about A_c and N_c: the frequencies of M and O are identical, as are L and P, K and Q, etc.

[25] Cf. subpar. 84*b* on p. 238 of *Military Cryptanalytics, Part II*.

(2) First, the cipher text of Fig. 64*a* (the true additive encipherment), under the assumption of encipherment by the three methods, with an arbitrary Ø as the first plaintext digit:

```
      Additive trial            Subtractive trial           Minuend trial
K:    0 7 7 9 7 7 9 8 9     K:    0 7 1 7 3 7 3 0 7     K:    0 3 9 3 7 3 7 0 3
C:  4 7 4 6 6 4 6 7 7 6     C:  4 7 4 6 6 4 6 7 7 6     C:  4 7 4 6 6 4 6 7 7 6
P:  0 7 7 9 7 7 9 8 9 7     P:  0 7 1 7 3 7 3 0 7 3     P:  0 3 9 3 7 3 7 0 3 7

    0 7 7 9 7 7 9 8 9 7         0 7 1 7 3 7 3 0 7 3         0 3 9 3 7 3 7 0 3 7
    1 6 8 8 8 6 0 7 0 6         1 8 2 8 4 8 4 1 8 4         1 4 0 4 8 4 8 1 4 8
    2 5 9 7 9 5 1 6 1 5         2 9 3 9 5 9 5 2 9 5         2 5 1 5 9 5 9 2 5 9
    3 4 0 6 0 4 2 5 2 4         3 0 4 0 6 0 6 3 0 6         3 6 2 6 0 6 0 3 6 0
    4 3 1 5 1 3 3 4 3 3         4 1 5 1 7 1 7 4 1 7         4 7 3 7 1 7 1 4 7 1
    5 2 2 4 2 2 4 3 4 2         5 2 6 2 8 2 8 5 2 8         5 8 4 8 2 8 2 5 8 2
    6 1 3 3 3 1 5 2 5 1         6 3 7 3 9 3 9 6 3 9         6 9 5 9 3 9 3 6 9 3
    7 0 4 2 4 0 6 1 6 0         7 4 8 4 0 4 0 7 4 0         7 0 6 0 4 0 4 7 0 4
    8 9 5 1 5 9 7 0 7 9         8 5 9 5 1 5 1 8 5 1         8 1 7 1 5 1 5 8 1 5
    9 8 6 0 6 8 8 9 8 8         9 6 0 6 2 6 2 9 6 2         9 2 8 2 6 2 6 9 2 6

        FIGURE 65a                  FIGURE 65b                  FIGURE 65c
```

The original plain text of course will appear on one of the generatrices of Fig. 65*a*, but no evidence of either it or its complement in Figs. 85*b* or *c*. Next, the cipher text of Fig. 64*b* (the true subtractive encipherment), under the assumption of encipherment by the three methods:

```
      Additive trial            Subtractive trial           Minuend trial
K:    0 9 9 5 1 1 9 2 7     K:    0 9 7 1 7 9 9 0 9     K:    0 1 3 9 3 1 1 0 1
C:  4 9 8 4 6 2 0 1 9 0     C:  4 9 8 4 6 2 0 1 9 0     C:  4 9 8 4 6 2 0 1 9 0
P:  0 9 9 5 1 1 9 2 7 3     P:  0 9 7 1 7 9 9 0 9 9     P:  0 1 3 9 3 1 1 0 1 1

    0 9 9 5 1 1 9 2 7 3         0 9 7 1 7 9 9 0 9 9         0 1 3 9 3 1 1 0 1 1
    1 8 0 4 2 0 0 1 8 2         1 0 8 2 8 0 0 1 0 0         1 2 4 0 4 2 2 1 2 2
    2 7 1 3 3 9 1 0 9 1         2 1 9 3 9 1 1 2 1 1         2 3 5 1 5 3 3 2 3 3
    3 6 2 2 4 8 2 9 0 0         3 2 0 4 0 2 2 3 2 2         3 4 6 2 6 4 4 3 4 4
    4 5 3 1 5 7 3 8 1 9         4 3 1 5 1 3 3 4 3 3         4 5 7 3 7 5 5 4 5 5
    5 4 4 0 6 6 4 7 2 8         5 4 2 6 2 4 4 5 4 4         5 6 8 4 8 6 6 5 6 6
    6 3 5 9 7 5 5 6 3 7         6 5 3 7 3 5 5 6 5 5         6 7 9 5 9 7 7 6 7 7
    7 2 6 8 8 4 6 5 4 6         7 6 4 8 4 6 6 7 6 6         7 8 0 6 0 8 8 7 8 8
    8 1 7 7 9 3 7 4 5 5         8 7 5 9 5 7 7 8 7 7         8 9 1 7 1 9 9 8 9 9
    9 0 8 6 0 2 8 3 6 4         9 8 6 0 6 8 8 9 8 8         9 0 2 8 2 0 0 9 0 0

        FIGURE 65d                  FIGURE 65e                  FIGURE 65f
```

The original plain text will of course appear on one of the generatrices of Fig. 65*c*, and the complements of the true plain on one of the generatrices of the minuend trial; the additive trial yields nothing. Finally the cipher text of Fig. 64*c* (the true minuend encipherment), under the assumption of the three enciphering methods:

Additive trial

```
K:   0 1 1 5 9 9 1 8 3
C: 6 1 2 6 4 8 0 9 1 0
P: 0 1 1 5 9 9 1 8 3 7

   0 1 1 5 9 9 1 8 3 7
   1 0 2 4 0 8 2 7 4 6
   2 9 3 3 1 7 3 6 5 5
   3 8 4 2 2 6 4 5 6 4
   4 7 5 1 3 5 5 4 7 3
   5 6 6 0 4 4 6 3 8 2
   6 5 7 9 5 3 7 2 9 1
   7 4 8 8 6 2 8 1 0 0
   8 3 9 7 7 1 9 0 1 9
   9 2 0 6 8 0 0 9 2 8
```

FIGURE 65*g*

Subtractive trial

```
K:   0 1 3 9 3 1 1 0 1
C: 6 1 2 6 4 8 0 9 1 0
P: 0 1 3 9 3 1 1 0 1 1

   0 1 3 9 3 1 1 0 1 1
   1 2 4 0 4 2 2 1 2 2
   2 3 5 1 5 3 3 2 3 3
   3 4 6 2 6 4 4 3 4 4
   4 5 7 3 7 5 5 4 5 5
   5 6 8 4 8 6 6 5 6 6
   6 7 9 5 9 7 7 6 7 7
   7 8 0 6 0 8 8 7 8 8
   8 9 1 7 1 9 9 8 9 9
   9 0 2 8 2 0 0 9 0 0
```

FIGURE 65*h*

Minuend trial

```
K:   0 9 7 1 7 9 9 0 9
C: 6 1 2 6 4 8 0 9 1 0
P: 0 9 7 1 7 9 9 0 9 9

   0 9 7 1 7 9 9 0 9 9
   1 0 8 2 8 0 0 1 0 0
   2 1 9 3 9 1 1 2 1 1
   3 2 0 4 0 2 2 3 2 2
   4 3 1 5 1 3 3 4 3 3
   5 4 2 6 2 4 4 5 4 4
   6 5 3 7 3 5 5 6 5 5
   7 6 4 8 4 6 6 7 6 6
   8 7 5 9 5 7 7 8 7 7
   9 8 6 0 6 8 8 9 8 8
```

FIGURE 65*i*

Here the original plain text will appear on one of the generatrices of Fig. 65*i*, as expected, while the complements of the true plain appear on one of the generatrices of the subtractive trial, with nothing manifested in the additive trial. In the foregoing cases, the original plain text will yield the reconstructed matrix of Fig. 66*a*, below, whereas the complementary plain text will yield the equivalent matrix of Fig. 66*b*.

	1	2	3	4	5
1	A	B	C	D	E
2	F	G	H	I	K
3	L	M	N	O	P
4	Q	R	S	T	U
5	V	W	X	Y	Z

FIGURE 66*a*

	9	8	7	6	5
9	A	B	C	D	E
8	F	G	H	I	K
7	L	M	N	O	P
6	Q	R	S	T	U
5	V	W	X	Y	Z

FIGURE 66*b*

h. Digital *ciphertext* autokey systems also have their restrictions in their analysis as regards their type of encipherment: here a subtractive system can only be solved as a subtractive system, but an additive system may be solved as a minuend system, or vice versa. As an example, let us examine three different encipherments of the same plain text by ciphertext autokey, by the additive method (Fig. 67*a*), the subtractive method (Fig. 67*b*), and the minuend method (Fig. 67*c*):

Additive encipherment

```
K: 0 4 7 8 3 4 7 0 4 7
P: 4 3 1 5 1 3 3 4 3 3
C: 4 7 8 3 4 7 0 4 7 0
```

FIGURE 67*a*

Subtractive encipherment

```
K: 0 4 9 2 3 8 5 8 6 7
P: 4 3 1 5 1 3 3 4 3 3
C: 4 9 2 3 8 5 8 6 7 6
```

FIGURE 67*b*

Minuend encipherment

```
K: 0 6 3 2 7 6 3 0 6 3
P: 4 3 1 5 1 3 3 4 3 3
C: 6 3 2 7 6 3 0 6 3 0
```

FIGURE 67*c*

(Here it will be noted that the additive and minuend encipherments are complementary.) Now we shall see what happens when the cipher text of each case is assumed to have been produced by the additive, subtractive, and minuend methods, in turn. First, the cipher text of Fig. 67*a* (the true additive encipherment), under the assumption of encipherment by the three methods:

	Additive trial		*Subtractive trial*		*Minuend trial*
K:	4 7 8 3 4 7 0 4 7	K:	4 7 8 3 4 7 0 4 7	K:	4 7 8 3 4 7 0 4 7
C:	4 7 8 3 4 7 0 4 7 0	C:	4 7 8 3 4 7 0 4 7 0	C:	4 7 8 3 4 7 0 4 7 0
P:	3 1 5 1 3 3 4 3 3	P:	1 5 1 7 1 7 4 1 7	P:	7 9 5 9 7 7 6 7 7
	FIGURE 68*a*		FIGURE 68*b*		FIGURE 68*c*

The original plain text appears in the additive trial, its complement in the minuend trial, and nothing in Fig. 68*b*. Next, the cipher text of Fig. 67*b* (the true subtractive encipherment), under the assumption of encipherment by the three methods:

	Additive trial		*Subtractive trial*		*Minuend trial*
K:	4 9 2 3 8 5 8 6 7	K:	4 9 2 3 8 5 8 6 7	K:	4 9 2 3 8 5 8 6 7
C:	4 9 2 3 8 5 8 6 7 6	C:	4 9 2 3 8 5 8 6 7 6	C:	4 9 2 3 8 5 8 6 7 6
P:	5 3 1 5 7 3 8 1 9	P:	3 1 5 1 3 3 4 3 3	P:	5 7 9 5 3 7 2 9 1
	FIGURE 68*d*		FIGURE 68*e*		FIGURE 68*f*

Here the original plain text comes out under the subtractive trial, but nothing in Figs. 68*d* or *f*. Finally, the cipher text of Fig. 67*c* (the true minuend encipherment), under the assumption of encipherment by the three methods:

	Additive trial		*Subtractive trial*		*Minuend trial*
K:	6 3 2 7 6 3 0 6 3	K:	6 3 2 7 6 3 0 6 3	K:	6 3 2 7 6 3 0 6 3
C:	6 3 2 7 6 3 0 6 3 0	C:	6 3 2 7 6 3 0 6 3 0	C:	6 3 2 7 6 3 0 6 3 0
P:	7 9 5 9 7 7 6 7 7	P:	9 5 9 3 9 3 6 9 3	P:	3 1 5 1 3 3 4 3 3
	FIGURE 68*g*		FIGURE 68*h*		FIGURE 68*i*

The original plain text here comes out under the minuend trial, its complement under the additive trial, and nothing in Fig. 68*h*. As before, the original plain text will yield the matrix of Fig. 66*a*, the complementary plain text the matrix of Fig. 66*b*.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

CHAPTER VI

SYSTEMS EMPLOYING LONG OR CONTINUOUS KEYS

36. Preliminary remarks.—*a*. In subpar. 1*d* it was stated that two procedures suggest themselves to cryptographers for eliminating the weaknesses introduced by periodicity of the type produced by simple, repeating-key methods. One of these, when studied, embraced some of the very simple methods of suppressing or destroying periodicity, by such devices as interrupting the key and using variable-length groupings of plain text. It was demonstrated that subterfuges of this simple nature are inadequate to eliminate the weaknesses referred to, and must be discarded in any system intended to afford real security. The other alternative alluded to in subpar. 1*d* therefore remains now to be investigated, *viz.*, that of lengthening the keys to a point where there would seem to be an insufficient amount of text to enable the cryptanalyst to solve the traffic. Attempts toward this end usually consist in extending the key to such a length that the enemy cryptanalysts will have only a very limited number of periods to work with. The key may, indeed, be lengthened to a point where it becomes as long as, or longer than, the text to be enciphered, so that the key is used only once in a message.

b. It is obvious that one of the simplest methods of lengthening the key to a message is to use a long phrase or even a complete sentence, provided that it is not too long to remember. In addition to the difficulties that would be encountered in practical military cryptography in selecting long mnemonic phrases and sentences which would have to be imparted to many clerks, there is the fact that the probable-word method of solution still remains as a powerful tool in the hands of the cryptanalyst. And if only a word or two of the key can be reconstructed as a result of a fortunate assumption, it might be possible that the cryptanalyst could guess the entire key from a fragment thereof, since any long phrase or sentence which is selected because it can easily be remembered is likely to be well known to many people.

c. There are, however, relatively simple ways of employing a short mnemonic key in order to produce a much longer key. Basically, any transposition method applied to a single alphabetic sequence repeated several times will yield a fairly long key, which, moreover, has the advantage of being unintelligible, thus approaching the appearance of a random selection of letters. For example, a numerical key may be derived from a word or a short phrase; this numerical key may then be applied as a columnar-transposition key for a rectangle within which the normal A–Z sequence has been repeated a previously agreed-upon number of times in straight horizontals starting at the upper left-hand corner, or in any other prearranged manner. The letters when transcribed from the transposition rectangle then become the successive letters for enciphering the plain text, using any desired type of primary components. Or, if a single transposition is not thought to be sufficiently secure, a double transposition will yield a still more mixed-up sequence of key letters. Other types of transposition may be employed for the purpose, including various types of geometric figures. Furthermore, some nontransposition methods of lengthening the keying sequence which at the same time introduce an irregularity, such as aperiodic interruption, have already been described (see subpar. 14*b*).

d. Another method of developing a long key from a short mnemonic one is that shown below. Given the key word CHRISTMAS, a numerical sequence is first derived and then the successive sections of this numerical key are written down, these sections terminating with the successive key numbers 1, 2, 3, . . . of the numerical key. Thus:

Mnemonic key: C H R I S T M A S
Numerical key: 2 3 6 4 7 9 5 1 8

Extended key: C H R I S T M A[1] | C[2] | C H[3] | C H R I[4] | C H R I S T M[5] | C H R[6] | C H R I S[7] |
C H R I S T M A S[8] | C H R I S T[9] |

Thus the original key of only 9 letters is expanded to one of 45 letters (1+2+3+ . . . +9=45). The longer key is also an interrupted key of the type noted under subpar. 14*a*, but if the message is long enough to require several repetitions of the expanded key, the encipherment becomes periodic and can be handled by the usual methods employed in solving repeating-key ciphers. If the basic key is fairly long,[1] so that the expanded key becomes a quite lengthy sequence, then the message or messages may be handled in the manner explained in par. 19.

e. One method for producing a rather long sequence of digits for keying purposes from a single key number is to select a number whose reciprocal when converted by actual division into its equivalent decimal yields a long series of digits. For example, the reciprocal of 49, or 1/49, yields a sequence of 42 digits beginning .0204081632 . . . . Such a number, coupled with a key word like CHRISTMAS, could be used for interrupted keying, the successive cipher alphabets being used for enciphering as many letters as are indicated by the successive digits. In the case of the example cited, the first digit is Ø; hence the C alphabet would not be used. The next digit is 2; the H alphabet would be used for enciphering the first and second letters. The third digit is again Ø; the R alphabet would not be used. The fourth digit is 4; the I alphabet would be used for enciphering the third, fourth, fifth, and sixth letters. And so on.

f. In the case of digital cryptosystems, various methods have been used to produce an expanded key from a shorter one. Several examples will now be cited.

(1) The method in subpar. *d*, above, may be adapted so that from the key word CHRISTMAS the basic numerical key 236479518 is expanded into the 45-digit sequence 2 3 6 4 7 9 5 1|2|2 3| 2 3 6 4|2 3 6 4 7 9 5|2 3 6|2 3 6 4 7|2 3 6 4 7 9 5 1 8|2 3 6 4 7 9. Or, this 9-digit basic key may be expanded into an 81-digit key by tying together 9 cycles of the basic key, beginning with 1 8 2 3 6 4 7 9 5|2 3 6 4 7 9 5 1 8|3 6 . . . and ending with |9 5 1 8 2 3 6 4 7. Or, again, the 9-digit key may be expanded into a 90-digit key by adding 1 to the digits of the basic key at the end of each cycle, so that the final key will be 2 3 6 4 7 9 5 1 8|3 4 7 5 8 0 6 2 9|4 5 . . . .

(2) Another method for producing digital key is by a "Fibonacci series," [2] which is a series or sequence generated by the successive addition (almost invariably mod 10, or, in certain specialized usages mod 2) of pairs or larger combinations of elements (usually adjacent), beginning with a specified initial key number. Thus, in its simplest form, if the initial key is the dinome 01, we can generate the Fibonacci sequence 0112358314 . . . , which has a cycle length of 60. If the initial key is the pentanome 00001 and the rule of combination is a+b=f (i.e., the sum of the first and second elements of the sequence

[1] For example, if the basic sequence were a 26-letter mixed sequence such as HBPYEQDFSRGTAJVUKWLMXINZCO based on HYDRAULIC, then the expanded key would read

H B P Y E Q D F S R G T A[1] | H B[2] | H B P Y E Q . . . C[3] | H B P etc.

The length of the expanded key is the sum of the first n numbers, given by the formula $\Sigma(n)=1+2+3+\ldots+n=\frac{n(n+1)}{2}$; since n=26, the total key length is 351. A possible alternative is the construction of a 676-long key by fitting together 26 cycles of the basic sequence, the successive cycles beginning with A, B, C, . . . (or H, Y, D, . . ., or any other scheme for including all 26 letters for the starts of the cycles).

[2] This series is named after Leonardo of Pisa (ca. 1170–1248), also known as Fibonacci (i.e., "filius Bonacci," or "son of Bonacci"), who first investigated its properties in connection with the proliferative activities of rabbits.

equals the sixth), at the end of the first 50 digits the sequence will read 1542869604 . . . , continuing for a cycle of 16,401 digits; on the other hand, with this same initial pentanome and the rule of combination a+c+d+e=f, the cycle of 19,344 digits is reached, maximum for this length of initial key.[3] The mathematics involved here is related to the mathematics of shift registers (see subpar. 58*h*); in fact, some shift register feedback rules are called "Fibonacci rules." The shift register cases all involve mod-2 arithmetic; the mod-10 version, as illustrated here, has additional complications.

g. In connection with the subject of extensive or lengthy keys is the cipher system known as the *running-key*, *continuous-key*, or *nonrepeating-key* system, in which the key consists of a sequence of elements which never repeats, no matter how long the message to be enciphered happens to be. Although any continuous text could be used, the most common and most practical source of such a key is that in which the plain text of a previously agreed-upon book serves as the source for successive key letters for encipherment;[4] or, in a numerical cryptosystem, the key could be obtained from various mathematical tables or even from a telephone directory. Even though in a running-key system the key for an individual message may be as long as the message and never repeats, nevertheless, if a large group of correspondents employ the same key sequence, it may happen that there will be several messages in the same key, and they will all begin at the same position in the keying sequence; or there might be several messages which will overlap one another with respect to the key, even if they begin at different points in the keying sequence. These situations form the basis of solution of systems of this genre.

h. In addition to the foregoing manual methods, there are a great many mechanical methods of producing a long key, such as those employed in mechanical or electrical cipher machines. In most cases these methods depend upon the interaction of two or more short, primary keys which jointly produce a single, much longer, secondary or resultant key (see par. 4). Only brief reference will be made at this point in the cryptanalytic studies to cases of this kind; the matter will be taken up in detail in Chapter IX.

[3] The mathematics of Fibonacci cycles involves complicated aspects of algebra and number theory, and there is no short cut to the determination of the cycle length, given any specified initial key and rule of combination. For the interested reader, the table below gives the cycle lengths for all rules of combination (always including the *a* digit) for pentanomic initial keys; the number in parentheses indicates the number of different cycles of that length. (For a stated rule of combination, the initial group 00001 always gives rise to a maximum cycle.)

a+b=f	a+c=f	a+d=f	a+e=f	a+b+c=f	a+b+d=f	a+b+e=f	a+c+d=f
16,401 (4)	744 (129)	6448 (15)	168 (500)	840 (80)	1560 (60)	6248 (8)	9372 (4)
5,467 (4)	124 (32)	208 (15)	84 (33)	168 (100)	312 (20)	3124 (12)	4686 (4)
2,343 (4)	31 (1)	124 (1)	42 (16)	140 (20)	30 (4)	1562 (4)	3124 (4)
781 (4)	24 (1)	31 (1)	28 (1)	120 (80)	15 (2)	781 (8)	2343 (8)
21 (1)	1 (1)	4 (1)	24 (500)	28 (24)	2 (4)	8 (2)	1562 (4)
7 (1)		1 (1)	21 (1)	24 (100)	1 (2)	4 (3)	781 (8)
3 (1)			12 (33)	20 (20)		2 (1)	12 (1)
1 (1)			7 (1)	14 (1)		1 (2)	6 (1)
			6 (15)	7 (2)			4 (1)
			5 (1)	4 (24)			3 (2)
			4 (1)	2 (1)			2 (1)
			3 (1)	1 (2)			1 (2)
			1 (2)				

a+c+e=f	a+d+e=f	a+b+c+d=f	a+b+c+e=f	a+b+d+e=f	a+c+d+e=f	a+b+c+d+e=f
1860 (48)	10,934 (4)	744 (129)	744 (129)	1240 (72)	19,344 (5)	4686 (16)
930 (4)	5,467 (8)	124 (32)	124 (32)	620 (6)	624 (5)	2343 (8)
372 (16)	1,562 (4)	31 (1)	31 (1)	248 (15)	124 (1)	781 (8)
62 (4)	781 (8)	24 (1)	24 (1)	124 (1)	31 (1)	6 (4)
60 (12)	14 (1)	1 (1)	1 (1)	40 (72)	4 (1)	3 (2)
15 (2)	7 (2)			31 (1)	1 (1)	1 (2)
12 (4)	2 (1)			20 (6)		
1 (2)	1 (2)			8 (15)		
				4 (1)		
				1 (1)		

[4] The key is either the plain text itself as key letters, or a transformation of the plaintext key into digits, say, by enciphering the plain text through a monome-dinome rectangle. See in this connection Appendix "A" (on pp. 250–256) of Alexánder Foote's *Handbook for Spies*, New York, 1949.

i. Finally, there must be mentioned certain devices in which, as in encipherment by the autokey method, the text itself serves to produce the variation in cipher equivalents, by controlling the selection of secondary alphabets, or by influencing or determining the sequence in which they will be employed. Naturally, in such cases the key is automatically extended to a point where it coincides in length with that of the text. An excellent example of such a device is the Wheatstone cipher device,[5] the solution of which will be treated in Chapter VIII.

37. Depth and its exploitation.—*a.* The concept of depth was introduced in the previous text;[6] it was pointed out that, unless special circumstances are present, the re-use of key, either within a message or among several messages, is a necessary condition to the solution of polyalphabetic ciphers The sole exception is the case wherein the components are known and the continuous, nonrepeating key is plain text (or has otherwise recognizable systematic or exploitable nonrandom properties); correct cribs in either the plain text or the key will enable a solution to be effected—but this of course in reality is tantamount to a depth of two[redacted].

b. The solution of a single message in a periodic polyalphabetic cipher involving unknown primary components rests on the ability of the cryptanalyst to superimpose cycles of the period into a cryptographic *depth,* i.e., so that all the cipher letters in the individual columns of the depth diagram belong to individual monoalphabetic distributions.[redacted]

d. We shall now take up the solution of cryptograms involving long or continuous keys under the two general cases: (1) those wherein the primary components are known sequences, and (2) those wherein they are unknown.

38. Solution of a single cryptogram involving known primary components and an unknown plaintext running key.—*a.* This is a trivial case, since what we have is really a depth of *two* (i.e., the plain text of the message, and the plain text constituting the key); if the primary components are known sequences, a correct assumption of plain text in either the cipher or the key will produce plain text in the other. Furthermore, if a novel or other book is used as key, upon the recovery of sufficient key text the cryptanalyst might be able to identify the particular book that is being used for the key, which would greatly simplify his subsequent efforts.

b. As an example, let us suppose that we have at hand the following short cryptogram, assumed to have been enciphered with direct standard alphabets and a plaintext running key:

```
EUGEM  NLLHG  BYDQV  SXPMF  AAGZJ  BKWAX
FRHFA  QAAHK  GSLIF  LXCWV  REKCQ  DYWMF
XKLAC  YDRLL  GICCE  XAVOQ  HRRHF  RPTNL
```

The procedure here is to try sliding probable words or other plaintext polygraphs through the cipher text, seeing what plausible key fragments are produced thereby, and expanding the placed crib and its

---

[5] Some writers classify and treat this method, as well as autokey methods, as forms of the running-key system, but it is preferable to consider the latter as being radically different in principle from the former types, because in the true running-key system the key is wholly *external to and independent of* the text being enciphered. This is hardly true of autokey systems or of the Wheatstone device.

[6] Cf. par. 65, *Military Cryptanalytics, Part II.*

[redacted]

[8] Cf. subpars. 14*b* and *c* of Chapter III.

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associated key elements. For instance, if we slide TION through the text, we would construct the diagram shown in Fig. 69, below (under the assumption of A_p as the index letter): [9]

```
   E U G E M N L L H G B Y D Q V S X P M F A A G Z J B K W A X F R H F A
T |L B N L T U S S O N I F K X C Z E W T M H H N G Q I R D H E M Y O M H
I |  M Y W E F D D Z Y T Q V I N K P H E X S S Y R B T C O S P X J Z X S
O |    S Q Y Z X X T S N K P C H E J B Y R M M S L V N W I M J R D T R M
N |      R Z A Y Y U T O L Q D I F K C Z S N N T M W O X J N K S E U S N

  (F R H F A)Q A A H K G S L I F L X C W V R E K C Q D Y W M F X K L A C
T |M Y O M H X H H O R N Z S P M S E J D C Y L R J X K F D T M E R S H J
I |X J Z X S I S S Z C Y K D A X D P U O N J W C U I V Q O E X P C D S U
O |R D T R M C M M T W S E X U R X J O I H D Q W O C P K I Y R J W X M O
N |S E U S N D N N U X T F Y V S Y K P J I E R X P D Q L J Z S K X Y N P

  (X K L A C)Y D R L L G I C C E X A V O Q H R R H F R P T N L
T |E R S H J F K Y S S N P J J L E H C V X O Y Y O M Y W
I |P C D S U Q V J D D Y A U U W P S N G I Z J J Z X J H L
O |J W X M O K P D X X S U O O Q J M H A C T D D T R D B F Z
N |K X Y N P L Q E Y Y T V P P R K N I B D U E E U S E C G A Y
```

FIGURE 69

Several candidates for plaintext key fragments are noted and are marked in the diagram, the best one perhaps being SARY, as part of the word NECESSARY. Key and plain text are expanded simultaneously, as shown in the following successive steps:

```
                25        30        35        40        45        50
K:                                  s n e c e s S A R Y f o r
C: . . . A A G Z J B K W A X F R H F A Q A A H K G S L I F L X C W V . . .
P:                                  i n d i c a T I O N s o f

                          * * * * *
                25        30        35        40        45        50
K:                        i t b e c o m e S N E C E S S A R Y F O R
C: . . . A A G Z J B K W A X F R H F A Q A A H K G S L I F L X C W V . . .
P:                        s e e n f r o m I N D I C A T I O N S O F

                          * * * * *
                25        30        35        40        45        50
K:                e n t s I T B E C O M E S N E C E S S A R Y F O R
C: . . . A A G Z J B K W A X F R H F A Q A A H K G S L I F L X C W V . . .
P:                f o r e S E E N F R O M I N D I C A T I O N S O F
```

At this point we assume a verb before the word FORESEEN, and we try IS (yielding the key YH), WAS ($=EGH_k$), ARE ($=APV_k$), BE ($=FV_k$), among others. The $E_p=V_k$ looks promising, however, and we suddenly recognize the key text as coming from the Constitution of the United States. The problem is now solved:

```
           5         10        15        20        25        30
K: W H E N I N T H E C O U R S E O F H U M A N E V E N T S I T
C: E U G E M N L L H G B Y D Q V S X P M F A A G Z J B K W A X . . .
P: I N C R E A S E D E N E M Y R E S I S T A N C E F O R E S E
```

Note that we could also have solved a problem such as this by assuming key text, and seeing what develops in the decipherment of the cipher text.

[9] Cf. subpar. 22*d* on pp. 41–42 of *Military Cryptanalytics, Part II.*

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(b)(3)-P.L. 86-36

[redacted]

(b)(1)
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(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
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(The digraphs obtained from reversals are shown in parentheses.) The cipher component may now be chained out and reconstructed in its original form, and the plain component can be recovered in a few moments by taking advantage of the now-known cipher component. The completion of the solution, including recovery of the key for the messages, is again left to the reader as an exercise.

41. Recovery of plain texts and the unknown primary components from a number of messages in flush depth.—*a*. In the previous text [11] it was noted that if there were available a sufficient number of messages (say 25–30, for English) all enciphered beginning at the same point in a long key, the messages would be in flush depth, and the frequency distributions of the columns of the superimposition diagram could be solved, without knowing the length of the key.[12] It was further noted that even if an extremely long key is employed and a series of messages beginning at different initial points is enciphered by such a key, this method of solution by superimposition can be employed, provided that the messages can be superimposed correctly—that is, so that the letters which fall in one column really belong to one cipher alphabet. Messages may be put into depth as a result of the solution of indicators, or by means of long ciphertext repetitions, or by means of the kappa (κ) test.

b. The kappa test has been treated at some length in the previous volume; a review at this time might not be amiss.[13] The kappa test, or its equivalent form, the kappa I.C., involves nothing more than counting hits between two superimposed messages, and evaluating the result. We know, for instance, that since the repeat rate for English (i.e., the κ_p) is .0667, if two 1000-letter messages are properly superimposed with respect to their keying sequence (i.e., so that both letters of a vertical pair have been enciphered by the same key letter), we should expect 66 or 67 hits between them if the underlying text is English; and if these same messages are not correctly superimposed, the repeat rate for random ($\kappa_r = 1/26 = .0385$) should prevail, so that the expected number of hits will then be 38 or 39. A demonstration of the application of the kappa test will now be given.

[redacted]

[11] Cf. par. 65 (on p. 157) of *Military Cryptanalytics, Part II*.

[12] Prob. 9 (on p. 608) of Section G, Appendix 9, *Military Cryptanalytics, Part II*.

[13] Cf. par. 98 on pp. 302–308 of *Military Cryptanalytics, Part II*.

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(b)(1)
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(b)(1)
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CHAPTER VII

CYLINDRICAL CIPHER DEVICES AND STRIP CIPHER SYSTEMS

45. General.—*a.* The cryptography of cylindrical cipher devices and strip cipher systems was treated briefly in the previous text.[1] In the first published description of a cylindrical cipher device,[2] the apparatus consisted of 20 numbered rotatable disks, each bearing a different alphabet engraved on its periphery. The disks were to be arranged in an agreed-upon order on a shaft, and rotated so that the first 20 letters of the message plain text appeared in a row; the cipher text was then formed by arbitrarily taking off *any other row*—thus the basic cryptoprinciple is that of polyalphabetic substitution with the added complication of a *variable generatrix* feature. The remaining letters of the message were to be treated in the same fashion, 20 letters at a time. In decipherment, the disks were first arranged on the shaft according to the agreed-upon order, the first 20 cipher letters set up across one row, and the other rows searched for the one and only one row that contained plain text all the way across; and so on for the rest of the decipherment.

b. Credit for the original invention of the cylindrical cipher device belongs to Thomas Jefferson, among whose papers in the Library of Congress (vol. 232, item 41575, Jefferson's Papers) there was found in 1922 a description of a device incorporating 36 disks, on each of which are placed "all the letters of the alphabet, not in their established order, but jumbled & without order so that no two shall be alike." The device was independently invented for the third time by Captain Parker Hitt, U.S. Army, who in 1914 conceived a device of 25 disks, and also a similar apparatus of flat strips, cryptographically equivalent to the disk device. Hitt's alphabets were not very thoroughly mixed and bore many relationships among them so as to enable their reconstruction from memory—for example, there were many repetitions of sequences such as AEIOU, BCDFG, and HNRST. Major Joseph O. Mauborgne (later to become Chief Signal Officer of the U.S. Army) in 1917 noted that, because the alphabets were not thoroughly scrambled, the cryptograms produced by Hitt's device were too easily solved; his modification of the device, prescribing that "each disk have on it 26 letters of the alphabet, nonrepetitive on any disk and as far as possible so arranged that the fewest number of repetitions of pairs of letters . . . would occur," resulted in the adoption by the U.S. Army in 1923 of the cylindrical cipher device known as the M–94.

c. A strip cipher system comprises a series of printed, random-alphabet strips to take the place of the disks of a cylindrical cipher device; these strips, bearing identifying numbers, slide freely in the channels of a strip board which may be made of metal, wood, or plastic. The strips are simpler to produce and more economical to replace than metal disks; but, most important of all, with the use of strip cipher systems it is possible to incorporate certain simple modifications of the Jefferson principle (i.e., the variable-generatrix principle) which result in a very considerable augmentation of the security of the system. First of all, in cylindrical cipher devices the number of disks must be limited, for the sake of practicability, and furthermore all n disks (20 in the Bazeries device, 25 in the M–94) are always used together, so that the latent period is always n. But with a strip cipher system, it is possible to employ, say, 100 or more strips in the basic system, and then choose, for example, 30 of these strips for a daily key; furthermore, it is possible to employ a variable keying element *for each message*, so that several of the strips are eliminated before encryption begins, avoiding the possibility of the encryption of two messages in exactly the same key, and in general resulting in a latent flexible periodicity. Finally, because of the ease of production of paper strips, it is possible to have different sets of strips for different groups of holders, or for various types of traffic, thus reducing the volume of traffic in any one cryptosystem.

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[1] See *Military Cryptanalytics, Part II*, pp. 443–444.

[2] Etienne Bazeries, *Les chiffres secrets dévoilés*, Paris 1901.

d. The analysis of cylindrical cipher devices and strip cipher systems involves two main aspects: (1) when the alphabets are unknown, and (2) when the alphabets are known, but their specific order is unknown. These cases will be discussed in the succeeding paragraphs.

46. Reconstruction of unknown cipher alphabets.—*a.* Let us assume that we have at hand the compromised plain text to a long cipher message, known to involve a cylindrical cipher device or strip cipher system of 25 alphabets.[3] The matched plain and cipher, written on a width of 25, is shown below:

1.	THEFO	LLOWI	NGALT	ERNAT	EPLAN	21.	AGENE	RALLY	PARAL	LELOF	ROADS
	EPWWZ	VIUFT	EVTJB	MCFIH	IBRSP		ZJAKJ	ADJTT	JXULT	BVMEE	WJITJ
2.	SOFDE	FENSE	FORTH	EREGI	MENTA	22.	TOONE	THREE	THREE	THREE	STOPO
	HEETJ	DFCMZ	WKUKR	ZGWAA	GYFKM		QUMWM	AXYFL	MQPGN	GEKKS	JYYXE
3.	LRESE	RVELI	NESAR	EHERE	BYSUB	23.	RDASH	TOEXT	ENDRE	GIMEN	TALDE
	PACCX	ZSSXE	WMVPM	PZZKJ	MSJSF		FGZRN	ZHPZK	UKVQM	OEKXZ	BJWLW
4.	MITTE	DCOLO	NONED	ASHAG	ENERA	24.	FENSI	VELIN	EBYTA	KINGU	PPOSI
	NPEFO	ENNQW	FBZKK	VLLCI	WYQWG		BREPN	WQBAB	YHXVQ	XXXOM	QRRBP
5.	LDEFE	NSEOF	REGIM	ENTAL	RESER	25.	TIONS	INDEF	ENSEO	FNORT	HWEST
	IUQVK	KAANS	VCRGU	BUCHJ	UXXJI		UDLQR	GJUIE	KWXTN	DINVC	AOXUS
6.	VELIN	ESCOM	MAWIT	HDEPL	OYMEN	26.	SLOPE	OFBUC	KHILL	ANDSO	UTHSL
	NPIWL	HZXXE	GKHMV	IOIVF	TQRKS		USXFP	JKUKT	PSQKU	LJRFC	HNXPI
7.	TOFTH	REERI	FLECO	MPANI	ESINL	27.	OPESO	FDISC	UMHIL	LSTOP	FOURD
	CAUGN	MUPHM	OIUOH	FYVPF	TGEBI		HVMNN	WRGCU	QVCWW	MFVDY	OMRCT
8.	INESU	PPORT	EDBYC	OMPAN	YHAND	28.	ASHIN	CASEO	FATTE	MPTED	ENEMY
	LJWUW	JWUGE	WJPQV	WWAIX	PYLQM		YTTQR	WVJXX	QTAFL	EACYQ	LOSTP
9.	USING	THERI	FLECO	MPANY	MACHI	29.	PENET	RATIO	NSTHR	OUGHL	EWISV
	VWZTX	EAQCW	RFFZX	IDHXL	JYJCK		TQBPF	ADQCK	AJDDN	RACOC	AMTVN
10.	NEGUN	SFROM	SELEC	TEDPO	SITIO	30.	ALLEY	DASHT	OCOUN	TERAT	TACKF
	YYQAH	CNHLI	ZUWPA	HIVNG	PXJWH		DGJGZ	JBXPZ	ZPNXF	RDIFY	KLXLR
11.	NSFRO	MTHIS	LINES	TOPTW	ODASH	31.	ROMEA	STSLO	PEOFB	UCKHI	LLORF
	LNSGF	JUGJN	RBOZB	ANNUP	VYJQA		EXCLI	GNHJZ	YQIIE	VVAKJ	KQJVM
12.	DEFEN	SEOFR	EGIME	NTALR	ESERV	32.	ROMSO	UTHWE	STSLO	PEOFH	ERRHI
	QODLJ	GOQBI	QKUVD	RPOMB	QNMVE		TIWTF	PUGNY	EHKHU	HMMAV	KIBLS
13.	ELINE	SSTOP	THREE	DASHI	NCASE	33.	LLSTO	PFIVE	DASHI	NCASE	OFATT
	VGRAA	EFPFU	PTCMW	YTQBS	XVVZZ		SUIYT	ITJED	YFYTS	TQMUH	YIWHA
14.	OFATT	EMPTE	DENVE	LOPME	NTOFO	34.	EMPTE	DENVE	LOPME	NTOFO	URLEF
	LIDNJ	TQJZL	IWCIN	XFLIS	KYYRE		UNXAF	SVXYH	PQIKF	WXRCH	TPPIC
15.	URRIG	HTFLA	NKDAS	HTOCO	UNTER	35.	TFLAN	KDASH	TOEXT	ENDOU	RDEFE
	EYZZL	LDJEZ	TUEGP	ZVDFR	EDXWY		PVGHJ	CACFV	HCBML	NHPHO	JJMMB
16.	ATTAC	KFROM	VICIN	ITYOF	RJONE	36.	NSELI	NEBYT	AKING	UPPOS	ITION
	NKRSH	FCEQN	XXNVE	GFKTG	HRFHO		KYAEA	GFTOB	YGOHA	OTDEQ	MQTMO
17.	THREE	THREE	THREE	MOVIN	GFROM	37.	ONNOR	THEAS	TSLOP	EOFSY	KESHI
	RLDGA	INGNG	PTCMW	XSFUG	FHFNT		ULGGK	JLWLH	KTPGQ	RKLWA	ZATUM
18.	RESER	VEASS	EMBLY	POSIT	IONWE	38.	LLORB	UTLER	HILLO	RTOCO	UNTER
	BLYJE	OVWZI	XCTMT	BZBPI	RFAED		GRISX	FPXWK	BLWWB	PGUQG	NTJPA
19.	STVIA	STONE	WALLC	OMMAS	OUTHW	39.	ATTAC	KFROM	BUTLE	RHILL	ORGEN
	BVKLK	MJVXJ	CHKRQ	AHBLM	FAHIR		QSHOU	NPOBK	UWIJA	QKPKG	WVYVP
20.	ESTVI	ASTON	EWALL	TOROA	DCOMM	40.	ERALL	YALON	GTHEM	ORLAY	TRAIL
	IBVPT	GYDIM	JONEJ	GFKJU	QOYVP		YHGPA	FIMGZ	LSTAK	NUFNN	RUTVS

[3] The diagnosis of a strip cipher system was made from the characteristic frequency distribution—made cumulatively on all the traffic—associated with a noncrashing system (see subpar. 48c); the number of strips involved was determined from the factors of intervals between long polygraphic repetitions found within or between messages.

b. Our first step is to make tabulations of the columnar plain-cipher equivalencies, as a prelude to the establishment of related lines of matched plain and cipher. In these tabulations the vertical plain-cipher digraphs have been recorded (under the reference alphabet) as single-letter entries under the earlier letter in the normal alphabetic sequence, so that, in col. 1 for example, TE and SH are recorded in the inverse form as ET and HS, while LP is recorded as LP. The tabulations for the first five columns of the 40 pairs of lines are shown below, where letters that have occurred at least twice in combination with the reference letter are ringed.

Col. 1:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	N	R	T	Q	T	R	L	S	(L)		N	P	N	V	U	(T)	T	(T)	U	U	V					
	Z	S			V			O	(L)			N		Y		(T)		(T)								
	Y	F			(U)							O														
	D				I							S														
	Q				R																					
					(U)																					
					Y																					

Col. 2:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	R	S		U	(O)	I	(L)	P	P	N	T	S	N	S	U	V		Y	W	V						
	O			G	P	V	J	L	O			U			X				(T)							
				I	V		(L)	R				N							Y							
					(O)							R							(T)							
					L																					
					R																					
					Q																					

Col. 3:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	D	N	E	F	(W)	U	Q	(T)	L	L	V	O	O		X	X		Z	Y	V						
	(E)		M	R	F	S	L	(T)	Z				W					T								
	Z				T		N		R																	
	(E)				Q				S																	
	G				(W)				O																	
					N																					
					M																					

Col. 4:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	U		S	T	(L)	W	T		W		N	P		(T)		V		(S)	U	Y						
	N				(G)	T	R		Z					(T)		S		(S)	T							
	S				J	V	O		L					W												
	T				P	P			Q					Q												
	H				(G)									S												
	O				(L)																					
					(L)																					

Col. 5:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	(E)	X	H		(J)	(O)	X	(N)	T	(N)	R	N		O	Z			S			W				Z	
	(E)		U		X	T	L	(N)	N	T				R	T											
	K				O	(O)		(N)		(N)																
	(I)				K																					
	(I)				R																					
	L				(J)																					
					M																					
					P																					
					F																					

* * * * * * *

c. The summary of related lines of matched plain and cipher from the foregoing five tabulations is given below:[4]

Col. 1:	EU	15, $\tilde{34}$
	PT	29, $\tilde{35}$
	RT	17, $\tilde{32}$
	IL	5, $\tilde{8}$
Col. 2:	EO	2, $\tilde{12}$
	GL	13, 30
	ST	28, $\tilde{39}$
Col. 3:	AE	21, 36
	EW	1, 8
	HT	28, $\tilde{39}$
Col. 4:	EL	12, 31, $\tilde{36}$
	EG	17, 30
	NT	9, $\tilde{14}$
	RS	23, $\tilde{38}$
Col. 5:	AE	13, 17
	AI	31, $\tilde{36}$
	EJ	2, 21
	FO	11, 32
	HN	7, $\tilde{10}$, 23
	JN	12, 35
	* * * * *	

A family of seven related lines (2, $\tilde{12}$, 21, 29, $\tilde{31}$, $\tilde{35}$, 36) is evident among the relationships set forth above; and several more families of related lines may also be built up from the foregoing data. If we examine all 40 lines, we will arrive at the following families:

Family "A": 2, $\tilde{12}$, 21, 29, $\tilde{31}$, $\tilde{35}$, 36
Family "B": 1, $\tilde{5}$, 8, $\tilde{28}$, 39
Family "C": 7, $\tilde{10}$, 23, 26, $\tilde{38}$
Family "D": $\tilde{9}$, 14, $\tilde{16}$, 20, 22
Family "E": $\tilde{11}$, 13, 17, 30, $\tilde{32}$
Family "F": $\tilde{15}$, 18, $\tilde{24}$, 34
Family "G": 3, $\tilde{25}$
Family "H": 19, $\tilde{27}$
Family "I": 37, $\tilde{40}$

Lines 4, 6, and 33 are isolated members of three different families: a maximum of 13 families in all is possible.

d. The relationship among the seven members of Family "A" is made clear when we examine the lines of matched plain and cipher:

```
 2.  S O F D E   F E N S E   F O R T H   E R E G I   M E N T A
     H E E T J   D F C M Z   W K U R R   Z G W A A   G Y F K M
12.  D E F E N   S E O F R   E G I M E   N T A L R   E S E R V
     Q O D L J   G O Q B I   Q K U V D   R P O M B   Q N M V E
21.  A G E N E   R A L L Y   P A R A L   L E L O F   R O A D S
     Z J A K J   A D J T T   J X U L T   B V M E E   W J I T J
29.  P E N E T   R A T I O   N S T H R   O U G H L   E W I S V
     T Q B P F   A D Q C K   A J D D N   R A C O C   A M T V N
31.  R O M E A   S T S L O   P E O F B   U C K H I   L L O R F
     E X C L I   G N H J Z   Y Q I I E   V V A K J   K Q J V M
35.  T F L A N   K D A S H   T O E X T   E N D O U   R D E F E
     P V G H J   C A C F V   H C B M L   N H P H O   J J M M B
36.  N S E L I   N E B Y T   A K I N G   U P P O S   I T I O N
     K Y A E A   G F T O B   Y G O H A   O T D E Q   M Q T M O
```

[4] The tilde (~) over the numbers indicates an inverse relationship between the lines involved; this point is clarified in subpar. *d*, below.

In this diagram, the identities of encipherment in the first five columns have been marked with a solid oval ring, while the inverse (i.e., reciprocal) relationships have been marked with a dotted oval ring. Thus it may be seen that lines 2, 21, 29, and 36 involve the same plain and cipher generatrices, while lines 12, 31, and 35 involve the same generatrices, but interchanged between plain and cipher. This means that for Family "A" we can establish the following two-letter chains as belonging to col. 1 (i.e., alphabet 1):

$\underline{2}$	$\underline{\widetilde{12}}$	$\underline{21}$	$\underline{29}$	$\underline{\widetilde{31}}$	$\underline{\widetilde{35}}$	$\underline{36}$
SH	QD	AZ	PT	ER	PT	NK

(Note that we have reversed the letters of the digraphs from lines 12, 31, and 35 which bear an inverse relationship to the other four lines.)

e. Since Families "A" through "F" contain the most members, we proceed to make chains for the individual alphabets. Below are given the chains for the first five alphabets, as an illustration:

Family "A"

1. SH QD AZ PT ER NK
2. XOEQ GJ VF SY
3. DFEA NB CM GL
4. DT LEP NK HA
5. EJN TF IA

* * * * *

Family "B"

1. TE IL YAQ
2. HP UD NJ TS
3. QEW TH
4. VFW SU QI AO
5. OZ KE CUW RN

* * * * *

Family "C"

1. TC YN RF SU GL
2. OA YE RLS DG
3. FU QG AZ IOX
4. TG AU SR PF
5. HN EP XB

* * * * *

Family "D"

1. VU OL NA EI TQ
2. WSB FI KT OU
3. ZI AD RTV OM
4. TNW SA VP
5. XG ITJ HC EM

* * * * *

Family "E"

1. LN EV TR AD
2. NS HLG IO
3. SF IRD LJ WM
4. EGR NA TS
5. FO EA YZ

* * * * *

Family "F"

1. EU RBF
2. YREL MN
3. ZR SY EN PX
4. ZI EJ PS TA
5. LG REF NI

* * * * *

We can now begin to amalgamate the chains within each alphabet in order to reconstruct the strips involved. In employing the graphical method of indirect symmetry, we arbitrarily assign to Family "A" the direction ↓, and to Family "D" the horizontal direction→. Thus in the initial reconstruction of the first five alphabets, we shall have the following partial diagrams, among lesser fragments:

Alph. 1	Alph. 2	Alph. 3	Alph. 4	Alph. 5
NA	X	AD	D	EM
KZ	OU	F	TNW	ITJ
	E	E	K	AFN
	Q	A		

Family "F" is tied into the diagrams through the observed relationship of REF in Alphabet 5 which is tied into the EF already in the diagram, establishing the direction of Family "F" as $\begin{smallmatrix}\hookleftarrow\!\rfloor 2\\ 1\end{smallmatrix}$. The other families are soon tied in; Alphabet 3 is reconstructed in its entirety, while the other four alphabets are recovered with only two or three blanks, after all the relationships present in the columns have been exhausted.[5]

f. The 25 alphabets recoverable from the 40 lines of matched plain and cipher are the following:[6] (The ringed letters could be recovered from other messages in the same key.)

```
 1. A B C E I G D ⓙ F V U Y M H T Q K Z O L R ⓧ S P ⓦ N
 2. A ⓒ D E H F I J K T L M O U V Y G ⓩ N P Q X R W S B
 3. A D K O M J U B G E P H S C Z I N X F Y Q R T V W L
 4. A E D C ⓑ I F G J H L K ⓜ R U O Q V P T N W Y ⓧ Z S
 5. A F N ⓠ U K ⓓ O P I T J B R H C Y S L W E M Z ⓥ X G
 6. A G P O C I ⓧ L U R N D Y Z H W ⓑ J S ⓠ F K V M E T
 7. A H X J E Z B N I K P V R O G S Y D U L C F M Q T W
 8. A I H P J O B W ⓚ C V F ⓩ L Q E R Y N S U M G T D X
 9. A J d S k Q O I V T Z E F H G Y u N L P M B X W C R
10. A K E L B D F J G H O N M T P R Q S V Z U X Y W I C
11. A L T M S X V Q P N O H U W D I Z Y C G K R F B E J
12. A M N F L H Q G C U J T B ⓨ P ⓩ K X I S ⓡ D V E W O
13. A N C ⓙ I L D H B ⓜ K G X U Z T S W Q Y V O R P F E
14. A O D W P K J V I U Q H Z C T X ⓑ L E G N Y R s M F
15. A P B V H I Y K S G U E N T C X O W F Q D R L J Z M
16. A Q ⓙ N U B T G I M W Z R V L X ⓒ ⓢ H D E O K F P Y
17. A R M Y O F T H E U S Z J X D P C W G Q I B K L N V
18. A S D M C N E Q B O Z P L G V J R K Y T F U I W X H
19. A T O J Y L F X N G W H V C M I R B S E K U P ⓓ ⓩ Q
20. A U T R Z X Q L Y I O V B P E S N H J W M D G F C K
21. A V N K H R G O X E Y B F S J M U D Q C L Z W T I P
22. A W V S F D L I E B H ⓚ N R J Q ⓩ G M X P U C O T Y
23. A X ⓚ W R E V ⓓ T U F O Y H M L S I Q N J C P G B ⓩ
24. A ⓨ J P X M V K B Q W U ⓖ L O S T E C H N Z F R I D
25. A Z D N B ⓤ H Y F W J L V G R C ⓠ M P S O E ⓧ T K I
```

[5] It would be excellent practice for the reader to reconstruct the first five alphabets—and, for that matter, all 25 of them—following the procedures exemplified in the foregoing subparagraphs.

[6] These are actually the alphabets of the U.S. Cipher Device, Type M-94, in their original order and at the correct decimation.

47. Analysis of cryptograms involving known alphabets but with unknown keys.—*a*. After the alphabets have been reconstructed, all messages which are in the same key (i.e., using the same order of the strips as just recovered) can now be read automatically. Subsequent key changes involve a permutation in the order of the strips, and for the analysis of messages in other key periods we must first construct 26 charts called "synoptic tables" which will facilitate our work. These tables are nothing more than generatrix diagrams based on the original recovered strips arranged vertically, in which the top rows of the tables consist of the strips aligned to all A's, all B's, etc., as shown in fragmentary form for the first two tables below: [7]

Synoptic Table for A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
1	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	C	D	K	D	N	P	X	H	D	E	T	N	C	D	B	J	M	D	O	T	N	V	K	J	D
3	E	E	O	C	Q	O	J	P	S	L	M	F	J	W	V	N	Y	M	J	R	K	S	W	P	N
4	I	H	M	B	U	C	E	J	K	B	S	L	I	P	H	U	O	C	Y	Z	H	F	R	X	B
5	G	F	J	I	K	I	Z	O	Q	D	X	H	L	K	I	B	F	N	L	X	R	D	E	M	U
									*	*	*	*	*	*	*										
23	P	W	V	X	V	M	Q	T	W	W	B	E	P	S	J	F	L	W	D	F	T	O	G	R	T
24	W	S	W	Z	X	E	T	D	C	I	E	W	F	M	Z	P	N	X	Z	C	I	T	B	I	K
25	N	B	L	S	G	T	W	X	R	C	J	O	E	F	M	Y	V	H	Q	K	P	Y	Z	D	I

Synoptic Table for B

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
1	C	A	G	I	R	J	N	W	X	D	E	Y	M	L	V	T	K	O	S	P	F	H	Z	Q	U
2	E	C	E	F	H	S	I	K	W	F	J	P	K	E	H	G	L	Z	E	E	S	K	A	W	H
3	I	D	P	G	C	Q	K	C	C	J	A	Z	G	G	I	I	N	P	K	S	J	N	X	U	Y
4	G	E	H	J	Y	F	P	V	R	G	L	K	X	N	Y	M	V	L	U	N	M	R	K	G	F
5	D	H	S	H	S	K	V	F	A	H	T	X	U	Y	K	W	A	G	P	H	U	J	W	L	W
									*	*	*	*	*	*	*										
23	W	R	M	E	I	Z	J	P	L	K	K	U	L	C	M	J	G	N	M	I	X	L	C	M	Z
24	N	W	J	D	T	H	E	J	P	E	R	J	D	T	A	N	Q	E	I	O	E	I	P	V	D
25	A	S	U	C	J	W	Z	O	M	L	F	T	H	X	P	U	I	Q	R	V	Y	E	G	K	N

(The small numbers on top are the strip numbers, while the large numbers at the side are the generatrix numbers.)

b. Let us now assume that we have at hand the following cryptogram enciphered by a different permutation of the strips that we have recovered, and that the first word of the underlying plain text is DIVISION:

```
C T Y N O   J L T D P   L R S Z J   Z U U W H   I O W S Y
J Q J Z G   O Z T S H   E X P E O   V D E Y A   I L Q V M
L Z U I I   W D C K W   S O J I W   Z G M S Z   R H G Y D
G Z R V K   M Y S Z R   T P W Y E   Q Q C T F   F B V L S
```

Our first step will be to determine the possible generatrices for the encipherment of DIVISION as CTYNOJLT. If we refer to the synoptic table for the letter D, we can see that D may be enciphered as C on generatrices 1, 2, and 3, but not by 5 through 8, nor by 10, 15, 18, 21, or 24. Similarly, referring to the synoptic table for the letter I, we find that $I_p = T_c$ is possible on all generatrices except 4, 5, 9, and 21.

[7] The complete synoptic tables are given in Appendix 2.

This process is repeated for all eight plain-cipher equivalencies of the encipherment of the putative crib; the results are shown in Fig. 73, below, wherein the x's indicate the generatrix possibilities for the eight encipherments. It will be seen that the correct generatrix must be either 1, 12, 19, or 25, since all the letters of the encipherment of the initial crib must necessarily come from a single generatrix.

	D	I	V	I	S	I	O	N
	C	T	Y	N	O	J	L	T
1	x	x	x	x	x	x	x	x
2	x	x	x	x		x		x
3	x	x	x		x	x	x	x
4	x		x	x	x	x	x	
5			x	x	x		x	x
6		x	x		x		x	x
7		x	x	x		x		
8		x			x		x	x
9	x		x	x		x	x	x
10		x		x	x	x	x	
11	x	x	x		x	x	x	
12	x	x	x	x	x	x	x	x
13	x	x		x			x	
14	x	x			x	x		x
15		x		x				x
16	x	x	x		x	x	x	
17	x	x	x	x	x		x	x
18		x	x			x		x
19	x	x	x	x	x	x	x	x
20	x	x		x	x	x		x
21			x	x	x	x	x	x
22	x	x	x	x	x		x	x
23	x	x	x	x			x	
24		x			x	x	x	
25	x	x	x	x	x	x	x	x

FIGURE 73

c. The next step is to consider each generatrix in turn, and determine which strip or strips can give rise to the particular encipherment on that generatrix. For example, in taking generatrix 1 into consideration, we find from the synoptic tables the following possibilities for strips:

D	I	V	I	S	I	O	N
C	T	Y	N	O	J	L	T
4	5	(2)	3	25	(2)	1	15

Generatrix 1 is impossible, because we cannot have strip number 2 in two positions since the permutation of strips does not admit of any repeated elements. Likewise, generatrix 25 is ruled out because of repeated unique strip numbers:

D	I	V	I	S	I	O	N
C	T	Y	N	O	J	L	T
2	21	(13)	7	(24)	(13)	(24)	4

The strip permutations (with multiple possibilities) for generatrices 12 and 19 are shown below:

```
           D  I  V  I  S  I  O  N
           C  T  Y  N  O  J  L  T
Gen. 12:   3  17 16 2  8  6  9  20
              19    14          22

           D  I  V  I  S  I  O  N
           C  T  Y  N  O  J  L  T
Gen. 19:   6  12 5  ~~5~~  10 18 ~~10~~ 11
           24 20    19          16 ~~19~~
                                17
```

Note that in the strip possibilities for generatrix 19 we have crossed out a 5, 10, and 19 from some columns since these numbers are uniquely represented in three other positions.

d. Generatrices 12 and 19 both admit of 8 possibilities of strip arrangements, so we shall try the former first. We first try the strip order 3–17–16–2–8–6–9–20 and, searching in the other three lines of cipher text for plausible plaintext fragments, we arrive at the results shown in Fig. 74*a*, below. The

```
3 17 16 2 8 6 9 20     3 17 16 2 8 6 9 22     3 17 16 14 8 6 9 22    3 19 16 14 8 6 9 22
C T Y N O J L T        C T Y N O J L T        C T Y N O J L T        C T Y N O J L T
D I V I S I O N        D I V I S I O N        D I V I S I O N        D I V I S I O N

J Q J Z G O Z T        J Q J Z G O Z T        J Q J Z G O Z T        J Q J Z G O Z T
D P P U N T O C        D P P U N T O P        D P P I N T O P        D U P I N T O P

L Z U I I W D C        L Z U I I W D C        L Z U I I W D C        L Z U I I W D C
E B L G F T F Y        E B L G F T F L        E B L E F T F L        E N L E F T F L
I R E R Y I L E        I R E R Y I L N

G Z R V K M Y S        G Z R V K M Y S        G Z R V K M Y S        G Z R V K M Y S
N Q E R R I W F        N Q E R R I W K        J E M P O F F A        J U M P O F F A
J E M M O F F B        J E M M O F F A

      (a)                    (b)                    (c)                    (d)
```

FIGURE 74

cipher JQJZGOZT yields DDPUNTOC as an interesting possibility; the cipher LZUIIWDC yields EBLGFTFY and IRERYILE that contain plausible plaintext fragments, while cipher GZRVKMYS yields NQERRIWF and JEMMOFFB among other generatrices containing fragments of what may be plain text. We now change the strip number 20 in the last position to its other variant, 22, and we see what the strip order 3–17–16–2–8–6–9–22 gives us; the results are shown in Fig. 74*b*. Changing the fourth strip (number 2) into its variant, 14, improves matters, as shown in Fig. 74*c*; and finally when we change the strip in the second position (number 17) into its variant, 19, so that the strip order now reads 3–19–16–14–8–6–9–22, we get valid plain text all the way across the generatrices, as shown in Fig. 74*d*.

e. The continuation of the third row of Fig. 74*d* seems to be the word FLANK; when this is assumed, the word POSITION emerges in the second row of the cipher text, as is shown in the decryption below:

```
3 19 16 14 8   6 9 22 1 24   10 4 20 17 15
C T  Y  N  O   J L T  D P    L  R S  Z  J    Z U U W H   I O W S Y
D I  V  I  S   I O N  r e    s  e r  v  e

J Q  J  Z  G   O Z T  S H    E  X P  E  O    V D E Y A   I L Q V M
D U  P  I  N   T O P  o s    i  T I  O  N

L Z  U  I  I   W D C  K W    S  O J  I  W    Z G M S Z   R H G Y D
E N  L  E  F   T F L  A N    K  s t  o  p

G Z  R  V  K   M Y S  Z R    T  P W  Y  E    Q Q C T F   F B V L S
J U  M  P  O   F F A  t n    o  o n  a  s
```

At this point it might be noticed that the key exhibits isomorphism beginning with the 12th element: this is the result of the use of an 11-element basic key (with a literal key probably underlying the key numbers) expanded to the required 25 elements. Thus the complete key may be recovered as in the following diagram:

3	19	16	14	8	6	9	22	1	24	10
4	20	17	15	11	7	12	23	2	25	13
5	21	18								

When this key is tried, it is found that the entire message can be read. A numerical key of the numbers of the top row of the foregoing diagram is made, and when this simplified basic key of 2–9–8–7–4–3–5–10–1–11–6 is analyzed,[8] it is found to be based upon the word CONFEDERATE, and the method of key production by the cryptographer was probably the following:

				5					10					15					20					25
C	O	N	F	E	D	E	R	A	T	E	C	O	N	F	E	D	E	R	A	T	E	C	O	N
3	19	16	14	8	6	9	22	1	24	10	4	20	17	15	11	7	12	23	2	25	13	5	21	18

f. The preceding case exemplified the exploitation of a crib whose position was known or assumed. In the next example we shall assume that the cryptogram below contains the word HEADQUARTERS, but that its position in the message is unknown:

AENLV DCBJQ JMTFE BJGLP AQKOT
ZPCRU JJZPN VXPTC ZJEFK HSLIL
WRZGQ YIYSQ RRDHV SCLHZ CWGLW
ZPRTQ KBLKK NNAIS WEAFZ AOAOA

Our first step is to determine the location of the crib. We could take advantage of the noncrashing properties of a strip cipher system, but there still would be too many possibilities for the placement of the crib; the property of the *limitations* of the cipher equivalents for plaintext letters on each generatrix will therefore be used. Since we do not know the generatrix involved, we shall first assume that it is number 13, and then, if this is unsuccessful, we shall try 14, 12, 15, 11 . . . in turn, until we have determined the correct generatrix.

g. In the alphabets of the M–94 there is a considerable limitation in the cipher equivalents of plaintext letters on all generatrices, except for generatrix numbers 1 and 25.[9] For example, in the synoptic table for A, we find on the 13th generatrix no occurrence of the letters A, B, D, E, F, I, J, K, M, N, or Q. If we were to make diagrams of the impossible cipher equivalents for the letters in the word HEADQUARTERS, on let us say, generatrix numbers 13 and 20, they would be the following:

Generatrix 13

H	E	A	D	Q	U	A	R	T	E	R	S
C	A	A	A	A	C	A	B	G	A	B	C
H	E	B	B	D	D	B	C	H	E	C	D
J	I	D	D	G	F	D	H	L	I	H	E
K	L	E	I	H	G	E	K	N	L	K	F
N	M	F	N	M	H	F	L	T	M	L	G
P	N	I	P	O	L	I	O	U	N	O	K
Q	P	J	Q	Q	M	J	R	Z	P	R	N
R	S	K	S	U	O	K	X		S	X	O
T	V	M	U	Z	Q	M			V		P
U	Y	N			T	N			Y		S
X		Q			U	Q					
					X						

Generatrix 20

H	E	A	D	Q	U	A	R	T	E	R	S
B	B	A	D	C	B	A	A	B	B	A	A
E	D	B	F	G	C	B	C	C	D	C	C
H	E	G	G	H	E	G	D	L	E	D	D
M	F	H	J	I	G	H	J	O	F	J	H
Q	G	S	K	M	K	S	K	P	G	K	M
W	J	T	L	N	L	T	L	T	J	L	P
Y	N	W	M	Q	N	W	M	V	N	M	R
	R	X	O	U	O	X	N	X	R	N	S
		Y	Q	W	S	Y	Q	Z		Q	X
		Z	T		T	Z	R			R	
			U		U		U			U	
			W				Y			Y	

[8] For the method of derivation of literal keys from numerical keys, see pp. 427–429 of *Military Cryptanalytics, Part II*.

[9] These alphabets have been specially constructed so that the smallest number of repetitions of pairs of letters would occur; in the 25 sequences, the only digraphs appearing twice are DE, YA, and UB. Nevertheless, there are manifold occurrences of skip digraphs (A–B, A—B, etc.) at wider intervals.

If we now start with the first letter of the cipher message, we can rule out every possible placement of HEADQUARTERS on the 13th generatrix, by tracing the successive letters of the cipher text against our *limitation diagram and finding conflicts.*

(1) Having eliminated the 13th generatrix for the encipherment of our crib, we now try the 14th generatrix and we get only one possible placement, at position 77:

```
            80            85            90
. . . Z P R T Q   K B L K K   N N A I S . . .
        H E A D   Q U A R T   E R S
```

This placement admits of the following possibilities for strip arrangements for the 14th generatrix:

	H	E	A	D	Q	U	A	R	T	E	R	S
Gen. 14:	P	R	T	Q	K	B	L	K	K	N	N	A
	6	9	1	4	2	7	16	13	10	23	11	3
			14	25	15	22						

None of the 16 possible arrangements yields plain text in corresponding positions of the other lines of 25 cipher letters; therefore the apparent placement of the crib at that position in the cipher and with the 14th generatrix must have been an accident.

(2) The word HEADQUARTERS could fit at position 39 on the 11th generatrix giving a multiple key of 24 possible strip arrangements, but this placement too is an accident. Unique strip arrangements are then obtained for the placement of the crib on the 25th generatrix at positions 21, 26, and 88; and we also obtain a unique strip arrangement for the placement on the 1st generatrix at position 62. Since none of these keys yields valid plain text on the other cipher lines, all of them must be accidents. On generatrix 3, the crib fits at position 85 with the following multiple key which yields a total, not of 288, but of 72 permissible arrangements:

	H	E	A	D	Q	U	A	R	T	E	R	S
Gen. 3:	K	N	N	A	I	S	W	E	A	F	Z	A
	1	7	16	19	20	13	~~14~~	11	8	16	15	14
	~~13~~	8	25				23	12	21	21	22	
	15	24						~~18~~	25			
								~~19~~				

None of these bears fruit, so we are left with the unmistakable conclusion that, having made a large number of trials thus far, if the word HEADQUARTERS is present in the cipher text at all it must be divided between two lines on two different generatrices.

(3) If HEADQUARTERS is divided between two lines, it must begin somewhere between the 15th and 25th positions of each line of the message except the last. It cannot begin at position 15, because

```
      15        20        25
. . . E B J G L P A Q K O T Z . . .
      H E A D Q U A R T E R S
```

of the crashing plain and cipher text, nor for the same reason can it begin in 12 other positions in the range specified; thus there are only 20 positions left to try. The first trial, assuming that the crib begins at position 16, is shown in Fig. 75*a*, below, and it is seen that generatrix numbers 14 and 25 are the

Crib at position 16

	H	E	A	D	Q	U	A	R	T	E	R	S
	B	J	G	L	P	A	Q	K	O	T	Z	P
1	x	x	x	x	x		x	x	x	x	x	x
2	x			x	x					x		
3				x			x		x	x	x	
4	x	x			x			x	x	x	x	x
5	x	x	x		x	x	x	x				x
6	x	x	x			x	x		x	x		x
7	x	x	x		x		x	x	x			x
8	x	x	x	x	x	x			x	x	x	
9	x	x	x	x	x		x	x			x	
10		x		x		x	x	x	x		x	x
11	x	x	x							x	x	
12	x	x	x	x		x		x		x		x
13	x	x	x	x	x	x			x	x	x	
14	x	x	x	x	x	x	x	x	x	x		x
15	x	x		x	x	x	x		x			
16		x	x	x	x	x	x		x	x	x	x
17		x	x			x						x
18			x			x	x	x		x	x	
19		x	x				x				x	x
20					x	x	x			x	x	
21	x			x		x		x			x	x
22	x	x	x	x	x	x		x	x	x	x	
23			x		x		x	x	x		x	x
24	x			x	x			x	x	x	x	x
25	x	x	x	x	x	x	x	x	x	x	x	x

FIGURE 75*a*

Crib at position 20

	H	E	A	D	Q	U	A	R	T	E	R	S
	P	A	Q	K	O	T	Z	P	C	R	U	J
1	x	x	x	x	x	x	x	x	x	x	x	x
2		x		x	x	x			x			x
3		x	x	x	x	x		x				x
4				x	x	x	x	x		x	x	x
5	x	x	x		x	x	x	x	x	x	x	
6	x	x	x	x		x		x		x	x	
7		x	x			x		x				
8	x	x		x	x			x		x	x	
9	x	x	x	x	x							x
10	x		x	x	x		x	x	x	x		
11	x	x			x		x		x	x	x	x
12	x	x		x			x	x	x	x		
13				x			x	x	x	x	x	x
14	x		x				x	x	x	x	x	x
15	x	x	x	x	x		x	x	x	x	x	x
16	x		x		x	x	x		x		x	x
17		x			x	x	x		x	x	x	x
18		x	x	x	x	x		x	x	x	x	
19	x		x		x	x	x	x		x		x
20	x	x	x		x			x				x
21	x	x		x			x	x				x
22	x	x		x		x	x		x	x		
23	x	x	x	x	x	x		x	x	x		
24		x				x	x	x	x		x	x
25	x	x	x	x	x	x	x	x	x	x	x	x

FIGURE 75*b*

only ones with x's all the way across to the vertical line marking the end of a line of 25 cipher letters of the message.[10] Fig. 75*b* shows the second trial, assuming that the crib begins at position 20 with the letters being split 50–50 on two cipher lines; the first six letters could involve generatrix numbers 1, 23, or 25, while the last six letters could involve generatrix numbers 1, 13, 14, 15, or 25. Both of the first two trials come to naught, since the keys derived do not yield plain text in other parts of the message.

[10] As mentioned in the previous footnote, these particular alphabets have been constructed so as to achieve the maximum variety of consecutive digraphs among the 25 sequences, therefore in the M–94 alphabets generatrix numbers 1 and 25 will almost always be present as possible candidates. If the alphabets had been constructed by a truly random process, these generatrices would be present no more frequently than any other generatrix.

(4) After very considerable and laborious experimentation involving several hundred trials, we arrive at the placement of the crib at position 72 as shown in Fig. 75*c*, below:

	H	E	A	D	Q	U	A	R	T	E	R	S
	W	G	L	W	Z	P	R	T	Q	K	B	L
1	x	x	x	x	x	x	x	x	x	x	x	x
2	x	x			x					x	x	
3	x	x	x	x	x	x	x	x	x	x	x	x
4			x	x		x	x	x	x			x
5	x	x	x	x	x	x	x	x	x	x		
6		x	x			x		x	x		x	
7		x	x	x	x			x			x	x
8		x		x	x							
9		x		x	x	x	x	x			x	
10	x	x	x		x	x		x	x	x		
11	x		x	x	x			x	x	x	x	x
12	x	x	x		x	x	x	x			x	x
13	x	x	x	x		x	x	x	x	x		x
14	x		x				x		x	x		x
15			x				x				x	x
16	x	x				x	x	x		x	x	
17	x		x	x		x		x	x		x	
18	x	x	x	x	x	x		x			x	x
19	x		x	x	x			x	x	x	x	
20			x		x	x	x	x	x	x	x	x
21		x		x		x	x		x			
22			x	x	x	x	x		x	x	x	
23		x	x	x	x		x	x	x	x		x
24	x	x		x	x			x	x			x
25	x	x	x	x	x	x	x	x	x	x	x	x

FIGURE 75*c*

Since the longer fragment QUARTERS has been enciphered on one generatrix, at the beginning of the fourth line of the cipher, we shall consider this portion first. The generatrix possibilities are generatrix numbers 1, 3, 20, and 25, so we derive the multiple keys shown below:

Gen. 1:

Q	U	A	R	T	E	R	S
Z	P	R	T	Q	K	B	L
22	(19)	17	3	1	(19)	(19)	5

Gen. 3:

Q	U	A	R	T	E	R	S
Z	P	R	T	Q	K	B	L
10 ~~18~~ 21	9	20	18	10 13	~~22~~ 25	2	22

Gen. 20:

Q	U	A	R	T	E	R	S
Z	P	R	T	Q	K	B	L
3 ~~9~~	~~9~~ 10 16	1 12	1 16	6	21	7 14 17 23	8 14

Gen. 25:

Q	U	A	R	T	E	R	S
Z	P	R	T	Q	K	B	L
19	22	9	20	7	10	5	23

Generatrix 1 is eliminated at once, and the possible strip arrangements for generatrices 3 and 25 do not decipher the text; this leaves generatrix 20 as the sole candidate, albeit in 28 possible strip arrangements.

(5) After several trials, it is found that the order of the first eight strips is 3–10–12–16–6–21–7–14, yielding the following decipherments on the 8th, 4th, 13th, and 20th generatrices (as measured from plain to cipher):

```
3 10 12 16 6  21 7 14
A E N L V   D C B J Q   J M T F E   B J G L P   A Q K O T
F U R T H   E R I

Z P C R U   J J Z P N   V X P T C   Z J E F K   H S L I L
P O L I C   Y W I

W R Z G Q   Y I Y S Q   R R D H V   S C L H Z   C W G L W
H E N E X   T F I                                 H E A D

Z P R T Q   K B L K K   N N A I S   W E A F Z   A O A O A
Q U A R T   E R S
```

The rest of the text is easily read, the complete order of strips established, and the basic 14-letter literal key recovered, as shown below:

				5					10					15					20					25
C	H	I	N	E	S	E	L	A	U	N	D	R	Y	C	H	I	N	E	S	E	L	A	U	N
3	10	12	16	6	21	7	14	1	23	17	5	20	25	4	11	13	18	8	22	9	15	2	24	19

h. The example in subpar. *b* was solved by means of a placed crib. If instead of a crib we had a pair of isologs available, we could treat the first line of 25 letters of one message as if it were the plain text of the other and arrive at an easy solution. Even if we had isologous beginnings of a pair of messages, solution could be effected in the manner of a placed crib. As an illustration, let us assume that we have at hand the message given in subpar. *b* and another message, both of which are suspected of beginning with the same unknown initial stereotype of unknown length. The two message beginnings are shown below:

```
"A":  C T Y N O   J L T D P   L R S Z J   Z U U W H   I O W S Y . . .
"B":  P Z K L H   H Y U Y R   X W I H V   Y M N I C   N A F U G . . .
```

We now construct a diagram for establishing generatrix possibilities, and we test the message beginnings to see if they could contain identical underlying plain text and for how many positions; this is shown in Fig. 76, below:

					5					10					15	
"A":	C	T	Y	N	O	J	L	T	D	P	L	R	S	Z	J	. . .
"B":	P	Z	K	L	H	H	Y	U	Y	R	X	W	I	H	V	. . .
1	x	x	x	x	x	x	x	x	x	x						
2		x		x	x	x		x	x	x						
3	x	x	x		x	x		x	x	x						
4		x		x		x	x	x								
5		x	x		x	x	x		x	x						
6	x	x	x		x	x	x		x	x						
7	x	x		x	x	x		x	x	x						
8	x	x		x	x	x		x		x						
9	x		x	x	x			x								
10			x	x	x	x	x	x	x							
11	x	x		x	x		x			x						
12		x	x	x	x	x	x		x	x						
13	x				x				x	x						
14		x			x		x			x						
15	x	x	x						x							
16	x		x	x	x	x	x		x	x						
17	x	x	x	x		x			x							
18	x	x		x	x		x		x	x						
19	x		x	x		x	x	x	x	x						
20	x			x	x		x	x	x	x						
21	x			x	x	x		x		x						
22				x		x	x	x		x						
23	x	x	x	x	x	x	x	x		x						
24	x	x				x	x	x								
25	x	x	x	x	x	x	x	x	x	x						

FIGURE 76

It may be seen that, if the messages do contain identical beginnings, these beginnings are probably no more than 8 letters in length, because in generatrix 23 there is only this number of consecutive x's before an impossibility arises. The following thus are the possibilities for strip numbers for the three generatrices that come into question:

	C	T	Y	N	O	J	L	T
	P	Z	K	L	H	H	Y	U
Gen. 1:	(23)	(9)	15	(9)	11	4	20	(23)

	C	T	Y	N	O	J	L	T
	P	Z	K	L	H	H	Y	U
Gen. 23:	3	19	16	6 14 ~~19~~	8 ~~16~~ 21	2 6	7 9 23	8 15 22

	C	T	Y	N	O	J	L	T
	P	Z	K	L	H	H	Y	U
Gen. 25:	(17)	13	18	(17)	10	(20)	19	(20)

Generatrices 1 and 25 are eliminated immediately because of repeated unique occurrences of strip numbers, leaving only generatrix 23 to be considered. The solution from here proceeds as in subpars. *d* and *e*, except that instead of only 8 possibilities of strip arrangements, there may be 45 possible trials before the correct arrangement is found.

48. Further remarks.—*a*. The Bazeries device incorporated 20 systematic 25-letter alphabets, including the normal and reversed normal sequences, and 14 keyword-mixed sequences. The alphabets of Hitt's device, as already indicated, bore many interrelationships, constituting a cryptographic weakness. For example, in the synoptic table for A, no generatrix contains more than 10 different letters, and several contain as few as four; moreover, some of the generatrices contain one letter 20 times: all this greatly facilitates crib placement. The M–94 alphabets devised by Mauborgne were intended to be as diverse as possible, but the diversity did not extend beyond digraphs; and since the only digraphs occurring twice are DE, YA, and UB, he originally called his system the DEYAUB cipher.

b. In Bazeries' device 24 cipher generatrices were possible, but he prescribed that the first two and last two should not be used; thus there were only 20 generatrices available. In the M–94, which used a guide bar along which to align the plaintext letters, the 1st generatrix was completely hidden, while the 2d generatrix was half hidden, so that these generatrices could not be used; conversely, the 24th and 25th generatrices were not to be used, because these could not be seen when the cipher text was set up along the guide bar. With strip cipher systems, no limitation of generatrices exists–except of course for the 26th generatrix, which is the same as the Ø or plaintext generatrix.

c. The cryptograms produced by a cylindrical cipher device or a strip cipher system are noncrashing, i.e., a plaintext letter can never be enciphered by itself. Since the 25 cipher equivalents of any plaintext letter are equiprobable, the theoretical ciphertext frequencies for English underlying text can be computed, resulting in the following distribution:

A 3.70	G 3.94	L 3.86	Q 3.99	V 3.94
B 3.96	H 3.86	M 3.90	R 3.70	W 3.94
C 3.88	I 3.70	N 3.68	S 3.76	X 3.98
D 3.83	J 3.99	O 3.70	T 3.63	Y 3.92
E 3.48	K 3.99	P 3.89	U 3.90	Z 4.00
F 3.88				100.00

The γ I.C. of the cipher text is most easily computed by the formula $\gamma=1+\frac{\beta_p}{(c-1)^2}$, where β_p is the bulge of the I.C. of the plain text (i.e., $\gamma-1.00$) and c is the number of categories; therefore $\gamma=1+\frac{.73}{(26-1)^2}=$ 1.0012. A property of such systems is that, if the distribution of the cipher letters is made in the descending frequency order of the plaintext letters of the language, the slope of frequencies of the ciphertext letters will be that of a gradual ascent; thus:

E 3.48	I 3.70	C 3.88	Y 3.92	X 3.98
T 3.63	S 3.76	F 3.88	G 3.94	Q 3.99
N 3.68	D 3.83	P 3.89	W 3.94	K 3.99
R 3.70	L 3.86	U 3.90	V 3.94	J 3.99
O 3.70	H 3.86	M 3.90	B 3.96	Z 4.00
A 3.70				100.00

The slope is so gradual that usually about 8000 letters are required for a positive determination of a noncrashing system.

d. In the example given in subpar. 46*a* the unknown alphabets were reconstructed by means of a long cipher message and its compromised plain text. The alphabets could also be recovered if we had available a long pair of isologs, even if the underlying plain text were unknown—in fact, even if the language were totally unknown! In this case, the cipher of Message "A" could be treated as if it were the plain of Message "B", and solution would proceed as outlined in par. 46.

e. In the absence of either matched plain and cipher or of isologs, the *modus operandi*, albeit very laborious, is as follows: First, lines of cipher text enciphered by the same generatrix must be established, initially on the basis of polygraphic repetitions of at least tetragraphs between them. Second, further lines of cipher are added, on the basis of coincidences between the lines under consideration and the family of lines already established. For example, if we are testing a new line against a homogeneous family of six lines, we should expect $.0667 \times 25 \times 6 = 9.9$ coincidences if the new line really belongs to that family, as against $.0385 \times 25 \times 6 = 5.7$ for random. Third, some plain text must be assumed, either for the repetitions or for possible initial stereotypes. Finally, the assumed plain text is tested against the other lines of the same family.

f. The first solution of a cylindrical cipher device was accomplished by the Marquis Gaëtan Henri Léon de Viaris, who in 1893 described his analysis of the Bazeries device in what is one of the most brilliant feats of cryptanalysis in history, in view of the state of the art at the time.[11] De Viaris assumed possession of the device, a premise granted by the inventor, and he asked Bazeries for three test situations: (1) a cryptogram with the generatrix numbers indicated for each line of cipher; (2) another cryptogram in which one word of the plain text would be given; and (3) three cryptograms with the only information that they are in the same key. The first test case was academic, since in practice a cryptanalyst would never know the generatrix numbers, and de Viaris solved this easily by means of synoptic tables of which he was the inventor. In the second problem he dragged the crib OFFICIER through the cipher, placing the crib correctly on the basis of ciphertext limitations, testing all 20 possible generatrices, and recovering the order of all the disks.[12] In the third situation he pondered the problem, toying with the idea of dragging possible cribs such as the endings EMENT and ERAIENT through the cipher texts, until he hit upon the thought that perhaps one of the messages began with the definite article LES. When he tried LES at the beginning of the second message, he obtained DES and LAV as the beginnings of the first and third messages. He expanded LAV first into LA VUE and then into LA VITESSE, which yielded DES PARCS A . . . and LES APPROV . . . in the other two messages; he expanded the latter into LES APPROVISIONNEMENTS which yielded DES PARCS AEROSTATIQUES and LA VITESSE D'UNE COLONNE and the problem came to a quick end. De Viaris not only obtained the numerical keys for the three test cases, but he also devised the procedure for recovering the literal keys on which the numerical keys were based.

g. The method of expanding a short key into a longer one was proposed by Bazeries for his device, and this method of key construction was prescribed in the instructions for the use of the M-94. The weakness of such a scheme can be demonstrated in connection with the partial key of eight elements recovered in subpar. 47*d*: 3-19-16-14-8-6-9-22. If we had encountered difficulties in assuming further plain text, and if an expanded key had been used, we could recover the entire key in the following way. If the basic key length is 9, then key number 1 must of course be in position 9, and we can also insert in the second and third cycles the key numbers shown:

				5					10					15					20					25
3	19	(16)	14	(8)	6	9	22	1]	4	20	17	15	10	7	11	23	2]	5	21	18	(16)	12	(8)	13

Since conflicts (indicated by the ringed numbers) arise, the key length cannot be 9. If the key length is 10, we know that the digit 1 must be in either position 9 or 10, and we can fill in the following mandatory values:

				5					10					15					20					25
3	19	(16)	14	8	6	9	22	.	.]	4	20	17	15	.	7	.	.	.	.]	5	21	18	(16)	.

[11] Marquis de Viaris, *L'art de chiffrer et déchiffrer les dépêches secrètes*, Paris 1893.

[12] De Viaris suggests that 20 clerks be employed for the generatrix testing, one for each of the generatrices permissible in the Bazeries device ("Notons en passant que ce travail d'essai pourrait être fait par 20 secrétaires examinant chacun l'hypothese d'une des 20 génératrices du cylindre"). Actually, if we consider the amount of work we had to do in solving the case given in subpars. 47*f* and *g*, this might not be such a bad idea. (The De Viaris suggestion antedates the contemporary "1,000,000 Chinamen" allusion.)

Since there is a conflict between the 16's at positions 3 and 24, we rule out a key length of 10 and try a length of 11, filling in the automatic values:

```
                5                   10                  15                  20                  25
3  19  16  14   8   6   9  22   .   .   .]  4  20  17  15   .   7   .   .   .   .   .]  5  21  18
```

Since no conflicts developed, we can take it that the basic key is probably 11 elements long, and that therefore the first cycle must contain the numbers (1, 10, 23), (1, 10, 24), (1, 12, 23), or (1, 12, 24) in one of their six possible permutations. We could now try the 24 possible keys on the rest of the cipher message to discover which is the correct one, or we could reduce the first cycle of each of the 24 keys to a basic key and recover a key word from the correct one. The only key from which a plausible key word can be derived is

Key:	3	19	16	14	8	6	9	22	1	24	10
Basic:	2	9	8	7	4	3	5	10	1	11	6

which is found to be based upon the word CONFEDERATE, so the entire key must be

```
            5               10              15              20              25
C   O   N   F   E   D   E   R   A   T   E   C   O   N   F   E   D   E   R   A   T   E   C   O   N
3  19  16  14   8   6   9  22   1  24  10   4  20  17  15  11   7  12  23   2  25  13   5  21  18
```

h. In subpar. *d*, above, it was stated that alphabets could be reconstructed through the use of isologs, even if the underlying text were in a language totally unknown. Furthermore, once we are in possession of the alphabets, even if their order is initially unknown, isologs would make it possible to recover the plain text by establishing the order *in toto* of the strips, following the procedure demonstrated in subpar. 47*h*. We would then search the generatrices for the one row in each set that gives evidence of being plain text of some kind, either by the distribution of vowels and therefore the pronounceability of the text, or by repetitions or other attributes expected of a language. We shall give below a striking demonstration of this possibility of analysis, involving the encipherment of an intermediate text of unknown composition.

(1) The following two intercepted messages were known to have been enciphered with the M–94:

Message "A"

```
D I I W Z   L E X L U   D T F N S   L M A U E   X G Y X O
S E L V A   I C I J D   K K V J Q   M K G W V   K V I D V
C A M M Y   Q E V E F   B I C N D   J M K Z H   J E G P T
L W P N C   I O C F H   V N R Y L   F V J G H   S H Q G A
```

Message "B"

```
G D O S F   V V R A S   B S F P H   W E M F W   H O A Y W
U Y P O D   W M Y B D
```

In the absence of any probable words to try, solution may be very difficult indeed—unless we are lucky and, for example, the messages contain identical beginnings in the underlying text. We observe a hit between the two messages at their 13th positions, so if they do have similar beginnings these could not be more than 12 letters in length.

(2) We first construct a diagram similar to that of Fig. 73 to test for similar beginnings, and we find that this situation is possible on generatrix numbers 7 or 25. We then establish the order of the disks for these two possibilities, as shown below:

```
          D   I   I   W   Z   L   E   X   L   U   D   T
Gen. 7:   G   D   O   S   F   V   V   R   A   S   B   S
          3  10  12  16   6   4   7  14   1  23  17   5
                         25  15          ~~1~~
                             19
                             21

          D   I   I   W   Z   L   E   X   L   U   D   T
Gen. 25:  G   D   O   S   F   V   V   R   A   S   B   S
          ①  ⑪   9  13   ⑧  16  12   ①  ⑪   ⑧  10  24
```

Generatrix 25 is ruled out at once because of conflicts, and we are left with the 8 possibilities of disk orders on generatrix 7. We now run down the generatrices on the beginning of Message "A" for the positions that have unique keys, as shown in Fig. 77*a*, below:

	3	10	12	16			7	14	1	23	17	5
	D	I	I	W	Z	L	E	X	L	U	D	T
1	K	C	S	Z			Z	B	R	F	P	J
2	O	A	R	R			B	L	X	O	C	B
3	M	K	D	V			N	E	S	Y	W	R
4	J	E	V	L			I	G	P	H	G	H
5	U	L	E	X			K	N	W	M	Q	C
6	B	B	W	C			P	Y	N	L	I	Y
7	G	D	O	S			V	R	A	S	B	S
8	E	F	A	H			R	S	B	I	K	L
9	P	J	M	D			O	M	C	Q	L	W
10	H	G	N	E			G	F	E	N	N	E
11	S	H	F	O			S	A	I	J	V	M
12	C	O	L	K			Y	O	G	C	A	Z
13	Z	N	H	F			D	D	D	P	R	V
14	I	M	Q	P			U	W	J	G	M	X
15	N	T	G	Y			L	P	F	B	Y	G
16	X	P	C	A			C	K	V	Z	O	A
17	F	R	U	Q			F	J	U	A	F	F
18	Y	Q	J	J			M	V	Y	X	T	N
19	Q	S	T	N			Q	I	M	K	H	Q
20	R	V	B	U			T	U	H	W	E	U
21	T	Z	Y	B			W	Q	T	R	U	K
22	V	U	P	T			A	H	Q	E	S	D
23	W	X	Z	G			H	Z	K	V	Z	O
24	L	Y	K	I			X	C	Z	D	J	P
25	A	W	X	M			J	T	O	T	X	I

FIGURE 77*a*

Generatrix numbers 1, 2, 24, and 25 are ruled out as possibilities for Message "A" because of the idiosyncrasies of the M–94, and likewise generatrix numbers 5 through 9 for Message "B". To our astonishment, however, even though we have proved the existence of similar beginnings we are unable to see plain text on any of the generatrices. The "plain text," then, must be some kind of intermediate text. In order to assist our analysis, we make generatrix diagrams for the other four lines of cipher text under the already recovered key; these are shown in Figs. 77*b–e* on the next page.

(3) In Fig. 77*e* (the diagram for the last line of Message "B") we note the letters A Q C X . . Y X X X on generatrix 20; if this generatrix is valid, perhaps null X's have been used to complete the last group of five letters, which would then make this message 32 letters in length. This observation, coupled with the length of Message "A" (100 letters) and the length of the similar beginnings (12 letters), leads us to the assumption that the underlying text consists of a 4-letter code. We now note on the 21st generatrix in Fig. 77*d* (the diagram for the last line of Message "A") the sequence of letters Q)V U P .) . I I E)T I V which, on the basis of 4-letter code groups, might indicate the presence of another VUPT which had already occurred on generatrix 22 of Fig. 77*a*. Of the two key possibilities (6 and 25) for the 5th column, only disk number 6 will produce a T_p out of C_c on the 21st generatrix. One more column of possible decipherments can now be added to each of the diagrams of Figs. 77*a–e*.

(4) We now take a close look at our diagrams and we examine the putative plain text we have just considered:

In Fig. 77*a*, generatrix 22: V U P T)R . A H)Q E S D
In Fig. 77*d*, generatrix 21: Q)V U P T) . I I E)T I V
In Fig. 77*e*, generatrix 20: A Q C)X I . Y)X X X

	3	10	12	16			7	14	1	23	17	5
	S	E	L	V	A	I	C	I	J	D	K	K
1												
2												
3	I	D	G	C			Q	H	U	F	V	P
4	N	F	C	S			T	Z	Y	O	A	I
5	X	J	U	H			W	C	M	Y	R	T
6	F	G	J	D			A	T	H	H	M	J
7	Y	H	T	E			H	X	T	M	Y	B
8	Q	O	B	O			X	B	Q	L	O	R
9	R	N	Y	K			J	L	K	S	F	H
10	T	M	P	F			E	E	Z	I	T	C
11	V	T	Z	P			Z	G	O	Q	H	Y
12	W	P	K	Y			B	N	L	N	E	S
13	L	R	X	A			N	Y	R	J	U	L
14	A	Q	I	Q			I	R	X	C	S	W
15	D	S	S	J			K	S	S	P	Z	E
16	K	V	R	N			P	M	P	G	J	M
17	O	Z	D	U			V	F	W	B	X	Z
18	M	U	V	B			R	A	N	Z	D	V
19	J	X	E	T			O	O	A	A	P	X
20	U	Y	W	G			G	D	B	X	C	G
21	B	W	O	I			S	W	C	K	W	A
22	G	I	A	M			Y	P	E	W	G	F
23	E	C	M	W			D	K	I	R	Q	N
24												
25												

FIGURE 77*b*

	3	10	12	16			7	14	1	23	17	5
	C	A	M	M	Y	Q	E	V	E	F	B	I
1												
2												
3	N	L	L	R			N	Q	D	H	N	B
4	X	B	H	V			I	H	J	M	V	R
5	F	D	Q	L			K	Z	F	L	A	H
6	Y	F	G	X			P	C	V	S	R	C
7	Q	J	C	C			V	T	U	I	M	Y
8	R	G	U	S			R	X	Y	Q	Y	S
9	T	H	J	H			O	B	M	N	O	L
10	V	O	T	D			G	L	H	J	F	W
11	W	N	B	E			S	E	T	C	T	E
12	L	M	Y	O			Y	G	Q	P	H	M
13	A	T	P	K			D	N	K	G	E	Z
14	D	P	Z	F			U	Y	Z	B	U	V
15	K	R	K	P			L	R	O	Z	S	X
16	O	Q	X	Y			C	S	L	A	Z	G
17	M	S	I	A			F	M	R	X	J	A
18	J	V	S	Q			M	F	X	K	X	F
19	U	Z	R	J			Q	A	S	W	D	N
20	B	U	D	N			T	O	P	R	P	Q
21	G	X	V	U			W	D	W	E	C	U
22	E	Y	E	B			A	W	N	V	W	K
23	P	W	W	T			H	P	A	D	G	D
24												
25												

FIGURE 77*c*

	3	10	12	16			7	14	1	23	17	5
	L	W	P	N	C	I	O	C	F	H	V	N
1												
2												
3	K	A	X	T			Y	B	Y	S	M	K
4	O	K	I	G			D	L	M	I	Y	D
5	M	E	S	I			U	E	H	Q	O	O
6	J	L	R	M			L	G	T	N	F	P
7	U	B	D	W			C	N	Q	J	T	I
8	B	D	V	Z			F	Y	K	C	H	T
9	G	F	E	R			M	R	Z	P	E	J
10	E	J	W	V			Q	S	O	G	U	B
11	P	G	O	L			T	M	L	B	S	R
12	H	H	A	X			W	F	R	Z	Z	H
13	S	O	M	C			A	A	X	A	J	C
14	C	N	N	S			H	O	S	X	X	Y
15	Z	M	F	H			X	D	P	K	D	S
16	I	T	L	D			J	W	W	W	P	L
17	N	P	H	E			E	P	N	R	C	W
18	X	R	Q	O			Z	K	A	E	W	E
19	F	Q	G	K			B	J	B	V	G	M
20	Y	S	C	F			N	V	C	D	Q	Z
21	Q	V	U	P			I	I	E	T	I	V
22	R	Z	J	Y			K	U	I	U	B	X
23	T	U	T	A			P	Q	G	F	K	G
24												
25												

FIGURE 77*d*

3	10	12	16			7	14	1	23	
U	Y	P	O	D	W	M	Y	B	D	
										1
										2
E	C	X	P			W	M	I	F	3
P	A	I	Y			A	F	G	O	4
H	K	S	A			H	A	D	Y	5
S	E	R	Q			X	O	J	H	6
C	L	D	J			J	D	F	M	7
Z	B	V	N			E	W	V	L	8
I	D	E	U			Z	P	U	S	9
N	F	W	B			B	K	Y	I	10
X	J	O	T			N	J	M	Q	11
F	G	A	G			I	V	H	N	12
Y	H	M	I			K	I	T	J	13
Q	O	N	M			P	U	Q	C	14
R	N	F	W			V	Q	K	P	15
T	M	L	Z			R	H	Z	G	16
V	T	H	R			O	Z	O	B	17
W	P	Q	V			G	C	L	Z	18
L	R	G	L			S	T	R	A	19
A	Q	C	X			Y	X	X	X	20
D	S	U	C			D	B	S	K	21
K	V	J	S			U	L	P	W	22
O	Z	T	H			L	E	W	R	23
										24
										25

FIGURE 77*e*

SECRET

It certainly looks as if the underlying code groups have the consonant-vowel pattern of CV—. On this basis, the correct values for the blank 6th column of our diagrams (with multiple keys of 4, 15, 19, and 21) can now be determined, since the second code group at the beginning of Message "A", R–AH, should have a vowel in the second position. The only disk that satisfies this condition is number 21; thus we are able to arrive at the following correct decipherments of the lines of cipher in the two messages:

Message "A"

```
3 10 12 16 6 21 7 14 1 23 17 5
D I I W Z L E X L U D T F N S L M A U E X G Y X O
V U P T)R U A H)Q E S D)       )       )       )

S E L V A I C I J D K K V J Q M K G W V K V I D V
I D G)C O V Q)H U F V)P      )       )       )

C A M M Y Q E V E F B I C N D J M K Z H J E G P T
W N)B E M K)S E T C)T E    )       )       )

L W P N C I O C F H V N R Y L F V J G H S H Q G A
Q)V U P T)C I I E)T I V  )       )       )       )
```

Message "B"

```
3 10 12 16 6 21 7 14 1 23 17 5
G D O S F V V R A S B S F P H W E M F W H O A Y W
V U P T)R U A H)Q E S D)       )       )       )

U Y P O D W M Y B D
A Q C)X I U Y)X X X
```

FIGURE 78

(5) At this point we have only nine code groups established in the code of the underlying text, as follows: BEMK, CIIE, COVQ, HUFV, QESD, RUAH, SETC, VUPT, and XIUY. Armed with this small sample of the total code population, we are nevertheless able, with a little experimentation, to construct the permutation table which gave rise to the these code groups; this table is shown in Fig. 79 on the next page.[13] We can now decipher the remaining code groups practically automatically, as will be demonstrated. Referring to Fig. 78, the group TIV–in the fourth line of Message "A" is shown by the permutation table to be TIVC, and the only disk number that can be involved which will also give a vowel at position 13 of the second line of the message is disk number 2. This yields in the third line the group TEI–, which must be TEIR from the permutation table. This general procedure is repeated, recovering the key for a column at a time, until the entire key is obtained. The key, on further analysis, is found to be based on CHINESE LAUNDRY, expanded to 25 letters.

---

[13] Actually, we made a slight mistake in the reconstruction of this table. After deciphering all the code groups in the two messages and encountering the group ZYAZ and ZYDC which do not fit in our table, we realize that we must have made an error in its reconstruction. We now modify it so that the middle portion at the junction of the two matrices looks as follows:

```
W|- - - - - - - - - - - - - - - - - A E I O U Y  |
X|- - - - - - - - - - - - - - - - - - A E I O U Y|
Z|Y - - - - - - - - - - - - - - - - - - A E I O U|

 |A B C D E F G H I K L M N O P Q R S T U V W X Y|Z
 |B C D E F G H I K L M N O P Q R S T U V W X Y Z|A
 |C D E F G H I K L M N O P Q R S T U V W X Y Z A|B
```

The bottom matrix is a 24×25 rectangle, and not a 25×25 square as we originally reconstructed it.

B	A	E	I	O	U	Y																			
C	-	A	E	I	O	U	Y																		
D	-	-	A	E	I	O	U	Y																	
F	-	-	-	A	E	O	O	U	Y																
G	-	-	-	-	A	E	I	O	U	Y															
H	-	-	-	-	-	A	E	I	O	U	Y														
J	-	-	-	-	-	-	A	E	I	O	U	Y													
K	-	-	-	-	-	-	-	A	E	I	O	U	Y												
L	-	-	-	-	-	-	-	-	A	E	I	O	U	Y											
M	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y										
N	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y									
P	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y								
Q	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y							
R	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y						
S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y					
T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y				
V	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y			
W	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y		
X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y	
Z	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	A	E	I	O	U	Y

A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	Z
B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	A
C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	B
D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	C
E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	D
F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	E
G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	F
H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	G
I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	H
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	I
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	V
X	Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	W
Y	Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	X
Z	A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Y

FIGURE 79

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

CHAPTER VIII

SYSTEMS EMPLOYING GEARED DISK CRYPTOMECHANISMS

49. Introduction.—*a.* The earliest cipher devices involved concentric disks rotated by hand in a prearranged manner to achieve a polyalphabetic substitution, usually periodic. In some of these devices the alphabets, usually standard sequences, were unchangeable; in others, the order of the letters in the sequences could be changed at will by the correspondents. At the beginning of this century the hand-operated disk cipher devices were improved by the incorporation of gear mechanisms to accomplish the polyalphabetic substitution automatically.

b. The more primitive of these devices produced periodic polyalphabetic substitution—in fact, one device marketed commercially produced cryptograms having a period of three (!). Others yielded the encipherments of progressive alphabet systems, ciphertext autokey systems, and plaintext autokey systems; still others used variable pin wheels to govern the displacement of the components in an irregular (but periodic) manner. The more sophisticated and interesting devices employing geared disk cryptomechanisms are those typified by the Wheatstone cipher device and the Kryha cipher machine, and these deserve extensive treatment.

50. The Wheatstone cipher device.—*a.* This device, invented by Sir Charles Wheatstone[1] in 1867, is a little more than four inches in diameter, and consists of a dial with two hands, as shown in Fig. 80, below. The dial is composed of two independent circles of letters. In the outer circle, which constitutes the plain component, the letters progress clockwise in normal alphabetic sequence, but there is an extra character between the Z and the A which is used as a word separator, making a total of 27 characters; furthermore, the digits 1–Ø are inscribed together with the letters A–J and also N–W, for enciphering numbers. In the inner circle, which constitutes the cipher component, the letters are arranged in a mixed sequence and are inscribed either on a surface which permits erasure, or on a detachable cardboard circle which can be removed and replaced by another circle bearing a different sequence.

b. The two hands are pivoted concentrically, after the fashion of the hour and minute hands of a clock. In a clock, when the minute hand makes a complete revolution, the hour hand makes $\frac{1}{12}$ of a complete revolution; the action in the case of this device, however, is somewhat different. The short hand is free to be set independently of the long one, although the motion of the latter affects the former. Since the outer circle has 27 spaces and the inner one only 26, by a simple gear assembly each complete revolution of the long hand causes the short hand to make $1\frac{1}{26}$ revolutions, thus causing the short hand to point one place in advance of where it pointed at the end of the preceding revolution of the long hand. For example in Fig. 80, when the long hand is over B of the outer circle and the short hand points to R of the inner circle, if the long hand is pushed clockwise around the dial, making a complete revolution, the short hand will also make a complete revolution clockwise plus one space, thus pointing to D.

[1] Credit for the conception of the system described here belongs not to Sir Charles Wheatstone, as is commonly thought, but to an American, Decius Wadsworth, who in 1817 constructed a device identical in principle with that described on pp. 342–348 of the *Scientific Papers of Sir Charles Wheatstone*, published by the Physical Society of London in 1879. The Wheatstone device used a 27-element outer alphabet and a 26-element inner alphabet; the Wadsworth device used a 33-element outer alphabet (26 letters and the digits 2–8, inclusive) and a 26-element inner alphabet. Furthermore, whereas in the Wheatstone device only the cipher component could be varied, in the Wadsworth device both components could be varied.

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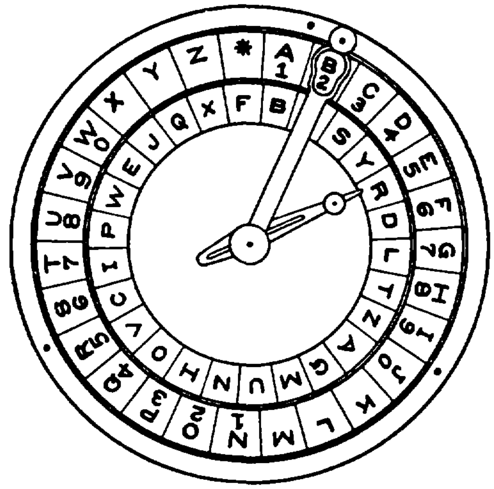


FIGURE 80

c. To encipher a message, the long hand and the short hand are first set to prearranged initial positions, or set according to an indicator procedure; furthermore, by previous agreement, the long hand is invariably to be moved in the same direction, usually clockwise. Suppose the message to be enciphered is SEND AMMUNITION. The long hand is moved clockwise until it is directly over S on the outer sequence; the letter to which the short hand points is the cipher equivalent of S and is written down. Then the long hand is moved clockwise to a position over E, and the letter to which the short hand points is noted and written down; the next two letters are enciphered in the same manner. After the encipherment of each word, the long hand is moved clockwise to the asterisk, and the cipher equivalent of this word separator is recorded; when a doubled letter occurs in the plain text, as in the case of the doubled M of AMMUNITION, some infrequently used letter such as Q_p must be substituted for the second occurrence of the letter, and encipherment proceeds as before. To decipher a message, the hands are first set to their prearranged initial position, and then the long hand is moved clockwise until the short hand points to the first cipher letter; the long hand is then directly over the plaintext equivalent. The process is continued until the message is completely deciphered.

d. The principle of the Wheatstone device can be duplicated with a pair of sliding alphabet strips— a 27-element plain component and a doubled-length cipher component. After the two strips are set to their prearranged initial juxtaposition, the cipher equivalent is found for the first plain-text letter. The cipher component strip is moved one position to the left for succeeding encipherments *whenever a letter in the plain component is located to the left of the immediately preceding plaintext letter of the message.* What makes the Wheatstone device particularly interesting is that the motion of the cipher component is highly irregular and unpredictable, depending as it does upon the particular sequence of the letters of the plain text and upon the particular arrangement of the letters in the plain component. At the time of its invention, the Wheatstone device was of course considered by the cryptologic world as "absolutely indecipherable."

51. Analysis of the Wheatstone cipher device.—*a.* In order to understand the mechanics of Wheatstone encipherment, let us study a cipher message with its compromised plain text, as given below:

5	10	15	20	25	30
S L Y Y I	R L M H T	Q W O A X	D C B Z R	I I J Z X	F I R I Q
R E F E R	Q I N G *	T O * Y O	U R * M E	S Q A G E	* N U M B
35	40	45	50	55	60
U R K L L	I R O T B	X M X P V	J Q S I V	C I S Z U	V I W W K
E R * O N	E * E I G	H T * T W	O * D A T	E D * T W	O * O N E
65	70	75	80	85	90
A V F Q H	H H G F W	O I B R J	T S B T X	Z U R E E	D J B G E
* J A N U	A R Y * C	O M Q A *	G E N E R	A L * L U	N S F O R
95	100	105	110	115	120
E Q X B B	V S D F T	B B Z F S	G X R B F	I L C E J	H D F X M
D * W I L	Q * B E *	A R Q I V	I N G * A	T * Y O U	R * H E A
125	130	135	140	145	150
K B V V V	F T X G R	Q J V F F	B Z P Y W	C K E S A	M O N F N
D Q U A R	T E R S *	A T * O N	E * S E V	E N * H U	N D R E D

(The enciphering components used in this example were the following,

```
             5         10        15        20        25  27
P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z *
C: Q U E S T I O N A B L Y C D F G H J K M P R V W X Z
```

although at this stage we are not supposed to be privy to this information, which is being given for surreptitious reference only.)

(1) We note that whenever there is a ciphertext doublet, the plaintext digraphic equivalent *always* consists of a pair of consecutive letters in the plain component, but in reverse; thus we can establish **EF**, **QS**, **NO**, and **DRAULI** as chains from the original plain component. We also note that whenever there is an **A–A** doublet in the plain text, the cipher equivalent will be a pair of consecutive letters (without reversal) in the cipher component; thus we can establish **LYC**, **IO**, **MP**, **VW**, **ST**, **UE**, and **FG** as chains from the original cipher component.

(2) We now note, at position 5, that $RQI_p = IRL_c$; and since we have determined that the letters **DRAULI** are in sequence in the plain component, it is obvious that in enciphering the plaintext fragment RQI_p, the cipher component must have moved just once. If the juxtaposition of the plain and cipher components at this point is $R_p = I_c$, then we can arrive at the following graphic representation for the distance between I_c and L_c (adding the O_c and YC_c from the chains already established in the cipher component):

```
At pos. 5, R Q Ip,  ∴  D R A U L I
           I R Lc      I O . . . L y c
```

Likewise, at positions 16, 80, and 102 we have the following relationships and recoveries of fragments of the cipher component:

```
At pos. 16,  U Rp,   ∴  D R A U L I
             D Cc         C D

At pos. 80,  R A Lp,  ∴  D R A U L I
             X Z Uc         X Z . U e

At pos. 102, R Q Ip,  ∴  D R A U L I
             B Z Fc         B . . . . F g
```

(3) Now since $ER_p = YI_c$ (at pos. 4) $= UR_c$ (31) $= TX_c$ (79), we can establish (R . . X Z . U E S T) as one of the chains from the cipher component, in addition to (I O . . . L Y C D) and (B . . . . F G) already recovered. Other relationships are exploited, as follows:

N G_p=X R_c (107)=M H_c (8), ∴ (H . . M)
N E_p=L I_c (35)=W K_c (59)=B T_c (78), ∴ (K . . . . V W)
T $*_p$=M X_c (42), ∴ (M . . . . X)
At. pos. 115, U R_p, ∴ D R A U L I; at pos. 147, D R_p, ∴ D R A U L I
J H_c H J O N_c O N
G $*_p$=H T_c (9)=R B_c (108), tying two of our chains together.

At this point the several chains can be amalgamated into one sequence,

H J K M . R V W X Z . U E S T I O N . B L Y C D F G

and the missing values can be recovered from other relationships in the matched plain and cipher.

(4) In reconstructing the plain component, we begin with the fragment (D R A U L I) and add the following relationships:

S L_c=R E_p (1), ∴ (D R A U L I . . E f)
T B_c=I G_p (39)=* A_p (100), ∴ (* . . D R A U L I . . E F G)
R I_c=E S_p (20)=U M_p (28), ∴ (* . . D R A U L I . . E F G . . M . . . . S)

We now add the following relationships:

F I_c=* N_p (26); M H_c=N G_p (8); X D_c=O U_p (15); X M_c=H T_p (41);
A X_c=Y O_p (14); V F_c=J A_p (62); S G_c=V I_p (105); X B_c=W I_p (93);
I Q_c=M B_p (29); F W_c=* C_p (69)

These yield the following recoveries in the plain component:

* H Y D R A U L I C B E F G J . M N O . Q S T V W . .

(5) A different method for deriving the cipher component from the foregoing matched plain and cipher will now be demonstrated. First, we make an abridged digraphic distribution of the plain text, as shown in Fig. 81, below, in which repeated letters in the columns (representing repeated plaintext digraphs with the reference letter at the top) have been circled.

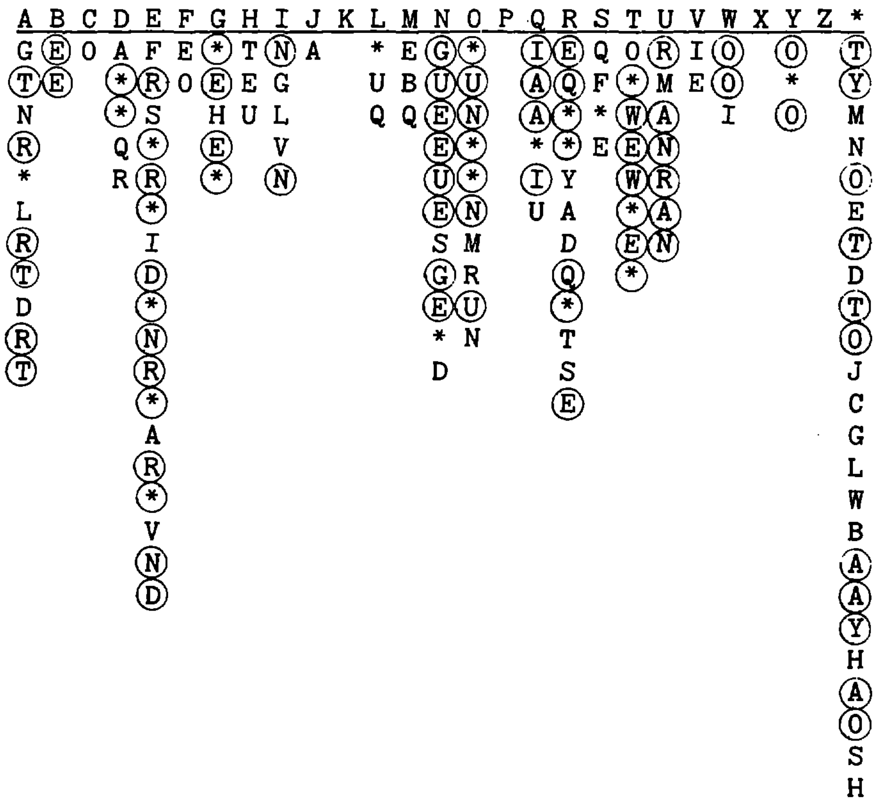


FIGURE 81

Referring to the matched plain and cipher, we make a table of the cipher equivalents of the repeated plaintext digraphs; this is shown in Fig. 82*a*, below:

1.	AR	HH BB VV	19.	QI	RL ZF
2.	AT	IV FI QJ	20.	RE	SL NF
3.	BE	QU DF	21.	RQ	IR BZ
4.	D*	IS EQ	22.	R*	CB RK HD
5.	ED	CI FN	23.	TE	VC FT
6.	EN	SB CK	24.	TW	PV ZU
7.	ER	YI UR TX	25.	T*	MX IL JV
8.	E*	XF IR KA FT BZ	26.	UA	HH VV
9.	GE	ZX TS	27.	UN	ED AM
10.	G*	HT RB	28.	UR	DC JH
11.	IN	LM GX	29.	WO	VJ UV
12.	NE	LI WK BT FB	30.	YO	AX CE
13.	NG	MH XR	31.	*A	TB BF RQ
14.	NU	IR QH	32.	*H	DF ES
15.	ON	LL WW FF	33.	*O	KL IW VF
16.	OU	XD EJ	34.	*T	TQ XP SZ
17.	O*	WO JQ VI	35.	*Y	OA LC
18.	QA	IJ BR			

FIGURE 82*a*

(6) The fragmentary chains for both the plain and cipher components are now consolidated in Fig. 82*b*, below:

1.		UAR	ON	HH	BB	VV	LL	WW	FF
2.			AT	FIV	QJ	OW			
3.		*H	BE	QU	DF	ES			
4.			D*	IS	EQ				
5.			ED	CI	FN	LS			
6.			EN	SB	CK				
7.			ER	YI	UR	TX			
8.	RQ	NU	TE*	XFT	IR	KA	QH	BZ	VC
9.			GE	ZX	TS				
10.			G*	HT	RB	JI			
11.			IN	LM	GX				
12.			NE	LI	WK	FBT	XM	VJ	QR
13.			NG	MH	XR				
14.			OU	XD	EJ				
15.			QI	RL	ZF				
16.			R*	CB	RK	HD			
17.			TW	PV	ZU				
18.			UN	ED	AM				
19.			UR	DC	JH				
20.			WO	UVJ					
21.			YO	AX	CE				
22.			*O	KL	IW	VF			
23.			*T	TQ	XP	SZ			
24.			*Y	OA	LC				

FIGURE 82*b*

Chaining of the cipher component by the graphical method of indirect symmetry is begun, using line 12 as the horizontal and line 2 as the vertical, followed by the addition of line 8, and in short order the following sequence is reconstructed:

M F B T Z P G L I Q R H Y O U V J C N E W K D A S X

(7) With this second method, there is no guarantee that the component recovered is the original cipher component. Using a blank strip for the plain component, if we try to crib in the plaintext values from the initial word REFERQING, we run into a conflict:

```
                2       4 3                         1
P:              E       E F                         R
C:  M F B T Z P G L I Q R H Y O U V J C N E W K D A S X
```

When we decimate this sequence at the correct decimation, however, we obtain the following values in the plain component,

```
P:        R       I     E F G       N     Q           *
C:  Q U E S T I O N A B L Y C D F G H J K M P R V W X Z
```

and the original plain component can be quickly reconstructed.

b. For a second case, let us assume that we have at hand the following short cryptogram, and that we know from past experience that the enemy has used a Wheatstone system, among others:

M L Y B A   F M Q R Q   W D W U H   Q P G M K   T N Z G A   C N X J X

On the assumption of direct standard alphabets, we complete the plain component sequence as is shown in Fig. 83*a*, below, for the first 15 letters:

M	L	Y	B	A	F	M	Q	R	Q	W	D	W	U	H
N	M	Z	C	B	G	N	R	S	R	X	E	X	V	I
O	N	A	D	C	H	O	S	T	S	Y	F	Y	W	J
P	O	B	E	D	I	P	T	U	T	Z	G	Z	X	K
Q	P	C	F	E	J	Q	U	V	U	A	H	A	Y	L
R	Q	D	G	F	K	R	V	W	V	B	I	B	Z	M
S	R	E	H	G	L	S	W	X	W	C	J	C	A	N
T	S	F	I	H	M	T	X	Y	X	D	K	D	B	O
U	T	G	J	I	N	U	Y	Z	Y	E	L	E	C	P
V	U	H	K	J	O	V	Z	A	Z	F	M	F	D	Q
W	V	I	L	K	P	W	A	B	A	G	N	G	E	R
X	W	J	M	L	Q	X	B	C	B	H	O	H	F	S
Y	X	K	N	M	R	Y	C	D	C	I	P	I	G	T
Z	Y	L	O	N	S	Z	D	E	D	J	Q	J	H	U
A	Z	M	P	O	T	A	E	F	E	K	R	K	I	V
B	A	N	Q	P	U	B	F	G	F	L	S	L	J	W
C	B	O	R	Q	V	C	G	H	G	M	T	M	K	X
D	C	P	S	R	W	D	H	I	H	N	U	N	L	Y
E	D	Q	T	S	X	E	I	J	I	O	V	O	M	Z
F	E	R	U	T	Y	F	J	K	J	P	W	P	N	A
G	F	S	V	U	Z	G	K	L	K	Q	X	Q	O	B
H	G	T	W	V	A	H	L	M	L	R	Y	R	P	C
I	H	U	X	W	B	I	M	N	M	S	Z	S	Q	D
J	I	V	Y	X	C	J	N	O	N	T	A	T	R	E
K	J	W	Z	Y	D	K	O	P	O	U	B	U	S	F
L	K	X	A	Z	E	L	P	Q	P	V	C	V	T	G

FIGURE 83*a*

M	L	Y	B	A	F	M	Q	R	Q	W	D	W	U	H
M	K	X	*	Y	C	J	N	O	M	S	Z	R	O	A
N	L	Y	A	Z	D	K	O	P	N	T	*	S	P	B
O	M	Z	B	*	E	L	P	Q	O	U	A	T	Q	C
P	N	*	C	A	F	M	Q	R	P	V	B	U	R	D
Q	O	A	D	B	G	N	R	S	Q	W	C	V	S	E
R	P	B	E	C	H	O	S	T	R	X	D	W	T	F
S	Q	C	F	D	I	P	T	U	S	Y	E	X	U	G
T	R	D	G	E	J	Q	U	V	T	Z	F	Y	V	H
U	S	E	H	F	K	R	V	W	U	*	G	Z	W	I
V	T	F	I	G	L	S	W	X	V	A	H	*	X	J
W	U	G	J	H	M	T	X	Y	W	B	I	A	Y	K
X	V	H	K	I	N	U	Y	Z	X	C	J	B	Z	L
Y	W	I	L	J	O	V	Z	*	Y	D	K	C	*	M
Z	X	J	M	K	P	W	*	A	Z	E	L	D	A	N
*	Y	K	N	L	Q	X	A	B	*	F	M	E	B	O
A	Z	L	O	M	R	Y	B	C	A	G	N	F	C	P
B	*	M	P	N	S	Z	C	D	B	H	O	G	D	Q
C	A	N	Q	O	T	*	D	E	C	I	P	H	E	R
D	B	O	R	P	U	A	E	F	D	J	Q	I	F	S
E	C	P	S	Q	V	B	F	G	E	K	R	J	G	T
F	D	Q	T	R	W	C	G	H	F	L	S	K	H	U
G	E	R	U	S	X	D	H	I	G	M	T	L	I	V
H	F	S	V	T	Y	E	I	J	H	N	U	M	J	W
I	G	T	W	U	Z	F	J	K	I	O	V	N	K	X
J	H	U	X	V	*	G	K	L	J	P	W	O	L	Y
K	I	V	Y	W	A	H	L	M	K	Q	X	P	M	Z
L	J	W	Z	X	B	I	M	N	L	R	Y	Q	N	*

FIGURE 83*b*

Nothing seems to be in evidence. But we are a little uneasy about our method, so on second thought we first decipher the cipher letters according to the Wheatstone rule of motion, at the setting $A_p = A_c$, and we then complete the 27-character plain-component sequence. This is shown in Fig. 83*b*, wherein the plain text appears on one generatrix. (In referring back to Fig. 83*a*, we see that we could have spotted the plain text on staggered generatrices, had we not been so hasty.) The generatrix method, then, is the one to use when the components are either known or are assumed—but we should take into consideration the mechanics of the particular cryptography involved if we are to achieve a facile solution.

c. For the next case, let us assume that we have the following cryptogram at hand, known to be in a Wheatstone system with mixed alphabets:

```
G E T E I   P L K U C   C F G K H   H U F Q J   R P Y L Y   I J M K A
O R O T H   E F M Q O   O I F V X   B E Q O M   P A J F E   T H K Z U
C J S T T   F E L B P   S H X H Y   W T I R I   K Q F D L   L R C F T
T D R P N   V I U U Q   N K I X A   F L Y S E   Q N O P U   L J I F Y
W B F T X   L Q Z T T   W E J J Y   W A S F M   M G O S U   U F G I Z
C M I N R   F J J S Q   O T Z P M   M W R Z J   U B P M A   K I N R B
O T P D G   R L L E Y
```

(1) In Wheatstone systems, different encipherments of the same underlying plain text will be isomorphic; we therefore search for isomorphs and find the following sets

```
a. K U C C F G K H H U F
b. F D L L R C F T T D R
c. S F M M G O S U U F G

d. Q O O I F V X B E Q O M P A J F E
e. P M M W R Z J U B P M A K I N R B
```

The chains derived from the foregoing are the following:

```
a–b: KFR   UD   GCL   HT
a–c: KS   HUFGO   CM
b–c: DFS   LM   RG   CO   TU
d–e: QPK   OMAIW   FR   VZ   XJN   EBU
```

Employing the graphical method of indirect symmetry beginning with the a–b and d–e chains as the horizontal elements and the a–c chains as the vertical, we arrive at the following reconstruction fragments:

```
   HT
  EBUD
 QPKFR          VZ          XJN
   SGCL
    OMAIW
```

There are no tie-ins between these fragments, so we shall have to see whether we can extend the isomorphic passages. We therefore copy the isomorphs together with their five preceding and following letters, as shown below:

```
a. t e i [p l K U C C F G K H H U F q j r] p y
b. i r i [k q F D L L R C F T T D R p n v] i u
c. j j y [w a S F M M G O S U U F G i z c] m i

d. t h e f m [Q O O I F V X B E Q O M P A J F E t h] k z u
e. s q o t z [P M M W R Z J U B P M A K I N R B o t] p d g
```

The solid brackets show the outer limits of the isomorphism. No new values are picked up, but the vertical digraph LQ between lines a–b now permits us to establish a long chain,

O M A I W S G C L Q P K F R V Z E B U D . . . . H T,

with only two placements possible for the fragment XJN and the missing letter Y, either XJNY or YXJN. We shall start with the XJNY possibility.

(2) If this is a decimation of a systematically mixed sequence, the original cipher component might be recovered by trial decimations to uncover the method by which it was constructed; otherwise we would be forced to assume decimations of -1, $+3$, -3, $+5$ . . . -11, in turn, "deciphering" the cipher text in the Wheatstone fashion, until the δ I.C. of the converted text showed us that we had arrived at the correct decimation. For example, using the sequence given above as the cipher component, and the normal sequence followed by a 27th character for the plain component (at the setting $A_p = O_c$), the following is the conversion of the first 60 letters of the text and their frequencies:

```
G E T E I   P L K U C   C F G K H   H U F Q J   R P Y L Y   I J M K A
G Q Z P B   I F I P D   C H A F S   R K D * L   C Z L W K   Q H N X N

O R O T H   E F M Q O   O I F V X   B E Q O M   P A J F E   T H K Z U . . .
K X J H F   X S G O E   D G P R X   T R J * A   J A T J N   W U G K N
```

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	*
3	1	2	3	1	3	4	3	2	4	4	2		4	1	3	2	3	2	2	1		2	4		2	2

The I.C. of this converted text is $\frac{27(126)}{60 \cdot 59} = 0.96$, somewhat less than miserable; the conversions using the next six decimations of the cipher component, through +7, also give negative results. The −7 decimation of our recovered sequence is the following:

```
O D F S H B P I N Z L M X R G T U K W Y E Q A J V C
```

When we use this for the cipher component we arrive at the following conversion of the text and its accompanying distribution:

```
G E T E I   P L K U C   C F G K H   H U F Q J   R P Y L Y   I J M K A
O U O T F   D H O M V   U X I L Y   X I U M O   D W I Z H   V K Y D I

O R O T H   E F M Q O   O I F V X   B E Q O M   P A J F E   T H K Z U . . .
M Z L * O   D L U D I   H O I D R   J Y Z D O   I Y Z D V   P D Q H O
```

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	*
			9		1		4	7	1	1	3	3		8	1	1	1		1	4	3	1	2	4	4	1

This gives us an I.C. of $\frac{27(238)}{60 \cdot 59} = 1.82$, so we know we have hit the answer.

(3) The entire text can now be converted to monoalphabetic terms and solved. The decipherment of the first 60 letters is shown below, together with the plaintext values of the monoalphabetically converted text:

```
  C: G E T E I   P L K U C   C F G K H   H U F Q J   R P Y L Y   I J M K A
"P": O U O T F   D H O M V   U X I L Y   X I U M O   D W I Z H   V K Y D I
  P: E N E M Y   * R E C O   N Q A I S   Q A N C E   * P A T R   O L S * A

  C: O R O T H   E F M Q O   O I F V X   B E Q O M   P A J F E   T H K Z U . . .
"P": M Z L * O   D L U D I   H O I D R   J Y Z D O   I Y Z D V   P D Q H O
  P: C T I V E   * I N * A   R E A * J   U S T * E   A S T * O   F * G R E

    P:       *   Y   R A U L I C   E F G J   M N O P Q S T V
  "P": A B C D E F G H I J K L M N O P Q R S T U V W X Y Z *
```

The original plain and cipher components are the following:

```
P:  H Y D R A U L I C B E F G J K M N O P Q S T V W X Z *
C:  A J V C O D F S H B P I N Z L M X R G T U K W Y E Q
```

The original cipher component, a numerical-key columnar transposition-mixed sequence based upon HYDRAULIC, could have been recovered earlier, but this was glossed over for the sake of the exposition which has just been presented.

d. We have seen in subpar. *c*, above, that when we know or correctly assume the cipher component, the cipher text can be converted to monoalphabetic terms and solved. When only the plain component is known in a Wheatstone system this is still a useful bit of information, since the placing of cribs is greatly facilitated. For example, if the plain component is the normal A–Z sequence followed by the 27th character for the word separator, the encipherment of the word *ENEMY* will always produce an A–B–BA– idiomorphic pattern, *regardless of the composition of the cipher component.* We can therefore make up idiomorphic lists of frequent words and word combinations to assist us in cribbing, such as the following patterns which are based upon a direct-standard plain component.

ENEMY	*CROSQROADS*	*COUNTERATQACK*
A-B-BA-	--A-B-BAC--C	--ABAB----BC--C
DIVISION	*ROADJUNCTION*	*COMQUNICATION*
-A-----BBA	--------A--BBA	--A--------ABB-
ARTILQERY	*HEADQUARTERS*	*RECONQAISQANCE*
-ABC--B-CA-	--A-A-B--B--B-	---ABBCA----C---
BATQALION	*INTELQIGENCE*	*REINFORCEMENTS*
-AA-B---BB-	-A--B-C-DAC-DB	-AB--C--BC--ADDB

(1) Let us now suppose that we have available the cryptogram given below, known to have been enciphered in a Wheatstone system involving a normal plain component and a mixed cipher component:

```
        5          10         15         20         25         30
M O L V L  E L X H G  M M S U K  C F D U E  Q Y I I H  P S B E E
        35         40         45         50         55         60
X N J T L  C B S X F  G I P H F  Z K B P G  W E J D P  O E F O I
        65         70         75         80         85         90
O J P M V  O O K G D  T I H Q H  A E I L U  D Q M F W  V V M C Q
        95         100
O W G I V  R U L R P
```

We search through our idiomorph list and find that the word *ROADJUNCTION* will fit at position 75:

```
               75        80        85        90
. . . T I H Q H A E I L U D Q M F W V V M C Q . . .
              * R O A D J U N C T I O N *
```

We set a blank strip against one bearing the normal sequence plus an asterisk, and we enter the cipher values from our crib on the blank strip, moving it to the left one position whenever we have to go to the left on the plain component. This yields the following partial sequence for the cipher component:

```
H . . I . . L M . . . . U . W . E Q A . V . . D F .
```

Since the three letters to the left of the crib are on our cipher component, they can now be deciphered, and the word NEAR is confirmed by the word separator (D_c) which is correctly deciphered with our

```
              70        75        80        85        90
. . . O O K G D T I H Q H A E I L U D Q M F W V V M C Q . . .
                  E A R * R O A D J U N C T I O N *
```

partial component: this adds a T in our sequence immediately to the left of the U.

(2) We now scan the cipher text, taking note of those letters which have already been placed in our cipher component, as shown by the capital letters in the diagram below:

```
        5         10        15        20        25        30
M o L V L E L x H g M M s U k c F D U E Q y I I H p s b E E . . .
```

Since there are two substantial clusters (of five letters each) at positions 3 and 17, we align our components arbitrarily at $A_p = H_c$, decipher the cipher according to the Wheatstone rule of motion and complete the plain-component sequence. This is shown in Figs. 84*a* and *b*, below:

. . . L	V	L (5)	E	L . . .
G	U	F	P	E
H	V	G	Q	F
I	W	H	R	G
J	X	I	S	H
K	Y	J	T	I
L	Z	K	U	J
M	*	L	V	K
N	A	M	W	L
O	B	N	X	M
P	C	O	Y	N
Q	D	P	Z	O
R	E	Q	*	P
S	F	R	A	Q
T	G	S	B	R
U	H	T	C	S
V	I	U	D	T
W	J	V	E	U
X	K	W	F	V
Y	L	X	G	W
Z	M	Y	H	X
*	N	Z	I	Y
A	O	*	J	Z
B	P	A	K	*
C	Q	B	L	A
D	R	C	M	B
E	S	D	N	C
F	T	E	O	D

FIGURE 84*a*

. . . F	D	U	E (20)	Q . . .
Y	W	K	O	P
Z	X	L	P	Q
*	Y	M	Q	R
A	Z	N	R	S
B	*	O	S	T
C	A	P	T	U
D	B	Q	U	V
E	C	R	V	W
F	D	S	W	X
G	E	T	X	Y
H	F	U	Y	Z
I	G	V	Z	*
J	H	W	*	A
K	I	X	A	B
L	J	Y	B	C
M	K	Z	C	D
N	L	*	D	E
O	M	A	E	F
P	N	B	F	G
Q	O	C	G	H
R	P	D	H	I
S	Q	E	I	J
T	R	F	J	K
U	S	G	K	L
V	T	H	L	M
W	U	I	M	N
X	V	J	N	O

FIGURE 84*b*

The plaintext fragments REQ*P and CAPTU suggest THREQ*P and *CAPTURED*, and the beginning of the text is quickly deciphered:

```
        5         10        15        20        25
M O L V L E L X H G M M S U K C F D U E Q Y I I H . . .
T H R E Q * P R I S O N E R S * C A P T U R E D *
```

This adds new values to our cipher component, making 20 recoveries in all:

```
H . . I . . L M X . G T U K W Y E Q A . V C O D F S
```

The rest of the cipher text is now deciphered in similar fashion, and the entire cipher component is found to be

```
H B P I N Z L M X R G T U K W Y E Q A J V C O D F S
```

52. The Kryha cipher machine.—*a.* This machine, invented by Alexander von Kryha in Germany in 1924, is a spring-operated polyalphabetic cipher machine which has as its principle the irregular displacement of two concentric disks which comprise the plain and cipher components. (Actually, the cipher component is a 52-element disk while the plain component is in the shape of a semicircular frame juxtaposed against the revolving cipher component.) The letters of the two components are printed on small metal tabs which are inserted in slots on the two disks so that the sequence of the letters may be varied according to prearranged keys. The displacement of the alphabets occurring after each encipherment is accomplished through a selector wheel having on its periphery 17 toothed sectors consisting of from one to six teeth each, the sectors being designated by the numbers 1–17. These teeth serve to displace the components a distance equivalent to the number of teeth in the sector; however, owing to the manner of spacing between the toothed portions of the wheel, an additional displacement of four positions is added at each operation of the machine. The selector wheel has the following effective displacements between its 17 numbered positions:

No.: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 1
Displacement: 7 6 7 5 6 7 6 8 6 10 5 6 5 7 6 5 9

Since the sum of these displacements is 111 ($\equiv 7$, mod 26) it follows that after a complete revolution of the selector wheel the cipher component will be displaced 7 positions to the left from its original juxtaposition; and since this number 7, is prime to 26, there will be 26 series of 17 displacements each, making the period of the machine $17 \times 26 = 442$.

b. As an illustration of encipherment, let us assume that the components of the machine have been arranged as follows:

```
P:  A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C:  H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
```

If the initial juxtaposition of the components is that shown above (i.e., in the key of H), the first plaintext letter is enciphered in the H alphabet, and a stepping button is pressed to bring the next alphabet into position. If the setting of the selector wheel was at position 1 at the beginning of the encipherment, the next alphabet to be brought into play will be 7 to the right of the first or H alphabet (i.e., the 8th or I alphabet), and the one after that will be 6 places to the right of the I alphabet (i.e., the 14th or J alphabet), and so on. At the end of 17 encipherments the alphabets shall have been displaced 7 positions, mod 26, relative to the initial setting, so that the key series 1, 8, 14 . . . becomes the isomorphic equivalent 8, 15, 21 . . .; in other words, the elements in each succeeding row of 17 letters are 7 more than the corresponding elements in the preceding row. The first five rows of 17 elements of the key are shown in the diagram below:

```
 1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16   17
   7    6    7    5    6    7    6    8    6   10    5    6    5    7    6    5    9
 1    8   14   21   26    6   13   19    1    7   17   22    2    7   14   20   25
 8   15   21    2    7   13   20   26    8   14   24    3    9   14   21    1    6
15   22    2    9   14   20    1    7   15   21    5   10   16   21    2    8   13
22    3    9   16   21    1    8   14   22    2   12   17   23    2    9   15   20
 3   10   16   23    2    8   15   21    3    9   19   24    4    9   16   22    1 . . .
```

Translating the foregoing numbers into literal form as 1=A, 2=B, . . . 26=Z for typographic convenience, we obtain the complete key diagram as shown in Fig. 85, below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
7	6	7	5	6	7	6	8	6	10	5	6	5	7	6	5	9
A	H	N	U	Z	F	M	S	A	G	Q	V	B	G	N	T	Y
H	O	U	B	G	M	T	Z	H	N	X	C	I	N	U	A	F
O	V	B	I	N	T	A	G	O	U	E	J	P	U	B	H	M
V	C	I	P	U	A	H	N	V	B	L	Q	W	B	I	O	T
C	J	P	W	B	H	O	U	C	I	S	X	D	I	P	V	A
J	Q	W	D	I	O	V	B	J	P	Z	E	K	P	W	C	H
Q	X	D	K	P	V	C	I	Q	W	G	L	R	W	D	J	O
X	E	K	R	W	C	J	P	X	D	N	S	Y	D	K	Q	V
E	L	R	Y	D	J	Q	W	E	K	U	Z	F	K	R	X	C
L	S	Y	F	K	Q	X	D	L	R	B	G	M	R	Y	E	J
S	Z	F	M	R	X	E	K	S	Y	I	N	T	Y	F	L	Q
Z	G	M	T	Y	E	L	R	Z	F	P	U	A	F	M	S	X
G	N	T	A	F	L	S	Y	G	M	W	B	H	M	T	Z	E
N	U	A	H	M	S	Z	F	N	T	D	I	O	T	A	G	L
U	B	H	O	T	Z	G	M	U	A	K	P	V	A	H	N	S
B	I	O	V	A	G	N	T	B	H	R	W	C	H	O	U	Z
I	P	V	C	H	N	U	A	I	O	Y	D	J	O	V	B	G
P	W	C	J	O	U	B	H	P	V	F	K	Q	V	C	I	N
W	D	J	Q	V	B	I	O	W	C	M	R	X	C	J	P	U
D	K	Q	X	C	I	P	V	D	J	T	Y	E	J	Q	W	B
K	R	X	E	J	P	W	C	K	Q	A	F	L	Q	X	D	I
R	Y	E	L	Q	W	D	J	R	X	H	M	S	X	E	K	P
Y	F	L	S	X	D	K	Q	Y	E	O	T	Z	E	L	R	W
F	M	S	Z	E	K	R	X	F	L	V	A	G	L	S	Y	D
M	T	Z	G	L	R	Y	E	M	S	C	H	N	S	Z	F	K
T	A	G	N	S	Y	F	L	T	Z	J	O	U	Z	G	M	R
A	H	N	U	Z	. . .											

FIGURE 85

c. Kryha subsequently improved his machine by incorporating a selector wheel with 52 adjustable screws or "stops," each screw having the function of bringing into play a particular displacement, of at least 3, of the alphabets. Any combination of these 52 screws could be used to generate a series of successive displacements which summed to 179 ($\equiv$23, mod 26). Since 23 is prime, the period of the improved Kryha machine is 26 times the number of stops used, and therefore ranges between 26 (for 1 stop, which would raise the devil with the machine mechanically) and 1352 (when all 52 stops are used). Below is given the stepping pattern of the improved model of the Kryha machine:

Screw No.:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Skips:	4	3	4	3	3	3	4	3	4	4	3	3	3	3	4	4	3	3	4	3

Screw No.:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Skips:	3	4	3	3	4	3	3	4	4	4	3	3	4	3	3	3	4	3	4	3

Screw No.:	41	42	43	44	45	46	47	48	49	50	51	52	1
Skips:	3	4	5	3	3	3	3	4	3	3	4	5	

d. In the analysis of the Kryha machine we shall first treat the aspects of solution of the original 1924 model and then those applying to the improved model.

53. Analysis of the original Kryha cipher machine.—*a*. For a demonstration of the general method of solution of the original Kryha machine, we shall use as an example a test message submitted to the U.S. Army Signal Corps in 1933; this is shown in Fig. 86, below: [2]

```
XYICP NDEAM APDTR AXXPZ XHYRY TWQXF
HCDJK AHQUR ZPPPZ QOFUV KFEMN EAONG
TTXSV VUBDG JREJF HEOKV CQHFH ROKUP
MQPQW ACOJC RLMBM EVKRV JDYNN SXUDL
HNPFW MOCMJ FLGPM BKHAU XLIVV QSXUN

JZUKK OBAAE UQOYJ IZSZU HGWGW ATEJW
YDIVX PEIKE ECMCI RXXLA ZLAIN MJZXI
CIDKQ LMMTE LLFJT JUBQO LJAWM FEHEV
SYCAS KFONO ZUMPA DAPJY LPFNT RUITC
BWHJH MOLCV RDEPF QACIU HCZCB XTOKC

IXGOS GCMRF HJVXS VZNMU GJJSO QBJQH
BQNLH RTMEL YNHKU FXJDM JYCPA DPPWY
MGUWO IAIIG PTSFC SOKID GGTYO AAQDR
QRRMN TSHYN EXYVF CMJJK NXVTE FXAUT
SEZQS HLULY CYGXO NLAWQ TEJNB SMVTE

HSXUY NJKXF PEPGF CMMCW ZRPJY GOPZU
QNVXI AXZKQ MJEFW WMRQR TETPX RSUKC
DLHED LLCTJ KSZMQ MKNJU VPFLY HYFQR
EWNDZ MBMPB OJXEQ IZAXH NDBQQ WDIXQ
PIFAY JGQJO FWFCD BXYNX YTWYK EQCDP

DYDOZ HJFCZ UEDDJ BFXTT VFYGH CTBGO
FEHBU BZDQQ TIGDY AIYFD FHABS AHYGX
IBBLE CQOSE MOMZV KHQSJ CMJFF EVVTL
WTESL AYWFY CKOXP SVNAI GOCZZ KVVVJ
SOPEN YXDDX LDCYA XMWWO CWOII BNXTV

TLIVQ WXUET PSUHC SOYTP VYIKZ NFVIE
YPHKI NCGGV IKROO SOVMG HKUNU SUNYV
CFELO OWSAI YRREV NEXPE SEGRP ZNBMM
YUZFG SXRXW MNWTL RHVFH GSXMW VREAJ
DGOZA GRXKJ LDOGY PTYXN TMWQM YSQWL

XHNGZ QDMCW PYATG NZFJK WGDKA VSJMH
JGWJE CWTDB ZNMYT NAORV HARRP DXGCA
PHJNZ KTLQR QJJAF FZGDX LRFFS AWSZN
GLSAQ BMCDY JFMBL SXEOT LFJGG LGKKR
YYWDA LHHJV CGYVR LYSPJ VPKGW WXHFA

CMTRG UJEJW TAFSN ZXVVW IYWOO MTLUF
SBCAJ RNRMP IYLWI KAOKH TMXCN IMWTF
GTTDE HTDHM KKCDK EAPHI AXZYP
```

FIGURE 86

[2] The interesting circumstances surrounding this case are recorded in an article by the author entitled "Q.E.D.—2 Hours, 41 Minutes," published in the *NSA Technical Journal*, Vol. XVIII, No. 4, Fall 1973.

b. Now it is clear from the key diagram in Fig. 85 that if a message were encrypted starting at, say, position 1 of the selector wheel and "alphabet 1" of the enciphering components, the 1st, 9th, 33d, 41st, and 57th letters, for example, would be enciphered in the key of A, and would therefore belong to one monoalphabetic distribution; this is shown in the key fragment in Fig. 87*a*, below. It may also be seen that, owing to the isomorphism of the key, even if a different initial alphabet were used (take

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
7	6	7	5	6	7	6	8	6	10	5	6	5	7	6	5	9
Ⓐ	H	N	U	Z	F	M	S	Ⓐ	G	Q	V	B	G	N	T	Y
H	O	U	B	G	M	T	Z	H	N	X	C	I	N	U	Ⓐ	F
O	V	B	I	N	T	Ⓐ	G	O	U	E	J	P	U	B	H	M
V	C	I	P	U	Ⓐ	H	N	V	B	L	Q	W	B	I	O	T

FIGURE 87*a*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
7	6	7	5	6	7	6	8	6	10	5	6	5	7	6	5	9
Ⓑ	I	O	V	A	G	N	T	Ⓑ	H	R	W	C	H	O	U	Z
I	P	V	C	H	N	U	A	I	O	Y	D	J	O	V	Ⓑ	G
P	W	C	J	O	U	Ⓑ	H	P	V	F	K	Q	V	C	I	N
W	D	J	Q	V	Ⓑ	I	O	W	C	M	R	X	C	J	P	U

FIGURE 87*b*

any other letter in the first column of Fig. 85—for example, B), as long as the selector wheel was set at position 1 the 1st, 9th, 33d, 41st, and 57th letters would still be monoalphabetically distributed, as shown by the key fragment in Fig. 87*b*, above. Therefore in the course of analysis a total of 17 sets of distributions might have to be made, and the correct initial position of the selector wheel would be that in which *all 26* distributions of one of the 17 sets revealed a close approximation to the expected δ I.C. for the language involved.

c. One way of making the distributions is to write out the cipher text in a block consisting of rows of 17 letters, allocating the letters of column 1 into 26 distributions, each succeeding letter in the column being recorded in a distribution 7 away from its predecessor because of the +7 isomorphic progression of the key; thus the letters of column 1 would go into distributions 1, 8, 15, 22, 3. . . . Then the letters of column 2 would likewise be allocated into the same distributions, but starting with the proper distribution for the first letter of column 2 (which depends upon the selector position assumed for the first letter of column 1). For example, with selector position 1 the keying sequence is that shown in line (1) of Fig. 88 below, so the first letter of column 2 would be put into the 8th distribution, the first letter of column 3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	7	6	7	5	6	7	6	8	6	10	5	6	5	7	6	5	9
(1)	1	8	14	21	26	6	13	19	1	7	17	22	2	7	14	20	25
	8	15	21	2	7	. . .											
(2)		1	7	14	19	25	6	12	20	26	10	15	21	26	7	13	18
	1	8	14	21	26	6	. . .										
						* * * * * * *											
(17)																	1
	10	17	23	4	9	15	22	2	10	16	26	5	11	16	23	3	8
	17	24	4	11	. . .												

FIGURE 88

into the 14th distribution, and so on to the first letter of column 17 which would be placed into the 25th distribution. In testing for initial selector position 2, the keying sequence is that shown in line (2) of Fig. 88, so if the first letter of column 2 would be put into the 7th distribution, the first letter of column 3 into the 14th distribution, and so on to the first letter of the last column which would be placed into the 1st distribution. Finally, in testing for initial selector position 17 with the keying sequence shown in line (17) of Fig. 88, the first letter of column 1 would be put into distribution 1, while the first letter of the last column would go into distribution 3.

d. A much easier and quicker way of making the distributions presents itself, using a grille to be placed over the cipher text which is written out in a block 17 wide. Using the key diagram of Fig. 85 as a guide, we cut holes into our grille at the locations of any arbitrary key letter, say the letter A, as shown in Fig. 89, below. We then complete the grille as illustrated in Fig. 90, with the necessary cyclically offset portion shown in dotted lines, and the grille doubled in length for convenience in use. (The heavy circles show the index or reference position used in designating the placement of the grille over the cipher text.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
7	6	7	5	6	7	6	8	6	10	5	6	5	7	6	5	9
Ⓐ	H	N	U	Z	F	M	S	Ⓐ	G	Q	V	B	G	N	T	Y
H	O	U	B	G	M	T	Z	H	N	X	C	I	N	U	Ⓐ	F
O	V	B	I	N	T	Ⓐ	G	O	U	E	J	P	U	B	H	M
V	C	I	P	U	Ⓐ	H	N	V	B	L	Q	W	B	I	O	T
C	J	P	W	B	H	O	U	C	I	S	X	D	I	P	V	Ⓐ
J	Q	W	D	I	O	V	B	J	P	Z	E	K	P	W	C	H
Q	X	D	K	P	V	C	I	Q	W	G	L	R	W	D	J	O
X	E	K	R	W	C	J	P	X	D	N	S	Y	D	K	Q	V
E	L	R	Y	D	J	Q	W	E	K	U	Z	F	K	R	X	C
L	S	Y	F	K	Q	X	D	L	R	B	G	M	R	Y	E	J
S	Z	F	M	R	X	E	K	S	Y	I	N	T	Y	F	L	Q
Z	G	M	T	Y	E	L	R	Z	F	P	U	Ⓐ	F	M	S	X
G	N	T	Ⓐ	F	L	S	Y	G	M	W	B	H	M	T	Z	E
N	U	Ⓐ	H	M	S	Z	F	N	T	D	I	O	T	Ⓐ	G	L
U	B	H	O	T	Z	G	M	U	Ⓐ	K	P	V	Ⓐ	H	N	S
B	I	O	V	Ⓐ	G	N	T	B	H	R	W	C	H	O	U	Z
I	P	V	C	H	N	U	Ⓐ	I	O	Y	D	J	O	V	B	G
P	W	C	J	O	U	B	H	P	V	F	K	Q	V	C	I	N
W	D	J	Q	V	B	I	O	W	C	M	R	X	C	J	P	U
D	K	Q	X	C	I	P	V	D	J	T	Y	E	J	Q	W	B
K	R	X	E	J	P	W	C	K	Q	Ⓐ	F	L	Q	X	D	I
R	Y	E	L	Q	W	D	J	R	X	H	M	S	X	E	K	P
Y	F	L	S	X	D	K	Q	Y	E	O	T	Z	E	L	R	W
F	M	S	Z	E	K	R	X	F	L	V	Ⓐ	G	L	S	Y	D
M	T	Z	G	L	R	Y	E	M	S	C	H	N	S	Z	F	K
T	Ⓐ	G	N	S	Y	F	L	T	Z	J	O	U	Z	G	M	R
A	H	N	U	Z...												

FIGURE 89

e. We now place the grille so that the index position is over the first letter of the cipher text, as illustrated in Fig. 91*a* (only the top 40 of the 67 lines of cipher are shown here), and we make the following distribution of the letters exposed through the apertures:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
6		1	2		1	1	2	1	3	2	2	1		2		3	1	1		1	2	2	2	5	4

This distribution has a δ I.C. of 1.18, nothing to write home about. We then slide the grille over one position to the right, so that the index position is over the second letter of the cipher text, and we arrive at a distribution with an I.C. of 0.95. Sliding the grille so that the index position is over the third letter of the cipher, we obtain a distribution with an I.C. of 0.97. When, however, we slide the grille so that the index position is over the fourth letter of the cipher (as illustrated in fragmentary form in Fig. 91*b*) we obtain the following distribution:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
5		5		4	1	4		1	2	2			2		6		1	4		4				5	4

The I.C. of this distribution is 1.79 so we know that, barring the touch of a Mephistophelian (or rather, a Bernoullian) finger, the fourth letter of the text was enciphered at position 1 of the selector wheel; therefore the initial setting for the first letter must have been position 15. In any case, we shall prove or disprove this point very quickly.

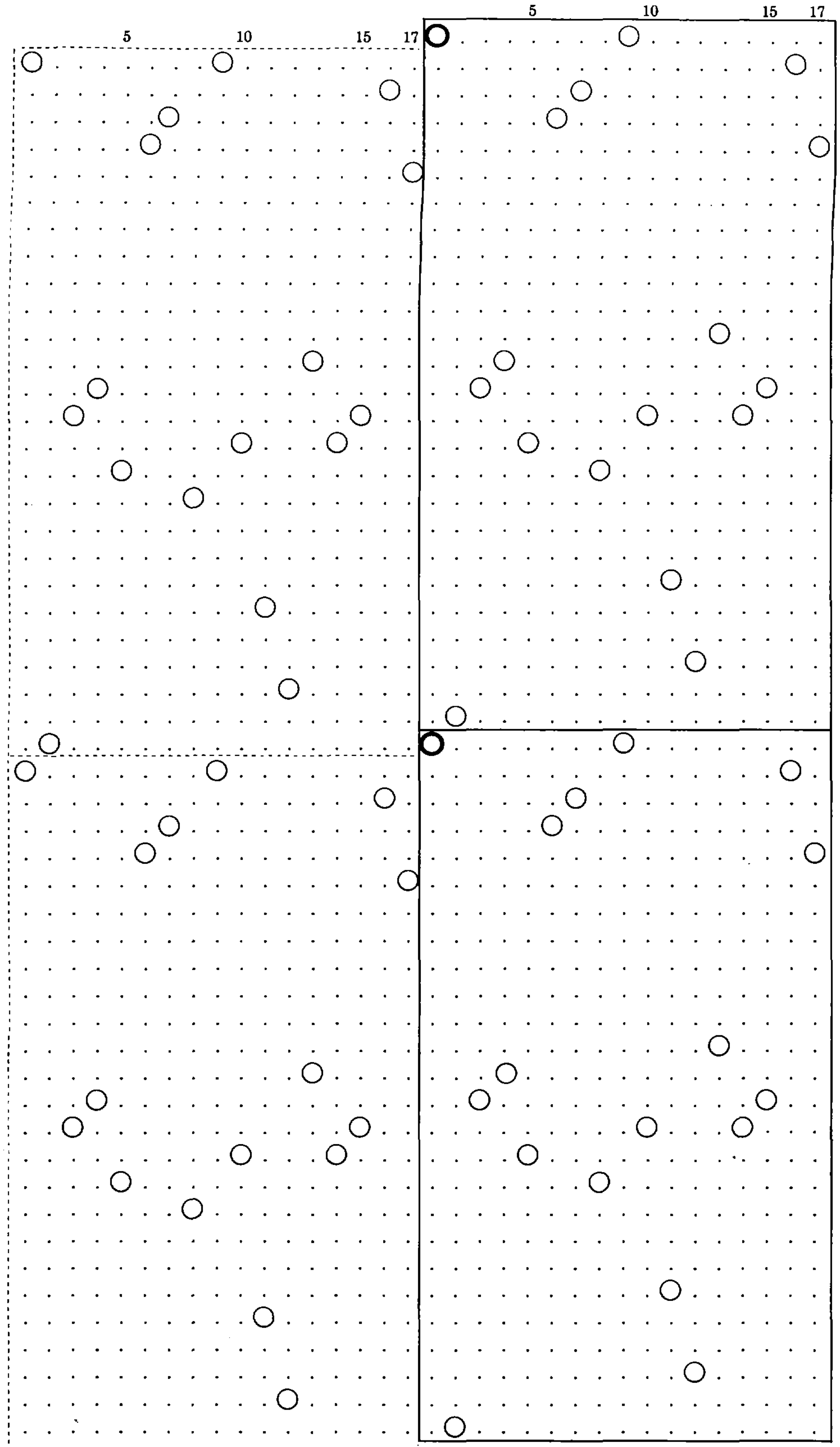


FIGURE 90

f. If this is the correct initial setting of the selector wheel and we drop the grille vertically one position, the letters exposed through the apertures will still be monoalphabetically distributed, but in an alphabet +7 away from that which we have just obtained above; in other words, referring to Fig. 85, if the letters exposed in Fig. 91*b* belonged to the A or 1st alphabet, then sliding the grille down one position would reveal the letters that belonged to the H or 8th alphabet. Sure enough, the I.C. of the distribution of these letters is 1.92, so we know that we are on the right track. By successively sliding

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	(X)	Y	I	C	P	N	D	E	(A)	M	A	P	D	T	R	A	X
	X	P	Z	X	H	Y	R	Y	T	W	Q	X	F	H	C	(D)	J
	K	A	H	Q	U	R	(Z)	P	P	P	Z	Q	O	F	U	V	K
	F	E	M	N	E	(A)	O	N	G	T	T	X	S	V	V	U	B
5	D	G	J	R	E	J	F	H	E	O	K	V	C	Q	H	F	(H)
	R	O	K	U	P	M	Q	P	Q	W	A	C	O	J	C	R	L
	M	B	M	E	V	K	R	V	J	D	Y	N	N	S	X	U	D
	L	H	N	P	F	W	M	O	C	M	J	F	L	G	P	M	B
	K	H	A	U	X	L	I	V	V	Q	S	X	U	N	J	Z	U
10	K	K	O	B	A	A	E	U	Q	O	Y	J	I	Z	S	Z	U
	H	G	W	G	W	A	T	E	J	W	Y	D	I	V	X	P	E
	I	K	E	E	C	M	C	I	R	X	X	L	(A)	Z	L	A	I
	N	M	J	(Z)	X	I	C	I	D	K	Q	L	M	M	T	E	L
	L	F	(J)	T	J	U	B	Q	O	L	J	A	W	M	(F)	E	H
15	E	V	S	Y	C	A	S	K	F	(O)	N	O	Z	(U)	M	P	A
	D	A	P	J	(Y)	L	P	F	N	T	R	U	I	T	C	B	W
	H	J	H	M	O	L	C	(V)	R	D	E	P	F	Q	A	C	I
	U	H	C	Z	C	B	X	T	O	K	C	I	X	G	O	S	G
	C	M	R	F	H	J	V	X	S	V	Z	N	M	U	G	J	J
20	S	O	Q	B	J	Q	H	B	Q	N	L	H	R	T	M	E	L
	Y	N	H	K	U	F	X	J	D	M	(J)	Y	C	P	A	D	P
	P	W	Y	M	G	U	W	O	I	A	I	I	G	P	T	S	F
	C	S	O	K	I	D	G	G	T	Y	O	A	A	Q	D	R	Q
	R	M	R	N	T	S	H	Y	N	E	X	(Y)	V	F	C	M	J
25	J	K	N	X	V	T	E	F	X	A	U	T	S	E	Z	Q	S
	H	(L)	U	L	Y	C	Y	G	X	O	N	L	A	W	Q	T	E
	(J)	N	B	S	M	V	T	E	(H)	S	X	U	Y	N	J	K	X
	F	P	E	P	G	F	C	M	M	C	W	Z	R	P	J	(Y)	G
	O	P	Z	U	Q	N	(V)	X	I	A	X	Z	K	Q	M	J	E
30	F	W	W	M	R	(Q)	R	T	E	T	P	X	R	S	U	K	C
	D	L	H	E	D	L	L	C	T	J	K	S	Z	M	Q	M	(K)
	N	J	U	V	P	F	L	Y	H	Y	F	Q	R	E	W	N	D
	Z	M	B	M	P	B	O	J	X	E	Q	I	Z	A	X	H	N
	D	B	Q	Q	W	D	I	X	Q	P	I	F	A	Y	J	G	Q
35	J	O	F	W	F	C	D	B	X	Y	N	X	Y	T	W	Y	K
	E	Q	C	D	P	D	Y	D	O	Z	H	J	F	C	Z	U	E
	D	D	J	B	F	X	T	T	V	F	Y	G	H	C	T	B	G
	O	F	E	H	B	U	B	Z	D	Q	Q	T	(I)	G	D	Y	A
	I	Y	F	(D)	F	H	A	B	S	A	H	Y	G	X	I	B	B
40	L	E	(C)	Q	O	S	E	M	O	M	Z	V	K	H	(Q)	S	J

* * * * *

FIGURE 91*a*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
	X	Y	I	(C)	P	N	D	E	A	M	A	(P)	D	T	R	A	X	
	X	P	Z	X	H	Y	R	Y	T	W	Q	X	F	H	C	D	J	○
	K	(A)	H	Q	U	R	Z	P	P	(P)	Z	Q	O	F	U	V	K	
	F	E	M	N	E	A	O	N	(G)	T	T	X	S	V	V	U	B	
5	D	G	J	R	E	J	F	H	E	O	K	V	C	Q	H	F	H	○
	R	O	(K)	U	P	M	Q	P	Q	W	A	C	O	J	C	R	L	
	M	B	M	E	V	K	R	V	J	D	Y	N	N	S	X	U	D	
	L	H	N	P	F	W	M	O	C	M	J	F	L	G	P	M	B	
	K	H	A	U	X	L	I	V	V	Q	S	X	U	N	J	Z	U	
10	K	K	O	B	A	A	E	U	Q	O	Y	J	I	Z	S	Z	U	
	H	G	W	G	W	A	T	E	J	W	Y	D	I	V	X	P	E	
	I	K	E	E	C	M	C	I	R	X	X	L	A	Z	L	(A)	I	
	N	M	J	Z	X	I	(C)	I	D	K	Q	L	M	M	T	E	L	
	L	F	J	T	J	(U)	B	Q	O	L	J	A	W	M	F	E	H	○
15	(E)	V	S	Y	C	A	S	K	F	O	N	O	(Z)	U	M	P	(A)	
	D	A	P	J	Y	L	P	(F)	N	T	R	U	I	T	C	B	W	
	H	J	H	M	O	L	C	V	R	D	(E)	P	F	Q	A	C	I	
	U	H	C	Z	C	B	X	T	O	K	C	I	X	G	O	S	G	
	C	M	R	F	H	J	V	X	S	V	Z	N	M	U	G	J	J	
20	S	O	Q	B	J	Q	H	B	Q	N	L	H	R	T	M	E	L	
	Y	N	H	K	U	F	X	J	D	M	J	Y	C	(P)	A	D	P	
	P	W	Y	M	G	U	W	O	I	A	I	I	G	P	T	S	F	
	C	S	O	K	I	D	G	G	T	Y	O	A	A	Q	D	R	Q	
	R	M	R	N	T	S	H	Y	N	E	X	Y	V	F	(C)	M	J	
25	J	K	N	X	V	T	E	F	X	A	U	T	S	E	Z	Q	S	
	H	L	U	L	(Y)	C	Y	G	X	O	N	L	A	W	Q	T	E	
	J	N	B	(S)	M	V	T	E	H	S	X	(U)	Y	N	J	K	X	
	F	P	E	P	G	F	C	M	M	C	W	Z	R	P	J	Y	G	○
	O	(P)	Z	U	Q	N	V	X	I	(A)	X	Z	K	Q	M	J	E	
30	F	W	W	M	R	Q	R	T	(E)	T	P	X	R	S	U	K	C	
	D	L	H	E	D	L	L	C	T	J	K	S	Z	M	Q	M	K	○
	N	J	(U)	V	P	F	L	Y	H	Y	F	Q	R	E	W	N	D	
	Z	M	B	M	P	B	O	J	X	E	Q	I	Z	A	X	H	N	
	D	B	Q	Q	W	D	I	X	Q	P	I	F	A	Y	J	G	Q	
35	J	O	F	W	F	C	D	B	X	Y	N	X	Y	T	W	Y	K	
	E	Q	C	D	P	D	Y	D	O	Z	H	J	F	C	Z	U	E	
	D	D	J	B	F	X	T	T	V	F	Y	G	H	C	T	B	G	
	O	F	E	H	B	U	B	Z	D	Q	Q	T	I	G	D	(Y)	A	
	I	Y	F	D	F	H	(A)	B	S	A	H	Y	G	X	I	B	B	
40	L	E	C	Q	O	(S)	E	M	O	M	Z	V	K	H	Q	S	J	○

* * * * *

FIGURE 91*b*

the grille down one position at a time in Fig. 91*b*, we obtain the 26 distributions shown in Fig. 92, below, given together with the numerical designations of these alphabets (in parentheses) and their I.C.'s. The average I.C. of these distributions, 1.88, makes us very happy.

	Alph.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	I.C.
1.	(21)	5		5		4	1	3		1	2	2			2		6		1	3		3				6	1	1.79
2.	(2)	2	3	2	1	6				2				2		1	5				5		1	6	3		5	1.92
3.	(9)				1	3	1	4	6	1	1	1	3		1	2		7	1	3	1	1	1	1	1		3	1.58
4.	(16)	3			4		2	1		1	4		2	1	2	1	2	3	1			2		4	8	1		1.72
5.	(23)		6	7	2	4	1			1	4			4	2	1	1	1	3	3	1		1					1.87
6.	(4)		1	2	1	1			1	1	7		4	2			2	1	1	2		2	6	4	4			1.78
7.	(11)	1		2		4			2	1				2	4	6	3	5		1	1	2	5	1		3		1.64
8.	(18)					1	8	7	4	1	3	1				2	5	3			1		2		3	1	1	2.19
9.	(25)	1		3	3	2	5	6			1						1	2	1	1		5		6	3	3		1.84
10.	(6)	2	1		4		4				5	2			1			1	4	6			5	3	5			1.96
11.	(13)	4	1		2	1		4			2	3		5		4	5	1	2	4					2	2	1	1.50
12.	(20)	2		1		9	4	1	4		4		2	4		1		1	3		3		1	3		1		1.95
13.	(1)	1	1	4	3	1			2		1		2	6	1	2				4			1		7	7	3	1.96
14.	(8)	2					5	3	1		1		1		3		3	2	3		8		6		4	2	1	1.94
15.	(15)				4	4	3				5	6	1	1	2	7		1	1		6		1			4		1.73
16.	(22)	6	1	10			4		3	2	1	3	1	1	1	1			3	2	1	1			3		1	2.10
17.	(3)	2		2	3		1		1			4		4			2	5	1			1	6			6	6	2.01
18.	(10)	3			2			3		3	6		3	1	3	7			5				3				3	2.05
19.	(17)			4	2		4		1	1		3	3	7	6	2				1	1	1		1	3		3	1.79
20.	(24)	3	5		2		3		8	1		1	6	2	1			2	1		2		2	2		2		1.90
21.	(5)		1		2						3	3			5	4	1		2	5		6		2		8		2.36
22.	(12)	2	2	3		4	3	2	1	4		1	2	2			1	1	2	1	4	5			2		1	1.18
23.	(19)	3			4	2	1	6	5	5		5		2	2			2				1			3		2	1.63
24.	(26)	7	1			1		3	3	6	1	1	2		2	3			2					3	1	5	3	1.76
25.	(7)		2					1		1			2	7	4		1	2		3	13	3		3	1	1	2	2.96
26.	(14)		2	2	3				3	5	2	4	6		2		6		1	2	1		2	4				1.68

FIGURE 92

g. The next step is to rewrite the successive columns of Fig. 92 as rows of a new matrix for convenience in the matching process on which we are about to embark. This new matrix is shown in Fig. 93, below, in which the cipher letters are at the left, and tally totals for each row at the right. The frequencies within each row represent the frequencies of the successive plaintext equivalents of the cipher letter designated at the left, and therefore the method of solution bears a close resemblance to that of the classic progressive alphabet cipher in which we match the cipher-letter rows to obtain a statistical reconstruction of the cipher component, regardless of the identity of the plain component.[3]

[3] Cf. pp. 162–168, *Military Cryptanalytics, Part II*.

	21	2	9	16	23	4	11	18	25	6	13	20	1	8	15	22	3	10	17	24	5	12	19	26	7	14	
A	5	2	–	3	–	–	1	–	1	2	4	2	1	2	–	6	2	3	–	3	–	2	3	7	–	–	49
B	–	3	–	–	6	1	–	–	–	1	1	–	1	–	–	1	–	–	–	5	1	2	–	1	2	2	27
C	5	2	–	–	7	2	2	–	3	–	–	1	4	–	–	10	2	–	4	–	–	3	–	–	–	2	47
D	–	1	1	4	2	1	–	–	3	4	2	–	3	–	4	–	3	2	2	2	2	–	4	–	–	3	43
E	4	6	3	–	4	1	4	1	2	–	1	9	1	–	4	–	–	–	–	–	–	4	2	1	–	–	47
F	1	–	1	2	1	–	–	8	5	4	–	4	–	5	3	4	1	–	4	3	–	3	1	–	–	–	50
G	3	–	4	1	–	–	–	7	6	–	4	1	–	3	–	–	–	3	–	–	–	2	6	3	1	–	44
H	–	–	6	–	–	1	2	4	–	–	–	4	2	1	–	3	1	–	1	8	–	1	5	3	–	3	45
I	1	2	1	1	1	1	1	1	–	–	–	–	–	–	–	2	–	3	1	1	–	4	5	6	1	5	37
J	2	–	1	4	4	7	–	3	1	5	2	4	1	1	5	1	–	6	–	–	3	–	–	1	–	2	53
K	2	–	1	–	–	–	–	1	–	2	3	–	–	–	6	3	4	–	3	1	3	1	5	1	–	4	40
L	–	–	3	2	–	4	–	–	–	–	–	2	2	1	1	1	–	3	3	6	–	2	–	2	2	6	40
M	–	2	–	1	4	2	2	–	–	–	5	4	6	–	1	1	4	1	7	2	–	2	2	–	7	–	53
N	2	–	1	2	2	–	4	–	–	1	–	–	1	3	2	1	–	3	6	1	5	–	2	2	4	2	44
O	–	1	2	1	1	–	6	2	–	–	4	1	2	–	7	1	–	7	2	–	4	–	–	3	–	–	44
P	6	5	–	2	1	2	3	5	1	–	5	–	–	3	–	–	2	–	–	–	1	1	–	–	1	6	44
Q	–	–	7	3	1	1	5	3	2	1	1	1	–	2	1	–	5	–	–	2	–	1	2	–	2	–	40
R	1	–	1	1	3	1	–	–	1	4	2	3	–	3	1	3	1	5	–	1	2	2	–	2	–	1	38
S	3	–	3	–	3	2	1	–	1	6	4	–	4	–	–	2	–	–	1	–	5	1	–	–	3	2	41
T	–	5	1	–	1	–	1	1	–	–	–	3	–	8	6	1	–	–	1	2	–	4	–	–	13	1	48
U	3	–	1	2	–	2	2	–	5	–	–	–	–	–	–	1	1	–	1	–	6	5	1	–	3	–	33
V	–	1	1	–	1	6	5	2	–	5	–	1	1	6	1	–	6	3	–	2	–	–	–	–	–	2	43
W	–	6	1	4	–	4	1	–	6	3	–	3	–	–	–	–	–	–	1	2	2	–	–	3	3	4	43
X	–	3	1	8	–	4	–	3	3	5	2	–	7	4	–	3	–	–	3	–	–	2	3	1	1	–	53
Y	6	–	–	1	–	–	3	1	3	–	2	1	7	2	4	–	6	–	–	2	8	–	–	5	1	–	52
Z	1	5	3	–	–	–	–	1	–	–	1	–	3	1	–	1	6	3	3	–	–	1	2	3	2	–	36

FIGURE 93

(1) A tabulation is now made of the rows in descending order of total number of tallies, as shown below:

53	52	50	49	48	47	45	44	43	41	40	38	37	36	33	27
J	Y	F	A	T	C	H	G	D	S	K	R	I	Z	U	B
M					E		N	V		L					
X							O	W		Q					
							P								

We begin by matching the J and M rows which have the most tallies, and we arrive at the highest ξ I.C., 1.89, for the following juxtaposition:

```
J  [2 - 1 4 4 7 - 3 1 5 2 4 1 1 5 1 - 6 - - 3 - - 1 - 2
M    - - - 5 4 6 - 1 1 4 1 7 2 - 2 2 - 7 -[- 2 - 1 4 2 2
```

The next three heavy distributions, X, Y, and F, are easily added, since the ξ I.C.'s with the J row yield 1.84, 1.82, and 1.83, respectively, for the best matches, as shown below together with the columnar sums of frequencies, labelled Σ(α). (The line at the top shows the relative placement of the five letters in the cipher component.)

```
           J . X . . . . . . . . . . . . . . . Y . M . . F . . .
        J [2 - 1 4 4 7 - 3 1 5 2 4 1 1 5 1 - 6 - - 3 - - 1 - 2
        M  - - - 5 4 6 - 1 1 4 1 7 2 - 2 2 - 7 -[- 2 - 1 4 2 2
        X  1 -[- 3 1 8 - 4 - 3 3 5 2 - 7 4 - 3 - - 3 - - 2 3 1
        Y  - 2 1 7 2 4 - 6 - - 2 8 - - 5 1 -[6 - - 1 - - 3 1 3
        F  1 - - 8 5 4 - 4 - 5 3 4 1 - 4 3 - 3 1 - - -[1 - 1 2
Σ(α):      4 2 2 27 16 29 — 18 2 17 11 28 6 1 23 11 — 25 1 — 9 — 2 10 7 10
```

(2) We now use the summation row, $\Sigma(\alpha)$, as a means of strengthening our results in matching the next five rows, yielding I.C.'s of 1.71, 1.96, 1.92, 1.85, and 1.78 for the highest scores, and we obtain a new summation row which we label $\Sigma(\beta)$. This matching is shown below, with the letters in the top row representing, as before, the placement of the letters thus far recovered in the cipher component.

	J	.	X	E	.	.	.	.	.	.	.	.	.	.	A	.	T	Y	H	M	.	.	F	.	.	C
$\Sigma(\alpha)$:	4	2	2	27	16	29	—	18	2	17	11	28	6	1	23	11	—	25	1	—	9	—	2	10	7	10
A	1	2	-	6	2	3	-	3	-	2	3	7	-	-	[5	2	-	3	-	-	1	-	1	2	4	2
T	-	3	-	8	6	1	-	-	1	2	-	4	-	-	13	1	[-	5	1	-	1	-	1	1	-	-
C	2	-	-	7	2	2	-	3	-	-	1	4	-	-	10	2	-	4	-	-	3	-	-	-	2	[5
E	1	-	-	[4	6	3	-	4	1	4	1	2	-	1	9	1	-	4	-	-	-	-	-	-	4	2
H	-	-	-	4	2	1	-	3	1	-	1	8	-	1	5	3	-	3	[-	-	6	-	-	1	2	4
$\Sigma(\beta)$:	8	7	2	56	34	39	—	31	5	25	17	53	6	3	65	20	—	44	2	—	20	—	4	14	19	23

Using the $\Sigma(\beta)$ as a base, we are able to match correctly the remaining rows, with ξ I.C.'s ranging between 1.64 and 2.13, being helped of course by the fact that no two rows may match flush. The final matching of all the rows is shown in Fig. 94, below:

	J	Q	X	E	P	S	W	G	I	U	B	N	L	Z	A	K	T	Y	H	M	R	D	F	O	V	C
J	[2	-	1	4	4	7	-	3	1	5	2	4	1	1	5	1	-	6	-	-	3	-	-	1	-	2]
M	-	-	-	5	4	6	-	1	1	4	1	7	2	-	2	2	-	7	-	[-	2	-	1	4	2	2
X	1	-	[-	3	1	8	-	4	-	3	3	5	2	-	7	4	-	3	-	-	3	-	-	2	3	1
Y	-	2	1	7	2	4	-	6	-	-	2	8	-	-	5	1	-	[6	-	-	1	-	-	3	1	3
F	1	-	-	8	5	4	-	4	-	5	3	4	1	-	4	3	-	3	1	-	-	-	[1	-	1	2
A	1	2	-	6	2	3	-	3	-	2	3	7	-	-	[5	2	-	3	-	-	1	-	1	2	4	2
T	-	3	-	8	6	1	-	-	1	2	-	4	-	-	13	1	[-	5	1	-	1	-	1	1	-	-
C	2	-	-	7	2	2	-	3	-	-	1	4	-	-	10	2	-	4	-	-	3	-	-	-	2	[5
E	1	-	-	[4	6	3	-	4	1	4	1	2	-	1	9	1	-	4	-	-	-	-	-	-	4	2
H	-	-	-	4	2	1	-	3	1	-	1	8	-	1	5	3	-	3	[-	-	6	-	-	1	2	4
G	-	-	2	6	3	1	-	[3	-	4	1	-	-	-	7	6	-	4	1	-	3	-	-	-	3	-
N	1	-	3	6	1	5	-	2	2	4	2	[2	-	1	2	2	-	4	-	-	1	-	-	1	3	2
O	1	1	-	6	2	-	-	4	1	2	-	7	1	-	7	2	-	4	-	-	3	-	-	[-	1	2
P	-	-	1	6	[6	5	-	2	1	2	3	5	1	-	5	-	-	3	-	-	2	-	-	-	1	1
D	1	-	-	3	4	2	-	3	-	4	-	3	2	2	2	2	-	4	-	-	3	[-	1	1	4	2
V	1	-	1	6	5	2	-	5	-	1	1	6	1	-	6	3	-	2	-	-	-	-	-	2	[-	1
W	2	-	-	3	3	4	[-	6	1	4	-	4	1	-	6	3	-	3	-	-	-	-	-	-	1	2
S	1	-	-	3	2	[3	-	3	-	3	2	1	-	1	6	4	-	4	-	-	2	-	-	1	-	5
K	-	-	-	6	3	4	-	3	1	3	1	5	1	-	4	[2	-	1	-	-	-	-	1	-	2	3
L	1	1	-	3	3	6	-	2	-	2	2	6	[-	-	3	2	-	4	-	-	-	-	-	2	2	1
Q	-	[-	-	7	3	1	1	5	3	2	1	1	1	-	2	1	-	5	-	-	2	-	1	2	-	2
R	-	-	1	4	2	3	-	3	1	3	1	5	-	1	2	2	-	2	-	1	[1	-	1	1	3	1
I	1	1	-	4	5	6	1	5	[1	2	1	1	1	1	1	1	-	-	-	-	-	-	-	2	-	3
Z	1	-	1	6	3	3	-	-	1	2	3	2	-	[1	5	3	-	-	-	-	1	-	-	1	-	3
U	-	1	-	6	5	1	-	3	-	[3	-	1	2	-	2	2	-	5	-	-	-	-	-	-	1	1
B	-	-	-	5	1	2	-	1	2	2	[-	3	-	-	6	1	-	-	-	1	1	-	1	-	-	1
	18	11	11	136	85	87	2	81	19	68	35	105	17	10	131	56	0	89	3	2	39	0	9	27	40	53

FIGURE 94

(The numbers at the bottom of the columns represent the frequencies of the letters, as yet unidentified, of the underlying plain text.) The cipher component as recovered is

J Q X E P S W G I U B N L Z A K T Y H M R D F O V C,

but knowing that there is a decimation of 7 involved, the original cipher component must have been

J N F G H E A C B D W Y X Z V U R S T Q L O I M P K.

h. Since we know both the cipher component and its motion, we are now able to reduce the cipher text to monoalphabetic terms, using an arbitrary A–Z sequence for the plain component at the initial setting as shown below:

```
P: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C: J N F G H E A C B D W Y X Z V U R S T Q L O I M P K
```

The first cipher letter of the message, X_c, is deciphered as M_p; the second letter, Y_c, involves a shift of the cipher component of 6 positions to the left and is therefore deciphered as F_p; and so on for the reduction of the rest of the text. The reduction of the first 10 of the 67 lines of 17 letters is shown in Fig. 95, together with the distribution of the monoalphabetically converted text.

```
        15 16 17 1 2 3 4 5 6 7 8 9 10 11 12 13 14
         6 5 9 7 6 7 5 6 7 6 8 6 10 5 6 5 7
  C:   X Y I C P N D E A M A P D T R A X
"P":   M F L N X U V M H R U E J I B L M

  C:   X P Z X H Y R Y T W Q X F H C D J
"P":   F L V L W X V L M X A L V N L H T

  C:   K A H Q U R Z P P P Z Q O F U V K
"P":   L M F L A V L R L E N L H E M F L

  C:   F E M N E A O N G T T X S V V U N
"P":   H E R M J E M N J R L X W J E Z N

  C:   D G J R E J F H E O K V C Q H F H
"P":   H V N U C R M J E N L S F H N F C

  C:   R O K U P M Q P Q W A C O J C R L
"P":   H G F M O H W W L V L E M H J M L

  C:   M B M E V K R V J D Y N N S X U D
"P":   H M W V X C M F L N J R L R H E T

  C:   L H N P F W M O C M J F L G P M B
"P":   X B T H E G M F L U R L X W M F L

  C:   K H A U X L I V V Q S X U N J Z U
"P":   V U R R H J E R L J B O L N H O L

  C:   K K O B A A E U Q O Y J I Z S Z U . . .
"P":   O I Z D U O G L J E O V L S R H E
```

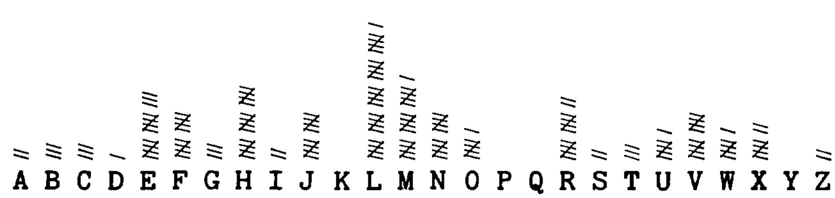


FIGURE 95

The simple substitution is now easily solved, beginning with the identification of L_c of the converted text as E_p and the initial trigraph MFL_c as THE_p. The decipherment of the first two rows is shown in the fragment below:

	15	16	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
	6	5	9	7	6	7	5	6	7	6	8	6	10	5	6	5	7	
C:	X	Y	I	C	P	N	D	E	A	M	A	P	D	T	R	A	X	
"P":	M	F	L	N	X	U	V	M	H	R	U	E	J	I	B	L	M	
P:	T	H	E	C	O	U	R	T	I	S	U	N	A	B	L	E	T	
C:	X	P	Z	X	H	Y	R	Y	T	W	Q	X	F	H	C	D	J	. . .
"P":	F	L	V	L	W	X	V	L	M	X	A	L	V	N	L	H	T	
P:	H	E	R	E	F	O	R	E	T	O	P	E	R	C	E	I	V	

The recovered plain component, set in position against the cipher component for decipherment of the first letter of the text, is as follows:

```
P:  P L M J N H G I B A K E T C D . . S W V U R F O . Y
C:  J N F G H E A C B D W Y X Z V U R S T Q L O I M P K
```

Since the sequences were made up at random (but not too thoroughly mixed at that) and since three letters did not occur in the first 170 letters of the plain text, we cannot be certain of the placement of the missing letters in the plain component. (An X_p does occur, however, at the 539th and 647th positions of the text, so this letter could be inserted in its proper place in the plain component, immediately to the left of the S.)

i. In subpar. *e* we arrived at the correct grille placement by sliding it through successive positions of the cipher until a favorable I.C. was reached. The theoretical I.C. of the cipher letters at an incorrect setting of the grille, however, is not 1.00 as might be supposed. For example, if we place our grille (with the "A" positions cut out) over the key diagram of Fig. 85, we shall expose 17 A's, of course; but if we move the grille one position over to the right, instead of a random assortment of equiprobable key letters we obtain a severely limited distribution of 4 F's, 6 G's, 4 H's, and one each of the letters I, J, and K. The distributions of key letters for the 17 positions of the grille are shown in the diagram below:

Pos.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1.	17																									
2.						4	6	4	1	1	1															
3.												4	2	5	3	1	2									
4.																	1		6	1	3	4	1		1	
5.	1	6		2	1																			2	3	2
6.				2		3	3	3	3	2		1														
7.									1			1	3	5	4	1	1	1								
8.																		1	2	7	1	3	3			
9.	4	6	1	1																					2	3
10.					1	1	6	4	3	2																
11.												3	3	1	7	2	1									
12.																	1	1	1	4	5	3	1			1
13.	3	3	3		2																		1		2	3
14.				1	2		6	1	2	3	2															
15.										1		1	4	3	1	6		1								
16.																		2	1	3	5	2	4			
17.	4	6	4																					1	1	1

It may be seen that the number of different key letters in the wrong cases is 6, 7, or 8, with biased distributions which are symmetrically disposed between 2 and 17, 3 and 16, 4 and 15, etc. This may result in situations wherein, if the number of letters in the distribution is too small a sample, a wrong grille position might yield an I.C. even in excess of the expected 1.73; that is why, in subpar. *f*, we reserved judgment until we made additional distributions at our assumed setting to confirm its validity. In order to establish the theoretical I.C. of the cipher text at a particular wrong setting of the grille, we must calculate the bulge of the γ I.C. (i.e., $\gamma-1$, or the excess over the expected 1.000 for random), which is given by the formula $\beta_c=\frac{\beta_k\times\beta_p}{c-1}$, where β_c is the bulge of the cipher text, β_k the bulge of the key, β_p the bulge of the plain text (for English, .73), and *c* is the number of categories (in this case, 26). Thus for selector position 2 with a distribution of (4, 6, 4, 1, 1, 1) the γ I.C. of the key[4] is $\frac{26[6^2+2(4^2)+3(1^2)]}{17^2}=$ 6.40, the bulge $\beta_k=6.40-1.00=5.40$, and the bulge of the cipher $\beta_c=\frac{(5.40)(.73)}{25-1}=0.16$; therefore the expected I.C. of the cipher text is $1.00+.16=1.16$. The complete table of values is shown below:

	γ_k	β_k	β_c	Exp. I.C.		γ_k	β_k	β_c	Exp. I.C.
1.	26. 00	25. 00	0. 73	1. 73	10.	6. 03	5. 03	0. 15	1. 15
2.	6. 40	5. 40	0. 16	1. 16	11.	6. 58	5. 58	0. 16	1. 16
3.	5. 30	4. 30	0. 13	1. 19	12.	4. 94	3. 94	0. 12	1. 12
4.	5. 85	4. 85	0. 14	1. 14	13.	4. 06	3. 06	0. 09	1. 09
5.	5. 30	4. 30	0. 13	1. 13	14.	5. 30	4. 30	0. 13	1. 13
6.	4. 06	3. 06	0. 09	1. 09	15.	5. 85	4. 85	0. 14	1. 14
7.	4. 94	3. 94	0. 12	1. 12	16.	5. 30	4. 30	0. 13	1. 13
8.	6. 58	5. 58	0. 16	1. 16	17.	6. 40	5. 40	0. 16	1. 16
9.	6. 03	5. 03	0. 15	1. 15					

The average of the theoretical I.C.'s for the 16 wrong settings of the grille is 1.14.

j. Once we have recovered the components, the solution of additional messages using the same components is a simple matter. For example, let us assume that we have recovered the components as

```
P: H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
C: Q U E S T I O N A B L Y C D F G H J K M P R V W X Z
```

and that the following message is at hand:

```
C N C A X   V H N Q O   J O C R Y   I C K O Q   J I A F L   Q N M O V
U A K G O   B E U S G   B B V F D   A R C C O
```

The simplest procedure here is to prepare a diagram such as that illustrated below, in which the first row of letters represents the "decipherments" of

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1	8	14	21	26	6	13	19	1	7	17	22	2	7	14	20	25	8	15	21	2
C	G	U	Z	P	J	I	H	S	G	L	V	O	F	L	Z	Q	K				
N		H	S	J	C	D	T	M	I	Y	O	G	L	Y	S	K	B	H			
C			Z	P	J	I	H	S	G	L	V	O	F	L	Z	Q	K	U	X		
A				K	B	R	V	N	C	D	P	J	I	D	T	M	E	Y	S	K	
X					Z	Q	G	L	X	P	C	R	W	P	F	U	H	O	E	A	W

[4] Since we are dealing with the entire key population in the various cases, it is the γ I.C. given by $\frac{c\Sigma f^2}{N^2}$ which is involved here and not the δ I.C. given by $\frac{c\Sigma f(f-1)}{N(N-1)}$.

the first letter of the cipher text, C_c (using the components at the initial setting $H_p = Q_c$), at 17 consecutive positions of the key (starting with selector position 1). The second row represents the 17 decipherments (starting with selector position 2) of the second letter of the cipher, N_c; and so on for the decipherments of the first five letters of the cipher text. One of the diagonals in the diagram must represent the correct keying of the selector wheel, but at an arbitrary juxtaposition of the components; therefore the next step is to complete the plain component sequence for, at most, the 17 diagonals. This is shown in Fig. 96, below. In the generatrix diagram for position 10 we see the plaintext fragment **AMMUN**, so we know that the message was enciphered starting at that position of the selector wheel. The rest of the message can now easily be read.

Pos. 1	Pos. 2	Pos. 3		Pos. 10
G H Z K Z	U S P B Q	Z J J R G	***	L O O I P
J Y H M H	L T Q E S	H K K A J		I P P C Q
K D Y N Y	I V S F T	Y M M U K		C Q Q B S
M R D O D	C W T G V	D N N L M		B S S E T
N A R P R	B X V J W	R O O I N		E T T F V
O U A Q A	E Z W K X	A P P C O		F V V G W
P L U S U	F H X M Z	U Q Q B P		G W W J X
Q I L T L	G Y Z N H	L S S E Q		J X X K Z
S C I V I	J D H O Y	I T T F S		K Z Z M H
T B C W C	K R Y P D	C V V G T		M H H N Y
V E B X B	M A D Q R	B W W J V		N Y Y O D
W F E Z E	N U R S A	E X X K W		O D D P R
X G F H F	O L A T U	F Z Z M X		P R R Q A
Z J G Y G	P I U V L	G H H N Z		Q A A S U
H K J D J	Q C L W I	J Y Y O H		S U U T L
Y M K R K	S B I X C	K D D P Y		T L L V I
D N M A M	T E C Z B	M R R Q D		V I I W C
R O N U N	V F B H E	N A A S R		W C C X B
A P O L O	W G E Y F	O U U T A		X B B Z E
U Q P I P	X J F D G	P L L V U		Z E E H F
L S Q C Q	Z K G R J	Q I I W L		H F F Y G
I T S B S	H M J A K	S C C X I		Y G G D J
C V T E T	Y N K U M	T B B Z C		D J J R K
B W V F V	D O M L N	V E E H B		R K K A M
E X W G W	R P N I O	W F F Y E		A M M U N
F Z X J X	A Q O C P	X G G D F		U N N L O

FIGURE 96

k. One of the properties of the Kryha machine is that causal isomorphs may occur, but only at multiples of 17 for the original machine (or at multiples of the number of active screws for the improved model). As an example, let us study the following message known to have been enciphered with the original Kryha machine:

```
        5           10          15    17
Y R H O Q M A H N W X U X J V S U
Q X R W X T I L A G S D A Q M F T
D K Z L Q J D K H Y L A M S H X B
Y G Z F J X G H E K M F R X X X K
G K B O X V F B Q I H Y S X L D Z
W Q W K Y I H S Y D R E X S K Z J
U Y O V H B X O W D F J F N C M J
H F Y Z D X K T P T V C L B M D G
W P O P Z I Z D H C M Q M V N Z L
V O U J Y T M U K A B C B E N F C
Y B O X K P Q G S F S V K Z N B D
G M S K Z E R P O V W K Z E E E V
M V D L E Q K D T B P C B C A E V
C R F L
```

(1) Several isomorphs have been found and these are listed below:

```
a. R H O Q M A H N W X U X J V S U Q X R
b. Y O V H B X O W D F J F N C M J H F Y
c. O U J Y T M U K A B C B E N F C Y B O
d. H E K M F R X X X K G K B O X V F B Q I H
e. P O V W K Z E E E V M V D L E Q K D T B P
```

The derived chains are the following:

a–b: RY QHOVC SMB AXF UJNWD
a–c: ROJE HUC QY WAMT VNK XB SF
b–c: HYOU VJCNE XMFBT WK DA
d–e: HP XEOL FKVQT GMW RZ IBD

It is but a simple matter to amalgamate these partial chains into a complete equivalent primary component,

V J C N E W K D A S X M F B T Z P G L I Q R H Y O U,

which when decimated at an interval of +5 yields the original component:

Q U E S T I O N A B L Y C D F G H J K M P R V W X Z

(2) Since we are in possession of the cipher component, we can assume an arbitrary A–Z sequence for the plain component in order to reduce the cipher text to monoalphabetic terms. But since we do not know the starting position of the selector wheel, we must be prepared to take all 17 positions in turn, making trial reductions of the text to find the correct one as revealed by the expected δ I.C. for the language. The conversion of the text starting at position 1 of the selector wheel is shown below:

```
        1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
   K:   7 6 7 5 6 7 6 8 6 10 5  6  5  7  6  5  9
   C:   Y R H O Q M A H N W  X  U  X  J  V  S  U
 "P":   L O D M B O W Y H R  I  G  X  L  J  K  D

   C:   Q X R W X T I L A G  S  D  A  Q  M  F  T . . .
 "P":   T K B W S S M L B C  G  L  A  N  Z  O  Z
```

The I.C. of the entire converted text at this initial position of the selector wheel is 1.15. When, however,

```
        11 12 13 14 15 16 17  1  2  3  4  5  6  7  8  9 10
  K:     5  6  5  7  6  5  9  7  6  7  5  6  7  6  8  6 10
  C:     Y  R  H  O  Q  M  A  H  N  W  X  U  X  J  V  S  U
"P":     L  Q  F  Q  D  Q  A  Z  J  T  N  L  C  O  N  M  E

  C:     Q  X  R  W  X  T  I  L  A  G  S  D  A  Q  M  F  T . . .
"P":     T  M  D  A  U  U  Q  M  D  E  L  Q  F  Q  D  Q  A
```

we reach initial selector position 11, we find that the I.C. is 1.68.[5] The monoalphabetic substitution is now solved,

```
        11 12 13 14 15 16 17  1  2  3  4  5  6  7  8  9 10
  K:     5  6  5  7  6  5  9  7  6  7  5  6  7  6  8  6 10
  C:     Y  R  H  O  Q  M  A  H  N  W  X  U  X  J  V  S  U
"P":     L  Q  F  Q  D  Q  A  Z  J  T  N  L  C  O  N  M  E
  P:     D  I  V  I  S  I  O  N  H  E  A  D  Q  U  A  R  T

  C:     Q  X  R  W  X  T  I  L  A  G  S  D  A  Q  M  F  T . . .
"P":     T  M  D  A  U  U  Q  M  D  E  L  Q  F  Q  D  Q  A
  P:     E  R  S  O  F  F  I  R  S  T  D  I  V  I  S  I  O
```

and the plain component is found to be the HYDRAULIC . . . XZ sequence.

---

[5] For the statistically curious, the following are the I.C.'s of the trial reductions to monoalphabetic terms of the entire cipher text at the various selector positions:

1. 1.15	6. 1.14	10. 1.20	14. 1.18
2. 1.31	7. 1.15	11. 1.68	15. 1.10
3. 1.31	8. 1.14	12. 1.22	16. 1.20
4. 1.19	9. 1.31	13. 1.07	17. 1.35
5. 1.16			

The standard deviation of the δ I.C. is given by the formula $\sigma=\frac{\sqrt{2(c-1)}}{\sqrt{N(N-1)}}$, which for this 225-letter message is 0.03, so the range of I.C.'s in the wrong cases, 1.07 to 1.35, might indicate astonishing deviations of 2.3σ to 11.7σ. But this apparent paradox is easily explained, since we know from subpar. *i* that the expected I.C. in a random case for the original Kryha machine is 1.14, and the formula given above for the standard deviation of the I.C. applies *only* to those situations wherein the expected I.C. for random is 1.00.

l. For the last example of solution of the original Kryha machine, let us assume that we have a cipher message and its compromised plain text. This is shown below, written on a width of 17.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
B	V	T	B	L	H	T	K	P	A	Y	U	V	M	X	H	K
R	E	F	E	R	R	I	N	G	T	O	Y	O	U	R	M	E
N	D	T	K	X	L	B	X	X	T	T	J	M	N	E	S	L
S	S	A	G	E	N	U	M	B	E	R	E	I	G	H	T	T
P	S	J	W	O	A	N	Q	X	I	D	X	R	M	Z	Z	H
H	R	E	E	F	I	V	E	D	A	T	E	D	O	N	E	S
A	I	H	T	L	K	N	O	D	Y	B	H	J	U	A	Q	O
I	X	D	E	C	E	M	B	E	R	S	T	O	P	P	A	R
C	Q	X	I	U	G	M	Z	L	K	O	C	H	P	Y	D	D
A	G	R	A	P	H	S	T	H	R	E	E	B	A	K	E	R
M	Y	T	M	N	U	B	M	J	Q	O	G	R	N	R	R	R
A	N	D	F	O	U	R	C	H	A	R	L	I	E	O	F	A
X	J	H	I	T	W	A	R	E	X	J	C	I	M	R	O	G
D	M	I	N	I	S	T	R	A	T	I	V	E	M	E	M	O
O	D	O	X	B	Z	L	T	P	M	U	Q	M	U	E	B	V
R	A	N	D	U	M	N	U	M	B	E	R	O	N	E	F	O
G	M	W	N	N	U	L	H	V	N	U	F	S	Q	O	R	F
U	R	H	A	V	E	B	E	E	N	R	E	S	C	I	N	D
U	Z	X	I	B	H	J	H	L	O	M	M	N	I	D	E	T
E	D	S	T	O	P	B	R	O	C	K	C	O	L	I	N	F

(1) The first thing we do is to record the cipher equivalents of identical plaintext letters in the columns, at intervals of 1 row, 2 rows, . . . 9 rows which correspond to intervals of 1, 2, . . . 9 multiples of 7 on the cipher component. This is shown in the diagram below:

Interval N:	1	2	3	4	5	6	7	8	9
7N (mod 26):	7	−12	−5	2	9	−10	−3	4	11
	C M	H T	B T	I N	K U	S M	B O	– –	V N
	W T	B W	R H	N B	D V	Q H	T U		
	L J	X C	I Q	T O		A X	J F		
	Y K	M N	O U	C F		X F	V M		
	J X	Z D	J C	M R		J N			
	R E	R B	V J	J M		H O			
	O D								
	G V								

In the first column we have the chain LJX, which should be spaced on the cipher component as

```
          5         10        15        20        25 26
L . . . . . . J . . . . . . X . . . . . . . . . . . .
```

Adding to our sequence the XC from the second column, the JM from the fourth column, and AXF and JN from the sixth column, we obtain

```
L . C . F . . J . M . . . . X . . . . . . . . . N A .
```

Incorporating relationships from the other columns, we are able to complete the reconstruction of the cipher component:

L Y C D F G H J K M P R V W X Z Q U E S T I O N A B

(2) For the reconstruction of the plain component, we make a 26×26 matrix with the recovered cipher component at the bottom, and we distribute properly the plain-cipher relationships of column 1 of our matched plain and cipher, starting with $R_p=B_c$ in the top row, $S_p=N_c$ seven rows down in row 8, $H_p=P_c$ in row 15, etc.; this is shown in Fig. 97, below, wherein the first four entries have been ringed.

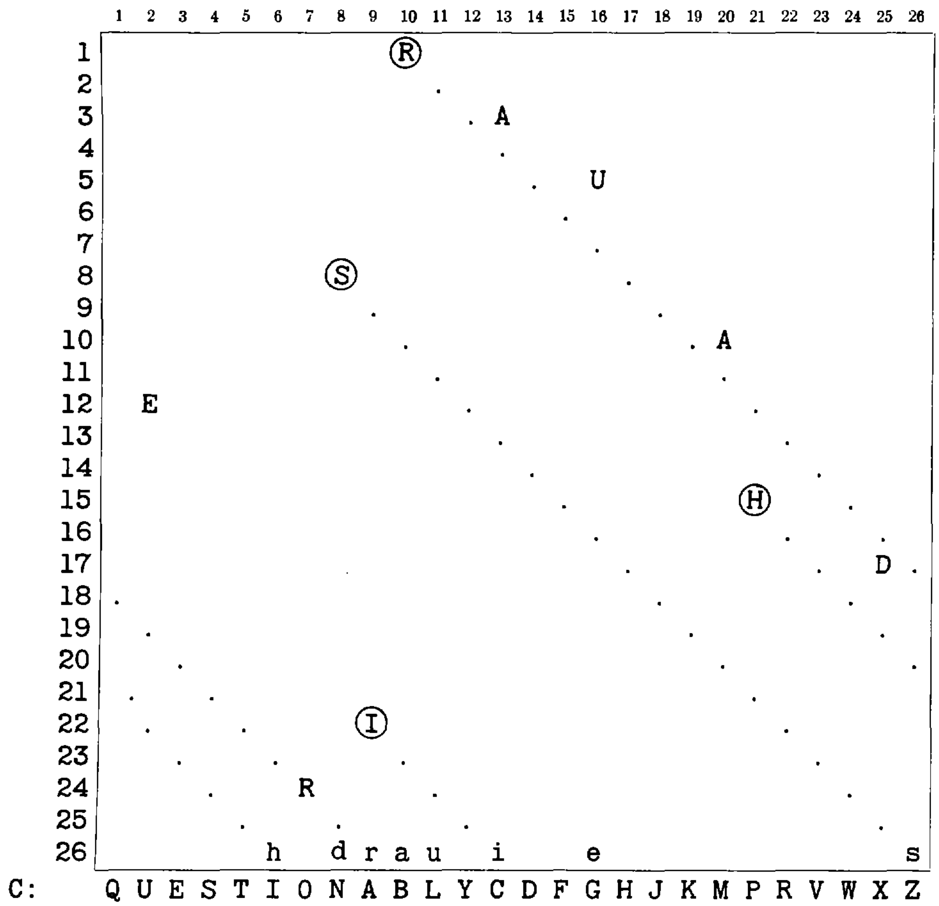


FIGURE 97

The partial plain component reconstructed from the values in the first column is

. . . . . H . D R A U . I . . E . . . . S

With the addition of the information in the other columns, the plain component is reconstructed as follows:

T V . Z X H Y D R A U L I C B E F G . K M N O P . S

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

206-687 O - 77 - 14

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

55. Concluding remarks.—*a.* Some words on the diagnosis of systems employing the Wheatstone device or the Kryha machine might be apropos. In both cases, the expected monographic I.C. is 1.00 since every key letter has an equal chance of occurring. As for the expected digraphic I.C., in a Wheatstone system there are only 26 ways in which a plaintext digraph may be enciphered; therefore since the digraphic I.C. of English plain text is 4.66, the expected digraphic I.C. can be approximated by 1.00 plus 1/26 of the 3.66 bulge, or 1.14. The expected digraphic I.C. in a Kryha system (1.034 for the original machine, [redacted] is considerably less than that of the Wheatstone since there are in effect several variant possibilities for the encryption of a digraph after the first letter is determined—6 ways for the original Kryha, and 3 ways for the improved model.[10] Cryptograms enciphered by the Wheatstone device would be replete with isomorphs at no consistent factorable interval; causal polygraphic repetitions within or between messages (at no consistent factorable intervals) would occur with the same frequency as those of a ciphertext autokey system with a single-letter introductory key (1/26 of the plaintext rate), since there is only 1 chance in 26 that a plaintext word, for example, would be enciphered both times at the same setting of the components; and since the Wheatstone produces aperiodic encipherment, no depth would be possible and therefore the kappa test would be inapplicable. Cryptograms enciphered by the Kryha machine could manifest isomorphs only at intervals of multiples of 17 [redacted] causal polygraphic repetitions within a message would be rare, at intervals of 442 [redacted]

b. The cryptographic principle of the Wheatstone was first employed in the device invented by Decius Wadsworth, 50 years before Wheatstone, as mentioned in an earlier footnote. (The Wadsworth device is in the possession of the Hamden Historical Society of Hamden, Connecticut.) A number of modifications of this principle have appeared, among which may be mentioned the devices invented by J. S. Beeman (1905) and William F. Friedman (1938). In the Beeman device (the first patented device embodying the Wheatstone principle) two disks, each composed of several members for varying the sequences of the two components, are mounted side-by-side within a frame and are relatively rotatable through gears containing 36 and 39 teeth, respectively. In the Friedman device, two gears are meshed side-by-side, the letters of the plain and cipher components being inscribed on the gears themselves.

c. The original Kryha machine was manufactured in three versions: the "standard" model, 10″×8″×4½″; the "Liliput" (sic) model, a miniature version of the standard machine resembling a large pocket watch and intercommunicable with the standard model; and an electric model, in which a cipher unit is interposed between two electric typewriters. The "new" standard model, sold commercially since 1934, incorporated the selector wheel with the 52 adjustable screws. Subsequent Kryha patents covered three other versions: one in which the plain and cipher components were carried on two endless metal tapes; one in which the selector wheel was composed of various interchangeable sectors, each having a different number of gear teeth or irregularly spaced stop holes so that the user could assemble his selector wheel according to different specific keys; and one in which there were two selector wheels, one controlling the irregular displacement of the plain component and the other controlling the displacement of the cipher component.

d. The principle of the Kryha machine is found, with slight variations, in several other devices, among which may be mentioned the ones invented by the following: Peter G. G. Beyer (1931) of Copenhagen, whose device was sold commercially by The Danish Cipher Machine Co., Ltd.; Beue Tann (1934) of Chungking and Washington, D.C., whose device was called the "Comet" machine; Ernesto Cristofani (1935) of Rome, Italy; and Jean François Joseph Dupouy (1935) of France. Some of these were only slightly larger than a pocket watch, similar to the "Liliput" model sold by Kryha; others,

[10] The way these digraphic I.C.'s are calculated may be of interest. For the original Kryha machine, the 17 key skips have a distribution of four 5's, six 6's, four 7's, and one each of 8, 9, and 10. The γ I.C. of this distribution is $\frac{6[6^2+2(4^2)+3(1^2)]}{17^2}=1.47$; dividing the bulge .14 for the general case (as in the Wheatstone) by 6/1.47 gives .034, so the digraphic I.C. is approximately 1.034. For the improved Kryha machine, the 52 key skips have a distribution of 31 3's, 19 4's, and two 5's. The γ I.C. is $\frac{3[31^2+19^2+2^2]}{52^2}=1.47$ again; dividing the bulge .14 by 3/1.47 gives .069, so the digraphic I.C. here is approximately 1.069.

(b) (1)
(b) (3)-18 USC 798
(b) (3)-50 USC 3024(i)
(b) (3)-P.L. 86-36

like the Kryha electric model, connected two solenoid-operated typewriters which printed the plaintext and ciphertext copies simultaneously. Some were nonprinting, while others printed on a tape or on a sheet of paper. In the Beyer device (one was a pocket model and another a larger, electromechanical machine) the plain and cipher components are irregularly displaced relative to one another according to a key sequence generated by two selector wheels, the one for the plain component having 58 teeth and 9 fixed stop pins, the one for the cipher component having 62 teeth and 10 fixed stop pins. Both components are displaced twice between encipherments, before and after the depression of an operating plunger, the amount of displacement being proportionate to the distance between two adjacent pins on a selector wheel. The "Comet" machine incorporated two mixed cipher sequences of 13 letters each which are slid relative to one another and against a mixed plain component in accordance with a key generated by a pair of 19- and 23-sector cam wheels. The period is either 13 times or 26 times the least common multiple of the product of the number of sectors in the two cams. The Cristofani Cryptograph incorporated a set of 10 small variable-pin wheels of different sizes (the patent drawing shows two each of sizes 8, 9, 10, 11, and 12) arranged in a circle around the cipher disk. All the pin wheels advance one step between encipherments, and the cipher disk is displaced from zero to five positions at each operation. The Dupouy tape-printing device incorporated a keying disk of 95 segments, any one of which could be removed to form an irregular pattern of stopping points for the rotating cipher component.

e. One further point might be brought out in connection with the analysis of the Kryha machine: the descriptive brochure accompanying it suggested the use of plaintext interruptors as an added security measure.[11] With the machine were furnished two sets of 26 black-on-white metal tab inserts for making up the plain and cipher components, and another set of 26 red-on-white tabs from which a letter (or letters) could be selected for the plaintext interruption. After enciphering the designated letter, an extra displacement would be imparted to the cipher component, thus interrupting the cycle. Causal isomorphs could still be produced, however, and through them the cipher component could be reconstructed.

(1) Let us now suppose that we have the following message at hand, known to have been enciphered on the original Kryha machine and suspected of beginning with the opening stereotype "REFERENCE YOUR MESSAGE NUMBER . . .":

```
SHVTT  JTNHD  YIDTI  PUJTY  XQGGW  VRFGR
CLIFP  ZNNCW  TSCPO  SDHKA  FSMBG  PXTLV
JGUOD  GBOHD  OOCJW
```

Let it further be assumed that we have recovered the cipher component through isomorphs and found it to be the QUESTIONABLY keyword-mixed sequence:

```
Q U E S T I O N A B L Y C D F G H J K M P R V W X Z
```

Since two long isomorphs within a single message were at an interval not a multiple of 17, this indicates the possibility of the use of a plaintext interruptor procedure. Now if there is a single plaintext interruptor and that letter is T_p, for example (or for that matter any other letter not in the assumed crib), the matched plain and cipher would be written out as shown in Fig. 110*a*, below. If the interruptor letter is E_p, the write-out would be as shown in Fig. 110*b*.

```
        5         10        15  17
R E F E R E N C E Y O U R M E S S
S H V T T J T N H D Y I D T I P U

A G E N U M B E R
J T Y X Q G G W V
```

FIGURE 110*a*

```
        5         10        15  17
R E - F E - R E - N C E - Y O U R
S H   V T   T J   T N H   D Y I D

M E - S S A G E - N U M B E - R
T I   P U J T Y   X Q G G W   V
```

FIGURE 110*b*

[11] The mention of an added security measure is strange indeed, since the machine was already touted as having an incredibly high (i.e., infinite) security. Dr. Georg Hamel, Professor of Mathematics at the Technische Hochschule in Berlin-Charlottenburg, wrote a paper entitled "The Kryha Coding Machines: A Mathematical Opinion," in which he gave the number of ways in which a message could be enciphered as 2.29×10^{82}; "Thus, if every person on earth (say 2,000,000,000 people) bought the machine then each of them could have 1.14×10^{73} independent keys without two persons ever having the same system." By comparison, the number of atoms in the universe, 3×10^{74} (personally vouched for by Sir Arthur Eddington), is only 26 times as large.

(2) In Fig. 110*a* we note the cipher equivalents of E_p in REFERENCE:

1 2 3 4 5 6 7 8 9
R E F E R E N C E
S H V T T J T N H

The interval on the known cipher component between H_c (at position 2) and T_c (at pos. 4) is 14, certainly possible when two skips are involved as may be seen by referring to the skip pattern of the original Kryha machine (at skip nos. 7, 8, or 16):

No: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 1
Skip: 7 6 7 5 6 7 6 8 6 10 5 6 5 7 6 5 9

(It may be seen that for two skips, the minimum interval is 11, the maximum 16; for three skips, the minimum is 16, the maximum 24.) The interval between T_c (pos. 4) and J_c (pos. 6) is 13, again quite possible for two skips; but the interval between J_c (pos. 6) and H_c (pos. 9) is 25, an impossible figure when only three skips are involved. Thus not only is T_p rejected as an interruptor, but it means that the interruptor letter must be one of those in the word REFERENCE. We now note in Fig. 110*b* the cipher equivalents of E_p in REFERENCE, on the trial that E_p is the interruptor letter.

1 2 3 4 5 6 7 8 9 10 11 12
R E - F E - R E - N C E
S H V T T J T N H

The interval between H_c (pos. 2) and T_c (pos. 5) is 14, and there is no way to arrive at this number with three skips; in the next pair of letters, T_c (pos. 5) and J_c (pos. 8) are at an interval of 13, which likewise is impossible to make with three skips; thus E_p is also eliminated as a possible plaintext interruptor. By continuing this process, all letters are eliminated as interruptors except C_p and N_p.

(3) The write-outs of the matched plain and cipher for the assumptions of C_p and N_p are shown in Figs. 110*c* and *d*, below

5 10 15 17
R E F E R E N C - E Y O U R M E S
S H V T T J T N H D Y I D T I P

S A G E N U M B E R
U J T Y X Q G G W V

FIGURE 110*c*

5 10 15 17
R E F E R E N - C E Y O U R M E S
S H V T T J T N H D Y I D T I P

S A G E N - U M B E R
U J T Y X Q G G W V

FIGURE 110*d*

In Fig. 110*c* the interval between Y_c (col. 4) and W_c (col. 9) is 12 on the cipher component with five skips involved; but since the 17 sets of five consecutive skips on the Kryha selector wheel total to 3, 5, 6, 7, 8, 9, and 11 (mod 26), C_p is ruled out as the interruptor. In Fig. 110*d*, the interval between H_c and W_c in col. 10 should be +7 on the cipher component which it is, thus proving that N_p is the interruptor letter. Now the cipher equivalents of SS_p at cols. 17 and 1, P_c and U_c, are at an interval of 7 on the component; since there are only four 7's in the keying sequence (at positions 1, 3, 6, and 14 of the selector wheel) the correct key must be one of the following, where position 1 of the keying sequence has been ringed:

5 10 15 17
(1) 6 7 5 6 7 6 8 6 10 5 6 5 7 6 5 9 (7)
(3) 5 6 7 6 8 6 10 5 6 5 7 6 5 9 (7) 6 7
(6) 6 8 6 10 5 6 5 7 6 5 9 (7) 6 7 5 6 7
(14) 6 5 9 (7) 6 7 5 6 7 6 8 6 10 5 6 5 7
R E F E R E N - C E Y O U R M E S
S H V T T J T N H D Y I D T I P

S A G E N - U M B E R
U J T Y X Q G G W V

The cipher equivalents of the two R's in REFERENCE, S_c and T_c, are at an interval of 1 on the cipher component; and since the sums (mod 26) of the first four key values in the four key streams above are 24, 24, 4, and 1, the last key (6 5 9 7 . . .) is obviously the correct one. From the initial crib we are able

. Y . . R A U . . C B E F G . . M N O . . S T . . . .,

to piece together a partial plain component, which enables us, since we know both the keying sequence and the identity of the plaintext interruptor, to decipher portions of the message as follows:

```
             5              10             15    17
 6  5  9  7  6  7  5  6  7  6  8  6 10  5  6  5  7
 S  H  V  T  T  J  T     N  H  D  Y  I  D  T  I  P
 R  E  F  E  R  E  N  -  C  E  Y  O  U  R  M  E  S

 U  J  T  Y  X     Q  G  G  W  V  R  F  G  R  C  L
 S  A  G  E  N  -  U  M  B  E  R  S           E  R

 I  F  P  Z  N  N  C  W     T  S  C  P  O  S  D  H
 O  S     O     G  E  N  -  E  R  A     C     A

 K  A  F  S  M  B  G  P  X  T  L  V  J  G  U  O
       C                 A  R  R        E     N  -

 D  G  B  O  H  D  O     O  C  J  W
 F        E     A  N  -  U  A  R  Y
```

From here on we can easily decipher the rest of the message and recover the plain component in its entirety.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

CHAPTER IX

FUNDAMENTALS OF KEY ANALYSIS

Paragraph

56. Convenient sources of key.—*a.* In literal and digital cryptosystems of the additive family (i.e., true additive, subtractive, and minuend systems) using long keys the key is sometimes derived from a source easily available in the public domain, either in order to lessen the problems of key production and distribution or to avoid the possession of possibly incriminating materials when used by secret agents. Digital key from open sources has been taken from telephone directories (in this case an indicator might show the starting page, or the starting name), from statistical tables such as those found in ordinary almanacs, from tables of mathematical functions,[1] or from published tables of random numbers; these might be used with an indicator to show the starting point for the key. Literal key from open sources has often been taken from the pages of a novel or other book, again with an indicator procedure to show the starting point for the key to be used. The enciphering components in these cases have usually been direct or reversed standard alphabets.) Other convenient sources of digital or literal key that have been used are old plaintext messages, and old cipher or code messages.

b. The plain text used as key might be subject to variation: for instance, only every other word might be taken, or one- and two-letter words might be ignored, or only the first two or three letters of each word might be used; or, in the case of key from a book, only, say, the first five letters from each line might be used, or only the first five *columns* of letters on each page (as measured from the left-hand margin). In a specialized case of running plain text used as key only the 20 consonants were used, dropping the six vowels. When mathematical or other tables are used as key, the normal direction of reading the groups might be changed; or every 5th group, say, might be deleted. In the case of old cipher or code messages, every other group might be taken as key, or the columns of groups of a standard message format (e.g., ten groups per line) might be taken off vertically from right to left.

c. The scrambled sequence of code groups in the encode section of a two-part code book has been used as key, with an indicator to show the starting point in the sequence. The code groups have been read in the normal fashion down the column, or sometimes up the column; and sometimes the individual code groups have been reversed.

d. Other keys have either been generated by some deterministic method, manual or machine, or have been produced in a random fashion in bulk and put together in key books of convenient size for the users. Some of these methods will now be discussed.

57. Manual key generation methods.—*a.* There are many schemes for the generation by hand of long literal keys. For instance, in a basic 26-letter mixed sequence the letters in the sequence up to A might be taken, followed by the letters up to B, C . . . Z as in the following example:

H Y D R A/H Y D R A U L I C B/H Y D R A U L I C/H Y D/. . . .

This yields a key of length $\frac{26(26+1)}{2}=351$.

[1] If these are 4-digit tables, a simple expedient that has been encountered for producing 5-digit key is by repeating the last digit.

b. Another scheme involves the 26 slides of a basic sequence,

A U L I C...Z H Y D R/B E F G J...A U L I C/C B E F G...R A U L I/D R A...,

yielding a key of length 676. A simple variation of this idea is reversing the sequence of letters in the B, D, F . . . segments:

A U L I C...Z H Y D R/B C I L U...K J G F E/C B E F G...R A U L I/D Y H . . .

c. In another method for producing a 676-element key,

```
H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
I Z E S B V M J D C F G H K L N O P Q R T U W X Y A
J A F T C W N K E D G H I L M O P Q R S U V X Y Z B
K B G U D X O L F E H I J M N P Q R S T V W Y Z A C . . .,
```

a basic 26-letter mixed sequence is succeeded by 25 isomorphic changes, either on the normal sequence as in the example above, or on the same mixed sequence (which amounts to nothing more than a simple slide),

```
H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
Y D R A U L I C B E F G J K M N O P Q S T V W X Z H
D R A U L I C B E F G J K M N O P Q S T V W X Z H Y
R A U L I C B E F G J K M N O P Q S T V W X Z H Y D . . .,
```

or on an unrelated sequence,

```
H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
J C F V B E Y O D L S G H K M P A N R U T I W X Z Q
K D G W L S C N F Y T H J M P R B A V E I O X Z Q U
M F H X Y T D A G C I J M P R V L F W S O N Z Q U E . . . .
```

d. In the following method,

```
H Y D R U L I C B...X Z/H Y D R A U L I C E F...X Z/H Y D R A U L
I B E F...X Z/H Y R A U L I C...X Z/H Y D R A U L I C B F . . .,
```

a basic 26-letter sequence is transformed into 26 sequences of 25 letters each, by first dropping out A, then B, C . . . Z, yielding a key of $26 \times 25 = 650$ letters. Here again a possible variation is the reversal of the sequence of letters in the 2d, 4th, 6th . . . segments.

e. Another method that has been proposed is writing the normal alphabetic sequence (or an agreed-upon mixed sequence) a fixed number of times under a transposition key, and taking off the columns of letters according to the numerical key. An even better idea is a double transposition, as in the following example:

B	E	R	N	E	S	W	I	T	Z	E	R	L	A	N	D
2	4	11	9	5	13	15	7	14	16	6	12	8	1	10	3
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Y	Z														

T1

M	A	N	C	H	E	S	T	E	R	E	N	G	L	A	N	D
11	1	12	3	9	5	16	17	6	15	7	13	8	10	2	14	4
N	D	T	J	Z	P	F	V	A	Q	G	W	M	C	S	I	Y
P	F	V	L	B	R	H	X	B	R	H	X	N	D	T	J	Z
E	U	K	A	Q	G	W	M	K	A	Q	G	W	M	C	S	H
X	N	D	T	J	Z	P	M	C	S	I	Y	O	E	U	D	T
J	Z	P	F	V	L	O	E	U	K	A	Q	G	W	C	S	I
Y	O	E	U	K	L	B	R	H	X	N	D	T	F	V	L	B
R	H	X	N	I	Y	O	E	U	K	A	Q	G	W	M	C	S
I	Y	O	J	Z	P	F	V	L	B	R						

T2

The final key (D F U N Z . . .) is much more thoroughly disarranged than that of the first transposition alone (N D T J Z . . .) which consists of slides of two 13-letter sequences owing to the even length of the transposition key.

f. Digital keys can be generated in even more simple ways than literal keys. The simplest example of self-generated key that has been encountered is a stream composed of an ascending sequence of numbers as in the two following examples:

1011 1213 1415 1617 1819 2021 2223 2425 2627 2829 . . .
2502 5125 2253 2542 5525 6257 2582 5926 0261 2622 . . .

Another procedure employed the file time, or the file time plus a sum check, to yield four-digit and five-digit numbers which were used in a fashion similar to the examples above.

g. When digital key is derived from plain text, a particularly popular method involves the use of the following diagram for the conversion:

1	2	3	4	5	6	7	8	9	0
A	B	C	D	E	F	G	H	I	J
K	L	M	N	O	P	Q	R	S	T
U	V	W	X	Y	Z				

Other schemes frequently encountered entail the use of a monome-dinome rectangle, or a bipartite matrix, such as in the following examples:

	9	4	8	1	2	7	6	5	0	3
–	R	E	P	U	B	L	I	C	[redacted]	
0	A	D	F	G	H	J	K	M	N	O
3	Q	S	T	V	W	X	Y	Z		

	6	7	8	9	0
1	R	E	P	U	B
2	L	I	C	A	D
3	F	G	H	K	M
4	N	O	Q	S	T
5	V	W	X	Y	Z

h. In another scheme for converting literal text from a book into digits, use is made of the following widely known memory training device:

1	2	3	4	5	6	7	8	9	0
T	N	M	R	L	Ŝ	K	F	P	S
D					Ĉ	C̄	V	B	Ç
TH					Ĝ	CK			Z
T̄H̄					Ĵ	G			
						Q			
						NG			

(W, H, and vowels have no value)

The conversion of plain language into digits is by *sounds*, and not by the alphabetical characters, as may be seen in the example below:[2]

TH	E	CH	A	R	G	E	O	F	TH	E	L	I	G	H	T	B	R	I	G	A	D	E	W	A	S
1		6		4	6			8	1		5				1	9	4		7		1				0

W	R	I	TT	E	N	B	Y	A	L	F	R	E	D	L	O	R	D	T	E	NN	Y	S	O	N
	4		1		2	9			5	8	4		1	5		4	1	1		2		0		2

[2] Note that the Ĉ and Ĝ sounds in the word CHARGE have a value of 6, while the G in LIGHT is not counted since it is not pronounced, and the G in BRIGADE has the value of 7.

i. The methods described in subpars. *a–e*, above, may also be adapted for the production of digital key. As for isomorphic key, using a basic 25-digit key as an example, nine subsequent rows of 25 digits each may be produced by adding a 1 or some other constant to each element of a row,

```
2 1 5 4 1 8 3 6 7 6 8 5 2 0 9 4 5 2 3 4 1 7 3 9 0
3 2 6 5 2 9 4 7 8 7 9 6 3 1 0 5 6 3 4 5 2 8 4 0 1
4 3 7 6 3 0 5 8 9 8 0 7 4 2 1 6 7 4 5 6 3 9 5 1 2
5 4 8 7 4 1 6 9 0 9 1 8 5 3 2 7 8 5 6 7 4 0 6 2 3 . . .,
```

or a 10-digit mixed sequence may be used to generate nine further rows,[3]

```
2 1 5 4 1 8 3 6 7 6 8 5 2 0 9 4 5 2 3 4 1 7 3 9 0
4 6 8 9 6 2 1 7 5 7 2 8 4 3 0 9 8 4 1 9 6 5 1 0 3
9 7 2 0 7 4 6 5 8 5 4 2 9 1 3 0 2 9 6 0 7 8 6 3 1
0 5 4 3 5 9 7 8 2 8 9 4 0 6 1 3 4 0 7 3 5 2 7 1 6 . . .,
```

or a long series of substitution alphabets such as

	1	2	3	4	5	6	7	8	9	0
(1)	5	8	2	0	1	6	9	7	3	4
(2)	6	1	9	5	7	2	0	3	8	4
(3)	6	4	7	1	5	0	2	8	3	9 . . .

might be used to generate a large number of rows from the basic sequence as in the following example:

```
2 1 5 4 1 8 3 6 7 6 8 5 2 0 9 4 5 2 3 4 1 7 3 9 0
8 5 1 0 5 7 2 6 9 6 7 1 8 4 3 0 1 8 2 0 5 9 2 3 4
1 6 7 5 6 3 9 2 0 2 3 7 1 4 8 5 7 1 9 5 6 0 9 8 4
4 6 5 1 6 8 7 0 2 0 8 5 4 9 3 1 5 4 7 1 6 2 7 3 9 . . . .
```

j. A frequently encountered key-generation method, especially for agent use, entails the use of Fibonacci treatment involving an introductory group of length n, and then (most usually) summing the $a+b$ digits to yield the digit in the $(n+1)$ position, summing the $b+c$ digits to yield the digit in the $(n+2)$ position and continuing this process to derive a key as long as necessary with which to encipher the message. The indicator in such cases is often the introductory priming group disguised by an additive (usually taken from one of the text groups of the cipher message); in some cases, however, the indicator is the introductory group left in the clear, as in the following examples employing 5-digit priming groups:

Ind.	10 15 20	Message key
31419	4 5 5 0 3 9 0 5 3 2 9 5 8 5 1	4 3 3 6 5 7 6 9 1 2 . . .
27188	9 8 9 6 7 7 7 5 3 4 4 2 8 7 8	6 0 5 5 4 6 5 0 9 0 . . .

In these two examples the developed key up to the 20th position was discarded as a safety measure, and the actual key used commenced with the 21st position.

k. Fibonacci variations are many. For example, if the introductory group consists of five digits, the formulas $a+e=f$ or $a+b+c=f$ might be used; or each succeeding five-digit group might be generated by a recursion procedure of $a+b=f$, $b+c=g$, $c+d=h$, $d+e=i$, but $e+a=j$ (instead of the more usual $e+f=j$). Sometimes a row of n digits, say 25, might be generated by a Fibonacci treatment, and a second row established underneath the first one, either by shifting the basic sequence against itself at some offset

```
3 1 4 1 9 4 5 5 0 3 9 0 5 3 2 9 5 8 5 1 4 3 3 6 5
1 9 4 5 5 0 3 9 0 5 3 2 9 5 8 5 1 4 3 3 6 5 3 1 4
```

[3] The 10-digit mixed sequence in this example, 2490316758, generates nine further rows, but it could be used to generate 99 further rows by successive changes of the mixed sequence to 3501427869, 4612538970 . . ., that is, by adding to the digits of the mixed sequence after each set of ten rows, yielding a key totalling 2500 digits.

or by adding a constant to the basic sequence,

3 1 4 1 9 4 5 5 0 3 9 0 5 3 2 9 5 8 5 1 4 3 3 6 5
4 2 5 2 0 5 6 6 1 4 0 1 6 4 3 0 6 9 6 2 5 4 4 7 6

or by continuing the Fibonacci treatment for another 25 digits,

3 1 4 1 9 4 5 5 0 3 9 0 5 3 2 9 5 8 5 1 4 3 3 6 5
7 6 9 1 2 3 5 0 3 5 8 5 3 8 3 3 8 1 1 6 1 9 2 7 7.

From here on a *vertical* Fibonacci treatment is applied to the columns, to yield a long key. Using the case just illustrated,

3 1 4 1 9 4 5 5 0 3 9 0 5 3 2 9 5 8 5 1 4 3 3 6 5
7 6 9 1 2 3 5 0 3 5 8 5 3 8 3 3 8 1 1 6 1 9 2 7 7
0 7 3 2 1 7 0 5 3 8 7 5 8 1 5 2 3 9 6 7 5 2 5 3 2

the length of the key would be 60×25=1500 digits before it begins to repeat.

l. Key has sometimes been produced by means of an assembly of strips. The system entails a set of, say, 50 strips containing 50 digits each, from which a selection of 20 strips is made, aligned flush with each other or at a specified alignment, and the key groups are read in the order in which they occur. As an example, in Fig. 111*a* below is illustrated a set of ten strips, and in Fig. 111*b* is the selection of strips *b*, *f*, *c*, and *a* at the alignment 3204. (Note that, to avoid possible ambiguity, the top ten digits on each strip consist of a scramble of the digits 0 through 9.)

a	b	c	d	e	f	g	h	i	j
1	2	0	4	9	6	9	4	4	2
8	3	3	0	6	4	8	1	6	5
4	0	4	9	5	7	7	6	9	1
3	1	9	2	0	0	1	8	7	7
9	8	8	7	8	2	6	5	0	6
2	7	5	3	7	5	0	2	3	0
6	9	7	8	2	1	3	9	1	9
7	5	1	6	4	9	2	0	5	3
5	6	2	5	1	3	5	3	2	4
0	4	6	1	3	8	4	7	8	8
7	0	7	9	9	5	4	7	2	0
8	5	6	3	6	0	2	4	5	3
0	2	3	6	1	9	9	1	9	6
4	8	0	8	8	6	5	0	8	2
9	7	4	5	5	1	1	2	3	8
1	4	5	7	3	3	3	6	7	9
2	3	9	0	2	4	8	5	6	5
5	6	1	1	7	7	6	8	0	4
6	1	8	2	4	2	7	9	1	7
3	9	2	4	0	8	0	3	4	1
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
9	4	1	4	1	9	8	7	9	0
7	2	5	6	8	8	1	4	0	7
1	5	0	2	4	2	0	3	2	5
0	0	9	0	3	0	5	2	8	1
2	8	6	3	2	1	2	5	4	6
5	3	7	8	5	7	4	9	7	2
4	7	2	5	9	5	3	0	6	8
6	9	3	1	0	3	9	8	1	3
8	1	8	7	6	4	7	6	3	4
3	6	4	9	7	6	6	1	5	9

FIGURE 111*a*

	f		
	6		
	4		a
b	7		1
2	0	c	8
3	2	0	4
0	5	3	3
1	1	4	9
8	9	9	2
7	3	8	6
9	8	5	7
5	5	7	5
6	0	1	0
4	9	2	7
0	6	6	8
5	1	7	0
2	3	6	4
8	4	3	9
7	7	0	1
4	2	4	2
3	8	5	5
⋯	⋯	⋯	⋯
2	1	1	1
5	7	5	0
0	5	0	2
8	3	9	5
3	4	6	4
7	6	7	6
9		2	8
1		3	3
6		8	
		4	

FIGURE 111*b*

The key groups would be taken off starting with 3204 0533 and ending with 3464 7676. If more key is needed, a new selection and alignment would be made. (The process could have been facilitated in this case by selecting eight strips, taking off two groups at a time.)

m. A method that has achieved prominence is the use of a stencil with which to extract key from a key page, as illustrated in the example given in Fig. 112, below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				20					25					30					35
a.	0	4	4	9	3	5	2	4	9	4	7	5	2	4	6	3	3	8	2	4	4	5	8	6	2	5	1	0	2	5	6	1	9	6	2
b.	0	0	5	4	9	9	7	6	5	4	6	4	0	5	1	8	8	1	5	9	9	6	1	1	9	6	3	8	9	6	5	4	6	9	2
c.	3	5	9	6	3	1	5	3					8	9	8	0					3	3	3	5	1	3	5	4	6	2	7	7	9	7	4
d.	5	9	8	0																					9	4	9	7	0	0	1	3	0	2	1
e.	4	6	0	5						6	0	1	3			9	2	2	8						1	4	4	0	7	7	9	3	9	1	0
f.	3	2	1	7																					7	0	7	0	0	7	8	6	7	4	3
g.	6	9	2	3		6	1	4	0			0	1	1	7					4	1	5	9		6	6	0	0	0	0	1	8	7	4	3
h.	1	9	5	6																					9	2	1	5	8	5	6	6	6	7	4
i.	4	5	1	5			4	9	3	8				7	6	0	7				4	3	6	6	7	9	4	5	4	3	5	9	0	4	7
j.	9	4	8	6																					8	8	1	5	5	3	0	1	5	4	0
	9	8	0	8					2	6	4	5						4	0	4	4				9	0	8	8	9	6	3	9	0	9	4
	3	3	1	8																					5	4	8	5	8	1	8	8	6	9	5
	8	0	9	5	1	0	0	4						8	2	7	0								1	2	3	3	8	7	2	5	0	1	6
	7	9	7	5	2	4	9	1	4	0	7	1	9	6	1	2	8	2	9	6	6	9	8	6	1	0	2	5	9	1	7	4	8	5	2
	1	8	6	3	3	3	2	5	3	7	9	8	1	4	5	0	6	5	7	1	3	1	0	1	0	2	4	6	7	4	0	5	4	5	5
	7	4	0	2	9	4	3	9	0	2	7	7	5	5	7	3	2	2	7	0	9	7	7	9	0	1	7	1	1	9	5	2	5	2	7
	5	4	1	7	8	4	5	6	1	1	8	0	9	9	3	3	7	1	4	3	0	5	3	3	5	1	2	9	6	9	5	6	1	2	7
	1	1	6	6	4	4	9	8	8	3	5	2	0	7	9	8	4	8	2	7	5	9	3	8	1	7	1	5	3	9	0	9	9	7	3
	4	8	3	2	4	7	7	9	2	8	3	1	2	4	9	6	4	7	1	0	0	2	2	9	5	3	6	8	7	0	3	2	3	0	7
	6	9	0	7	4	9	4	1	3	8	8	7	6	3	7	9	1	9	7	6	3	5	5	8	4	0	4	4	0	1	1	0	5	1	8

FIGURE 112

The key page in this instance consists of 20 rows of 35 random digits each, and the index position (the upper left-hand corner) of the stencil may be positioned at any one of the first 16 digits in the first ten rows of the key page (in this example, it is placed at position *c*5). The apertures of the stencil expose 15 four-digit groups,[4] and it may be used in any of the four positions, two obverse and two reverse. The advantage of such a system is the prolongation of life of key pages by means of frequent changes of the stencil (size, shape, and number of apertures per line) for different key periods; furthermore, it allows considerable flexibility in the choice of a message-to-message keying variable.

n. As a final comment, keys have been encountered which were produced by a person writing down random characters by hand, or with a typewriter. As may be imagined, these "random characters" are really not random, since it is inevitable that psychological factors enter into the "arbitrary" selection of characters.

58. Mechanical and electronic key generators.—*a.* Perhaps the most primitive "mechanical" generation of digital key is accomplished by means of printing slugs consisting either of single-digit type faces or of 4- or 5-digit groups. The slugs are assembled by hand into a chase or printing frame, and two or more copies are printed of several pages simultaneously. For example, in the case of 4-digit slugs there would be available a collection of one each of the 10,000 groups from 0000 to 9999, and a printing frame of four or eight pages of 100 groups each would be made up at the same time. After the requisite number of copies is printed, the used slugs would then be put back into the total collection, scrambled thoroughly, and another printing frame of four or eight pages would be made up.

[4] Cases have been encountered wherein the apertures of the stencil expose single digits, these digits comprising a periodic repeating key.

b. Old teleprinter tapes, either of plain text or cipher text, have been used as keys; or two or more tapes have been added together and the resultant used as a composite key. For that matter, even a single tape could be read at two different positions and the characters added together to produce key in plaintext autokey fashion.

c. Punched-card equipment has been used to produce key. The simplest method, used for digital key, is to take a deck of cards on which is punched either plain language or random alphabetical text, and then list the cards *but omitting the zone punch* (i.e., the Ø, "x", and "y" punches). This results in the following conversion as a consequence of the card punching code:

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
	S	T	U	V	W	X	Y	Z

Note that a Ø cannot be produced because all letters are coded by a combination of a digit punch from 1 to 9 plus an "x", "y", or "Ø" zone punch.

d. The more usual punched-card method, however, is to start with a single "supermaster card" punched in all 80 columns, perhaps with exactly eight occurrences each of the digits 1 through 0. Then by means of a plugging, 79 more cards are produced from the "supermaster card." These cards are now thoroughly shuffled, and 40 or 50 columns are selected by means of a plugging and printed to constitute 80 lines of key. When the 80 cards have been listed, either the plugging is changed or the cards are shuffled anew, or both, to produce further sets of 80 lines of key. Sometimes the "supermaster card" is used to to generate a total of 80 "master cards," and these "master cards" are then used to produce a second generation of 6400 cards which are thoroughly shuffled and from which a selection of, say, 2500 cards is made, after which columns are selected and listed as before.

e. Mechanical key generators have often employed a multiplicity of gears or variable-pin wheels with which to produce a long key. The best example of this type of key generator is the Hagelin C–38 cipher machine, which has six wheels of mutually prime lengths (26, 25, 23, 21, 19, and 17) which step once with each operation, so that the cycle is the product (101,405,850) of the wheel sizes. The same idea is applicable in the case of key generators employing several teleprinter tapes of mutually prime lengths. Other geared devices have used the principle of the Kryha cipher machine, involving a gear which moves irregularly, displacing a pair of enciphering components from 5 to 10 steps; after a cycle of the gear wheel (which can be varied, usually in the vicinity of 15 to 35) the sum of the displacements is prime to 26 so that the length of the key produced is 26 times the gear cycle. Still other generators have incorporated plaintext or ciphertext autokeying, or the principle of the Wheatstone cipher device which achieves a highly irregular motion of the pair of enciphering components because the stepping is predicated not only on the plain text being enciphered but also on the particular sequence of the letters in the plain component.

f. Often mechanical key generators are employed "off line," so to speak, to produce prefabricated key in advance of actual message encryption. For instance, a Hagelin C–38 machine may be used to encipher a constant plaintext letter, say A_p, which results in the production of pure Hagelin key; or, by prearrangement, plain text from a book or other source can be enciphered, yielding a highly complex resultant as key. Furthermore, in mechanical key generators the key produced is sometimes *cumulative* in nature, i.e., each key element is the sum of the key generated at that particular position plus all the preceding key elements.

g. One device that has appeared on the open market consisted of an array of print-wheels containing ten digits each which printed one page at a time, and after two (or more) copies have been printed the wheels advance to new positions and another page is printed. In Fig. 113, below, is illustrated a simplified sample of two successive pages (as printed) of ten rows of five 4-digit groups each. It will be noted that the numbers in the first and fifth groups advanced by 1 between the two pages, the numbers in

Page 63					Page 64				
0918	2009	3282	3952	0422	0919	2008	3285	3949	0423
9004	8549	5198	5065	9493	9005	8548	5201	5062	9494
7318	5020	4767	2626	6229	7319	5019	4770	2623	6230
7576	7649	2097	8774	9042	7577	7648	2100	8771	9043
5401	4405	6628	3100	0068	5402	4404	6631	3097	0069
0835	6991	7854	4278	1366	0836	6990	7857	4275	1367
2830	0326	8133	1059	4051	2831	0325	8136	1056	4052
5380	8623	8159	1362	5121	5381	8622	8162	1359	5122
9175	5374	6161	6226	5026	9176	5373	6164	6223	5027
8941	9269	0039	5839	1260	8942	9268	0042	5836	1261

FIGURE 113

the second group have retrogressed by 1, the numbers in the third group have advanced by 3, while the numbers in the fourth group have retrogressed by 3. (In the actual machine some of the wheels had repeated digits; the turnover points were not necessarily 9; and irregular motion was imparted to some of the wheels.) In order to disguise the relationship between successive pages, the pages were scrambled after a sufficient number was produced, and the pages were bound in individual key books.

h. Electronic key generators have now come to the fore, as might be expected. Some of them are weak in cryptoprinciple; some are merely electronic versions of pin-wheel or gear mechanisms. The more sophisticated of them, however, are electronic shift registers which produce binary key for the encryption of teleprinter characters, facsimile signals, or coded speech. These shift registers consist of a number of cells or stages, initially loaded with 0's and 1's for the start of the generation. As each binary key element is produced, the sequence of 0's and 1's is shifted to the right, dropping out the rightmost bit, and the leftmost stage is loaded with a new bit by some convention in the circuitry. The maximum cycle of such a register is 2^n-1, where n is the number of stages involved. In Fig. 114, below, is a sample register of 16 stages.

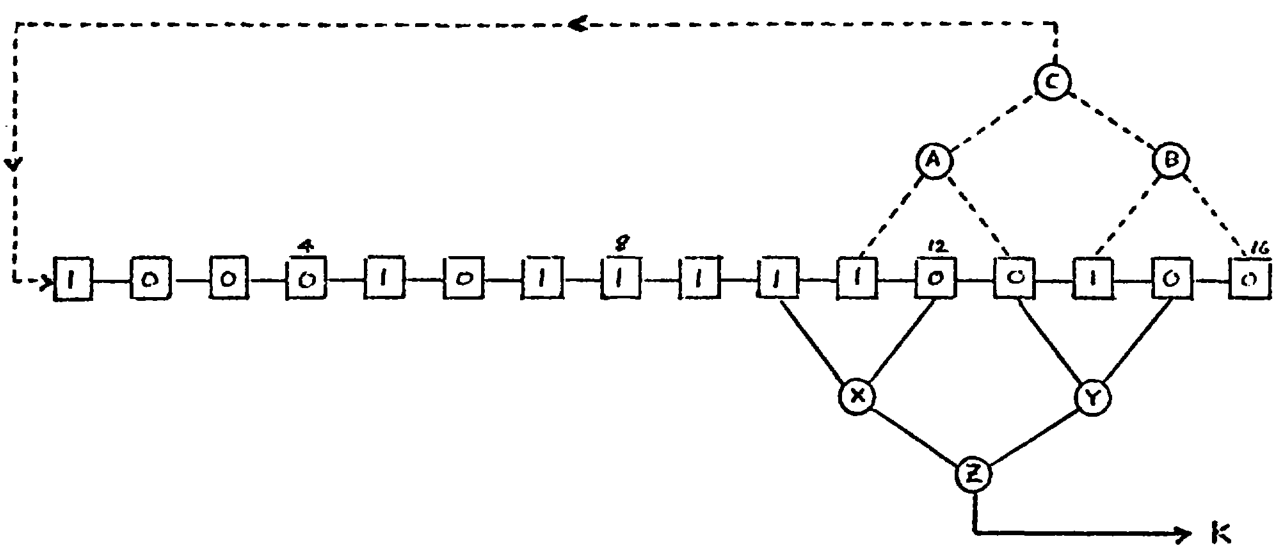


FIGURE 114

The initial fill is 1000 1011 1110 0100; the feedback to stage 1 is the combination of stages 11, 13, 14, and 16 by some rule (either mod 2, Vernam, Boolean, or multiplication)[5]; and the key is the combination of stages 10, 12, 13, and 15 by some rule. In this example, if all the combinations are mod 2 the key produced will be a 1, while the feedback to stage 1 will be a 0, making the new fill 0100 0101 1111 0010.

i. Just as manually produced key may be the result of psychological random by pencil or typewriter, likewise there are mechanical and electronic counterparts in nondeterministic key production. There have been key generators marketed which operate with ball bearings falling into slots or with balls of two different sizes; or the key generated may be produced by the firing of a gas tube, or by emissions from a radioactive source. Computer programs with complex jump orders may also be used for the production of digital or alphabetical key that approximates random key.

59. General analytic approaches.—*a*. The general methods and procedures of cryptanalysis that apply to encrypted text also apply in the analysis of key. Moreover, in key analysis there may be exploited properties of its source, generation, or production which are not present in the cryptanalysis of encrypted text. When the method of key production is deterministic, solution of the method is possible; but when the method is nondeterministic such as psychological random, or involving the firing of a gas tube, all that can be done is to apply the principles of astrology with a wet finger held in the air, capitalizing in whatever way possible on any key bias or tendencies observed.

b. Frequency distributions of various kinds, and searches for repetitions and evidence of isomorphism are a must. But ahead of these, the recovered key must first be *looked* at with both eyes and an open mind, which makes the derivation of the following key fragment

```
AAJVC  ODFSH  BAJVC  AJVCO  DAJVC  ODFSH . . .
```

recognizable in an instant,[6] and the derivation of the key fragment below

```
HBPYE  QDFSR  GTAJV  UKWLM  XINZC  OYEQD
FSRGT  AJVUK  WLMXI  NZCOH . . .
```

apparent after a very brief examination. In the next example of the key recovered from a 50-letter message,

```
BSJOF  WDULQ  HFWNI  ZQKBS  MDUEV  MNEVA
RICTK  LCTPG  XJARG  XOHYP
```

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1

its frequency distribution reveals superflat randomness and suggests a possible method of generation, confirmed by noting the plethora of digraphic repetitions and exploiting them to solution, recovering the complete key population in the process. Had the method of extraction been different, as for example in the following key,

```
BSJAR  IZOFW  NEVDU  LCTKQ  HYPGX  FWNEV
MIZQH  YPKBS  JARMD  ULCTE
```

complete solution would have been achieved by capitalizing on the long polygraphic repetitions manifested without the necessity of making a frequency distribution.

c. The common method of converting literal text to digital key through the use of the diagram below, with the set of equivalents as shown,

1	2	3	4	5	6	7	8	9	0
A	B	C	D	E	F	G	H	I	J
K	L	M	N	O	P	Q	R	S	T
U	V	W	X	Y	Z				

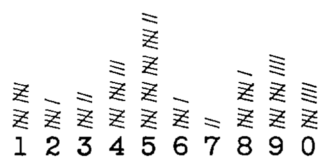
---

[5] The rules for combining bits are explained in footnote 5 of chapter X (p. 267).

[6] Or two.

yields a distinct set of probabilities of the digits, shown below together with the theoretical distribution in tally-mark form (for English underlying text):

1	.1027
2	.0614
3	.0710
4	.1265
5	.2246
6	.0560
7	.0199
8	.1097
9	.1347
0	.0935
	1.0000



The appearance of this distribution (and its equivalent in the language of the particular problem with which we might be concerning ourselves) should be memorized, together with its γ I.C. (in this case, for English, $\gamma = 1.28$), for its quick identification and interpretation when it is encountered. Once a numerical key is recognized as having arisen by means of this particular conversion process, the original plain text may easily be recovered by reading on simple generatrices, as in the following example:

```
        5         10        15        20        25        30
6 5 1 8 9 3 5 8 5 1 4 4 9 5 2 5 4 5 5 1 8 9 1 7 5 5 1 8 6 1 . . .
F E A H I C E H E A D D I E B E D E E A H I A G E E A H F A
P O K R S M O R O K N N S O L O N O O K R S K Q O O K R P K
Z Y U     W Y   Y U X X   Y V Y X Y Y U     U   Y Y U   Z U
```

d. Cases have been encountered in which key has been generated by enciphering plain text from a book through a Nihilist square which consists of a 5×5 square inscribed with the normal A–Z sequence (I and J combined), and coordinates 1–5 in normal order for both rows and columns. This yields the following sets of probabilities and theoretical distribution:

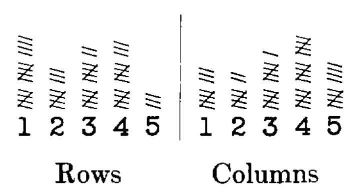
Rows:

1	.2865
2	.1567
3	.2426
4	.2584
5	.0558
	1.0000

Columns:

1	.1572
2	.1422
3	.2099
4	.3040
5	.1867
	1.0000

Composite distribution:



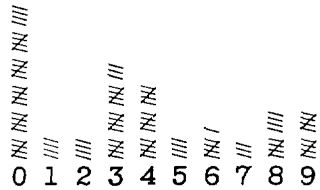
The nature of the dinomic key text, revealing plaintext properties underlying, makes it very easy to recover the original literal text from which the key was produced. Other schemes for obtaining digital key from plain text, such as encipherment with monome-dinome rectangles, are transparent and readily solvable if there is a sufficient sample of key available. Furthermore, there have been many cases in which key was derived from plain text enciphered by means of the same cryptosystem as was used to encipher the message.[7] For example, if the enemy is using the following monome-dinome rectangle,

	9	4	8	1	2	7	6	5	0	3
–	R	E	P	U	B	L	I	C		
0	A	D	F	G	H	J	K	M	N	O
3	Q	S	T	V	W	X	Y	Z		

[7] For an example of such a system used by secret agents in World War II, see pp. 206–210 of Alexander Foote, *Handbook for Spies*, New York, 1949.

this yields, for English plain text, the probabilities of the ciphertext digits (with γ I.C. of 1.63) and the theoretical distribution in tally-mark form shown below:

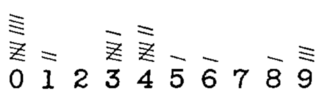
0	.2880
1	.0363
2	.0372
3	.1808
4	.1468
5	.0354
6	.0602
7	.0268
8	.0923
9	.0962
	1.0000



If, in solving an additive encipherment involving the foregoing rectangle, we recover the first 30 digits of key to be

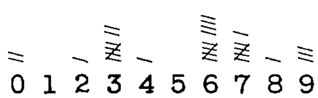
08031 93450 39409 00043 44314 00364

we would recognize from the appearance of the key that it must be derived from the same population as the theoretical distribution given above; this is attested by the distribution of the key:



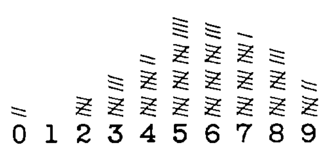
Even if we had recovered the following key,

68670 97326 79369 66637 33703 66743



we would still recognize from its appearance that it must be based upon plain text, and, because of its γ I.C. of 1.75 (close enough to the γ I.C. of 1.63), probably enciphered through the same monome-dinome matrix but with different coordinates. This is a hypothetical case, of course, but it might be apropos to include here the digit probabilities and theoretical distribution of cipher text produced by the true Nihilist system, involving an additive key produced by enciphering plain text through the same square as is used for the encryption of the message:

0	.0190
1	.0000
2	.0450
3	.0753
4	.1218
5	.1897
6	.1834
7	.1617
8	.1327
9	.0715
	1.0001



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e. The punched-card method of producing digital key by suppressing the zone punch of alphabetical material yields characteristic probabilities which are easy to recognize and interpret. In Fig. 115*a*, below, are the probabilities

0	.0000
1	.0753
2	.0739
3	.1590
4	.0931
5	.2248
6	.1192
7	.0477
8	.0567
9	.1503
	1.0000

FIGURE 115*a*

0	.0000
1	.0769
2	.1154
3	.1154
4	.1154
5	.1154
6	.1154
7	.1154
8	.1154
9	.1154
	1.0001

FIGURE 115*b*

of the digits when the data punched on the cards is English plain text, and in Fig. 115*b* are the probabilities when the data consists of random alphabetical characters. In tally-mark form the theoretical distributional appearance of these two cases is quite striking:

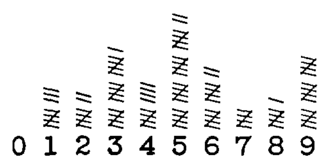


FIGURE 115*c*

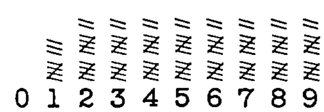


FIGURE 115*d*

The original plain text from which the key was derived may be easily recovered with a modification of the generatrix diagram shown in subpar. *c*, above.

f. Delta techniques play an important part in key analysis, as will now be shown. For example, if key recovered on true base from a cipher text were that given below,

3201 3404 3598 3784 3962 4133 4298 4456 4609 4757 . . .,

this appears to have been derived by some sort of mathematical progression or from a table of mathematical functions. A search in common mathematical tables reveals the source: in a table of 4-place logarithms, these are the logs of 209, 219, 229 . . . . But if the key were not on true base, so that the groups as recovered were

4435 4638 4832 5018 5196 5367 5532 5690 5843 5991 . . .,

this sequence could not be found in any of our available tables. We therefore take first differences (with *borrowing* subtraction) of the groups, and also further differences as shown in the diagram below:

	4435	4638	4832	5018	5196	5367	5532	5690	5843	5991 . . .
Δ_1:	203	194	186	178	171	165	158	153	148	
Δ_2:	9	8	8	7	6	7	5	5		
Δ_3:	−1	0	−1	−1	1	−2	0			

The oscillation of the final differences is indicative of derivation from a mathematical table of some sort, with round-off error because of incomplete information. We therefore make a search in available mathematical tables for groups with successive differences of 203, 194, 186 . . . , which leads us to solution, discovering the addition (with carrying arithmetic) of a constant 1234. Note that if we had taken our first differences mod 10, we would have had the following:

	4435	4638	4832	5018	5196	5367	5532	5690	5843	5991 . . .
Δ_1:	203	204	1286	188	271	275	168	253	158	

These fluctuating differences would have caused us to try normal arithmetical differences instead of mod 10 ones, leading to solution as before.

g. In Figs. 116*a* and *b*, below, are given two further examples of key and differences:

K:	2323	2530	2755	2998	3259	3538	3835	4150	4483	4834 . . .
Δ_1:	207	225	243	261	279	297	315	333	351	
Δ_2:	18	18	18	18	18	18	18	18		
Δ_3:	0	0	0	0	0	0	0			

FIGURE 116*a*

K:	5818	6215	6684	7231	7862	8583	9400	0319	1346	2487 . . .
Δ_1:	397	469	547	631	721	817	919	1027	1141	
Δ_2:	72	78	84	90	96	102	108	114		
Δ_3:	6	6	6	6	6	6	6			
Δ_4:	0	0	0	0	0	0				

FIGURE 116*b*

The fact that the last differences are all 0's shows that the numerical data are self-contained and complete. Now it is a well-known mathematical fact that the second differences of a series of squares (or any decimation of the series) will be constant, as will be the third differences of a series of cubes, as may be seen from the following:

N	N^2	Δ_1	Δ_2
1	1		
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	

FIGURE 116*c*

N	N^3	Δ_1	Δ_2	Δ_3
1	1			
2	8	7	12	
3	27	19	18	6
4	64	37	24	6
5	125	61	30	6
6	216	91		

FIGURE 116*d*

It therefore follows that the key in Fig. 116*a* is based on polynomials of the second degree, so we can solve a set of four simultaneous equations in four unknowns to arrive at an answer, as follows:

$$\begin{aligned}
&(1)\ ax^2 + bx + c = 2323\\
&(2)\ a(x+1)^2 + b(x+1) + c = 2530\\
&(3)\ a(x+2)^2 + b(x+2) + c = 2755\\
&(4)\ a(x+3)^2 + b(x+3) + c = 2998
\end{aligned}$$

$$\begin{aligned}
&(2)\ (ax^2+2ax+a) + (bx+b) + c = 2530\\
&(1)\ \underline{ax^2 \qquad + bx \qquad + c = 2323}\\
&(5)\ \qquad 2ax+a \qquad +b \qquad = 207
\end{aligned}$$

$$\begin{aligned}
&(3)\ (ax^2+4ax+4a) + (bx+2b) + c = 2755\\
&(2)\ \underline{(ax^2+2ax+\ a) + (bx+\ b) + c = 2530}\\
&(6)\ \qquad 2ax+3a \qquad +b \qquad = 225
\end{aligned}$$

$$\begin{aligned}
&(6)\ 2ax+3a+b = 225\\
&(5)\ \underline{2ax+\ a+b = 207}\\
&(7)\ \qquad 2a \quad = 18,\ \therefore\ a = 9
\end{aligned}$$

At this point we are in a quandary, since there is no clear-cut way to recover the values of x and b—indeed, there is an infinite set of values for x and b that will satisfy the equations. But let us try a bit of practical skulduggery. We see from (5) that $18x+9+b=207$, so that $18x+b=198$. Now since 18 is a divisor of 198, is it not possible that $x=11$, making $b=0$? We then have the first several elements of key reduced as follows:

1. $2323 = 9\cdot 11^2 + 1234$
2. $2530 = 9\cdot 12^2 + 1234$
3. $2755 = 9\cdot 13^2 + 1234$

Even though there is an infinite set of values of x and b that would satisfy the equations, there is little doubt, based on psychological considerations, that the values are $a=9$, $b=0$, $c=1234$, and $x=11$. As for the key of Fig. 116*b*, we must solve the following set of five simultaneous equations in five unknowns:

$$\begin{aligned}
&(1)\ ax^3 + bx^2 + cx + d = 5818\\
&(2)\ a(x+1)^3 + b(x+1)^2 + c(x+1) + d = 6215\\
&(3)\ a(x+2)^3 + b(x+2)^2 + c(x+2) + d = 6684\\
&(4)\ a(x+3)^3 + b(x+3)^2 + c(x+3) + d = 7231\\
&(5)\ a(x+4)^3 + b(x+4)^2 + c(x+4) + d = 7862
\end{aligned}$$

Here too there is an infinity of answers, but the psychologically most plausible one consists of the values $a=1$, $b=0$, $c=0$, $d=4487$, and $x=11$, simplifying the polynominal to x^3+4487.

h. For the next case to be considered, we have the following stretches of key recovered from reading three shallow depths (the message indicators are shown enclosed within parentheses):

Key "A"

(14342) 21900 30109 41280 71216 24400 93326 46025 41671 . . .

Key "B"

(09414) 28071 21624 40093 32646 02541 67110 51010 31452 . . .

Key "C"

(16715) 10510 10314 52188 22534 11141 87119 82119 71041 . . .

It is apparent from the relationships of the three keys that the sequence of 5-digit groups may be used at a slide. On closer examination, however, it is observed that if the keys are divided into tetranomes, the *a* digits are limited to 0, 1, or 2: this could mean that the enemy is using the encode section of one of his two-part code books as source of key. As for the indicators, they designate the group (with a sum-checking digit added) immediately preceding the first code group to be used as key. The beauty of this scheme to the cryptanalyst is that, if the code is still in use and has not been recovered to any extent, the sequence of key groups recovered above had the effect of transforming portions of the two-part code into a *one-part* code.[8]

i. In the next example let us assume that we have the following three keys available for study:

(1)	3204	0533	1149	8992	7386	9857	5575	6010	4927	0668 . . .
(2)	2601	3438	0744	1093	8289	7552	9176	5917	6325	4860 . . .
(3)	3871	9508	2727	6159	7215	5696	0734	7680	8355	0002 . . .

We note, between keys (1) and (2), that the *a* digits of (2) are the same as the *a* digits of (1), but at a slide of 1; that the *b* digits of (2) are the same as the *b* digits of (1) at a slide of 4; that the *c* digits of (1) and (2) are identical; and that the *d* digits of (2) are the same as the *d* digits of (1) at a slide of 2. Likewise, it may be seen that the *a* digits of (1) are related to the *d* digits of (3); the *b* digits of (1) are related to the *c* digits of (3); the *c* digits of (1) are related to the *b* digits of (3); and the *d* digits of (1) are related to the *a* digits of (3). All this points to a system of generation with sliding strips, such as that illustrated in subpar. 57*l*, so that after the strips are synthesized the key for a new message could be extrapolated after a few groups have been recovered.

j. In another example let us study the following three keys recovered from the initial analysis of a cryptosystem:

(1)	3153	8980	3335	6013	9228	6140	0117	4159	4938	7607 . . .
(2)	6315	6898	4333	3601	0922	4614	2011	0415	1493	4760 . . .
(3)	3072	0935	5135	3909	8677	0620	7452	9566	8194	7246 . . .

We immediately note that the *abc* digits of (1) are the same as the *bcd* digits of (2), and that the *d* digits of (1) are the same as the *a* digits of (3). At this point we might have conjectured a strip-generated system such as that of the preceding subparagraph, but the fact that no slides of the basic sequences between groups is noted suggests a stencil system, upon which we are able to synthesize the following partial sequences of the key page:

. . . 63153072 . . .	. . . 68980935 . . .	. . . 43335135 . . .
. . . 36013909 . . .	. . . 09228677 . . .	. . . 46140620 . . .
. . . 20117452 . . .	. . . 04159566 . . .	. . . 14938194 . . .
. . . 47607246 . . .		

As more key is recovered it can be fitted into our partial reconstruction, and we would eventually arrive at the key page and stencil shown in Fig. 112. In this case the solution of the indicators used would materially aid in the correct interleaving of the odd and even rows of the key page, since the stencil itself has its apertures two rows apart.

[8] In a related use of code books as sources of key, cases have been encountered in which the indicator in a transposition system consists of a code group (for example, LYUF, which might have the meaning "Mission accomplished," providing a 19-element key for the transposition). When the literal key is recovered from the numerical key used in the transposition, we also have an incidental code recovery for the group in question, if we did not have it before.

k. The key recovered from multiple anagramming of several identical-length transposition messages in the same key yields a partial cipher-to-plain (C→P) sequence which gives clues as to the nature of the transposition system. Let us take the following four 50-letter messages as an example:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Msg No. 1:	N	U	R	I	L	O	R	R	H	M	H	T	D	T	E	E	O	T	G	A	R	U	U	D	E	I	D	I	N	O
Msg No. 2:	I	P	S	L	T	O	L	L	N	O	X	T	R	E	I	U	R	N	H	L	Y	S	T	E	N	A	O	T	T	A
Msg No. 3:	T	Y	T	P	O	I	E	R	R	I	E	E	N	R	U	N	S	P	N	R	O	N	H	B	A	I	R	G	O	A
Msg No. 4:	E	O	N	S	R	S	L	U	F	E	U	T	N	T	T	E	R	O	S	L	R	N	U	P	O	A	F	A	T	R

	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Msg No. 1:	H	N	V	D	M	E	Z	I	I	N	A	C	A	E	T	A	T	S	E	R
Msg No. 2:	E	R	U	C	S	C	I	E	Q	E	H	E	E	O	I	L	R	Y	I	T
Msg No. 3:	E	T	S	W	G	N	R	S	L	E	E	K	P	O	N	T	A	A	O	T
Msg No. 4:	T	R	O	W	Q	P	I	E	P	E	R	S	I	O	I	O	O	Y	S	T

In the initial anagramming, 25 columns have been assembled, as shown below:

	41	10	35	23	1	4	14	26	17	29	45	7	20	38	32	48	42	11	36	24	2	5	15	27	18	
Msg No. 1:	A	M	M	U	N	I	T	I	O	N	T	R	A	I	N	S	C	H	E	D	U	L	E	D	T	. . .
Msg No. 2:	H	O	S	T	I	L	E	A	R	T	I	L	L	E	R	Y	E	X	C	E	P	T	I	O	N	. . .
Msg No. 3:	E	I	G	H	T	P	R	I	S	O	N	E	R	S	T	A	K	E	N	B	Y	O	U	R	P	. . .
Msg No. 4:	R	E	Q	U	E	S	T	A	R	T	I	L	L	E	R	Y	S	U	P	P	O	R	T	F	O	. . .

The partial C→P sequence recovered in the foregoing diagram is the following:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
41	10	35	23	1	4	14	26	17	29	45	7	20	38	32	48	42	11	36	24	2	5	15	27	18

It is clear from the isomorphic repetition manifested in the key beginning at the 17th position, that, unless blank cells are encountered later on, the system is a single columnar transposition involving 16 columns with the numerical key as shown in the diagram below, constructed from the recovered portion of the C→P sequence:

14	4	12	8	1	2	5	9	6	10	15	3	7	13	11	16
41	10	35	23	1	4	14	26	17	29	45	7	20	38	32	48
42	11	36	24	2	5	15	27	18							

If, in another example, the portion of the recovered C→P sequence had been the following,

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
10	24	29	19	35	6	15	41	46	11	1	25	30	20	36	16	42	47	2	26	31	21	37	7	17,

the interrupted isomorphism observed is indicative of *blank cells* in the matrix, and the numerical key together with the blank pattern thus far recovered for the matrix of 10 columns is shown in the following diagram:

3	1	6	7	5	8	2	4	9	10
10	●	24	29	19	35	6	15	41	46
11	1	25	30	20	36	●	16	42	47
●	2	26	31	21	37	7	17		

If, as a third example, the C→P sequence recovered in the process of multiple anagramming had been the following,

17	56	40	06	30	82	98	80	20	86	60	12	46	70	44	93	50	24	75	01	34	90	65	05	97
39	55	16	54	29	59	69	85	19	79	09	11	23	33	49	92	43	63	74	96	15	04	64	89	27
38	68	78	58	28	53	100	84	32	42	22	10	08	72	48	14	88	95	73	62	36	03	77	52	67
37	26	18	57	41	07	31	83	99	81	21	87	61	13	47	71	45	94	51	25	76	02	35	91	66

the isomorphic repetition beginning at the 78th position shows that double transposition with keys of lengths 7 and 11 must be involved, since 77 is an implausible key-length for single transpositions.

l. Slug key is usually produced by selection from a universe, say 10,000 for 4-digit slugs, comprising in this case all the groups from 0000 to 9999. When several pages are made up at the same time for printing, it is obvious that there will be no repetitions of groups between the pages since the population consists of unique slugs, all different. This absence of repetition between a set of pages permits us to conjecture the slug method of production, as may be seen by referring to the table below:

Probability of 0 tetranome repetitions			
N	1 chance in	N	1 chance in
50	*	300	93
100	**	350	483
150	3	400	3, 257
200	7	450	28, 492
250	23	500	323, 857

(*=88 chances in 100; **=61 chances in 100)

If, for example, we have four sets of 100 tetranomes each and we observe no group repetitions among them, and since this phenomenon is expected, by chance, only once in 3257 times, it is obvious that some kind of selection *without replacement* is involved, and slug key comes to mind, with four pages being assembled at a time.[9] The slugs are then replaced by the clerk, and four more pages are assembled. Sometimes, however, the slugs are not replaced after the first four pages, and another set of four pages is made up from the remainder, which makes the phenomenon even more striking. Again, sometimes the slugs from four pages are not replaced into the population, but are scrambled among themselves to yield four new pages, consisting of the 400 groups comprising the previous set of four pages; although this key is nondeterministic, nevertheless it is of considerable help to the cryptanalyst, who needs to consider only a finite number of possible keys during exploitation instead of the total population of 10,000.

m. In the usual punched-card method for key production, a permutation is applied to the elements of a master card or cards to produce secondary cards. As an example, let us consider the following master card, which for simplicity's sake has only 30 columns punched,

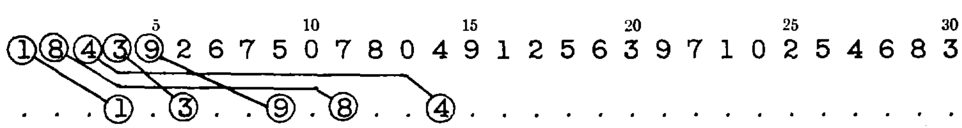
```
        5         10        15        20        25        30
1 8 4 3 9 2 6 7 5 0 7 8 0 4 9 1 2 5 6 3 9 7 1 0 2 5 4 6 8 3
```

and let us assume that the plugging for the punching permutation is

```
01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20
04 11 14 06 09 26 30 25 16 01 23 28 19 18 20 10 03 22 05 29

21 22 23 24 25 26 27 28 29 30
27 07 21 13 02 12 15 08 17 24
```

The first five elements of the master card would be permuted as follows on the secondary card:



[9] The possibility of generation by punched-card methods, with the cards containing one group each, should not be ignored.

The total number of cards generated from the permutation above (including the master card) is 30, and these are shown in Fig. 117, below:[10]

					5					10					15					20					25					30
(1)	1	8	4	3	9	2	6	7	5	0	7	8	0	4	9	1	2	5	6	3	9	7	1	0	2	5	4	6	8	3
(2)	0	2	2	1	6	3	7	6	9	1	8	5	0	4	4	5	8	4	0	9	1	5	7	3	7	2	9	8	3	6
(3)	1	7	8	0	0	1	5	8	6	5	2	2	3	2	9	9	3	4	0	4	7	4	8	6	6	3	1	5	9	7
(4)	5	6	3	1	0	0	4	5	0	9	7	3	6	8	1	6	9	2	3	9	8	4	2	7	8	1	7	2	4	5
(5)	9	8	9	5	3	1	4	2	0	6	6	1	7	3	7	0	4	8	6	1	2	2	7	5	5	0	8	3	9	4
(6)	6	5	4	9	6	5	2	3	3	0	8	0	5	9	8	0	9	3	7	7	7	8	6	4	2	1	2	1	1	4
(7)	0	2	9	6	7	9	8	1	6	0	5	1	4	4	2	3	1	9	5	8	6	3	8	4	3	5	7	0	7	2
(8)	0	3	1	0	5	6	3	0	7	3	2	5	4	9	7	6	7	4	4	2	8	9	5	2	1	9	6	1	8	8
(9)	3	1	7	0	4	0	9	1	5	6	3	9	2	1	6	7	8	9	4	7	5	4	2	8	0	6	8	5	2	3
(10)	6	0	8	3	4	0	4	5	4	7	1	6	8	7	8	5	2	1	2	6	2	9	3	3	1	0	5	9	7	9
(11)	7	1	2	6	2	3	9	9	4	5	0	0	3	8	5	4	7	7	8	8	3	1	1	9	5	0	2	6	6	4
(12)	5	5	7	7	8	6	1	6	2	4	1	0	9	2	2	4	6	8	3	5	1	7	0	4	9	3	3	0	8	9
(13)	4	9	6	5	3	7	7	0	8	4	5	3	4	7	3	2	8	2	9	2	0	8	1	9	6	6	1	0	5	1
(14)	4	6	8	4	9	5	8	0	3	2	9	6	9	6	1	8	5	7	4	3	1	2	5	1	0	7	0	3	2	7
(15)	2	0	5	4	4	4	2	3	9	8	6	7	1	8	0	3	2	6	9	1	5	7	9	7	0	5	1	6	3	8
(16)	8	0	2	2	9	4	7	6	4	3	0	5	7	5	1	9	3	8	1	0	9	6	6	8	3	4	5	7	1	2
(17)	3	3	3	8	1	2	6	7	9	9	0	4	8	2	5	4	1	5	7	1	6	8	0	2	6	4	9	5	0	7
(18)	9	6	1	3	7	8	8	5	1	4	3	4	2	3	9	9	0	2	8	5	0	5	0	7	7	2	6	4	1	6
(19)	4	7	0	9	8	3	5	4	7	9	6	2	7	1	6	1	1	3	2	9	0	2	3	6	5	8	0	4	5	8
(20)	9	5	1	4	2	9	2	4	8	1	7	8	6	0	0	7	5	1	7	6	3	3	6	8	4	3	0	2	9	5
(21)	1	4	5	9	7	4	3	2	2	7	5	3	8	1	0	8	9	0	6	0	6	1	7	5	4	9	3	8	6	2
(22)	7	4	9	1	6	9	1	8	7	8	4	9	5	5	3	2	6	1	8	0	7	0	5	2	2	4	6	3	0	3
(23)	8	2	6	7	8	1	0	3	6	2	4	4	2	9	6	7	0	5	5	3	5	1	4	3	8	9	7	9	0	1
(24)	2	8	0	8	5	7	1	9	8	7	2	9	3	6	7	6	0	9	2	6	4	5	4	1	3	1	5	4	3	0
(25)	7	3	0	2	2	8	5	4	5	6	8	1	1	0	5	8	3	6	3	7	4	9	2	0	9	7	4	9	6	1
(26)	6	9	3	7	3	2	9	9	2	8	3	7	0	0	4	5	6	0	1	5	2	6	8	1	4	8	4	1	7	5
(27)	8	4	6	6	1	7	6	1	3	5	9	8	1	3	4	2	7	0	0	4	8	0	3	5	9	2	2	7	5	9
(28)	5	9	7	8	0	6	0	7	1	2	4	2	5	6	2	3	5	3	1	4	3	0	9	9	1	7	8	8	4	6
(29)	2	1	5	5	1	8	0	8	0	3	9	7	9	7	8	1	4	6	5	2	9	3	4	6	7	6	3	2	4	0
(30)	3	7	4	2	5	5	3	2	1	1	1	6	6	5	3	0	4	7	9	8	4	6	9	0	8	8	9	7	2	0

FIGURE 117

[10] The permutation given yields a complete chain (01, 04, 06, 26, 12, . . .) of 30 elements, so a total of 30 distinct cards will be produced. If the permutation had been, say

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
16	08	30	26	12	14	23	27	03	17	21	01	25	24	15	02	28	07	04	05	29	10	09	06	20	22	13	19	18	11

this yields chains of 10, 9, 7, 3, and 1, so the number of different cards would in this case be the least common multiple of these numbers, or 210. The "optimum" partition of 30 is (11, 7, 5, 4, 3) with a least common multiple of 4620—but this number, far from being a cryptographic strength, is a decided weakness.

If we assume that for a particular cryptosystem we need key in lines of only 10 digits each, we could establish a print plugging, say columns 3–12–25–8–17–2–19–13–21–5, yielding the listing shown in Fig. 118*a*, below. When the 30 cards have been listed, the print plugging could be changed to, let us say, 5–30–17–1–4–12–19–23–11–7, producing the listing shown in Fig. 118*b*. Since, however, there are undesirable properties of direct symmetry manifested in the columns of these diagrams, it would have been better if we had scrambled the rows of Fig. 117, say to a permutation of 10–4–27–30–1–8–7–3–5–29 . . . 28–6–2–9–26, so that the pluggings for the column selection would have yielded the keys in Figs. 118*c* and *d*.

	3	12	25	8	17	2	19	13	21	5
(1)	4	8	2	7	2	8	6	0	9	9
(2)	2	5	7	6	8	2	0	0	1	6
(3)	8	2	6	8	3	7	0	3	7	0
(4)	3	3	8	5	9	6	3	6	8	0
(5)	9	1	5	2	4	8	6	7	2	3
(6)	4	0	2	3	9	5	7	5	7	6
(7)	9	1	3	1	1	2	5	4	6	7
(8)	1	5	1	0	7	3	4	4	8	5
(9)	7	9	0	1	8	1	4	2	5	4
(10)	8	6	1	5	2	0	2	8	2	4
	*	*	*	*	*					
(26)	3	7	4	9	6	9	1	0	2	3
(27)	6	8	9	1	7	4	0	1	8	1
(28)	7	2	1	7	5	9	1	5	3	0
(29)	5	7	7	8	4	1	5	9	9	1
(30)	4	6	8	2	4	7	9	6	4	5

FIGURE 118*a*

	5	30	17	1	4	12	19	28	11	7
(1)	9	3	2	1	3	8	6	6	7	6
(2)	6	6	8	0	1	5	0	8	8	7
(3)	0	7	3	1	0	2	0	5	2	5
(4)	0	5	9	5	1	3	3	2	7	4
(5)	3	4	4	9	5	1	6	3	6	4
(6)	6	4	9	6	9	0	7	1	8	2
(7)	7	2	1	0	6	1	5	0	5	8
(8)	5	8	7	0	0	5	4	1	2	3
(9)	4	3	8	3	0	9	4	5	3	9
(10)	4	9	2	6	3	6	2	9	1	4
	*	*	*	*	*					
(26)	3	5	6	6	7	7	1	1	3	9
(27)	1	9	7	8	6	8	0	7	9	6
(28)	0	6	5	5	8	2	1	8	4	0
(29)	1	0	4	2	5	7	5	2	9	0
(30)	5	0	4	3	2	6	9	7	1	3

FIGURE 118*b*

	3	12	25	8	17	2	19	13	21	5
(10)	8	6	1	5	2	0	2	8	2	4
(4)	3	3	8	5	9	6	3	6	8	0
(27)	6	8	9	1	7	4	0	1	8	1
(30)	4	6	8	2	4	7	9	6	4	5
(1)	4	8	2	7	2	8	6	0	9	9
(8)	1	5	1	0	7	3	4	4	8	5
(7)	9	1	3	1	1	2	5	4	6	7
(3)	8	2	6	8	3	7	0	3	7	0
(5)	9	1	5	2	4	8	6	7	2	3
(29)	5	7	7	8	4	1	5	9	9	1
	*	*	*	*	*					
(28)	7	2	1	7	5	9	1	5	3	0
(6)	4	0	2	3	9	5	7	5	7	6
(2)	2	5	7	6	8	2	0	0	1	6
(9)	7	9	0	1	8	1	4	2	5	4
(26)	3	7	4	9	6	9	1	0	2	3

FIGYRE 118*c*

	5	30	17	1	4	12	19	28	11	7
(10)	4	9	2	6	3	6	2	9	1	4
(4)	0	5	9	5	1	3	3	2	7	4
(27)	1	9	7	8	6	8	0	7	9	6
(30)	5	0	4	3	2	6	9	7	1	3
(1)	9	3	2	1	3	8	6	6	7	6
(8)	5	8	7	0	0	5	4	1	2	3
(7)	7	2	1	0	6	1	5	0	5	8
(3)	0	7	3	1	0	2	0	5	2	5
(5)	3	4	4	9	5	1	6	3	6	4
(29)	1	0	4	2	5	7	5	2	9	0
	*	*	*	*	*					
(28)	0	6	5	5	8	2	1	8	4	0
(6)	6	4	9	6	9	0	7	1	8	2
(2)	6	6	8	0	1	5	0	8	8	7
(9)	4	3	8	3	0	9	4	5	3	9
(26)	3	5	6	6	7	7	1	1	3	9

FIGURE 118*d*

Now suppose that we had recovered the 300 digits of key of Fig. 118*a* and had further obtained the first 50 digits of key of Fig. 118*b*. It is clear that *if we had written the key out in lines of 10 digits each* we would have noticed not only that the second column of Fig. 118*a* is identical to the sixth column of Fig. 118*b*, but also that there exist properties of direct symmetry, as previously mentioned, between the columns, speeding up the recovery of the entire 30 basic columns of Fig. 117.[11] If on the other hand we had recovered the 300 digits of key of Fig. 118*c* plus the first 50 digits of key of Fig. 118*d*, and had again written out the recovered key in rows of 10 digits each, we could extend the first, third sixth, and seventh columns of Fig. 118*d* for a total of 30 positions. What this emphasizes is that writing out recovered key on trial widths may have a rewarding result for the alert cryptanalyst.[12]

n. For another case of analysis, let us assume that we have recovered the two following stretches of key:

Key "A"

0918	2009	3282	3952	0422	9004	8549	5198	5065	9493
7318	5020	4767	2626	6229	7576	7649	2097	8774	9042
5401	4405	6628	3100	0068	0835	6991	7854	4278	1366
2830	0326	8133	1059	4051	5380	8623	8159	1362	5121
9175	5374	6161	6226	5026	8941	9269	0039	5839	1260

Key "B"

0567	5311	1436	1095	1832	3813	3421	0599	9181	8372
5729	4534	9670	7495	4843	5298	2095	6406	7753	7472

It is noticed that if Key ' B" is slid three groups to the left of Key "A", that the *a* digits in corresponding positions are predominantly hitting, or differing by only 1:

Key "A":	0918	2009	3282	3952	0422	9004	8549	5198	5065	9493
Key "B":	1095	1832	3813	3421	0599	9181	8372	5729	4534	9670
Key "A":	7318	5020	4767	2626	6229	7576	7649	2097	8774	9042 . . .
Key "B":	7495	4843	5298	2095	6406	7753	7472			

It is further noticed that the differences (with borrowing subtraction) of the 1st, 5th, 6th, 10th, 11th, 15th, and 16th groups is 177, and that the differences between the 2d, 7th, 12th, and 17th groups is −177. This suggests to us to write out the keys in rows of five groups each, as follows:

Key "A"

0918	2009	3282	3952	0422
9004	8549	5198	5065	9493
7318	5020	4767	2626	6229
7576	7649	2097	8774	9042
5401	4405	6628	3100	0068
0835	6991	7854	4278	1366
2830	0326	8133	1059	4051
5380	8623	8159	1362	5121
9175	5374	6161	6226	5026
8941	9269	0039	5839	1260

Key "B"

		0567	5311	1436
1095	1832	3813	3421	0599
9181	8372	5729	4534	9670
7495	4843	5298	2095	6406
7753	7472			

[11] An interesting observation may be made about the average I.C. of the rows of Fig. 118*a* (including the missing rows 11–25), and of Fig. 118*b*. For Fig. 118*a*, since one row has a ϕ count of 2, ten rows have ϕ's of 4, eleven rows have ϕ's of 6, six rows have ϕ's of 8, and two rows have ϕ's of 10, the δ I.C. $=\frac{(2/9)+10(4/9)+11(6/9)+6(8/9)+2(10/9)}{30}=0.6519$. For Fig. 118*b*, the δ comes out to be 0.7481. The property of an observed I.C. consistently *below random* is the result of the superflatness of the master card, which contains three each of the digits 0 to 9. A selection of any columns from the generated cards will always result in an expected average I.C. below 1.0000 for the rows; consequently, this phenomenon can be used to diagnose the method of generation of the key. In this case of 30 digits per card, the expected δ I.C. is 10(2/29) or 0.6897; if we approached this figure in testing a large number of row distributions, we would know that the master card generating the population must have contained 30 digits.

[12] It was emphasized previously that the general methods and procedures of cryptanalysis that apply to encrypted text also apply in the analysis of key. In this connection, if key text is written out on width *w* and the average I.C. for this array is considerably improved (let us say, doubled) over the over-all I.C. of the sample, it is indicative that the cycle of *w* has some significance in the generation of the key.

We now note that the groups in the third column of Key "B" are those of Key "A" but with an additive of 531 (=3·177) while the groups in the fourth column of Key "B" are those of Key "A" but with a subtractor of 531. This means that we have two pages of key 177 pages apart, produced by a key generator that operates on the following rule, after the 50 printing counters have been set to arbitrary initial positions: the counters in column 1 and column 5 advance one position (carrying arithmetic), the counters in column 2 retrogress by one position, the counters in column 3 advance three positions, and the counters in column 4 retrogress three positions. This conclusion is substantiated by the first three groups of Key "B", 0567 5311 1436, which are groups from the immediately preceding printed page, following the rule just given. If Key "A" were page 1 as printed, then pages 177 and 178 would be the following (page 177 is shown in fragmentary form):

Page 177

1094				
9180				
7494				
7752				
5577				
1011				
3006				
5556				
9351				
9117	9093	0567	5311	1436

Page 178

1095	1832	3813	3421	0599
9181	8372	5729	4534	9670
7495	4843	5298	2095	6406
7753	7472	2628	8243	9219
5578	4228	7159	2569	0245
1012	6814	8385	3747	1543
3007	0149	8664	0528	4228
5557	8446	8690	0831	5298
9352	5197	6692	5695	5203
9118	9092	0570	5308	1437

We can, of course, reproduce all the pages from 1 to 200 or more, if we desire, with which to read all the traffic in the system.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

60. Analysis of key in a double transposition cipher.—*a*. Each problem in key analysis is a distinct and unique experience, as is indeed any problem in cryptanalysis. Nevertheless, as an example illustrating techniques of analysis as applied to key from a manual cryptosystem, we shall study the steps involved in the analysis of key derived from multiple anagramming of several identical-length messages in the same pair of double transposition keys. But in order to understand the reasons for our steps, we shall first examine the mechanics of double transposition encipherment.

b. Let a 51-letter plaintext message be represented by the consecutive numbers from 01 to 51, as inscribed in the T1 matrix shown below. The columns of the T1 matrix are taken off in numerical-

6	2	7	1	5	3	8	4
01	02	03	04	05	06	07	08
09	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51					

T1

3	9	1	7	4	2	11	8	10	6	5
04	12	20	28	36	44	02	10	18	26	34
42	50	06	14	22	30	38	46	08	16	24
32	40	48	05	13	21	29	37	45	01	09
17	25	33	41	49	03	11	19	27	35	43
51	07	15	23	31	39	47				

T2

key order and inscribed in the rows of the T2 matrix, and the final cipher text, 20 06 48 33 15 . . . (the P→C sequence) is obtained by taking off the columns of the T2 matrix in numerical-key order.

Now the P→C sequence, which is an inversion of the C→P sequence obtained from anagramming several identical-length messages, is that shown below, together with the delta of the P→C sequence which we shall find very useful in a moment or two:

Term No.:	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
P→C:	20	06	48	33	15	44	30	21	03	39	04	42	32	17	51	36	22	13	49	31
Δ:	−14	42	−15	−18	29	−14	−9	−18	36	−35	38	−10	−15	34	−15	−14	−9	36	−18	3

Term No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
P→C:	34	24	09	43	26	16	01	35	28	14	05	41	23	10	46	37	19	12	50	40
Δ:	−10	−15	34	−17	−10	−15	34	−7	−14	−9	36	−18	−13	36	−9	−18	−7	38	−10	−15

Term No.:	41	42	43	44	45	46	47	48	49	50	51
P→C:	25	07	18	08	45	27	02	38	29	11	47
Δ:	−18	11	−10	37	−18	−25	36	−9	−18	36	

FIGURE 119*a*

There is one 4-element repetition in the delta stream (at term nos. 16–20 and 29–33), and three sets of 3-element repetitions. If we write down the fragments of the P→C sequence associated with the 4-element repetition and difference these fragments,

(Term nos. 16–20):	36	22	13	49	31
(Term nos. 29–33)	28	14	05	41	23
Differences:	8	8	8	8	8

we obtain a constant difference of 8, *which corresponds to the width of the T1 matrix.* If we difference the other corresponding fragments we shall likewise find a constant difference of 8.

c. Now that we have ascertained the width of the T1 matrix, we shall proceed to recover the T2 matrix. Let us digress for a moment, however, in order to explain the techniques to be employed. We will consider the *C→P sequence* obtained from the original multiple anagramming of the messages; this is shown in Fig. 119*b*, below:

Term No.:	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
C→P:	27	47	09	11	31	02	42	44	23	34	50	38	18	30	05	26	14	43	37	01

Term No.:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
C→P:	08	17	33	22	41	25	46	29	49	07	20	13	04	21	28	16	36	48	10	40

Term No.:	41	42	43	44	45	46	47	48	49	50	51
C→P:	32	12	24	06	45	35	51	03	19	39	15

FIGURE 119*b*

If we now treat the C→P sequence as if it were the plain text and encipher it through our double transposition keys we will have the following:

6	2	7	1	5	3	8	4
27	47	09	11	31	02	42	44
23	34	50	38	18	30	05	26
14	43	37	01	08	17	33	22
41	25	46	29	49	07	20	13
04	21	28	16	36	48	10	40
32	12	24	06	45	35	51	03
19	39	15					

T1

3	9	1	7	4	2	11	8	10	6	5
11	38	01	29	16	06	47	34	43	25	21
12	39	02	30	17	07	48	35	44	26	22
13	40	03	31	18	08	49	36	45	27	23
14	41	04	32	19	09	50	37	46	28	24
15	42	05	33	20	10	51				

T2

We see that the numbers in the *columns* of the T2 matrix are consecutive, and that if we were to take off these columns according to the numerical key and inscribe them into a matrix of the same dimensions as the T1 matrix, these numbers would be consecutive in rows and would represent the sequence of the original plain text:

01	02	03	04	05	06	07	08
09	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51					

d. Returning to our analysis thus far, we have already established the T1 matrix as having eight columns, so we inscribe the C→P sequence into a matrix of eight columns, as shown below:

27	47	09	11	31	02	42	44
23	34	50	38	18	30	05	26
14	43	37	01	08	17	33	22
41	25	46	29	49	07	20	13
04	21	28	16	36	48	10	40
32	12	24	06	45	35	51	03
19	39	15					

We now find the column containing the number 01 and set it down as follows:

```
|11 38 01 29 16 06|
```

We then look for the column containing the number 02 and set it down under the previous column so that the 02 is under the 01,

```
|11 38 01 29 16 06|
      |02 30 17 07 48 35|
```

and we continue this procedure with the columns containing the numbers 03, 04, and 05, arriving at the following diagram:

```
         |11 38 01 29 16 06|
               |02 30 17 07 48 35|
|44 26 22 13 40 03|
   |27 23 14 41 04 32 19|
            |42 05 33 20 10 51|
```

The next term in the sequence is 06, but it is already present in the diagram, as is also 07. In the column headed by 06 there is vacant space for 08 and 09, and in the column headed by 11 there is space for 12, so these segments are added to the diagram:

```
                     |11 38 01 29 16 06
|47 34 43 25 21 12 39|02 30 17 07 48 35|
      |44 26 22 13 40 03|31 18 08 49 36 45|
         |27 23 14 41 04 32 19|09 50 37 46 28 24 15|
                  |42 05 33 20 10 51|
```

It is clear that the diagram is only five rows deep, so we fill out our diagram as follows:

```
                  |11 38 01 29 16 06|47 34 43 25 21 12 39|
|47 34 43 25 21 12 39|02 30 17 07 48 35|44 26 22 13 40 03|
      |44 26 22 13 40 03|31 18 08 49 36 45|27 23 14 41 04 32 19|
         |27 23 14 41 04 32 19|09 50 37 46 28 24 15|
|09 50 37 46 28 24 15|42 05 33 20 10 51|
```

We can now draw the outline of the T2 matrix in this diagram, since it is clear from the location of the numbers 20 and 25 that the column with the terms 11–15 belongs at the extreme left, and that the column with the terms 21–24 must be at the extreme right. The transposition key for the T2 matrix is established directly from this matrix, the 1 in the key being over the column headed by 01, the 2 of the key being over the column headed by 06, and so forth:

						3	9	1	7	4	2	11	8	10	6	5					
						11	38	01	29	16	06	47	34	43	25	21	12	39			
	47	34	43	25	21	12	39	02	30	17	07	48	35	44	26	22	13	40	03		
			44	26	22	13	40	03	31	18	08	49	36	45	27	23	14	41	04	32	19
				27	23	14	41	04	32	19	09	50	37	46	28	24	15				
09	50	37	46	28	24	15	42	05	33	20	10	51									

At this point we inscribe the C→P sequence into the T1 matrix already known to have eight columns, and we recover the transposition key by following the sequence of rows in the T2 matrix; thus the terms 11 38 01 29 16 06 must be under key 1, the terms 47 34 . . . 12 39 under key 2, and so on. The recovered key is shown at the top of the T1 matrix below:

6	2	7	1	5	3	8	4
27	47	09	11	31	02	42	44
23	34	50	38	18	30	05	26
14	43	37	01	08	17	33	22
41	25	46	29	49	07	20	13
04	21	28	16	36	48	10	40
32	12	24	06	45	35	51	03
19	39	15					

e. Now let us apply these techniques to the solution of an unknown example. It is assumed that we have recovered a C→P sequence from the anagramming of several messages of identical length enciphered by the same pair of double transposition keys. The C→P sequence is shown below, as is the P→C sequence together with its delta:

Term No.:	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
C→P:	14	35	40	23	02	07	37	63	01	70	47	57	52	19	30	06	13	22	46	51
Term No.:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
C→P:	36	21	44	56	34	69	62	05	45	18	12	55	68	50	11	10	33	29	43	66
Term No.:	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
C→P:	25	54	61	17	32	42	67	16	49	28	04	09	39	15	31	65	60	27	08	41
Term No.:	61	62	63	64	65	66	67	68	69	70										
C→P:	59	58	03	24	48	53	20	26	38	64										

* * * * *

Term No.:	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
P→C:	09	05	63	51	28	16	06	59	52	36	35	31	17	01	54	48	44	30	14	67
Δ:	-4	58	-12	-23	-12	-10	53	-7	-16	-1	-4	-14	-16	53	-6	-4	-14	-16	53	-45
Term No.:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
P→C:	22	18	04	64	41	68	58	50	38	15	55	45	37	25	02	21	07	69	53	03
Δ:	-4	-14	60	-23	27	-10	-8	-12	-23	40	-10	-8	-12	-23	19	-14	62	-16	-50	57
Term No.:	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
P→C:	60	46	39	23	29	19	11	65	49	34	20	13	66	42	32	24	12	62	61	57
Δ:	-14	-7	-16	6	-10	-8	54	-16	-15	-14	-7	53	-24	-10	-8	-12	50	-1	-4	-14
Term No.:	61	62	63	64	65	66	67	68	69	70										
P→C:	43	27	08	70	56	40	47	33	26	10										
Δ:	-16	-19	62	-14	-16	7	-14	-7	-16											

f. From the differences of portions of the P→C sequence corresponding to the repetitions in the delta stream,

Term nos. 16–20:	48 44 30 14 67	Term nos. 26–30:	68 58 50 38 15
Term nos. 11–15:	35 31 17 01 54	Term nos. 31–35:	55 45 37 25 02
	13 13 13 13 13		13 13 13 13 13

the width of the T1 matrix is established as 13, so we inscribe the C→P sequence horizontally into a matrix of 13 columns:

14	35	40	23	02	07	37	63	01	70	47	57	52
19	30	06	13	22	46	51	36	21	44	56	34	69
62	05	45	18	12	55	68	50	11	10	33	29	43
66	25	54	61	17	32	42	67	16	49	28	04	09
39	15	31	65	60	27	08	41	59	58	03	24	48
53	20	26	38	64								

The column containing 01 is written horizontally, with the column containing 02 directly beneath it so that the 02 is under the 01:

```
|01 21 11 16 59|
|02 22 12 17 60 64|
```

We quickly add the columns containing 03, 04, and 05, properly juxtaposed:

```
            |01 21 11 16 59|
            |02 22 12 17 60 64|
|47 56 33 28 03|
   |57 34 29 04 24|
      |35 30 05 25 15 20|
```

At this point the column containing 06 seems to give conflicts, as does the column containing 07, so we add the columns containing 13 and 14 to go under our 12:

```
               |01 21 11 16 59|
               |02 22 12 17 60 64|
|47 56 33 28 03|23 13 18 61 65 38|
   |57 34 29 04 24|14 19 62 66 39 53|
      |35 30 05 25 15 20|
```

From here on it proceeds rapidly, and we arrive at the following diagram and the outline of the T2 matrix:

```
              9  2 10 12  7  6  1  5  3  4 13 14  8 11 15
             :40 06 45 54 31 26|01 21 11 16 59|63 36 50 67:41
 63 36 50 67:41|07 46 55 32 27|02 22 12 17 60 64|37 51 68:42 08
    37 51 68:42 08|47 56 33 28 03|23 13 18 61 65 38|52 69:43 09 48
       52 69:43 09 48|57 34 29 04 24|14 19 62 66 39 53|70:44 10 49 58
          70:44 10 49 58|35 30 05 25 15 20:
```

As before, the T2 key is derived by numbering the column headed by 01 as 1, the column headed by 06 as 2, and so forth to its complete recovery. The T1 key is obtained from our previous diagram of the C→P sequence inscribed into a matrix of 13 columns by following the breaks shown in the T2 matrix, above; thus the column 40 06 45 54 31 26 is given key number 1, the column 01 21 11 16 59 is given key number 2, and so on for the complete recovery of the T1 key.[14]

[14] The T1 and T2 keys are derived from literal keys. See in this connection subpar. 62*a*.

11	13	1	8	5	4	6	3	2	12	7	10	9
14	35	40	23	02	07	37	63	01	70	47	57	52
19	30	06	13	22	46	51	36	21	44	56	34	69
62	05	45	18	12	55	68	50	11	10	33	29	43
66	25	54	61	17	32	42	67	16	49	28	04	09
39	15	31	65	60	27	08	41	59	58	03	24	48
53	20	26	38	64								

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

g. There are several other methods for recovering double transposition keys from the C→P sequence, but the one just shown was deemed best from the standpoint of illustrative key analysis.

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(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

62. Concluding remarks.—*a.* In cryptographic practice 26-letter mixed sequences are often based upon key words. The recovery of these key words has been treated in par. 51 (on pp. 86–90) of *Military Cryptanalytics, Part I.* Likewise, numerical keys, especially those in transposition systems, are often based on underlying literal keys. The recovery of these underlying literal keys from the numerical keys has been treated on pp. 427–429 of *Military Cryptanalytics, Part II.*

b. A few final remarks on key analysis might be of assistance. When key is taken from the four-digit numbers in a telephone directory there is precious little to give it away, except for the observation that the number 0000 is never expected to appear; if it *is* encountered, the premise of a telephone directory as the source of the key is refuted. Furthermore, although there might be no discernible limitations in the directory of a large city, nevertheless cases have been encountered wherein, for instance, no telephone numbers began with the dinomes 07, 08, or 09, permitting easy identification of the particular directory. Psychological random, hand produced key, will have characteristics which can be identified as such; typewriter random key has similar attributes, including a very infrequent digit "1" when this digit is not on the number row but must be typed as a lower-case "L". Document analysis [21] enters into the picture when key has been captured or surreptitiously photographed: in a simple case, the inadvertent occurrence of an upside-down digit in the key proves manufacture by single-digit slugs, whereas an entire group upside-down proves that group-slugs are involved in the production. The manufacture of key by punched-card methods may be indicated when the particular type-styles are identified with commercial electric accounting machines. The particular source or origin of the key might be indicated by the type of paper used, the chemical composition of the ink, or even the manner of stapling or binding. These are aspects of key analysis that should not be overlooked. The field, then, is pretty well all-embracing, and requires versatility of outlook and breadth of experience to achieve a significant margin of success.

[21] In this connection, the following works may be of more than passing interest:
Wilson R. Harrison, *Suspect Documents: Their Scientific Examination*, New York, 1958.
Arthur J. Quirk, *Forged, Anonymous, and Suspect Documents*, London, 1930.
Albert S. Osborn, *Questioned Documents*, New York, 1929.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

CHAPTER X

TELEPRINTER KEY ANALYSIS

63. General.—In *Military Cryptanalytics, Part I* and *Part II* we were introduced to Baudot systems and their encipherment.[1] The international Baudot code is given here below for ready reference:

UPPER CASE: WEATHER SYMBOLS	↑	⊕	○	↗	3	→	↘	↓	8	↙	←	↖	•	⦶	9	Ø	1	4	🔔	5	7	⦶	2	/	6	+	−	⟨	≡			
UPPER CASE: COMMUNICATIONS	−	?	:	$	3	!	&	£	8	'	(	)	.	,	9	Ø	1	4	🔔	5	7	;	2	/	6	"	≀	⟨	≡			
LOWER CASE	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	BLANK	C.R.	L.F.	SPACE	LTR. SHIFT	FIG. SHIFT
1	●	●		●	●	●				●	●						●		●		●		●	●	●	●					●	●
2	●		●				●		●	●	●	●				●	●	●			●	●	●						●		●	●
3			●			●		●	●		●		●	●		●	●		●		●	●		●	●					●	●	
4		●	●	●		●	●			●	●		●	●	●			●				●		●				●			●	●
5		●					●	●				●	●		●	●	●			●		●	●	●	●	●					●	●

FIGURE 121

In Fig. 122*a* is given the Baudot combination table containing sums according to the rule that like impulses produce a "+", unlike a "−"; in Fig. 122*b* is the complementary table of Baudot sums combined according to the rule that *unlike* impulses produce a "+", like impulses a "−". In most cases it is immaterial which rule of combination is assumed; but once a convention is established, it must be continued in the particular problem under study. In this discussion we shall assume the convention that, unless otherwise specified, like impulses produce a "+", unlike a "−". In the Baudot combination tables, the character representing the carriage return is symbolized by the digit "3", the line feed by "4", the figure shift by "5", the blank by "7", the letter shift by "8", and the space by "9".

64. Teleprinter key generation methods.[2]—*a*. One of the simplest ideas for the electromechanical generation of teleprinter key is the employment of 5 cam wheels or pin wheels, one for each "level" of Baudot key. As an elementary example, suppose a machine had five wheels in its key-generator component of lengths 11, 9, 8, 7, and 5, with pin (or notch) configurations as shown in Fig. 123 ("x" denotes an active pin, "o" an inactive pin).

---

[1] It might be well at this point to review the material covered in par. 56 (pp. 99–101 of *Military Cryptanalytics, Part I*, and in par. 97 (pp. 294–302) of *Military Cryptanalytics, Part II*.

[2] See also par. 14 on pp. 471–474 of Appendix 6 ("Cryptographic Supplement"), *Military Cryptanalytics, Part II*.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	3	4	5	7	8	9
A	8	S	L	Y	X	Z	I	3	G	Q	W	C	7	T	9	R	J	P	B	N	5	4	K	E	D	F	H	V	U	M	A	O
B	S	8	3	K	U	J	M	L	7	F	D	H	G	R	V	T	Z	N	A	P	E	O	Y	5	W	Q	C	9	X	I	B	4
C	L	3	8	O	T	M	J	S	Q	G	V	A	F	X	D	U	I	5	H	E	P	K	4	N	9	7	B	W	R	Z	C	Y
D	Y	K	O	8	Q	5	N	4	T	X	B	9	R	G	C	7	E	M	W	I	Z	3	S	J	A	U	V	H	F	P	D	L
E	X	U	T	Q	8	W	9	R	O	Y	Z	N	4	L	I	3	D	H	5	C	B	7	F	A	J	K	P	M	S	V	E	G
F	Z	J	M	5	W	8	3	I	H	B	X	7	C	V	R	9	S	O	Q	4	Y	N	E	K	U	A	G	T	D	L	F	P
G	I	M	J	N	9	3	8	Z	A	C	R	Q	B	D	X	W	L	K	7	Y	4	5	P	O	T	H	F	U	V	S	G	E
H	3	L	S	4	R	I	Z	8	F	7	9	B	Q	U	W	X	M	E	C	5	N	Y	O	P	V	G	A	D	T	J	H	K
I	G	7	Q	T	O	H	A	F	8	L	P	J	S	Y	E	K	C	W	M	D	V	U	R	9	N	3	Z	5	4	B	I	X
J	Q	F	G	X	Y	B	C	7	L	8	5	I	3	O	N	4	A	V	Z	9	W	R	U	D	E	S	M	P	K	H	J	T
K	W	D	V	B	Z	X	R	9	P	5	8	4	N	M	3	I	U	G	Y	7	Q	C	A	F	S	E	O	L	J	T	K	H
L	C	H	A	9	N	7	Q	B	J	I	4	8	Z	E	Y	5	G	U	3	X	R	W	V	T	O	M	S	K	P	F	L	D
M	7	G	F	R	4	C	B	Q	S	3	N	Z	8	K	5	Y	H	D	I	W	9	X	T	V	P	L	J	E	O	A	M	U
N	T	R	X	G	L	V	D	U	Y	O	M	E	K	8	J	S	9	B	P	A	H	F	7	C	I	4	5	Z	3	W	N	Q
O	9	V	D	C	I	R	X	W	E	N	3	Y	5	J	8	Z	T	F	4	Q	7	B	H	G	L	P	K	S	M	U	O	A
P	R	T	U	7	3	9	W	X	K	4	I	5	Y	S	Z	8	V	A	N	B	C	Q	G	H	M	O	E	J	L	D	P	F
Q	J	Z	I	E	D	S	L	M	C	A	U	G	H	9	T	V	8	4	F	O	K	P	5	Y	X	B	7	R	W	3	Q	N
R	P	N	5	M	H	O	K	E	W	V	G	U	D	B	F	A	4	8	T	S	L	J	I	3	7	9	X	Q	C	Y	R	Z
S	B	A	H	W	5	Q	7	C	M	Z	Y	3	I	P	4	N	F	T	8	R	X	9	D	U	K	J	L	O	E	G	S	V
T	N	P	E	I	C	4	Y	5	D	9	7	X	W	A	Q	B	O	S	R	8	3	Z	M	L	G	V	U	F	H	K	T	J
U	5	E	P	Z	B	Y	4	N	V	W	Q	R	9	H	7	C	K	L	X	3	8	I	J	S	F	D	T	G	A	O	U	M
V	4	O	K	3	7	N	5	Y	U	R	C	W	X	F	B	Q	P	J	9	Z	I	8	L	M	H	T	D	A	G	E	V	S
W	K	Y	4	S	F	E	P	O	R	U	A	V	T	7	H	G	5	I	D	M	J	L	8	Z	B	X	9	C	Q	N	W	3
X	E	5	N	J	A	K	O	P	9	D	F	T	V	C	G	H	Y	3	U	L	S	M	Z	8	Q	W	R	7	B	4	X	I
Y	D	W	9	A	J	U	T	V	N	E	S	O	P	I	L	M	X	7	K	G	F	H	B	Q	8	5	4	3	Z	R	Y	C
Z	F	Q	7	U	K	A	H	G	3	S	E	M	L	4	P	O	B	9	J	V	D	T	X	W	5	8	I	N	Y	C	Z	R
3	H	C	B	V	P	G	F	A	Z	M	O	S	J	5	K	E	7	X	L	U	T	D	9	R	4	I	8	Y	N	Q	3	W
4	V	9	W	H	M	T	U	D	5	P	L	K	E	Z	S	J	R	Q	O	F	G	A	C	7	3	N	Y	8	I	X	4	B
5	U	X	R	F	S	D	V	T	4	K	J	P	O	3	M	L	W	C	E	H	A	G	Q	B	Z	Y	N	I	8	9	5	7
7	M	I	Z	P	V	L	S	J	B	H	T	F	A	W	U	D	3	Y	G	K	O	E	N	4	R	C	Q	X	9	8	7	5
8	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	3	4	5	7	8	9
9	O	4	Y	L	G	P	E	K	X	T	H	D	U	Q	A	F	N	Z	V	J	M	S	3	I	C	R	W	B	7	5	9	8

FIGURE 122*a*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	3	4	5	7	8	9
A	7	G	F	R	4	C	B	Q	S	3	N	Z	8	K	5	Y	H	D	I	W	9	X	T	V	P	L	J	E	0	A	M	U
B	G	7	Q	T	0	H	A	F	8	L	P	J	S	Y	E	K	C	W	M	D	V	U	R	9	N	3	Z	5	4	B	I	X
C	F	Q	7	U	K	A	H	G	3	S	E	M	L	4	P	0	B	9	J	V	D	T	X	W	5	8	I	N	Y	C	Z	R
D	R	T	U	7	3	9	W	X	K	4	I	5	Y	S	Z	8	V	A	N	B	C	Q	G	H	M	0	E	J	L	D	P	F
E	4	0	K	3	7	N	5	Y	U	R	C	W	X	F	B	Q	P	J	9	Z	I	8	L	M	H	T	D	A	G	E	V	S
F	C	H	A	9	N	7	Q	B	J	I	4	8	Z	E	Y	5	G	U	3	X	R	W	V	T	0	M	S	K	P	F	L	D
G	B	A	H	W	5	Q	7	C	M	Z	Y	3	I	P	4	N	F	T	8	R	X	9	D	U	K	J	L	0	E	G	S	V
H	Q	F	G	X	Y	B	C	7	L	8	5	I	3	0	N	4	A	V	Z	9	W	R	U	D	E	S	M	P	K	H	J	T
I	S	8	3	K	U	J	M	L	7	F	D	H	G	R	V	T	Z	N	A	P	E	0	Y	5	W	Q	C	9	X	I	B	4
J	3	L	S	4	R	I	Z	8	F	7	9	B	Q	U	W	X	M	E	C	5	N	Y	0	P	V	G	A	D	T	J	H	K
K	N	P	E	I	C	4	Y	5	D	9	7	X	W	A	Q	B	0	S	R	8	3	Z	M	L	G	V	U	F	H	K	T	J
L	Z	J	M	5	W	8	3	I	H	B	X	7	C	V	R	9	S	0	Q	4	Y	N	E	K	U	A	G	T	D	L	F	P
M	8	S	L	Y	X	Z	I	3	G	Q	W	C	7	T	9	R	J	P	B	N	5	4	K	E	D	F	H	V	U	M	A	0
N	K	Y	4	S	F	E	P	0	R	U	A	V	T	7	H	G	5	I	D	M	J	L	8	Z	B	X	9	C	Q	N	W	3
O	5	E	P	Z	B	Y	4	N	V	W	Q	R	9	H	7	C	K	L	X	3	8	I	J	S	F	D	T	G	A	0	U	M
P	Y	K	0	8	Q	5	N	4	T	X	B	9	R	G	C	7	E	M	W	I	Z	3	S	J	A	U	V	H	F	P	D	L
Q	H	C	B	V	P	G	F	A	Z	M	0	S	J	5	K	E	7	X	L	U	T	D	9	R	4	I	8	Y	N	Q	3	W
R	D	W	9	A	J	U	T	V	N	E	S	0	P	I	L	M	X	7	K	G	F	H	B	Q	8	5	4	3	Z	R	Y	C
S	I	M	J	N	9	3	8	Z	A	C	R	Q	B	D	X	W	L	K	7	Y	4	5	P	0	T	H	F	U	V	S	G	E
T	W	D	V	B	Z	X	R	9	P	5	8	4	N	M	3	I	U	G	Y	7	Q	C	A	F	S	E	0	L	J	T	K	H
U	9	V	D	C	I	R	X	W	E	N	3	Y	5	J	8	Z	T	F	4	Q	7	B	H	G	L	P	K	S	M	U	0	A
V	X	U	T	Q	8	W	9	R	0	Y	Z	N	4	L	I	3	D	H	5	C	B	7	F	A	J	K	P	M	S	V	E	G
W	T	R	X	G	L	V	D	U	Y	0	M	E	K	8	J	S	9	B	P	A	H	F	7	C	I	4	5	Z	3	W	N	Q
X	V	9	W	H	M	T	U	D	5	P	L	K	E	Z	S	J	R	Q	0	F	G	A	C	7	3	N	Y	8	I	X	4	B
Y	P	N	5	M	H	0	K	E	W	V	G	U	D	B	F	A	4	8	T	S	L	J	I	3	7	9	X	Q	C	Y	R	Z
Z	L	3	8	0	T	M	J	S	Q	G	V	A	F	X	D	U	I	5	H	E	P	K	4	N	9	7	B	W	R	Z	C	Y
3	J	Z	I	E	D	S	L	M	C	A	U	G	H	9	T	V	8	4	F	0	K	P	5	Y	X	B	7	R	W	3	Q	N
4	E	5	N	J	A	K	0	P	9	D	F	T	V	C	G	H	Y	3	U	L	S	M	Z	8	Q	W	R	7	B	4	X	I
5	0	4	Y	L	G	P	E	K	X	T	H	D	U	Q	A	F	N	Z	V	J	M	S	3	I	C	R	W	B	7	5	9	8
7	A	B	C	D	E	F	G	H	I	J	K	L	M	N	0	P	Q	R	S	T	U	V	W	X	Y	Z	3	4	5	7	8	9
8	M	I	Z	P	V	L	S	J	B	H	T	F	A	W	U	D	3	Y	G	K	0	E	N	4	R	C	Q	X	9	8	7	5
9	U	X	R	F	S	D	V	T	4	K	J	P	0	3	M	L	W	C	E	H	A	G	Q	B	Z	Y	N	I	8	9	5	7

FIGURE 122*b*

	1	2	3	4	5	6	7	8	9	10	11
Wheel A:	x	x	o	o	x	o	x	x	x	o	o
Wheel B:	x	o	x	x	o	x	o	x	o		
Wheel C:	x	o	x	o	o	x	o	x			
Wheel D:	o	x	x	o	x	o	o				
Wheel E:	o	x	o	x	o						

FIGURE 123

If these wheels are aligned at their initial positions as shown in Fig. 123, and if each wheel moves one step at each operation, the following would be the generation of the first 35 elements of key:[3]

```
                5         10         15         20         25         30         35
A:   x x o o x o x x x o o}x x o o x o x x x o o}x x o o x o x x x o o}x x
B:   x o x x o x o x o}x o x x o x o x o}x o x x o x o x o}x o x x o x o x
C:   x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x
D:   o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}
E:   o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}
Key: U B C L D I Z U X R 9 5 A H 4 F V E 8 E 4 P D 8 9 R Y 4 Z K J H C Z U. . .
```

FIGURE 124.—Machine "A"

[3] The braces in the diagram mark the cyclic repetitions of the five wheels.

Since the wheel sizes are relatively prime and each wheel moves one step at each operation, the length of the total key is $11\times9\times8\times7\times5=27{,}720$. Of the total of 40 pins on the five wheels, 20 are in the active position; nevertheless, the generated key is not flat, because in the A and B wheels there is a slight bias (6/11 and 5/9, respectively) in favor of a "+", while in the D and E wheels there is a slightly stronger bias (4/7 and 3/5) in favor of a "−". Thus the highest probability, that of a key of A (=**xxooo**), is $\frac{6\times5\times4\times4\times3}{27,720}=.0519$, as compared with the random expectancy of 1/32 or 0.03125; this same probability will also hold for a key of **U** (=**xxxoo**), since the probability of either a mark or a space in the third level is 0.5. On the other hand, the lowest probability is that of a key of **M** (=**ooxxx**) or **O**(=**oooxx**), which is $\frac{5\times4\times4\times3\times2}{27,720}=0.0173$. The gamma ($\gamma$) I.C. of this key is $\frac{32\Sigma f^2}{27720^2}=1.083$, showing that the key is anything but flat, as has already been surmised.[4] In its Baudot character form, **UBCLDIZUXR . . .**, nothing is discernible in the foregoing key; but if the component mark and space impulses are written out in the five levels as in Fig. 124, the cyclic repetitions may clearly be seen.

b. In order to increase the cryptosecurity of machine "A", the wheels might be used *in combination* to produce key for the five Baudot levels. In Fig. 125*a*, below, are the *wheel streams* of the "A" machine set at the same initial position and advancing one step after each operation. In Fig. 125*b* are the *key streams* for the five levels produced by adding wheels A and B together for the key of level 1, adding B and C for the key of level 2, C and D for level 3, D and E for level 4, and E and A for level 5. In this stretch of key, no cyclic repetitions are observable in the five levels: the first level has a period of $11\times9=99$; the second, 72; the third, 56; the fourth, 35; and the fifth, 55. The total cycle is, of course, 27,720, since the motion of the wheels in this machine is the same as that of the "A" machine in Fig. 124.

```
             5         10        15        20        25        30        35
A:   x x o o x o x x x o o}x x o o x o x x x o o}x x o o x o x x x o o}x x
B:   x o x x o x o x o}x o x x o x o x o}x o x x o x o x o}x o x x o x o x
C:   x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x
D:   o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}
E:   o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}
```

FIGURE 125*a*

```
               5         10        15        20        25        30        35
A+B:   x o o o o o o x o o x x x x o o o o x o o o o x x o o o o x x x o o x
B+C:   x x x o x x x x o o o o o o o o x x x x o x x x o o o o x x o o x x x
C+D:   o o x x o o x o x o o o x o x x x x x x x o o x o o o x x x o o x x o
D+E:   x x o o o x o x x o x x x o x o x x x x x o o x x o o x o o o o o o x
E+A:   o x x o o x x o x x x x o o x o o o x o x o o x x x x x x o o o x x o
Key:   J G P 9 4 G P J M T B B F E M 9 C C 8 C M 4 4 8 B T T M P U E E P P J . . .
```

FIGURE 125*b*.—Machine "B"

c. Since the wheels as combined in the "B" machine produce levels of periods 99, 72, 56, 35, and 55, it might be desirable to extend the periods where possible. This may be done by the simple expedient of combining the longest wheel, A, with the other four wheels for four of the levels, and combining wheel B with C for the fifth level. This produces the following key:

```
               5         10        15        20        25        30        35
A+B:   x o o o o o o x o o x x x x o o o o x o o o o x x o o o o x x x o o x
A+C:   x o o x o o o x x x o o o o x x o o x o x o o x o x x x o x o o o o x
A+D:   o x o x x x o o x o x x o x x x o o x o x x x x x o o x o x x x o o o
A+E:   o x x o o x x o x x x x o o x o o o x o x o o x x x x x x o o o x x o
B+C:   x x x o x x x x o o o o o o o o x x x x o x x x o o o o x x o o x x x
Key:   W M O I H M O W C R F F E S C I T T 8 T C H H 8 F R R C O Q S S O O W . . .
```

FIGURE 126.—Machine "C"

[4] Since we are concerned with the universe or total population of this key, the correct statistic to be computed is the gamma I.C., not the delta I.C., which is that for a sampling of a universe.

Now the periods of the five levels are 99, 88, 77, 55, and 72—an improvement in only two of the cycle lengths. A better idea might have been to combine *three* wheels for each level key, as in the following diagram wherein, for example, the first key level is produced by adding together the A, B, and C wheels:

```
             5         10        15        20        25        30        35
ABC: x x o x x o x x o x x o o x x o o x x x x o x x x x o x x x o x o x x
BCD: o x x x x o o o o o x o x x x o x o x o x o x x x o x x o x o x x o o
CDE: x o o x x x x x x x x o o o o o x o x o o o x x x x o o x o x o o x x
DEA: x x x x o o o x x x o x x x o o o x x x o x o x o x o o o o o x x o x
EAB: o o x o x x o o o x o x o x x x o x x x x o x x o x o x o o o x x o o
     ---------------------------------------------------------------------
Key: F J G K Q H S F N X U O R 5 W T I B 8 B W 3 Q 8 U X 4 W S A 9 5 G S F . . .
```

FIGURE 127.—Machine "D"

The periods of the five levels are now 792 (=11×9×8), 504, 280, 385, and 495, a considerable improvement in the lengths of the individual cycles.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

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(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

65. Analysis of combination streams.—*a*. Let us assume that we have [redacted] recovered the 35 elements of key of the "B" machine shown in Fig. 125*b*. Key level 1 is as follows:

```
                5         10        15        20        25        30        35
x o o o o o o x o o x x x x o o o o x o o o o x x o o o o x x x o o x
```

No cyclic phenomena are observable in this stream of binary impulses (also called binary digits or "bits"), and the conjecture is made that, unless a single wheel of size 34 or larger is involved, the stream might have been produced in a cipher teleprinter by the interaction of two regularly stepping wheels of different sizes.[10] Consequently, the stream of bits is written on various trial widths, commencing with a width of 5 (see Fig. 136*a*, below). Now in each one of the width write-outs we will add the first row of bits to

1	2	3	4	5
x	o	o	o	o
o	o	x	o	o
x	x	x	x	o
o	o	o	x	o
o	o	o	x	x
o	o	o	o	x
x	x	o	o	x

1	2	3	4	5	6
x	o	o	o	o	o
o	x	o	o	x	x
x	x	o	o	o	o
x	o	o	o	o	x
x	o	o	o	o	x
x	x	o	o	x	

1	2	3	4	5	6	7
x	o	o	o	o	o	o
x	o	o	x	x	x	x
o	o	o	o	x	o	o
o	o	x	x	o	o	o
o	x	x	x	o	o	x

1	2	3	4	5	6	7	8
x	o	o	o	o	o	o	x
o	o	x	x	x	x	o	o
o	o	x	o	o	o	o	x
x	o	o	o	o	x	x	x
o	o	x					

1	2	3	4	5	6	7	8	9
x	o	o	o	o	o	o	x	o
o	x	x	x	x	o	o	o	o
x	o	o	o	o	x	x	o	o
o	o	x	x	x	o	o	x	

FIGURE 136*a*

[10] If we had recovered enough key, we could have noticed that the KL1 stream repeated after 99 positions.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

the second, the second row to the third, and so on. Since in binary arithmetic addition is identical with subtraction, this produces what amounts to a delta or difference stream at the particular interval, e.g., a "delta 9" (abbreviated Δ_9). At the correct assumption of the length of one of the wheels, the length of the other wheel will be disclosed by a repetition in the delta stream.[11] In Fig. 136*b* we show the

```
1 2 3 4 5    1 2 3 4 5 6    1 2 3 4 5 6 7    1 2 3 4 5 6 7 8    1 2 3 4 5 6 7 8 9
o x o x x    o o x x o o    x x x o o o o    o x o o o o x o    o o o o o x x o x
o o x o x    o x x x o o    o x x o x o o    x x x o o o x o    o o}o o o o o x x
o o o x x    x o x x x o    x x o o o x x    o x o x x o o x    o x o o}o o o o
x x x x o    x x x x x x    x o x x x x o    o x o
x x x o x    x o x x o
o o x x x
```

FIGURE 136*b*

delta streams as derived from the width write-outs of Fig. 136*a*; in the width write-out of 9, an 11-long delta repetition may be seen, thus proving that the sizes of the wheels involved are 9 and 11.

b. Knowing that the lengths of the wheels are 9 and 11, we can now recover the wheel patterns. We write the KL1 key stream on either one of the two widths, say 9. The first key element, "x", is assumed to have arisen from a combination of an "x" with an "x" on the two wheels; on this assumption, an "x" is recorded in the A-wheel portion of our diagram of the key write-out, and also at the beginning of every cyclic repetition of the other wheel. This is shown in Fig. 137*a*, below:

```
       1 2 3 4 5 6 7 8 9      1 2 3 4 5 6 7 8 9      1 2 3 4 5 6 7 8 9
  A:   x                      x   x   o   o          x o x x o x o x o
KL1:   x o o o o o o x o      x o o o o o o x o      x o o o o o o x o
  B:   x                      x                      x x o o x o x x x

       o x x x x o o o o      o x x x x o o o o      o x x x x o o o o
          }x                     }x                  o o}x x o o x o x

       x o o o o x x o o      x o o o o x x o o      x o o o o x x o o
              }x                     }x              x x o o}x x o o x

       o o x x x o o x        o o x x x o o x        o o x x x o o x
                  }x                     }x          o x x x o o}x x
```

FIGURE 137*a*   FIGURE 137*b*   FIGURE 137*c*

The placement of the "x" impulses at the beginnings of the B-wheel streams enables us to derive three more values for the A wheel, as shown in Fig. 137*b*; the complete patterns of the two wheels are then quickly recovered, as shown in Fig. 137*c*. These are either the true patterns, or their inverse, since our assumption in Fig. 137*a* of an "x" on the two wheels to make the first "x" in the key stream was arbitrary—there is no way of proving this point.

c. The solution of the remaining wheels of the machine continues in similar fashion with the procedure just illustrated; or, if the cryptanalyst has a hunch that the 9- or 11-wheel has been used for another key level, perhaps level 2, he might try writing this level out under the two wheel patterns just recovered, as shown below:

```
       1 2 3 4 5 6 7 8 9 10 11            1 2 3 4 5 6 7 8 9
       x x o o x o x x x o  o             x o x x o x o x o
KL2:   x x x o x x x x o o  o      KL2:   x x x o x x x x o
       x x x x x o x x o x  x             x o x o o x o x}x

       o o o o o x x x x o  x             o o o o o o o x x
       o o x x o o x x x x  o             o x o o x o x}x o

       x x o o o o x x o o  x             x x o x x x o o o
       x x x x o x x x o x  x             x o o x o x}x o x

       x x                                o x x o o x x x
       x x                                o o x o x}x o x
```

FIGURE 138*a*   FIGURE 138*b*

[11] See also in this connection subpar. 97*i* on p. 301 of *Military Cryptanalytics, Part II*, with reference to the solution of a two-tape Baudot system.

It may be seen in Fig. 138*b* that upon "stripping off" the 9-wheel from the KL2 stream, we are left with a residue that cycles at 8, which is the length of the other wheel in the KL2 stream. We could now strip off this 8-wheel from the remaining key levels, achieving success with KL3 and recovering the 7-wheel; the 7-wheel stripped off from KL4 yields the 5-wheel; and finally the 5-wheel stripped off from KL5 produces the 11-wheel initially recovered.

d. If the cryptanalyst had the 35 elements of recovered key of the "B" machine and knew the sizes of the five wheels (11, 9, 8, 7, and 5) but not their patterns or combinations for the particular levels, there is another easier procedure for their recovery. With five wheel sizes to choose from, there are ten possible combinations of five things taken two at a time [12] to produce the key for a particular level: 11–9, 11–8, 11–7, 11–5, 9–8, 9–7, 9–5, 8–7, 8–5, and 7–5. In analyzing key level 1, we will make the assumption that is it composed of an 11-wheel and one of the other sizes. We will begin by constructing a rectangle of, say, 11×8, and inscribing KL1 in a cyclic diagonal pattern from the upper left-hand corner of the rectangle. This is shown in Fig. 139*a*, below, wherein the first diagonal ends in col. 8 and continues in the first row, col. 9 briefly, and then in the fourth row, col. 1:

	1	2	3	4	5	6	7	8	9	10	11
	x		x			o			o		o
x	x		x			o			o		o
	o	o		o			o			o	
		x	o		o			x			x
	x			o		o			o		
		x			o		o			o	
			x			o		x			o
	o			o			o		x		
		x			o			x		x	

FIGURE 139*a*

	1	2	3	4	5	6	7	8	9	10	11
	x		x			o			o		o
x	x		x			o			o		o
o	o	o		o			o			o	
		x	o		o			x			x
x	x			o		o			o		
		x	o		o		o			o	
			x			o		x			o
o	o			o			o		x		
		x			o			x		x	

FIGURE 139*b*

	1	2	3	4	5	6	7	8	9	10	11
	x	x	x	x		o	x		o	x	o
x	x		x			o			o		o
o	o	o		o			o			o	
		x	o		o			x			x
x	x			◎		o			o		
		x			o		o			o	
			x			o		x			o
o	o			o			o		x		
		x			o			x		x	

FIGURE 139*c*

Since the first element of the KL1 stream is an "x", we will assume it is the result of a combination of an "x" with an "x"; accordingly, we will place an "x" over col. 1 and an "x" to the left of the first row. Now that we have an "x" at the side of the first row (i.e., an element of our supposed 8-wheel), we can place four more values heading the columns (i.e., in our assumed 11-wheel); all this is shown in Fig. 139*a*. The "x" over col. 1 enables us to place three more values in our 8-wheel: this is shown in Fig. 139*b*. These latter placements enable us to add some more values in the columns (see Fig. 139*c*), until we notice a conflict (shown by the ringed value).[13] This is proof that KL1 is not composed of an 11-wheel and an 8-wheel.

---

[12] The formula for the number of combinations of n things taken r at a time is $C=\frac{n!}{r!(n-r)!}$: in the case just mentioned, $C=\frac{5!}{2!(5-2)!}=\frac{5\times4\times3\times2\times1}{(2\times1)(3\times2\times1)}=\frac{5\times4}{2\times1}=10$. The number of combinations of 5 things taken 3 at a time is also 10, since $C=\frac{5!}{3!(5-3)!}=\frac{5\times4\times3\times2\times1}{(3\times2\times1)(2\times1)}=\frac{5\times4}{2\times1}$.

[13] A quicker process of finding conflicts is to compare the top row (above the matrix) with each successive fragmentary row within the matrix: every "x" in the top row should go unambiguously to the same symbol (either "x" or "o") in a particular row. In Fig. 139*c* a conflict is seen when an "x" in the top row goes to both "x" and "o" in the fourth row within the matrix.

e. In Figs. 140*a* and *b*, below, are shown two more incorrect assumptions (11–5 and 11–7), with conflicts indicated by the ringed values—one conflict is all it takes. (Note that we do not require all 35 elements of the recovered key level: it will suffice for our purposes to use just enough of the key stream to insure two or three values in each of the columns.) Finally, we have in Fig. 140*c*, below, the correct assumption of wheel sizes (11–9); there are no conflicts, and the two wheel patterns have been completely recovered:

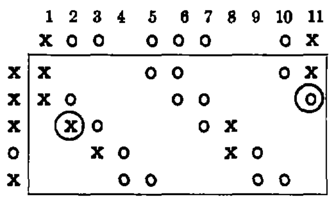

FIGURE 140*a*

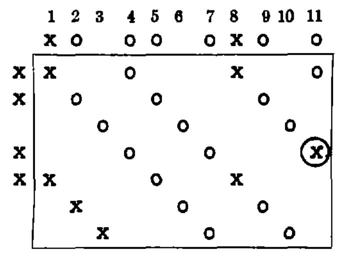

FIGURE 140*b*

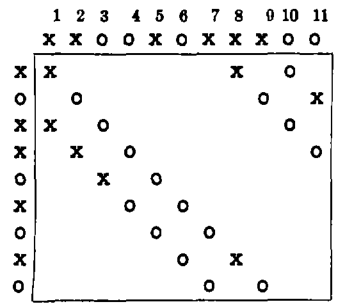

FIGURE 140*c*

f. As mentioned before, the wheels recovered will either be the true patterns, or their inverse: solution can be gotten either way. One thing that *is* proved, however, is the particular rule of combination, whether it is Vernam or mod 2. In subpars. *b*–*e* we assumed the Vernam rule, and it worked. But suppose we had used the mod 2 rule? As an example, let us take the situation of Fig. 137*a*, but with the incorrect rule. In Fig. 141*a* we assume that the first element of KL1, an "x", is the result of the combination of an "x" with an "o"; in Fig. 141*b* is the completed diagram with the patterns of the two wheels:

```
        1 2 3 4 5 6 7 8 9
   A:   x
 KL1:   x o o o o o o x o
   B:   o

        o x x x x o o o o
          }o

        x o o o o x x o o
            }o

        o o x x x o o x
                }o
```

FIGURE 141*a*

```
        1 2 3 4 5 6 7 8 9
   A:   x o x x o x o x o
 KL1:   x o o o o o o x o
   B:   o o x x o x o o o

        o x x x x o o o o
        x x}o o x x o x o

        x o o o o x x o o
        o o x x}o o x x o

        o o x x x o o x
        x o o o x x}o o
```

FIGURE 141*b*

It can be seen in Fig. 141*b* that, whereas *one* of the wheels (the 9-wheel) has the true pattern, the other has the inverse pattern, so it would be patently impossible to reconstruct the machine with an incorrect and incompatible "recovery". But the proof of Vernam addition has been evidenced from the start, had we been more observant—and had we known the earmarks—from the sample of 35 elements of the "B" machine key in Fig. 125*b* that we were analyzing: all the Baudot characters contain 1, 3, or 5 mark impulses, never 0, 2, or 4. Thus there is a maximum of only 16 different characters that can occur (there were actually only 13 in the sample). (Had mod-2 addition been involved, all the Baudot key characters would have been composed of 0, 2, or 4 marks.) The reason for this is not difficult to ascertain: because each wheel in our machine is present an *even* number of times during the formation of a Baudot character, it follows that the binary sum of the contribution of each wheel is an "x" (in the case of Vernam addition) or an "o" (in the case of mod-2 addition).[14] As an example, the first key character in Fig. 125*b*

[14] This observation holds true only if the wheels have *single* key-reading stations; if there are two or more reading stations it cannot be guaranteed (except in certain special cases) that the binary sum of the contribution of each wheel will be an "x".

is a J (=xxoxo); this is derived from the following wheel combinations and their contributions: A+B, B+C, C+D, D+E, E+A. Since in such a set of ten binary impulses there will *always* be an

```
A+B, B+C, C+D, D+E, E+A
 xx   xx   xo   oo   ox
```

even number of marks or spaces, the binary sum of these ten will always be a mark—therefore the final Baudot character must contain 1, 3, or 5 mark impulses. (Note that in Fig. 126, since each wheel is *not* present an even number of times, there is no such mark limitation; there is, however, another limitation which will be discussed later.)

g. Let us return to the "B" machine key as given in Fig. 125*b* for some further theoretical observations. Since KL1 is composed of the A wheel (length 11) and the B wheel (length 9), this level will cycle at $11\times9=99$. KL2, composed of the B and C wheels, will cycle at $9\times8=72$. If these two key levels are combined, as shown in the illustration below, the resultant stream will cycle at 88, the product of the A and C wheels, since the effect of the B wheel present in both key levels has been cancelled in the

```
              5         10        15        20        25        30        35
KL1:  x o o o o o o x o o x x x x o o o o x o o o o x x o o o o x x x o o x
KL2:  x x x o x x x x o o o o o o o o x x x x o x x x o o o o x x o o x x x
Sum:  x o o x o o o x x x o o o o x x o o x o x o o x o x x x o x o o o o x
```

addition process. This is really not of much help in analysis of this case, since it would be just as easy to attack KL2 directly and partition this key level into its two component 9- and 8-wheels; nevertheless, it demonstrates the cancellation of a wheel when it is present *an even number of times* in a summation stream.

(1) Now let us consider the 35 elements of key of the "D" machine given in Fig. 127, in which each key level is derived from a combination of *three* wheels. KL1 comes from wheels A, B, and C, and the product of these wheel sizes, $11\times9\times8$, may be written as $11(9\times8)$ or $9(11\times8)$ or $8(11\times9)$; therefore if KL1 were written on a width of 11 or 9 or 8, upon taking vertical differences on those widths (cf. subpar. *a*) we would get cyclic repetitions in the delta streams of lengths 72, 88, and 99, respectively (it may be seen that vertical differences on a width of 11 would take less material for solution). But let us combine KL1 and KL2 of Fig. 127, as shown below:

```
              5         10        15        20        25        30        35
KL1:  x x o x x o x x o x x o o x x o o x x x x o x x x x o x x x o x o x x
KL2:  o x x x x o o o o o x o x x x o x o x o x o x x x o x x o x o x x o o
Sum:  o x o x x x o o x o x x o x x x o o x o x x x x x o o x o x x x o o o
```

If extended far enough, the summation stream would be found to cycle at 77, since in the course of combining A+B+C and B+C+D the effect of the B and C wheels would be cancelled, leaving only the combination of the A and D wheels, with the resultant cycle of 77. But there is a remarkably easy procedure in the solution of the small amount of key of Fig. 127, as will now be shown.

(2) On the hypothesis that each level of the key of Fig. 127 is composed of three wheels in combination, we will derive summation streams by taking the 10 possible combinations of three levels, in addition to the summation of all five levels taken together. In Fig. 142*a*, below, we have the key streams for the five levels, while in Fig. 142*b* we have the 11 possible odd combinations of these levels, in the first row of which, for example, is the summation of KL1, KL2, and KL3. An astonishing phenomenon is now brought to light: in five of the rows of Fig. 142*b* we can clearly see the cyclic repetitions of the five wheels of our machine!

```
              5         10        15        20        25        30        35
KL1:  x x o x x o x x o x x o o x x o o x x x x o x x x x o x x x o x o x x
KL2:  o x x x x o o o o o x o x x x o x o x o x o x x x o x x o x o x x o o
KL3:  x o o x x x x x x x x o o o o o x o x o o o x x x x o o x o x o o x x
KL4:  x x x x o o o x x x o x x x o o o x x x o x o x o x o o o o o x x o x
KL5:  o o x o x x o o o x o x o x x x o x x x x o x x o x o x o o o x x o o
```

FIGURE 142*a*

```
Levels          5              10             15             20             25             30             35
123:   o o x x x x o o x o x o x o o o o x x x o o x x x o x o o o x o x o o
124:   o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}
125:   x o o o x x x x o o o x x x x x x o x o x o x x o o x x x o o x o x x
134:   x o x x o x o x o}x o x x o x o x o}x o x x o x o x o}x o x x o x o x
135:   o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}
145:   o o o o o x x o x x x o x x o x o x x x o x o x x x o o x x o x o x o
234:   o o o x o x x o o o o x o o x o o x x x x x o x o o x x x x x o o x o
235:   x x o o x o x x x o o}x x o o x o x x x o o}x x o o x o x x x o o}x x
245:   x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x
345:   o x o o o o x o o x x o x o x x x o x o x x o x x x o x x o x o o x o
12345: x x x o o o o x o o x o o o x x o x x x x x o x x o x x o o x o x o x
```

FIGURE 142*b*

Now let us examine what brought this about, since we know the answer from Fig. 127. The sum of levels 124 yielded the 5-wheel, so we shall set down the arithmetic involved:

```
KL1 is composed of wheels  A, B, C
KL2 is composed of wheels     B, C, D
KL4 is composed of wheels  A,       D, E
The sum, by cancellation:              E
```

It can be seen that by summing key levels 1, 2, and 4, four of the wheels are cancelled, leaving the E wheel alone in the residue. Similarly, the other four wheels are exposed uniquely in the residues of the following combinations of key levels:

```
KL1:  A,B,C          KL1:  A,B,C          KL2:    B,C,D        KL2:    B,C,D
KL3:      C,D,E      KL3:      C,D,E      KL3:      C,D,E      KL4:  A,    D,E
KL4:  A,    D,E      KL5:  A,B,    E      KL5:  A,B,    E      KL5:  A,B,    E
Sum:    B            Sum:        D        Sum:  A              Sum:      C
```

If we study the following diagram of all the combinations of five wheels taken three at a time, plus the one combination of all five wheels taken together,

```
 1.  A B C – –
 2.  A B – D –
 3.  A B – – E
 4.  A – C D –
 5.  A – C – E
 6.  A – – D E
 7.  – B C D –
 8.  – B C – E
 9.  – B – D E
10.  – – C D E
11.  A B C D E
```

it may be seen that if we add any *two* of these lines together, we shall have an *even* number of wheels in the answer; and if we add any *three* lines together, we shall have an *odd* number of wheels in the answer.[15] The reason for the inclusion of line 11 (summing all five wheels together) may be shown if we had to contend with the following combinations composing the level key streams, for example:

```
             1.  A B C – –
             2.  A B – D –
Key levels:  3.  A B – – E
             4.  A – C D –
             5.  – B C – E
```

[15] This answer is really a function of the number of reading stations involved, not the number of wheels, but in this case the two are synonymous.

Wheel A would be exposed by summing key levels 1, 2, and 4; wheel B, by summing 1, 3, and 5; wheel D, by 3, 4, and 5; wheel E, by 2, 4, and 5; but in order to expose wheel C, we would have to sum all five levels taken together.

(3) Now to get back to our analysis, without the benefit of foreknowledge. We have found out that the key levels of Fig. 142*a* are composed of combinations of three wheels of lengths 11, 9, 8, 7, and 5, but we don't know specifically which three wheels go to make up a particular key level. Let us consider the combination of levels 1, 3, and 4 which produced a unique wheel of length 9 in the answer. We know, first of all, that an even number of levels will produce a sum composed of an even number of wheels in the answer. Now if we make an arbitrary assumption that level 1 consists of wheels ABC, level 3 will consist of either (a) wheels ABD, or (b) wheels ADE, since the sum of KL1 and KL3 *must* be an *even* number of wheels. (The ABC, ABD, and ADE as used here are arbitrary designations assigned for solution and are not necessarily related to the designations originally assigned to machine "D".)

(4) Let us assume that KL3 consists of wheels ABD. If so, then the addition of these two levels will produce wheels CD:

```
                 5             10            15            20            25            30            35
KL1:  x x o x x o x x o x x o o x x o o x x x x o x x x x o x x x o x o x x
KL3:  x o o x x x x x x x x o o o o o x o x o o o x x x x o o x o x o o x x
Sum:  x o x x x o x x o x x x x o o x o o x o o x x x x x x o x o o o x x x
```

FIGURE 143*a*

Since the addition of level 4 produces a unique wheel, this level must perforce consist of wheels ACD, BCD, or CDE. Furthermore, since the addition of level 5 to the combination of KL1 and KL3 results in a different unique wheel (of length 7), this means that level 5 must have been produced by one of the three possibilities not used for level 4. KL4 contains the 9-wheel and an unknown pair, while KL5 contains the 7-wheel and the same unknown pair. Since the recovered wheels are of lengths 9 and 7, this unknown pair of wheels must be one of the remaining three combinations 11–8, 11–5, or 8–5. If we now strip the 9-wheel from KL4,

```
                 5             10            15            20            25            30            35
KL4:  x x x x o o o x x x o x x x o o o x x x o x o x o x o o o o o x x o x
 9W:  x o x x o x o x o}x o x x o x o x o}x o x x o x o x o}x o x x o x o x
Sum:  x o x x x o x x o x x x x o o x o o x o o x x x x x x o x o o o x x x
```

FIGURE 143*b*

only one trial should be necessary to determine the composition of the unknown pair—because if we difference out the 11-wheel, the residue must be the 8-wheel or the 5-wheel; if not, then the unknown pair must be the 8–5 combination. Accordingly, we difference out the 11-wheel from the summation stream above, as follows:

```
                 5             10            15            20            25            30            35
Sum:  x o x x x o x x o x x x x o o x o o x o o x x x x x x o x o o o x x x
11W:  x x o o x o x x x o o}x x o o x o x x x o o}x x o o x o x x x o o}x x
Res:  x o o o x x x x o o o x x x x x x o x o x o x x o o x x x o o x o x x
```

FIGURE 143*c*

Clearly, the residue is neither the 8-wheel nor the 5-wheel, so it must be the 8–5 combination. But we had better make sure; so we shall difference out the 8-wheel, which will yield the 5-wheel in the residue:

```
                 5             10            15            20            25            30            35
Sum:  x o x x x o x x o x x x x o o x o o x o o x x x x x x o x o o o x x x
 8W:  x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x o o x o x}x o x
Res:  x x x o o o o x o o x o o o x x o x x x x x o x x o x x o o x o x o x
```

FIGURE 143*d*

The 5-wheel is nowhere to be seen. What is wrong?—the basic assumption in the first sentence of this subparagraph is in error: the sum of KL1 and KL3 must consist of *two* pairs of wheels, and not one pair as we thought; therefore KL3 does not consist of wheels ABD, but rather ADE.

(5) Since we now know that KL3 is composed of wheels ADE (with respect to the arbitrary assumption of wheels ABC for KL1), this means that the sum of KL1 and KL3 must be *four* wheels rather than two:

```
KL1:  A, B, C
KL3:  A,       D, E
Sum:     B, C, D, E
```

Furthermore, the addition of KL4 to this sum exposes one of the wheels *already* in this sum, so KL4 must consist of three of the wheels in the sum. Likewise, the addition of KL5 to the sum exposes another one of the wheels already in the sum, so KL5 must consist of another combination of three wheels in the sum. This is shown graphically by the following example, in which the exposed wheels are arbitrarily designated as D and E:

```
Sum:  B, C, D, E          Sum:  B, C, D, E
KL4:  B, C, D             KL5:  B, C,    E
               E                        D
```

This means, then, that if we subtract the D and E wheels from the sum of KL1 and KL3, we shall be left with the combination of two other wheels, B and C. The two wheels already exposed are the 9-wheel and 7-wheel, so we first add these two together:

```
             5        10        15        20        25        30        35
9W:  x o x x o x o x o}x o x x o x o x o}x o x x o x o x o}x o x x o x o x
7W:  o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}o x x o x o o}
Sum: o o x o o o x o o x x x o x o o x x x x o o o x x x x o x x x x x x o
```

FIGURE 144*a*

If we now strip off the effect of the 9- and 7-wheels from the sum of KL1 and KL3 (which is a *four*-wheel sum), we will have left the sum of two of the remaining wheels: either 11–8, 11–5, or 8–5. In Fig. 144*b*, below, the top row is the sum of KL1 and KL3, the second row is the sum of the 9- and 7-wheels,

```
             5        10        15        20        25        30        35
(1): x o x x x o x x o x x x x o o x o o x o o x x x x x x o x o o o x x x
(2): o o x o o o x o o x x x o x o o x x x x o o o x x x x o x x x x x x o
(3): o x x o o x x o x x x x o o x o o o x o x o o x x x x x x o o o x x o
```

FIGURE 144*b*

and the third row is the result of the Vernam addition of the first two rows. On the assumption that the 11-wheel is involved, we will subtract it from the summation line of Fig. 144*b*, as shown below:

```
             5        10        15        20        25        30        35
(3): o x x o o x x o x x x x o o x o o o x o x o o x x x x x x o o o x x o
11W: x x o o x o x x x o o}x x o o x o x x x o o}x x o o x o x x x o o}x x
Res: o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}o x o x o}
```

FIGURE 144*c*

The 5-wheel is disclosed in the residue, so this now tells us that the sum of KL1 and KL3 consists of the 11-, 9-, 7-, and 5-wheels.

(6) The composition of KL4 and KL5 may now be determined exactly from the fact that when KL4 is added to the sum of KL1 and KL3 the residue is the 9-wheel, and when KL5 is added to the sum of KL1 and KL3 the residue is the 7-wheel. The wheel combinations for KL4 and KL5 must therefore be those in the following diagrams:

```
KL1 + KL3:  11 9 - 7 5        KL1 + KL3:  11 9 - 7 5
      KL4:  11 - - 7 5              KL5:  11 9 - - 5
  Residue:     9                Residue:        7
```

The determination of the composition of the remaining key levels is an equally simple matter. In Fig. 142*b* we saw that the combination of levels 2, 4, and 5 produced the 8-wheel uniquely, so in order to recover the combination for KL2 all that is required is to set down the wheel combination which when added to the sum of KL4 and KL5 will give a residue of the 8-wheel:

KL4:	11	–	–	7	5
KL5:	11	9	–	–	5
KL2:	–	9	8	7	–
Res:			8		

Having ascertained the composition of KL2, we can now consider the combination of KL2, KL3, and KL5 which produces a residue of the 11-wheel alone:

KL2:	–	9	8	7	–
KL5:	11	9	–	–	5
KL3:	–	–	8	7	5
Res:	11				

Finally, after the KL3 combination has been recovered, the composition of the remaining level (KL1) is obtained from the combination of levels 1, 3, and 5, which produces a residue of the 7-wheel:

KL3:	–	–	8	7	5
KL5:	11	9	–	–	5
KL1:	11	9	8	–	–
Res:				7	

The machine is now completely solved. In recapitulation, the key levels of the machine were produced by the following combinations of wheels:

Key levels:	11	9	8	7	5
1	x	x	x		
2		x	x	x	
3			x	x	x
4	x			x	x
5	x	x			x

h. One more aspect of wheel combination remains to be treated: that of Boolean addition. Let us refer to the key of the "E" machine given in Fig. 128, wherein KL5 is produced by the Boolean addition of the A- and E-wheel streams. If KL5 were extended to 70 positions, we would have the following:

				5					10					15					20					25					30					35
x	x	o	x	x	o	x	x	x	o	o	x	x	x	o	x	x	x	x	x	o	x	x	x	o	o	x	o	x	x	x	x	o	x	x

				40					45					50					55					60					65					70
o	x	x	x	x	x	x	o	x	x	x	x	o	x	o	x	x	x	x	o	x	x	o	x	x	o	x	x	x	o	o	x	x	x	o

In Fig. 145, below, we have written KL5 out on several trial widths, and at the bottom of each column we have recorded the *product* of the entries in the column, i.e., the combination according to the *multiplication rule* (oo, ox, xo = o); in other words, we record an "x" only if *every* impulse in the column is an "x", otherwise we record an "o".

```
1 2 3 4 5   1 2 3 4 5 6   1 2 3 4 5 6 7   1 2 3 4 5 6 7 8   1 2 3 4 5 6 7 8 9 10 11
x x o x x   x x o x x o   x x o x x o x   x x o x x o x x   x x o x x o x x x o o
o x x x o   x x x o o x   x x o o x x x   x o o x x x o x   x x x o x x x x x o x
o x x x o   x x o x x x   o x x x x x o   x x x x o x x x   x x o o x o x x x x o
x x x x x   x x o x x x   x x x o o x o   o o x o x x x x   x x o x x x x x x o x
o x x x o   o o x o x x   x x x x o x x   o x x o x x x x   x x x o x o x x x x o
o x o x x   x x o x x o   o x x x x x x   x x o x x x x o   x x o x x o x x x o o
x x o x x   x x x x x x   o x x x x o x   x o x x x x o x   x x x o
o x x x x   o x x x x o   o x x x x o x   x o x x o x x x   x x o o x o x x x o o
x x o x x   x o x x x x   x o x x o x x   o o x x x o
x x o x o   o x x o x x   x o o x x x o   o o o o o o o o
x x x x o   o x x x o o   o o o o o o o
x x o x x   x x x o
o x x x o   o o o o o o
o x x x o
o x o x o
```

FIGURE 145

It may be seen that in the write-outs of the correct widths (5 and 11), the patterns of the wheels are manifested; on any incorrect width, the sum of every column will be an "o", unless the amount of material available for analysis is too limited.

i. If only one wheel of a two-wheel Boolean summation stream had been recovered, the procedure is still simple. Suppose that in the preceding subparagraph we had neglected to try a width of 5, and that only the 11-wheel had been uncovered. We would write the cycles of the 11-wheel over the key stream, leaving a blank row in the middle for insertion of values from the other component wheel; these values are inserted in the unknown wheel stream ("?W") only when they are unambiguous on the Boolean assumption, i.e., when there is at least one "o" at a given position in the superimposed 11W and K streams, so that oo = o, and ox or xo = x. Thus:

```
             5         10        15        20        25        30        35
11W: x x o o x o x x x o o x x o o x o x x x o o x x o o x o x x x o o x x
 ?W:     o x   o       o o     x o   x       o x   o o     o         x o
  K: x x o x x o x x x o o x x x o x x x x x o x x x o o x o x x x x o x x

             40        45        50        55        60        65        70
11W: o o x o x x x o o x x o o x o x x x o o x x o o x o x x x o o x x o o
 ?W: o x   x       o x     x o   o       x o     o x   o       o o     x o
  K: o x x x x x x o x x x x o x o x x x x o x x o x x o x x x o o x x x o
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From the location of the "x" and "o" entries in the middle row, we are able to eliminate wheel sizes for the unknown wheel on the basis of conflicts. For example, sizes 2, 6, and 7 are ruled out because of the clash of the "x" bit at position 4 with the "o" bits at positions 6, 10, and 11. The tabulation of wheel sizes eliminated up to 20 is shown below:

W	*Positions*	*W*	*Positions*	*W*	*Positions*	*W*	*Positions*
2	4–6	7	4–11	12	10–22	17	4–21
3	22–25	8	6–14	13	26–39	18	21–39
4	22–26	9	17–26	14	3–17	19	14–33
5		10		15		20	
6	4–10	11	3–14	16	3–22		

The absence of conflicts at an interval of 5 and any of its multiples points to the size of the unknown wheel; this is substantiated when we write out the partial bit stream on a width of 5, the bits in the columns being combined by the multiplication rule to disclose the pattern of the wheel:

1	2	3	4	5
		O	X	
O				O
O			X	O
	X			
O	X			O
O		O		
	X	O		
O	X		X	
		O	X	
	X	O		O
			X	O
		O	X	
O				O
O			X	O
O	**X**	**O**	**X**	**O**

Notice that if we write the partial stream on widths of 10 or 15 the cyclic pattern of OXOXO is still clear in the product line, even though in the writeout on a width of 15 it is incomplete because of lack of sufficient material (this latter point must not be forgotten when dealing with limited amounts of data):

1	2	3	4	5	6	7	8	9	10
		O	X		O				O
O			X	O		X			
O	X			O	O		O		
	X	O			O	X		X	
		O	X			X	O		O
			X	O			O	X	
O				O	O			X	O
O	**X**	**O**	**X**	**O**	**O**	**X**	**O**	**X**	**O**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		O	X		O				O	O			X	O
	X				O	X			O	O		O		
	X	O			O	X		X				O	X	
	X	O		O				X	O			O	X	
O				O	O			X	O					
O	**X**	**O**	**X**	**O**	**O**	**X**		**X**	**O**	**O**		**O**	**X**	**O**

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CHAPTER XI

PRINCIPLES OF CRYPTODIAGNOSIS

71. General.—*a.* It never hurts to know what it is we're talking about, so let us begin with the definition of "diagnosis" as given in *Webster's Third New International Dictionary:*

> "*diagnosis*, n. 1. The art or act of identifying a disease from its signs and symptoms; also, the decisions reached. 2. A concise technical description of a taxonomic entity giving its distinguishing characters. 3. Investigation or analysis of the cause or nature of a condition, situation, or problem: a statement or conclusion about the nature or cause of a phenomenon."

From *Webster's New International Dictionary*, Second Edition, the following entries under "diagnosis" are of interest:

> "*diagnosis*, n. 1. . . . 2. Determination of a type or condition through case or specimen study. 3. Conclusion arrived at through critical perception or scrutiny; hence, keen understanding of appearances."

Finally, let us set down the definition of "diagnosis" as it appears in the NSA *Basic Cryptologic Glossary* (June 1971):

> "*diagnosis*, n. 1. In cryptanalysis, a systematic examination of encrypted text or key with a view to discovering the nature of the cryptosystem that produced it. 2. The result of such examination."

These definitions should clear the air and set the stage for the discussion which follows.

b. Let us first quote from a previous text [1] an extract which is particularly apropos, elaborating on the meaning of diagnosis:

> (1) "Except in the case of the more simple types of cryptograms, the step often referred to as *diagnosis*, that is, ascertaining the general system according to which a given cryptogram has been produced is usually a difficult, if not the most difficult, step in its solution. The reason for this is not hard to find."
>
> (2) "As will become apparent to the student as he proceeds with his study, *in the final analysis, the solution of every cryptogram involving a form of substitution depends upon its reduction to monoalphabetic terms, if it is not originally in those terms.* This is true not only of ordinary substitution ciphers, but also of combined substitution-transposition ciphers, and of enciphered code. If the cryptogram must be reduced to monoalphabetic terms, the manner of its accomplishment is usually indicated by the cryptogram itself, by external or internal phenomena which become apparent to the cryptanalyst as he studies the cryptogram. If this is impossible, or too difficult, the cryptanalyst must, by one means or another, discover how to accomplish this reduction, by bringing to bear all the special or collateral information he can get from all the sources at his command. If both these possibilities fail him, there is little left but the long, tedious, and often fruitless process of elimination. In the case of transposition ciphers of the

[1] *Military Cryptanalytics, Part I,* par. 17 (on pp. 20–21).

more complex type, the discovery of the basic method is often simply a matter of long and tedious elimination of possibilities. For cryptanalysis has unfortunately not yet attained, and may indeed never attain, the precision found today in qualitative analysis in chemistry, for example, where the analytic process is absolutely clear-cut and exact in its dichotomy. A few words in explanation of what is meant may not be amiss. When a chemist seeks to determine the identity of an unknown substance, he applies certain specific reagents to the substance and in a specific sequence. The first reagent tells him definitely into which of two primary classes the unknown substance falls. He then applies a second test with another specific reagent, which tells him again quite definitely into which of two secondary classes the unknown substance falls, and so on, until finally he has reduced the unknown substance to its simplest terms and has found out what it is. In striking contrast to this situation, cryptanalysis affords exceedingly few 'reagents' or tests that may be applied to determine positively that a given cipher belongs to one or the other of two systems yielding externally similar results. And this is what makes the analysis of an isolated, complex cryptogram so difficult. Note the limiting adjective 'isolated' in the foregoing sentence, for it is used advisedly. It is not often that the general system fails to disclose itself or cannot be discovered by painstaking investigation when there is a great volume of text accumulating from a regular traffic between numerous correspondents in a large organization. *Sooner or later* the system becomes known, either because of blunders and carelessness on the part of the personnel entrusted with the encrypting of the messages, or because the accumulation of text itself makes possible the determination of the general system by cryptanalytic, including statistical, studies. But in the case of a single or even a few isolated cryptograms concerning which little or no information can be gained by the cryptanalyst, he is often unable, without a knowledge of, or a shrewd guess as to the general system employed, to decompose the heterogeneous text of the cryptogram into homogeneous, monoalphabetic text, which is the ultimate and essential step in analysis. The only knowledge that the cryptanalyst can bring to his aid in this most difficult step is that gained by long experience and practce in the analysis of many different types of systems. In this respect the practice of cryptanalysis is analogous to the practice of medicine: correct diagnosis is the most important and often the most difficult first step toward success."

c. The parallels between cryptanalytic diagnosis—"cryptodiagnosis"—and medical diagnosis are very striking.[2] Medicine is far from an exact science—it really is an art—and the same is true of cryptanalysis. In medicine, complete diagnosis is often accomplished only after an autopsy; in cryptanalysis, complete diagnosis is sometimes possible only after a body of cryptomaterials is in the hands of the nevertheless-quite-competent analyst. Or another way of looking at it is that in medical as well as in cryptanalytic diagnosis, success may come only after the patient is dead or the system has expired; in either case, an autopsy could tell us what we *should* have done. Even with (or in spite of) the present state of medical knowledge, the causes of many diseases and their cures are still unknown; in cryptanalysis, there are many cipher systems for which no general solution exists or is known, even if we know the basic principles of the system in question.

d. Cryptanalysis may be thought of as consisting of two aspects, *diagnosis* and *exploitation*. Diagnosis of a cryptosystem is concerned with the discovery of (1) the *nature of the basic elements* (or tools) employed in the cryptosystem (e.g., the language, code book, additive key, transposition matrices, substitution alphabets), and (2) the *rules by which a cryptographer uses these elements* to convert plain text into encrypted text; diagnosis continues as long as any element or any enciphering principle remains unknown. Exploitation of a cryptosystem is concerned with (1) the *recovery of the basic elements* (e.g., the code book, additive key, transposition key, substitution alphabets) and (2) the subsequent *reading of the plain text*. Although the functions of diagnosis and exploitation are distinct, it is a mistake to assume that one finishes before the other commences. In general, some recovery of particular elements is incidental to the diagnostic process; similarly, a system can be partially exploited before diagnosis proper takes place (e.g., isolated spelling-group messages in a low- or medium-grade system are often read

[2] See in this connection Lambros D. Callimahos, "Cybernetics and Problems of Diagnostics; the Parallels Between Medicine and Cryptanalysis," appearing in the NSA *Technical Journal*, Vol. XIV, No. 1, Winter 1969.

before the mechanics of the system are fully understood—in fact, they can form the starting point of the diagnosis).

72. The basic steps in diagnosis.—*a.* The first attempt to formalize the discipline of diagnostics was made in 1563 by the Neapolitan physicist, physician, and cryptologist, Giovanni Battista della Porta; at the end of his treatise, *De Occultis Literarum Notis*, he appended a set of synoptic tables for cipher analysis, with bifurcated routes to be followed according to the particular phenomena displayed by the cipher under investigation. Three and a half centuries later there appeared, in 1918, a work by William F. Friedman entitled *Synoptic Tables for the Solution of Ciphers*, published by the Department of Ciphers of the Riverbank Laboratories at Geneva, Illinois, where Friedman was employed as a geneticist. This latter work consisted of nine tables, strictly dichotomous in nature, with which, it was hoped, an unknown system could be diagnosed. In the next attempt to devise synoptic tables for the analysis of cryptograms, Captain Roger Baudouin, in his *Éléments de Cryptographie* published in Paris in 1939, devised 17 tables which were to lay bare the cryptosystems of the moment. All of these attempts had something in common; they failed in their avowed purpose. The reason they failed is that, unfortunately, cryptodiagnosis does not lend itself to scientific rigor such as the taxonomy of qualitative analysis in inorganic chemistry. The construction of dichotomous charts, or variations thereof, is useless in the analysis across the board of systems that turn up in operational practice. So much for synoptic tables.

b. In *Military Cryptanalytics, Part I*, it was stated (on p. 18) that the art of cryptanalysis may be reduced to the following steps:

Procedures in cryptanalysis	*Requirements*
1. Arrangement and rearrangement of data to disclose nonrandom characteristics or manifestations (i.e., in frequency counts, repetitions, patterns, symmetrical phenomena, etc.).	Experience or ingenuity, and time (which latter may be appreciably lowered by the use of machine aids in cryptanalysis).
2. Recognition of the nonrandom characteristics of manifestations when disclosed.	Experience or statistics.
3. Explanation of the nonrandom characteristics when recognized.	Experience or imagination, *and* intelligence.

This was followed by the remark that "In all of the foregoing, the element of luck plays a very important part, as it is possible to side-step a large amount of labor and effort, in many cases if hunches or intuition lead the analyst forthwith to the right path. Therefore, the phrase 'or luck' should be added to each of the requirements above. In fact, it all boils down to the simple statement: 'Find something significant, and attach some significance thereto.' " All of this is of course an oversimplification; nevertheless it is an accurate description of the procedures of cryptanalysis—and, actually, *the steps of diagnosis.* But, in a more sophisticated or modern approach, cryptodiagnosis may be regarded as consisting of two aspects: (1) hypothesis formulation and (2) hypothesis testing; the formulation of a hypothesis will call for a particular diagnostic test or tests, which in turn will or will not produce the phenomena expected for that hypothesis, enabling the cryptanalyst to derive a measure of acceptance or rejection of the hypothesis. This will be elaborated upon in succeeding paragraphs.

73. The diagnostician and his attributes.[3]—*a.* What sort of person would we choose for the ideal diagnostician? What abilities, traits of character, and experience must he have? In order to answer these questions, we must look at the problem of diagnosis itself. In brief, the task of the diagnostician consists of collecting and organizing available material, searching for and recognizing phenomena, building up hypotheses and making every effort to knock them down again. Let us examine each stage and see what is required of our Compleat Diagnostician.

b. While the collecting and organizing of material and relevant information may frequently not be under the diagnostician's direct control, he must ensure that no useful evidence is beyond his reach. This means that he must be familiar with the collection process and must be able to specify the form in which the material is presented to him. With a fairly naïve encipherment system, or in any case if there is not too much material, he should normally start with the hard copies of the intercepted messages

[3] This paragraph ("The diagnostician and his attributes") is taken, with but minor changes, from D. A. Wilson's delightful paper, "Education in Diagnosis," appearing in the *NSA Technical Journal*, Vol. XIV, No. 1, Winter 1969.

themselves, and preferably do his own logging of traffic, at least initially. With more sophisticated and more voluminous systems, he will be forced to make use of data processing machinery or a computer for logging or "diarization" (i.e., the process of indicating the positions at which interesting features or passages occur in the messages). Under these circumstances the computer may deign to show him only those stretches of material which it finds interesting. He must then firmly insist on seeing periodically a selection of "normal" traffic, in order that he may both remind himself of what normal traffic looks like and with luck find new phenomena which the computer blindly ignores. He must either himself be able to program or at least be able to communicate sensibly with the programmer.

c. The diagnostic expert must also familiarize himself with the cryptographic background of the system. He must study both predecessor and contemporary systems which may have been conceived under the same roof, and he must know something about the users of the system and be able to deduce the likely degree of cryptographic sophistication and the required portability of the encipherment material. He must appreciate the value of external and nontextual features of the transmissions (the traffic analyst will probably be able to explain some, but not all of these), and he has to be familiar with the likely causes of busts, due both to faulty operators and, in the case of machine ciphers, to faulty machinery.

d. Up to now, our Compleat Diagnostician appears as a technical administrator. The main quality required is the ability to digest large amounts of information and retain the main gist in the front of his mind while taking care not to push the smallest scrap out at the back. He must be thorough and careful. He also needs to have some organizational capability, a technical knowledge of cryptographic systems, transmission systems, and probably programming.

e. Having put his material into a convenient form, the analyst enters the most critical stage of the diagnostic process. He has to search for, and find, phenomena. Without any phenomena to work with, diagnosis is impossible. The first thing to do is to look at the material. As I. J. Good remarks in his "Standard Reagents and Diagnostician's Dictionary," it may be plain text! Even if it is not, interesting features may spring to the eye. Long repeats, limitations, suspicions of rough frequency counts, excesses of doublets, indications of a significant width, depths, isologs, indicators, and many more phenomena may be found in a quick, not necessarily systematic, look at a few messages.

f. Unless enough evidence has now been found to have a first shot at establishing and testing a hypothesis, the diagnostician moves on to obtaining and examining descriptive statistics—numerical summaries of the material. Examples are frequency counts, repeat rates or indices of coincidence, pattern counts, and level and density counts of teleprinter text. Often these will be done on computers.

g. Throughout this search, the analyst is on the lookout for anything out of the ordinary, any departure from normal procedure or normal-looking traffic, and also for any consistent feature throughout the traffic which would not be expected in a random sequence of letters. His main task is to decide which of the features he sees (or thinks he sees) are important in one of these ways. He must recognize phenomena; he must also recognize when a suspected feature is a nonphenomenon.

h. The qualities possessed by our Compleat Diagnostician which make him an expert in finding phenomena are elusive. Certainly experience of phenomena which have proved helpful in the past is important. Knowledge of enciphering systems and their features, combined with the background information of subpar. *c* may help him to decide what to look for, but he does not blindly follow the path along which they lead: new systems usually have new features. It is not enough to say that he is observant; the word is so general that in this context it is almost devoid of meaning. His powers of observation are peculiarly imaginative. While looking at his material he must have in mind a multiplicity of possible features he would like to find, but frequently the most important observations are phenomena he could not have forecast. He must have an appreciation of pattern, and his mind works in a way which would probably give him a good score in a diagrammatic intelligence test. Possibly sheer physical ocular athleticism or a wide field of vision may contribute. If we could define that something implicit in the statement that he has a cryptanalytic "green thumb" we would be much nearer to understanding the crux of the diagnostic process. It is this something that separates our Compleat Diagnostician from the run-of-the-mill analyst who views diagnosis as merely the application of a sequence of standard statistical tests.

i. The assessment of the phenomena which have come to light requires a judgment of their value—a statistical common sense based on the concept, and frequently on the practical application, of significance tests. Furthermore, the diagnostician realizes that some of his significant features may really be only subphenomena, and is always ready to hunt for related features which will support a more general hypothesis. For instance, having observed a significant number of repeats at a certain distance he will want to know whether this is supported by a rough digraph count at that distance, or by repeats at multiples or factors of the original distance.

j. The next stage in the process is the setting up of hypotheses, to be followed by attempts to upset these same hypotheses. Each hypothesis should explain as many as possible of the observed phenomena. This explanation is then tested in two ways: first by deciding whether the remaining phenomena which have been seen conflict with the hypothesis, and second by predicting further phenomena which might be expected if the hypothesis is true and looking to see whether they exist.

k. In the more naïve systems the hypothesis will often be a complete tentative specification of the encipherment process, but this will rarely be the case with a sophisticated problem. Here the hypotheses will usually be either the restriction of the system to a general class (e.g., transposition of some kind, additive mod 26, binary linear recursive key), or a precise specification of some small detail (the existence of a Hagelin-type slide feature). The diagnosis of a sophisticated noncommercial machine may require the establishment of hundreds of viable partial hypotheses of this kind, gradually building up the complete picture from hosts of small details.

l. Here our Compleat Diagnostician needs the qualities of imagination, impartiality, deduction and ability to synthesize—in short, he is a good research scientist. He must be imaginative in his construction of hypotheses, deductive in predicting resultant expected phenomena, and impartial in his interpretation of the results of the tests of his hypothesis. The synthesis of many partial hypotheses brings in again his grasp of previous and contemporary systems—which must also have contributed to his selection of hypotheses in the first place. In machine systems a working knowledge of mechanics, electromechanics, or electronics is needed to diagnose faulty conditions and to predict other faults which are likely to occur.

m. Little has been said about the actual testing of hypotheses. Obviously our Compleat Diagnostician must be familiar with all the routine standard tests and conversant enough with testing theory to devise special tests where no appropriate ones are known. He must always ensure that his hypotheses can be tested—this will be a brake which keeps his imagination under control. Finally he remembers that the acid test of diagnosis is the reading of plain text, although the diagnostic process is seldom complete when the first sensible text emerges.

n. This concludes the list of qualities peculiarly appropriate to the diagnostician. Add to these the attributes we like to see in any senior cryptanalyst (perseverance, the ability to communicate and to keep a record of ideas and results, drive, supervisory capabilities, sheer common sense, etc.) and the list becomes formidable indeed. We begin to understand why so few really first-class diagnosticians emerge from the maelstrom of the cryptologic community.

74. Embarking on the unknown cryptosystem.—*a*. Let us suppose that we have been given traffic in an unknown cryptosystem and have been asked to solve the messages. In preparation for this likely event we will arm ourselves with information and facts, as outlined below.

b. The first step should be to find out all that is known, communications-wise and cryptologically speaking, about the origin of the messages. Is the target country known, say, Zendia, or is it undetermined? If it is Zendia, we (or someone in the research section) should tap all information sources about its past cryptologic history and national cryptographic habits, and we should obtain at least a general familiarity with its language, geography, political and military structure, industrial resources, and key personalities. If information on past cryptologic history is sparse, we take into account information on countries with which Zendia has an alliance, in case there is foreign influence in her communications.

c. We endeavor, through traffic analysis and other sources, to determine who the users of the cryptosystem might be—army, navy, air force, ambassadors, police, etc.—and in so doing, we make conjectures as to the probable content of the messages. We take note of the extent of use of the cryptosystem, and construct a cryptonet diagram.

d. If there is a predecessor system that has been read, we study decrypts and take note of the subject matter treated. Is there a high incidence of stereotyped beginnings or endings, or are there fairly rigid formats to some of the messages, and if so, between what stations or organizations? What procedures are there, if any, for the use of nulls, message bisection, and padding, and what conventions are there for the spelling out of digits? On what links or lanes have exploitable violations or weaknesses occurred in the past?

e. If the cryptosystem does not have a readable predecessor, does plain-language traffic intercepted on the links carrying the encrypted traffic yield any information that might be used as a break-in into the system? Is there any information of potential cryptanalytic value in the associated chatter? Do other intelligence sources yield any information of use? Do the formats of the messages (e.g., 5-digit, 4-letter), or the traffic volume, or the message lengths (e.g., a preponderance of messages exactly 32 groups long) give any suggestions as to the probable system? What are the probable cryptosecurity requirements of the system? If it is military traffic at corps or army level, the security of the system is expected to be high, possibly machine cipher, used under favorable operating conditions, with sufficiently rugged equipment, operated by personnel with adequate training. On the other hand, military traffic at, say, regimental level may be expected to be less secure, possibly a manual system, one which might be used under unfavorable operating conditions by personnel inadequately trained and under pressure of time or the exigencies of combat conditions. What is the probable reason for the introduction of the system; or, if it has a predecessor, the reason for the change of system?

f. Where the language of the encrypted messages is known or can be assumed, it goes without saying that we should equip ourselves with the necessary cryptolinguistic letter frequency and other data that will aid us in our analysis. All too frequently, especially in the rarer languages, such information has not been compiled in sufficient detail, or its existence is unknown, or worse yet, cannot be found.

75. Preliminary actions in attacking the unknown cryptosystem.—*a.* Let us assume that a pile of 5-letter traffic in an unknown cryptosystem has been placed on our desk. The messages bear an apparent discriminant, DICID, in the A1 position; therefore they may be presumed to be homogeneous.[4] The nationality of the target is uncertain, but the nets on which the traffic has been intercepted are presumed to be Zendian. We have taken into consideration the points discussed in par. 74, and we have rolled up our sleeves in preparation for the battle of wits: our cryptanalytic prowess against the enemy's cryptographic strength.

b. We begin by leafing casually, even unsystematically, through the messages, to see what we can see—and at this stage we really don't know what we expect to see—it suffices to keep an open, observant mind.[5] After a brief period of sampling the visible "flavor" of the messages, looking for anything that seems peculiar or out of the ordinary, we sort the traffic with the idea of assurance of maximum cryptographic homogeneity. If we have enough traffic, we can afford to restrict ourselves to sorting the traffic by net or even link, and within that by date; otherwise, we would be content with logging one or two months' messages, say, in date-time [6] order. The logs optimally should contain all of the elements of the preamble and, say, the first five and last five groups of text. (If the volume of the traffic is too great,

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[4] Homogeneity is a relative term, with different meanings in different contexts: in speaking of an enciphered code system, homogeneous traffic might be considered as embracing messages encrypted with the same code book and additive book; in a machine system, homogeneous traffic might be interpreted as consisting of messages enciphered with the same internal pin- and lug-settings, or the same rotor order; in a transposition system, homogeneous messages might be thought of as those enciphered with the same matrix *and* transposition key; in a Playfair system, since there is only one variable, *viz.*, the Playfair square, homogeneous messages would be those enciphered with the same square. These are anything but precise definitions; for example, if the enemy were using several types of code and enciphered code systems, and also Playfair systems, the several different Playfair systems taken together might be considered as a homogeneous grouping vis-à-vis the other nonrelated systems. In the case at hand, since the A1 groups of all the messages are the same 5-letter group, DICID, the traffic is homogeneous to that degree. But there are varying degrees of homogeneity: there might be another nontextual indicator among the groups of the message (or even in the preamble) which could further delimit the homogeneity; or only the traffic within a certain time period (e.g., month, week, day, 6 hours) might be homogeneous in the second degree; or only the traffic within a particular geographical area; or only traffic within a specified net or link, or even lane; or, for that matter, only traffic from a particular originator, regardless of addressee.

[5] A mistake, unfortunately all too common in operational practice, is disassociating plain-language messages and chatter from the encrypted traffic before it gets to the cryptanalyst. We should endeavor to examine the rolls of intercept copy as they come from the intercept operator, before they are cut up and the encrypted messages removed. Also, in sorting out duplicate messages, we should make sure that apparent "duplicates" are not really *bust messages:* fingers have been burnt in this situation more than once.

[6] *File* date-time, when this is in the preamble or postamble, and not intercept date-time.

this would call for the logs to be prepared by machine methods; otherwise, hand logging should suffice [7]—besides, we would become more intimately familiar with the traffic if the messages were logged by hand. Even if the logs are made by machine, it might not be a bad idea to take a limited amount of material, say an active link on a particular day or week, and log that much by hand for its instructional value.) The logs are then examined to see if anything of interest turns up; e.g., repeated groups between the messages near their beginnings or endings, limitation or association of characters near the beginnings and endings, or any correlation between preamble components and text groups. Columnar frequency counts of the *a*, *b*, *c*, *d*, and *e* positions of the A2 (and perhaps the A3) groups of the messages are taken; if the A2 (or A3) is an indicator group, such counts might reveal limitations or other phenomena often associated with indicators. Similar counts might be made with the Z0 and Z1 groups.

c. We now examine the messages themselves, assisted by our log. We take a good look at the messages having (1) the same serial number (i.e., station serial number, or message center number if there be one), (2) identical A2 groups (or if an indicator were not apparent as in this DICID system, identical A1 groups), (3) identical file date/times, (4) identical group counts, (5) sequential station serial numbers, (6) limitations in the beginning or ending groups, (7) polygraphic repetitions in the beginning or ending groups, either within or between messages, and (8) isomorphic beginnings or endings, which might arise from identical underlying plain text enciphered by different keys. Traffic emanating from each originator is compared, taking special note of messages going to the same addressee; and it might not be amiss, for that matter, to compare traffic going to each addressee, taking note of messages from the same originator. Traffic from both ends of a link is compared for possible polygraphic repetitions or other phenomena between messages which might earmark the use of identical keys (even including a mistake in the use of the wrong pad in a one-time-pad system which could then enable the reading of the resultant two messages in depth if the components are known or can be assumed). We also keep our eyes open for sets of messages that are likely to have similar—perhaps even proforma—content, which might be profitable to study separately. Practice traffic, if identified at this stage, should be removed from the message file. Suspected bust messages, resends, isologs, staggers, and any unusual situations should be worked on without delay; but we should be reasonable about the time spent on unsystematic fiddling at this stage. We study the group counts of the messages for evidence of key-length limitations and number of nontextual groups, and we do the "remainder test" for evidence of the length of the underlying cryptographic units.

76. First step: manipulating the data.—*a*. At the outset, it should be clear why diagnosis in cryptanalysis is at all possible: the plain text underlying cryptograms has pronounced properties of frequency variation and repetition, and these properties can be made to show through the cryptographic disguise by proper manipulation of the encrypted text. It is obvious that if the underlying text were flat, having no properties distinguishable from random, nothing would be manifested in its encryption, no matter what striking properties or characteristics were inherent in the application of the cryptosystem. In other words, flat "plain text" plus key, no matter how rough, will yield flat (i.e., random) cipher. But the converse is also true: rough plain text plus flat key will also yield flat cipher. Therefore, *as long as the cryptosystem has sufficiently nonrandom properties in its make-up, if the underlying plain text has a characteristic roughness, diagnosis and solution are possible.* But given the case of the encryption of plain text in a one-time-pad system in which the key is produced by a random process and has no limitations, characteristics, or patterns that are distinguishable from random, and where neither the key nor any portion of it is ever used again for the encryption of another message, solution is demonstrably impossible.

b. Even in the absence of cryptanalysis of messages, the patent characteristics of the encrypted texts and the manner in which traffic is passed from originator to addressee can yield information when there is either adherence to *or* departure from *established norms:* this is the essence of traffic analysis.[8] For example, three-letter traffic suggests a three-letter code system; the composition of a radio net and the manner of its working yield information on the relative subordination of the units or organizations involved; the relaying of a message from A to B to C discloses the existence of certain information channels, and if the message is reencrypted on the second leg of its journey an isolog will be produced;

[7] As we log each message, we look over the *entire* text, to catch anything that might be unusual.

[8] See in this connection par. 74 of Appendix 7 ("Communication Intelligence Operations"), *Military Cryptanalytics Part I*, and also Appendix 7 ("Introduction to Traffic Analysis"), *Military Cryptanalytics, Part II.*

a sudden increase in traffic volume on a particular link or net, which had been stable for some time, can be the prelude to an impending operation or activity.

c. When we manipulate data from the texts of messages, we are looking for characteristics of the underlying plain text to show through in some degree. When we examine our traffic logs, we look for signs that are characteristic, for example, of isologs, retransmissions, apparent mistakes of communications or cryptography; we find these because we know how these features are reflected in the logs. For example, in searching for isologs we look for messages of the same or nearly the same group count, sent within a relatively short time span (say, from less than an hour up to a day or two) from one originator to (preferably) the same addressee, or sent from an originator to two or more (possibly subordinate) addressees; and we also look for isologs on other legs of possible relays. Mistakes (so-called "busts") can be uncovered by astute observation, or we can also be helped immeasurably by operator chatter such as "You set the 47-wheel incorrectly, you blithering idiot!" which tells us that the cryptosystem is in all probability a machine system, and that the machine has a "47-wheel" (this might be either a 47-point wheel, or one with 47 marked positions on it, or simply a wheel marked "No. 47").

d. In this numbered paragraph, emphasis will be placed on the manipulation of data to uncover nonrandom characteristics or manifestations. Par. 77 deals with recognizing phenomena, and par. 78 with interpreting phenomena; but because cryptodiagnosis really does not keep entirely separate the steps of manipulating data and recognizing and interpreting phenomena, the discussion in pars. 76–78 perforce will overlap in these aspects.

e. The study of the group counts of messages can yield information of considerable importance to the cryptanalyst, as can be shown by the two examples which follow. In studying these group counts, we could make a simple tabulation in group-count order; we could group the data into classes such as all messages of group counts 21–25, 26–30, etc.; we could group them into two classes, those messages with an odd number of groups, and those with an even number; or we could study the group counts of messages (1) having a particular discriminant or indicator, (2) sent by a particular station, or (3) sent at a particular time of the day. The data manipulation in the two examples below consists merely of recording the group counts in a form suitable for study: in this case, in group count order.

(1) For the first example, the group counts of 100 messages have been tabulated; the traffic (apparently high-grade because of the importance of the users, and also because of the lack of visible properties) consists of messages sent in 5-digit groups, with a group count range of 21–99. In the diagram below, the columns labelled "GR" are the group counts, the columns labelled "Msgs" give the number of messages having the particular group count. The striking peaks at group counts of 32, 63, and 94 are noted. As a possible explanation of this phenomenon, we could arrive at the hypothesis (1) that there is involved a key book containing blocks of 30 key groups each, or a stencil system with 30 5-digit key-groups exposed by the apertures, (2) that the location of the first block is given by two indicator groups, and (3) that subsequent blocks require but one indicator group to designate their location. Under this hypothesis, then, it would be possible to have a 32-group message, but not 33, because the next greater length would have to be *34:* 32 groups for the first block including the indicators, and one more indicator plus one more text group. Likewise, the next message length after 63 groups must be 65, and the next after 94 must be 96.This hypothesis is strengthened by the absence of messages having group counts of 33, 64, and 95. A further hypothesis is made that the underlying text is cipher text rather than code text. The reasoning behind this assumption is that the cryptographic clerk, mindful of the key-length limitation of 30 groups, tries out of either laziness or economy not to go only one or two groups beyond the block limitation; thus, he might abbreviate a signature, or spell the last STOP or two as "STP," or make other slight changes—easier in a *cipher* system than in a *code* system—to avoid exceeding the imposed limitation. And as for the reason why the preponderance of plain texts happens to be groups of 30 and its multiples, perhaps it is because of the particular contents of the messages, or because of certain habits being followed as regards padding to fill out a convenient 30 groups—quién sabe? With further examination and analysis of the traffic, we should be able to prove or disprove our hypotheses—at least we have made a beginning, and we can pursue the trail we have hewn for ourselves.

GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs
21	1	31	1	41		51	1	61	4	71	1	81		91	2
22	1	32	6	42	2	52	1	62	1	72	2	82	2	92	2
23	1	33		43		53	2	63	8	73		83		93	
24		34	1	44	1	54		64		74	2	84	1	94	5
25	1	35	3	45	2	55	4	65	1	75		85	1	95	
26		36	3	46		56		66	1	76	2	86		96	
27	1	37	2	47	1	57	1	67		77	1	87	1	97	1
28		38	1	48		58	1	68	2	78	2	88		98	1
29	1	39		49	2	59	2	69		79		89	1	99	1
30	1	40		50	3	60	4	70	1	80	1	90	2	100	

Figure 166

(2) For the second example, we shall study the group counts of 200 messages from an actual operational situation. The traffic, presumably high-grade, consists of messages sent in 5-digit groups, and the group-count range is from 23 to 100 groups.

GR	Msgs	GR	Msgs	GR	Mscs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs	GR	Msgs
21		31	1	41	1	51	1	61	4	71	4	81	4	91	1
22		32	1	42	4	52		62	4	72	1	82	5	92	1
23	1	33		43	1	53	4	63	3	73	3	83	1	93	2
24		34	1	44	1	54	5	64	2	74	8	84	2	94	2
25		35	3	45	1	55	4	65	4	75	6	85	3	95	1
26	1	36	2	46	3	56	4	66	7	76	2	86	6	96	2
27		37	1	47	1	57	1	67	1	77	1	87	3	97	1
28	1	38	2	48	2	58	6	68	4	78	6	88	4	98	4
29		39	2	49	1	59	4	69	4	79	4	89	1	99	1
30		40	1	50	5	60	4	70	9	80	3	90	4	100	2

Figure 167

If nothing about the traffic were known, it would be wise to determine the size of the underlying cryptographic units, if possible, and also uncover any evidence as to the existence of nontextual groups that might be indicators. The procedure called for here is the *remainder test,* applicable when the last group has been padded with an indeterminate number of disguised nulls necessary to complete the group. If the text is sent in 5-character groups, the modular (remainder) counts after dividing by 2 (testing for underlying dinomic text), by 3 (for trinomic text), and by 4 (for tetranomic text) have the following expected percentages: [9]

Dinomic	*Trinomic*	*Tetranomic*
2N + 0 = 60%	3N + 0 = 40%	4N + 0 = 40%
2N + 1 = 40%	3N + 1 = 20%	4N + 1 = 20%
	3N + 2 = 40%	4N + 2 = 20%
		4N + 3 = 20%

If the underlying text is pentanomic, or if it consists of irregular-length units, the remainder test will be inconclusive. (Parenthetically, *tetranomic* text when examined on a *dinomic* basis also gives a distribution of 60% and 40% for remainders of 0 and 1, respectively, but there is no confusion since the tetranomic examination will give its proof. If, however, *dinomic* text is examined on a *tetranomic* basis, the distribution will be 30%, 20%, 30%, 20% for remainders of 0, 1, 2, and 3, respectively.) As an illustration, if the underlying text is, say, trinomic, the messages with group counts exactly divisible by 3 with no remainder will account for 40%, those with a remainder of 1 will account for 20%, and those with a remainder of 2 will account for 40%. If, however, there is one nontextual group present, the counts will be shifted cyclically one position downward, resulting in a distribution of 40%, 40%, 20% for remainders 0, 1, and 2; and if there are two nontextual groups present, the counts will be displaced cyclically two positions downward, resulting in a distribution of 20%, 40%, 40% for remainders 0, 1, and 2. In the

[9] The reason for these percentages is not difficult to see. Taking the example of underlying dinomic text, we might have to add 1, 2, 3, or 4 nulls to complete the final group, unless it is already a complete group of five; thus we have the following five cases:

a.	12121	21212			(2N+0)
b.	12121	21212	12XXX		(2N+1)
c.	12121	21212	1212X		(2N+1)
d.	12121	21212	12121	2XXXX	(2N+0)
e.	12121	21212	12121	212XX	(2N+0)

It can be seen that in three of the five cases (60%) the group counts of messages are exactly divisible by 2 with no remainder, and in two of the five cases (40%) there is a remainder of 1. In underlying trinomic text, we have the following five cases:

a.	12312	31231	23123				(3N+0)
b.	12312	31231	23123	123XX			(3N+1)
c.	12312	31231	23123	12312	3XXXX		(3N+2)
d.	12312	31231	23123	12312	3123X		(3N+2)
e.	12312	31231	23123	12312	31231	23XXX	(3N+0)

In two of the cases (40%) the group counts are exactly divisible by 3, in one case (20%) there is a remainder of 1, and in two cases (40%) there is a remainder of 2. In underlying tetranomic text, we have the following five cases:

a.	12341	23412	34123	41234					(4N+0)
b.	12341	23412	34123	41234	1234X				(4N+1)
c.	12341	23412	34123	41234	12341	234XX			(4N+2)
d.	12341	23412	34123	41234	12341	23412	34XXX		(4N+3)
e.	12341	23412	34123	41234	12341	23412	34123	4XXXX	(4N+0)

In two of the cases (40%) the group counts are exactly divisible by 4, and in the remaining three cases there is one case each (20%, 20%, 20%) of remainders of 1, 2, 3.

example given in Fig. 167, the group counts have the following remainders when examined on hypotheses of underlying dinomic, trinomic, and tetranomic text:[10]

Dinomic	*Trinomic*	*Tetranomic*
2N + 0 = 121 (60.5%)	3N + 0 = 67 (33.5%)	4N + 0 = 39 (19.5%)
2N + 1 = 79 (39.5%)	3N + 1 = 63 (31.5%)	4N + 1 = 36 (18.0%)
	3N + 2 = 70 (35.5%)	4N + 2 = 82 (41.0%)
		4N + 3 = 43 (21.5%)

It is clear that the underlying text must be tetranomic, and that there are two nontextual groups (probably indicators) present.[11]

f. An important phase of cryptanalysis, one which can be crucial especially in the early stages of analysis, is the study of indicators. In general, there are two kinds of indicators: (1) system indicators (also called "discriminants"), which define the sets of cryptomaterials used, and (2) message indicators, which define the specific application of the cryptomaterials or keys. Indicators may be recognized as such (1) if they are repeated in the message text; (2) if they possess distinctive patterns; (3) if they have limitations or characteristics in their elements; (4) if there are associations in the messages between the indicators and another group (which may be a text group, a second indicator, or an indicator check group); (5) if there are associations between the indicator and some element or elements in the preamble; or (6) if they have a roughness different from that of the message texts. These several aspects will now be illustrated.

(1) Below are exemplified sets of groups with patently obvious patterns and characteristics, different from the rest of the message texts, which categorize them as indicators:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
COBRA	BABAX	AIRBC	ZEBEC	DARIC	59329	81473	LAMIV
GRAVY	NONOX	EGQBC	ZEVUB	GOSUK	07726	04739	DCGBI
ALBUM	PIPIX	EFNBC	ZEMIN	JENIM	83573	35320	CVYML
NYLON	GAGAX	BKRBC	ZESOW	CUVOD	30317	52691	RBTTN
BONUS	ZUZUX	DGOBC	ZEDIG	MIWEH	92102	29903	VCYLK
EAGLE	MEMEX	AJLBC	ZELEM	BORAB	14230	75858	MIVFB
ROYAL	TUTUX	CFLBC	ZEWIZ	BOQAC	48754	18761	ZDDTX
YODEL	DODOX	BIPBC	ZEDAD	LAZUF	25186	93227	HSACD
GLORY	HAHAX	DJLBC	ZEZUF	HIXEB	31947	43042	QGXRP
LIVER	QEQEX	CHMBC	ZECOG	KUTIL	73820	56435	TPJLV

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[10] The easiest way to take remainder counts is with a desk calculator, "zoning" the keyboard into four (and then three) areas with the column markers, and recording the group counts four (and then three) at a time from the group-count record sheet: the accumulated totals will appear in the four (and then three) areas of the indicating dials. The remainder counts for the dinomic hypothesis are obtained easily from those of the tetranomic hypothesis, by adding together the counts of remainders 0 and 2, and those of 1 and 3: these are now the dinomic remainder totals for 0 and 1, respectively.

[11] Note, as an additional example, the remainder counts of the 100 messages given in Fig. 166:

Dinomic	*Trinomic*	*Tetranomic*
2N + 0 = 52	3N + 0 = 34	4N + 0 = 25
2N + 1 = 48	3N + 1 = 32	4N + 1 = 21
	3N + 2 = 34	4N + 2 = 27
		4N + 3 = 27

The remainder test here is inconclusive, showing that the underlying text either is pentanomic or consists of units of irregular length. Since in that example we had come to a conclusion that the underlying text was cipher rather than code, we can arrive at the tentative hypothesis that what we are faced with is probably an enciphered monome-dinome system. As a parenthetical aside, it is important when employing the remainder test (1) that the material under study be homogeneous, and (2) that the group counts are not biased in favor of a particular group count, which might give misleading results.

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In set (a) the indicators are self-evident, with 5-letter dictionary words used to convey the specific keying information; set (b) apparently embraces a potential 100 (or perhaps 120) keys or starting positions; set (c), 210 keys or starting positions; set (d), 100 keys or starting positions; and set (e), 2500 keys or starting positions. From sets (d) and (e) we may reconstruct the permutation tables [12] involved, supporting our contention as to the total number of possible indicators. Set (f) consists of sum-checking groups, and set (g) consists of groups self-summing to a constant 3, mod 10. Set (h) is harder to analyze, but on close examination it is seen that the middle letter is the mod 26 sum of the first two, and the last letter is the mod 26 sum of the third and fourth letters.

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(4) When indicators are enciphered, particularly in numerical cryptosystems, an additive key for encrypting the plaintext indicator has often been taken from a text group or groups near the beginning of the message or near the end. (Sometimes elements of the preamble, such as the group count, the message center number [13]—if there is one—or the file date/time have also been used as key.) These latent indicators

[13] The message center number (abbreviated "MCN") is a serial number assigned by a message center; this should not be confused with the station serial number (abbreviated "NR") which is that assigned by the radio operator to all messages in order of their transmission.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

are made patent by manipulation; subtracting all pairs of groups digit-by-digit, say A1 to A5 and Z4 to Z0, might uncover relevant phenomena. Let us study the following example of a log:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
DRW	WTZ	451	160752	62	30303	67470	82148	08651	32823	06647	09384
DRW	WTZ	452	160805	81	30303	21387	46095	50582	23172	53594	08128
DRW	WTZ	453	160817	65	30303	74582	98111	74502	84102	70193	85211
DRW	WTZ	454	160824	46	30303	82083	05559	64462	29489	54930	38196
DRW	WTZ	455	160830	84	30303	51756	74288	10975	66593	34461	28475

Since the A1 is patently a discriminant, there might be an indicator in the A2 position; and if a text group is used for encipherment of the indicator, testing various groups near the beginning or end as trial additives might uncover some phenomenon. In the case at hand, subtracting the A3 from the A2 groups yields the following:

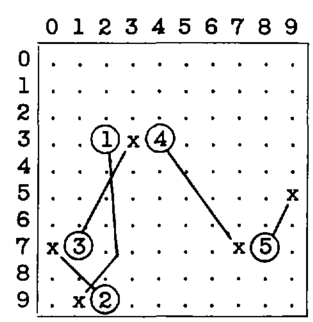
85332
85392
86471
87534
87578

The fact that the *a* and *b* digits sum to *c*, and that the numbers are in ascending sequence, is very interesting. Further study discloses an even more interesting phenomenon: since there are two nontextual groups (the discriminant and indicator), if we deduct 2 from each of the group counts, we can show that the messages are "tailing," with the system incorporating 10×10 additive pages of 100 groups each and the coordinates in normal order.[14] Note that we could have seen the relationship between the A2 and A3 groups by differencing: the vertical differences between the *a* positions of the groups in the A2 column match the differences of the *a* positions of the groups in the A3 column; furthermore, the vertical differences in the A2 and A3 groups of the top two messages have the first trinome in common, as is also the case with the vertical differences of the last two messages.

(5) In the next example,

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
CLD	SJO	827	171155	95	11033	14571	58536	64823	37867	83165	27120
CLD	SJO	828	171224	56	11033	38649	16650	19091	45648	56692	34603
CLD	SJO	829	171243	58	11033	81101	38012	48610	45432	66482	13393
RGF	SJO	361	171258	70	11033	79374	00559	60726	02491	41273	72458
RGF	SJO	362	171319	83	11033	23070	13078	70066	06315	58817	48815

[14] In the diagram below we have plotted the message starting points and their lengths. Message no. 451 began at coordinates 3–2 for a total of 60 *text* groups ending at position 9–1; no. 452 began with the very next key group (9–2) for 79 text groups, ending at position 7–0; no. 453 began at position 7–1 and ended at position 3–3; no. 454 began at 3–4 and ended at 7–7; and no. 455 began at 7–8 and must have ended at 5–9.



the plaintext indicators underlying the A2 groups are uncovered by using the Z1 groups as key, recognizing the indicators as such on the basis of the sum-checking digits in the *c* position; had there not been this property of the indicators, an additional trial of the Z0 groups as key for the A3 groups would reveal the following deciphered indicator groups:

31416	31416
82057	82057
25729	25729
38101	38101
75263	75263

Differencing techniques here too can reveal the existing relationships in the example above. The difference between a pair of A2 and A3 groups will be found to be the same as the difference between the Z1 and Z0 groups of the same message.

(6) The preceding example had the plaintext indicators repeated in the A2 and A3 groups. In the next example,

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
CTX	LFY	392	180806	81	55555	34464	40158	20920	96282	92540	91715
TIF	LFY	807	180821	24	55555	93215	83125	36436	78925	90360	01133
OLZ	LFY	496	180828	35	55555	66641	13894	05305	48820	46652	13841
FHJ	LFY	112	180845	60	55555	43193	79067	46951	94151	16094	33057
JRU	LFY	504	180905	54	55555	75541	32884	27036	57595	91953	09218

applying the Z0 groups as key for the A2 groups yields sum-checking *c* digits as the only noticeable property. But if we subtract the Z0 groups also from the A3 groups, we have the following:

43759	59443
92182	82092
53800	00053
10146	46010
76333	33676

The two separate indicator elements *ab* and *de* in the A2 groups have been transposed in the A3 groups to avoid a simple repetition of the indicator. In this example had we differenced the A2 and A3 groups, we would have uncovered the sum check in the *c* position, since the effect of the Z0 group would have been cancelled.

(7) For the next example we shall consider the following log:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
LXM	ZZA	739	191447	67	67890	27312	97444	61173	81932	61179	31051
LXM	OXL	624	191520	49	67890	56749	00750	18548	07446	23799	62749
LXM	NZM	124	191555	93	67890	03990	36773	56735	18857	52724	89122
LXM	DXY	368	191614	85	67890	05209	24516	79381	83011	94912	98336
LXM	KIR	902	191637	72	67890	50825	63650	73362	44065	66430	86021

This case is related to that in subpar. (6); the A2 and A3 groups contain the underlying indicators, and the keying groups are the A4 and Z0 groups. The second indicator is the same as the first, except that instead of summing the *ab* digits to *c*, the *cd* digits are summed to *e*, thus:

66249	66493
48201	48011
57265	57651
36928	36280
87563	87639

Here the relationships are harder to spot, but the difference between the *ab* digits of the A2 group and those of the A3 group will be the same as the difference between the *ab* digits of the A4 group and those of the Z0 group.

(8) Thus far in our examples the indicators have been in a fixed position in the body of the messages. The next case incorporates "floating indicators," with predetermined rules as to their location in the messages:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
MDX	HOU	751	210912	68	98765	69334	39494	63952	24737	19070	21798
MDX	HOU	751	220840	85	98765	60943	67038	70277	05392	17176	29317
MDX	HOU	751	230811	43	98765	67523	84674	43996	81846	76694	05132
MDX	HOU	752	230837	59	98765	00056	81271	15763	45263	56082	77857
MDX	HOU	753	230950	17	98765	71342	75778	56826	96091	73637	17872
MDX	HOU	754	231102	46	98765	14684	40901	85617	22495	34301	46549
MDX	HOU	751	240815	74	98765	58537	10507	92279	28474	68925	89235
MDX	HOU	752	241047	68	98765	42019	95611	21290	25709	21960	86403
MDX	HOU	753	241203	37	98765	44181	59813	62977	48329	47713	09960
MDX	HOU	754	241321	37	98765	51870	72113	49999	68212	99837	29780

This is more difficult of analysis, but the trial application of the Z0 group against the beginning groups of the messages yields the following at the indicated positions:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5
MDX	HOU	751	210912	68	98765	48646	.....	.....	.....
MDX	HOU	751	220840	85	98765	.....	48721	.....	.....
MDX	HOU	751	230811	43	98765	.....	.....	48864	.....
MDX	HOU	752	230837	59	98765	.....	.....	48916	.....
MDX	HOU	753	230950	17	98765	.....	.....	49054	.....
MDX	HOU	754	231102	46	98765	.....	.....	49178	.....
MDX	HOU	751	240815	74	98765	.....	.....	.....	49249
MDX	HOU	752	241047	68	98765	.....	.....	.....	49306
MDX	HOU	753	241203	37	98765	.....	.....	.....	49469
MDX	HOU	754	241321	37	98765	.....	.....	.....	49532

The rule for placing the floating indicator is not difficult to see: the units digit of the date governs the location of the indicator in the first 10 groups of the message, after the discriminant. Note also the last two 37-group messages, sent by an originator to the same addressee, a little over an hour apart; they may be isologs.

(9) The following example involves another case of floating indicators:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
ITV	SKG	201	250600	64	24680	49951	05973	17328	19856	16096	31859
ITV	SKG	202	250637	81	24680	20531	50244	59455	34690	83026	42522
ABS	SKG	751	250654	53	24680	30825	33446	99810	85035	26193	11881
DLL	SKG	401	250745	72	24680	71010	36895	00313	78387	52886	58753
ITV	SKG	203	250801	46	24680	25459	32083	81420	61717	76691	47303
KZN	SKG	601	250836	79	24680	59825	34904	28755	99683	46873	11595
USR	SKG	551	250855	78	24680	62863	88235	73903	37875	93751	95778
ITV	SKG	204	250916	81	24680	91342	18577	80532	17122	68066	13001
USR	SKG	552	250941	87	24680	12787	89220	86111	15909	21642	01989
ABS	SKG	752	251137	63	24680	38095	25720	05164	10654	85863	27886

This, too, is difficult of analysis, but the trial application of the Z0 group against the beginning groups of the messages yields the following at the indicated positions:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5
ITV	SKG	201	250600	64	24680	.....	.....	.....	88007
ITV	SKG	202	250637	81	24680	88019	.....	.....	.....
ABS	SKG	751	250654	53	24680	.....	.....	88039	.....
DLL	SKG	401	250745	72	24680	.....	88142	.....	.....
ITV	SKG	203	250801	46	24680	88156	.....	.....	.....
KZN	SKG	601	250836	79	24680	.....	.....	.....	88198
USR	SKG	551	250855	78	24680	.....	.....	88235	.....
ITV	SKG	204	250916	81	24680	88341	.....	.....	.....
USR	SKG	552	250941	87	24680	.....	88341	.....	.....
ABS	SKG	752	251137	63	24680	.....	.....	88388	.....

After a few moments' study we arrive at the rule for placing the floating indicator: the units digit of the group count, *mod 5*, governs the placement in the first five groups after the discriminant. Note also the two messages with the same indicator, 88341, which could be a possible depth:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	
ITV	SKG	204	250916	81	24680	(88341)	18577	80532	17122	. . .
USR	SKG	552	250941	87	24680	12787	(88341)	86111	15909	. . .

The *a* digits of the first three text groups (1, 8, 1) hit in the two messages, and the *b* digits of the three text groups yield even differences between the two messages; if there is a high hit rate among the *a* digits in the two messages as superimposed,[15] or if the even differences of the *b* digits prevail,[16] we shall have proof that the messages are in depth.

(10) As the final example of enciphered indicators, let us study the following log:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
CBI	BOW	002	010832	68	13579	59361	53381	82796	87762	82303	01952
CBI	BOW	001	030816	46	13579	03530	18529	68995	43932	77362	25994
CBI	BOW	002	040820	50	13579	13891	24972	17752	54294	83479	13151
CBI	BOW	005	070848	82	13579	55748	57242	45415	99143	06959	50829
CBI	BOW	001	110800	95	13579	53311	68617	27855	91717	88907	50983
CBI	BOW	001	160812	45	13579	81754	63746	49393	34151	19255	06040
CBI	BOW	003	190825	63	13579	09277	01671	13900	55676	98488	24012
CBA	BOW	001	220815	78	13579	85836	16035	63707	34246	66010	47101
CBI	BOW	004	230837	72	13579	81942	95559	61989	41353	46767	83744
CBI	BOW	001	270802	59	13579	94482	55379	77472	58894	68471	04047

After some experimentation with differencing various groups, we arrive at subtracting A5 from A2, yielding the results shown in Fig. 168*a*, below. Interesting properties are uncovered, but the backward progression of the groups as deciphered does not please us, so we subtract A2 from A5, as shown in Fig. 168*b*. We now have serial indicators in the last three deciphered digits (except for apparently two missed messages), but there is something wrong with the initial dinomes. If we now take vertical differences of the consecutive initial dinomes, we observe that the differences (2, 1, 3, 4, 5, 3, 3, 1, 4) correspond exactly with the vertical differences on the consecutive file dates, if the differencing is done with borrowing subtraction instead of the more usual mod-10 arithmetic; we therefore subtract the dates from the (A5–A2) groups, yielding the plaintext indicators as shown in Fig. 168*c*. This means that the cryptographer's

[15] Perhaps the underlying text is 5-digit code with a severe limitation in the *a* position.

[16] This could be the result of an underlying 5-digit code wherein the *b* digits of all the groups have the same parity, all odd or all even.

method of encryption was first to add the two digits of the date (with carrying addition) to the *ab* positions of the plaintext indicator, and then use the A2 group as an additive (mod 10) for the modified indicator, burying the enciphered indicator in the A5 position.

A2	A5	A2–A5
59361	87762	72609
03530	43932	60608
13891	54294	69607
55748	99143	66605
53311	91717	62604
81754	34151	57603
09277	55676	54601
85836	34246	51690
81942	41353	40699
94482	58894	46698

FIGURE 168*a*

A2	A5	A5–A2
59361	87762	38401
03530	43932	40402
13891	54294	41403
55748	99143	44405
53311	91717	48406
81754	34151	53407
09277	55676	56409
85836	34246	59410
81942	41353	60411
94482	58894	64412

FIGURE 168*b*

Date	A5–A2	Ind.
01	38401	37401
03	40402	37402
04	41403	37403
07	44405	37405
11	48406	37406
16	53407	37407
19	56409	37409
22	59410	37410
23	60411	37411
27	64412	37412

FIGURE 168*c*

g. Before leaving the subject of indicators there are still a few points that should be brought out; these are listed below.

(1) As already shown, indicators may be in the clear, or they may be enciphered. An indicator may be inserted in the message preamble, or it may be derived by selecting certain elements of the preamble. In some systems the indicator may be either the page, row, and column of the additive book, or it may be one of the actual key groups on a page of key. If it is the latter case the quoted key may be the one used to encipher the first group of the message, or it may be the one immediately preceding the key group used to begin the text encryption.

(2) In the preceding examples the discriminants were shown in the clear as the A1 group, but there is no reason why they cannot be enciphered and for that matter also hidden among the text groups (as indeed has occurred operationally). The remarks on indicator encryption and on floating indicators apply equally well to discriminants. The discriminant may not necessarily be an entire group: it could be part of a group, even a single character.

(3) Whether indicators are in the clear or are enciphered, sometimes a single indicator suffices. But more often, indicators (either in the clear or enciphered) are repeated as a check. For example, when the A2 is consistently repeated as the Z0, this is most probably an indicator (unless it is a control group as discussed below). Sometimes the indicator is confirmed by a check indicator; for example, the check indicator being such that the mod-10 sum of the indicator and its check is 0 0 0 0 0, or the five digits of the check indicator comprise the sums *ab*, *bc*, *cd*, *de* and *ea* of the indicator digits. Cryptographers usually prefer to vary the appearance of the plaintext check indicator when it is enciphered, as has been shown in some of our examples.

(4) Discriminant (and also indicator) encipherment is sometimes governed by *control* elements in the message text or preamble, or both; these elements are not the actual key elements, but rather designate where the actual keys (from a separate list) may be found. The control elements might be found in an individual group, or they can be scattered. For example, the units digit of the group count in conjunction with the *a* element of the A3 group might designate the page of a 100-page key book to be consulted, and the *de* elements of the A3 might designate the row-column position of the key group which is to be used to encrypt the discriminant or indicator. In analysis, messages with hitting groups in the suspected discriminant position are examined, to determine which textual or preamble elements also hit consistently and thus point to the control elements involved. In the sets of message beginnings below, in which there are hits in the A1 group (the suspected position of a discriminant), it is apparent that the only consistent control for the discriminant encipherment must be the units digit of the group count, the *c* position of the A4 group, the *b* position of Z1, and the *e* position of Z0. (The last set, which is in apparent conflict with our findings, must involve a *noncausal* repetition of the A1 group.)

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
DES	UAC	637	251435	83	53464	62080	46684	25906	94912	93313	67702
KCO	LRK	509	031102	53	53464	52104	75216	20969	66024	03580	81512
TVJ	UHW	225	171526	93	53464	25338	24300	35987	64073	73964	73262
IGR	BIU	618	131014	57	64291	92726	04269	82279	67823	54781	63600
IFZ	LPT	924	240819	47	64291	21641	21992	45263	15030	24618	29740
MOJ	SWT	036	291650	87	64291	74983	85054	94288	58692	64956	90970
KUD	RYU	492	161055	46	67629	21079	75093	02955	32116	53449	87202
NBW	CMV	578	180918	43	67629	25596	02364	80665	49911	98818	34797

(5) There are cases in which no discriminant is used, the appearance of the text being sufficient to denote the system; e.g., where a single unenciphered code system is being used, or a transposition system, or a bipartite system with self-evident row- and column coordinates. In these cases the only indicator will be a message indicator, if one is necessary.

(6) There have been instances when the key in certain low-grade systems was determined by the units digit of the group count (thus amounting to a single indicator digit), continuing in either an ascending or descending numerical sequence to produce a 10-digit cyclic key. In other instances the units digit (n) of the group count was the first key digit, the second key digit was (n+1), and thereafter the key was derived from the initial dinome by Fibonacci treatment, i.e., a+b=c, b+c=d, etc. Thus if the units digit of the group count were 2, the starting dinome would be 23, and the resulting Fibonacci sequence would be 2358314594 . . . which has a cycle of 60. (Other Fibonacci cycles, wherein a+b=c, have lengths of 20, 12, 4, 3, and the trivial length 1.) Additive keys for low-grade systems have sometimes also been taken from several elements appearing in the preamble of the messages. For example, the file date and time could constitute a 6-digit cyclic key; or the file date/time reversed plus the group count and the mod-10 sum of the latter could constitute a 9-digit additive key.

(7) In the study of indicator usage the analyst should be on the lookout for evidence both of tailing and trailing,[17] not only on the basis of a chart as exemplified earlier, but also on the basis of the sequence of settings of various machine cryptosystems with which he is familiar. (As a parenthetical aside, although one-time-pad systems are wont to have tailing indicators, the assumption of a one-time-pad system is the *last* assumption an analyst should make about an unknown cryptosystem; after all, the keys may be used serially, and then re-used from the beginning again. There have been too many cases of traffic put aside under the label of "one-time-pad system" that was subsequently solved as some *other* type of system.)

(8) It is important to examine indicators of sequential messages, whether these indicators are in the clear, or enciphered, or decrypted, because sometimes the indicators may be related in some fashion from message to message. As an elementary example, let us suppose that we have intercepted a string of five messages from one originator, as shown in the log extract below:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
GJB	YPO	314	281142	63	56156	86175	75356	63698	07426	54252	93651
GJB	YPO	315	281159	42	56156	47823	78625	51818	41757	46728	29160
GJB	YPO	316	281220	49	56156	15057	90977	77279	38000	81647	10762
GJB	YPO	317	281312	55	56156	65528	06001	61452	49192	17321	17383
GJB	YPO	318	281336	50	56156	10704	72147	72350	14144	19735	80114

[17] It might be well to define these two terms as given in the *Basic Cryptologic Glossary:*

"*tailing,* n. The practice of beginning the encipherment of one message with the elements of key immediately following those used to encipher the last textual element of the preceding message; equivalent in a machine cipher to starting the second message at the position reached by the machine on completion of the first."

"*trailing,* n. The practice of beginning the encipherment of one message with elements of key at a comparatively short interval after those used to encipher the last textual element of the preceding message; equivalent in a machine cipher to starting the second message at a machine position a few steps after that reached by the machine on completion of the first message."

The phenomena present in the A2 indicator groups make it possible to predict the indicators for succeeding messages, after it is noted that the Fibonacci addition of a + b, b + c, c + d, d + e, and e + a of the digits of one indicator form the digits of the indicator for the next message. In another example, we have a log extract of five sequential messages that have been intercepted:

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	Z0
WHM	PJY	428	051600	55	77788	86173	68548	16136	11573	52552	44611
WHM	PJY	429	051623	58	77788	57492	13347	57418	49468	43852	60739
WHM	PJY	430	051635	47	77788	33565	33239	07394	14333	45477	66794
WHM	PJY	431	051655	73	77788	80957	62416	86251	89835	69485	43263
WHM	PJY	432	051702	52	77788	13412	56209	92192	22184	27255	69611

In this example, the Fibonacci treatment has been applied *vertically* between messages in the column of A2 indicator groups, so that it becomes possible to predict the indicators for succeeding messages.[18]

(9) As a final note of caution on indicators, the number of possible different indicators in a particular indicator system does not necessarily reflect the number of possible distinctly different effective indicators. In an example given earlier, the first letters of the 5-letter groups were limited to the A–E range, the second letters to those in the F–K range, and the third letters to those in the L–R range (the last two letters appeared as a constant digraph, therefore possess no information value). The number of possible indicators is then $5 \times 6 \times 7$ or 210; this number is either the total number of keys, or the total number of starting points in the key, or a lesser number of either, if *variants* happen to be involved; for all we know, if A and E are variants, as are also F/J, G/K, L/P, M/Q, and N/R, the number of distinct indicators is reduced to $4 \times 4 \times 4$ or 64.

h. Having treated the diagnostics of message length characteristics and indicators, we shall now take up the manipulation of message texts. Both individual messages and sets of messages are subjected to study, in a manner calculated to make latent phenomena patent through manipulative processes (or, for that matter, to make patent phenomena more obvious). Depending on the volume of traffic at hand, we segregate messages by visible characteristics such as indicators, or we segregate messages sent within a relatively narrow time span; we compare messages sent by the same originator; or we compare those sent on the same day of the week (to uncover possible weekly re-use of keys), or on the same day of the month (to uncover possible monthly key rotas). As mentioned before, any phenomena uncovered may suggest subsequent steps to be taken. At this stage, the pertinency and utility of logging methods may be decisive. The manner of preparing worksheets and of summarizing statistical data play an important part in the timely solution of a cryptosystem. Sometimes the particular order of procedures and tests makes little difference, and at other times the order is of considerable importance.

i. In all of one's endeavors, the successful cryptanalyst's precept should be kept foremost in mind: "Try the simplest thing first." It is surprising how often this admonishment is disregarded, to the later chagrin of the cryptosinner. The comments by I. J. Good under the topic "rational behavior" are particularly germane to this discussion:[19]

> "The principle of rational behavior is to maximize 'expected utility' per unit time. An example is that some experiments are worth performing mainly because they are quick and easy. The reasons are: (a) being quick, the 'expectation' of (value of the test) ÷ (time expended) is liable to be large because the denominator is small; (b) being easy, the hypothesis behind the experiment is usually simple and therefore has a nonnegligible initial probability of being true; (c) if easy things were not the most important, human beings would *already* be extinct.
>
> "If of two hypotheses that are equally easy to test, one is much more useful if true, it may be more worth testing than the other, even if much less probable. This principle is perhaps more important in cryptanalysis than in other branches of science. Its relevance is sometimes obscured by the use of expressions such as 'I don't like this hypothesis,' which ought to mean 'I don't think its expected utility per unit testing time is large,' but is liable to mean 'I think its probability is low.' "

[18] The observant reader might note that the Z0 group in the two examples above is an indicator check; let him uncover the procedures involved.

[19] *Standard Reagents and Diagnostician's Dictionary* (p. 2), National Security Agency, 1966.

Taking a case in point, the process of completing the plain-component sequence using standard alphabets is not very likely to bear fruit in an operational problem; *but when it does, it's marvelous.* There isn't much effort involved in the procedure (especially if we possess already prepared alphabet strips), and the time it takes is very small compared with many other aspects of cryptanalysis. To illustrate the value of trying the process anyway, let us suppose that we have some traffic in an unknown cryptosystem, the messages being sent in 5-letter groups except for the A1 which is always a 5-digit number. No unusual statistical properties, repetitions, or other phenomena have been observed, and we might even suspect a high-grade system to be involved (if we know nothing about the cryptographic history of the target country). Nevertheless, if we try completing the plain-component sequence on the basis of direct standard alphabets, we would obtain a solution, as shown below for one of the message beginnings (the A1 here was 26996):

```
C:  U S V I Q J O Z Y F Q E G N O D A K I F E H A Y Y K N Z K D . . .
    V T W J R K P A Z G R F H O P E B L J G F I B Z Z L O A L E
    W U X K S L Q B A H S G I P Q F C M K H G J C A A M P B M F
    X V Y L T M R C B I T H J Q R G D N L I H K D B B N Q C N G
    Y W Z M U N S D C J U I K R S H E O M J I L E C C O R D O H
    Z X A N V O T E D K V J L S T I F P N K J M F D D P S E P I
    A Y B O W P U F E L W K M T U J G Q O L K N G E E Q T F Q J
    B Z C P X Q V G F M X L N U V K H R P M L·O H F F R U G R K
    C A D Q Y R W H G N Y M O V W L I S Q N M P I G G S V H S L
    D B E R Z S X I H O Z N P W X M J T R O N Q J H H T W I T M
    E C F S A T Y J I P A O Q X Y N K U S P O R K I I U X J U N
    F D G T B U Z K J Q B P R Y Z O L V T Q P S L J J V Y K V O
    G E H U C V A L K R C Q S Z A P M W U R Q T M K K W Z L W P
    H F I V D W B M L S D R T A B Q N X V S R U N L L X A M X Q
    I G J W E X C N M T E S U B C R O Y W T S V O M M Y B N Y R
    J H K X F Y D O N U F T V C D S P Z X U T W P N N Z C O Z S
    K I L Y G Z E P O V G U W D E T Q A Y V U X Q O O A D P A T
    L J M Z H A F Q P W H V X E F U R B Z W V Y R P P B E Q B U
    M K N A I B G R Q X I W Y F G V S C A X W Z S Q Q C F R C V
    N L O B J C H S R Y J X Z G H W T D B Y X A T R R D G S D W
    O M P C K D I T S Z K Y A H I X U E C Z Y B U S S E H T E X
    P N Q D L E J U T A L Z B I J Y V F D A Z C V T T F I U F Y
    Q O R E M F K V U B M A C J K Z W G E B A D W U U G J V G Z
    R P S F N G L W V C N B D K L A X H F C B E X V V H K W H A
    S Q T G O H M X W D O C E L M B Y I G D C F Y W W I L X I B
    T R U H P I N Y X E P D F M N C Z J H E D G Z X X J M Y J C
```

It can be seen that the encipherment is effected three letters at a time with a particular key letter, and the shifting of the components is apparently never greater than 9 positions. Knowing the mechanics of the system, we can now read all messages with comparative ease, even though the meaning of the A1 might not be understood for some time.[20] The chances of solving such a system by any other means are rather slim, yet the simplest technique of all gives a quick—and effortless—answer.

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[20] The method of key generation is as follows: Each correspondent has a secret fixed key which is used as a subtractor to encipher a randomly chosen plaintext indicator, and the enciphered form is sent as the A1 group of the message. The plaintext indicator consists of a 5-digit group wherein the *e* digit, as a safety factor, is the noncarrying sum of the first four digits; this indicator is then used as a priming key to generate a Fibonacci sequence by the formula a + b=f. Thus, in the example above, the secret key 12345 is added to the A1 group (26996) to yield the plaintext indicator 38231, from which is generated the sequence 105421596364599 . . . This sequence (omitting any zeroes that might occur in order to avoid enciphering more than three letters at one setting) is then used, starting with the initial alignment of the components at $A_p=A_c$, to give the successive displacements 1542159636 which will read the first 30 letters of the message.

j. In the manipulation of message texts we perform essentially only a few basic processes, those necessary to reveal hoped-for phenomena, just as the physician begins his first examination by looking at the patient's tongue, taking his temperature, and tapping his knee with his little hammer—all this in the hope of disclosing something untoward. The cryptanalyst's manipulation entails the manner of recording original and derived data, writing and rewriting data in various forms and formats most suitable for the particular purpose or study, and the transformation or reduction of data. Initially, the manipulation and examination is done by hand, at least for a few messages—there still is no computer equal to the human brain—while waiting for the results of machine examination to come back. We proceed with machine processing (a) if nothing of interest turns up, (b) if what turns up cannot be interpreted, and (c) if examining a volume of traffic. Emphasis on the initial importance of looking at the material does not belittle machine techniques, which are valuable and often essential; it is rather a plea not to use machine techniques too early, for the vital clue may not respond to any of the mechanized tests, even with a very large library of routines.

k. The cryptanalyst takes various kinds of frequency counts in the hope of disclosing a significant roughness which is then mash for his still; he indexes groups in the search for polygraphic repetitions; he examines patterns in the search for isomorphs; he writes out the texts and other data in various formats; he superimposes and compares messages, looking for an excess hit rate or other phenomena of significance; he derives secondary data such as differences, delta streams, or parity streams; and he compares sets of original and derived data. In the manipulative stage, much depends upon the cryptanalyst's experience and ingenuity if he is to avoid unnecessary work and achieve his results economically. We shall now give examples to illustrate these various points in as general a context as possible in order to show that their application is universal, whether manual or machine cryptosystems are involved, or whether the examination is performed by manual or machine methods.

l. We shall use as our first example the 200-letter message below, with which a number of points can be illustrated:

```
P Y L P P   N A F T H   T K S S F   Z P M Y X   X X J N U   M H K Z K
R U H B S   Q I E B S   F M V W C   F Y D T F   D F I J B   Y A Q D X
X O J O C   N X W N N   P R U J S   J I H H W   B N P V B   V C G K Y
H S T Z V   Y C F Y D   V N E U M   O O O Z G   B U Z P I   Y M H H J
W D Z D U   C C E F N   Q I K G F   N G C S J   U Q U K T   V X R D Q
Y Y M T M   Z R U C W   F I H Y L   E G G G A   X K L F Z   Z Q T O Q
G T R Y T   Z Y B R X   D Z I G M   S O M L Y
```

A cursory examination shows a rather high doublet rate of 15 when we expect 7.7, but when this is evaluated it turns out to be a deviation of 2.6σ, enough to be interesting, but not enough to be exciting.[21] We have not forgotten so soon the admonishment contained in subpar. *i*, above—standard alphabets just might be involved in some fashion or other; but the generatrices of this test show nothing. A uniliteral frequency distribution is taken,

[21] The calculation is performed as follows: the expected number of doublets in this sample is $\frac{199}{26}$ or 7.7; the standard deviation may be taken as the square root of the expected number, which is $\sqrt{7.7}=2.8$; the excess doublets over the expected is $15.0-7.7=7.3$; and the sigmage is obtained by dividing the standard deviation into the excess number, so $\frac{7.3}{2.8}=2.6\sigma$.

which to the unpracticed eye might look "a little rough," but the I.C. is actually lower (0.97) than the average expected for a random sample of this size.[22] In searching for polygraphic repetitions, we find just one repeated tetragraph, CYFD. In a sample of this size, the chance of a random tetragraphic repetition is small; in fact, it may be shown that if we had 1000 samples of 200 letters each of random text, we would expect to find a repetition in only 43 of them.[23] The repetition CYFD is at an interval of 52, an interesting number to a cryptanalyst since it has a factor of 26. When the cryptogram is written out on a trial width of 26, the columnar ϕ values sum to 80 as shown in the diagram below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
	P	Y	L	P	P	N	A	F	T	H	T	K	S	S	F	Z	P	M	Y	X	X	X	J	N	U	M	
	H	K	Z	K	R	U	H	B	S	Q	I	E	B	S	F	M	V	W	C	F	Y	D	T	F	D	F	
	I	J	B	Y	A	Q	D	X	X	O	J	O	C	N	X	W	N	N	P	R	U	J	S	J	I	H	
	H	W	B	N	P	V	B	V	C	G	K	Y	H	S	T	Z	V	Y	C	F	Y	D	V	N	E	U	
	M	O	O	O	Z	G	B	U	Z	P	I	Y	M	H	H	J	W	D	Z	D	U	C	C	E	F	N	
	Q	I	K	G	F	N	G	C	S	J	U	Q	U	K	T	V	X	R	D	Q	Y	Y	M	T	M	Z	
	R	U	C	W	F	I	H	Y	L	E	G	G	G	A	X	K	L	F	Z	Z	Q	T	O	Q	G	T	
	R	Y	T	Z	Y	B	R	X	D	Z	I	G	M	S	O	M	L	Y									
Σ	4	2	2	-	4	2	4	2	2	-	6	4	2	12	6	4	4	2	4	2	8	2	-	2	-	-	=80

[22] The delta I.C. is calculated by the formula $\delta = \frac{c\Sigma f(f-1)}{N(N-1)}$, where c is the number of categories, $\Sigma f(f-1)$ is the observed value of ϕ, and N is the sample size. The expected value of the δ I.C. for random is 1.0. In this example, $\delta = \frac{26(1478)}{200 \times 199} = 0.97$. The δ I.C. is the correct statistic to be used for samples from a population, since it compensates for sample size. Another statistic, the gamma (γ) I.C., with the formula $\gamma = \frac{c\Sigma f^2}{N^2}$, is the measure of roughness to be used when we have the universe or total population of a particular case.

[23] The expected numbers of two-, three-, and fourfold occurrences of trigraphs, tetragraphs, and pentagraphs in 26-category random text for samples up to 1000 letters are given in the following table:

No. of letters	Exp. no. of trigraphs E(2)	E(3)	E(4)	Tetragraphs E(2)	E(3)	Penta. E(2)
100	0. 269	0. 001		0. 010		
200	1. 10	0. 004		0. 043		0. 002
300	2. 48	0. 014		0. 096		0. 004
400	4. 40	0. 033		0. 171		0. 007
500	6. 85	0. 064		0. 270		0. 011
600	9. 81	0. 111	0. 001	0. 389		0. 015
700	13. 3	0. 175	0. 002	0. 530		0. 021
800	17. 3	0. 261	0. 003	0. 693		0. 027
900	21. 8	0. 371	0. 005	0. 877		0. 034
1000	26. 8	0. 505	0. 008	1. 08	0. 001	0. 042

Similarly, the expected numbers of two-, three-, and fourfold repetitions of trinomes, tetranomes, and pentanomes in 10-category random text for samples up to 1000 digits are given in the following table:

No. of digits	Exp. no. of trinomes E(2)	E(3)	E(4)	Exp. tetranomes E(2)	E(3)	E(4)	Pentanomes E(2)	E(3)
100	4. 49	0. 147	0. 004	0. 49	0. 002		0. 046	
200	16. 3	1. 08	0. 053	1. 95	0. 013		0. 199	
300	33. 3	3. 31	0. 246	4. 35	0. 043	0. 001	0. 447	
400	53. 6	7. 12	0. 707	7. 67	0. 102	0. 002	0. 795	
500	75. 8	12. 6	1. 57	11. 9	0. 197	0. 002	1. 24	0. 002
600	98. 8	19. 7	2. 95	16. 9	0. 337	0. 005	1. 79	0. 004
700	122	28. 3	4. 94	22. 8	0. 531	0. 009	2. 43	0. 006
800	144	38. 3	7. 64	29. 5	0. 785	0. 016	3. 17	0. 008
900	165	49. 4	11. 1	37. 0	1. 11	0. 025	4. 01	0. 012
1000	184	61. 3	15. 3	45. 2	1. 50	0. 038	4. 95	0. 017

Since there are 18 long columns (of 8 letters each) and 8 short columns (of 7 letters each), the I.C. of the text written on this width is $\delta=\frac{26(80)}{18(8\times7)+8(7\times6)}=1.55$, interestingly close (an acceptable deviation for a small sample) to 1.73 if the language of the underlying text is English. If the cryptogram is in a progressive alphabet system we can solve it by a statistical method[24] with additional traffic, or we can even solve this single message if there are isomorphs present; if, however, it is in a different kind of system employing 26 alphabets, it will take more traffic to enable a classic solution by frequency analysis and the probable word method.

m. Continuing with the analysis of the foregoing cryptogram, we conduct a search for possible isomorphs, using a simple method: each cipher letter is compared with, say the 5 letters following, and if there is a reoccurrence of that letter within that span, a numerical indication is placed under the reoccurrence of the letter equal to the interval of the monographic repetition. The cipher text of our example with the appropriate numerical indications is given below:

```
P Y L P P  N A F T H  T K S S F  Z P M Y X  X X J N U  M H K Z K
      3 1             2     1               1 1               2
R U H B S  Q I E B S  F M V W C  F Y D T F  D F I J B  Y A Q D X
                 5 5               5       4  3 2
X O J O C  N X W N N  P R U J S  J I H H W  B N P V B  V C G K Y
1     2          3 1               2     1             4  2
H S T Z V  Y C F Y D  V N E U M  O O O Z G  B U Z P I  Y M H H J
                 3                 1 1         4             1
W D Z D U  C C E F N  Q I K G F  N G C S J  U Q U K T  V X R D Q
      2      1                       3         2
Y Y M T M  Z R U C W  F I H Y L  E G G G A  X K L F Z  Z Q T O Q
  1     2                            1 1               1       3
G T R Y T  Z Y B R X  D Z I G M  S O M L Y
  4     3  3                         3
```

The foregoing method insures the easy findings of the following sets of isomorphs:

```
           3 1         2     1
(1)  P Y L P P N A F T H T K S S
(2)  N X W N N P R U J S J I H H
(3)                  D Z D U C C

           1 1
(4)  M Y X X X J N U M
(5)  U M O O O Z G B U
(6)  L E G G G A X K L

         2                 1     2
(7)  F D F I J B Y A Q D X X O J O
(8)  U Q U K T V X R D Q Y Y M T M
```

From these isomorphs the following partial chains may be derived:

```
1–2:  PNP  YX  LW  AR  FU  TJ  HSH  KI
1–3:  TD  HZ  KU  SC
2–3:  JD  SZ  IU  HC
4–5:  YMUB  XO  JZ  NG
4–6:  ML  YE  NXG  JA  UK
5–6:  UL  ME  OGX  ZA  BK
7–8:  FU  DQD  IK  JT  BV  YXY  AR  OM
```

[24] Cf. par. 69 (pp. 161–169) of *Military Cryptanalytics, Part II.*

By using the principles of indirect symmetry of position,[25] we can amalgamate these chains and construct a sequence which, when decimated at the proper interval, will yield the QUESTIONABLY . . . XZ keyword-mixed sequence, the original cipher component of the system. Having the cipher component at hand, we are now able to effect a *reduction to monoalphabetic terms* of the complex cipher text, if we know or can assume the motion of the cipher component; in the case of a progressive alphabet system, the motion is at a constant interval, either to the right or to the left. If we arbitrarily assume the normal sequence for the plain component, with the two components juxtaposed at the setting $A_p = Q_c$, and decipher the beginning of the cryptogram by moving the cipher component to the right after each decipherment,

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
C:	P	Y	L	P	P	N	A	F	T	H	T	K	S	S	F	Z	P	M	Y	X	X	X	J	N	U	M
"P":	U	M	M	X	Y	M	O	V	M	Z	O	D	P	Q	C	O	K	K	D	R	S	T	N	E	Z	S
C:	H	K	Z	K	R	U	H	B	S	Q	I	E	B	S	F	M	V	W	C	F	Y	D	T	F	D	F
"P":	Q	T	B	V	Z	G	W	Q	L	J	P	N	V	Q	C	I	M	O	E	H	F	I	A	L	L	N
C:	I	J	B	Y	A	Q	D	X	X	O	J	O	C	N	X	W	N	N	P	R	U	J	S	J	I	H
"P":	F	S	L	O	M	F	T	F	G	P	B	R	Y	U	M	M	X	Y	M	O	V	M	Z	O	D	P

the distribution of the resultant "plain text"

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

with its I.C. $= \frac{26(252)}{78 \times 77} = 1.09$ is not satisfactory. But when we decipher the beginning of the cryptogram by moving the cipher component to the *left* after each decipherment,

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
C:	P	Y	L	P	P	N	A	F	T	H	T	K	S	S	F	Z	P	M	Y	X	X	X	J	N	U	M
"P":	U	K	I	R	Q	C	C	H	W	H	U	H	R	Q	A	K	E	C	T	F	E	D	V	K	D	U
C:	H	K	Z	K	R	U	H	B	S	Q	I	E	B	S	F	M	V	W	C	F	Y	D	T	F	D	F
"P":	Q	R	X	P	R	W	K	C	V	R	V	R	X	Q	A	E	G	G	U	V	R	S	I	R	P	P
C:	I	J	B	Y	A	Q	D	X	X	O	J	O	C	N	X	W	N	N	P	R	U	J	S	J	I	H
"P":	F	Q	H	I	E	V	H	R	Q	X	H	V	A	U	K	I	R	Q	C	C	H	W	H	U	H	R

the distribution of this "plain text"

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

with its I.C. $= \frac{26(388)}{78 \times 77} = 1.68$ shows that this converted text has been reduced to a simple substitution which we can now readily solve, discovering in the process that the plain component is the HYDRAU-LIC . . . XZ keyword-mixed sequence.

n. There are certain situations, rarely predictable in advance, wherein it is profitable (nay, even mandatory) to derive a delta stream of the cipher text, typically by differencing the first element of text from the second, the second from the third, and so on, on a known or assumed modulus or scale. The value of this technique will be apparent in the examples which follow.

(1) For the first example, we shall assume that we are in possession of a small volume of traffic sent in 5-letter groups in an unknown cryptosystem. The over-all I.C. of the traffic is close to 1.0, as are the I.C.'s of the individual messages. A few polygraphic repetitions, both within and between messages, are in evidence, more than expected at random; the intervals between the repetitions have no common factor, but the closest ones are at an average of approximately 22 letters apart, and then at an average of approxi-

[25] Cf. Chapter VI, *Military Cryptanalytics, Part II.*

mately 45 letters apart. No evidence of depth exists between messages, although a higher-than-expected hit rate for random is observed in the first 5–6 letters or so between messages that contain common polygraphic repetitions within the body of the messages. A typical message with its distribution is given below:

```
D L H G I   H X N Y T   V P I Z M   B I F C S   H T K X M   T H K C S
L C C X Z   X D S O X   G F I O J   L F Y A X   Z M U X T   H J Z T B
U M Z E L   C L K S I   T O Q K T   R X M I A   X N C L L   G T Z D L
M B J A C   T A G I F   R I X W V   G B A S F   F F Y O I   A Z Z C C
X F X H C   W F E H N   I K E X Z   W Y Y A F   S X O P A   V U M Z Z
Z S I C U   J S T P I   O M E U R   N T H H H   W E D G E   Q U S N Z
F N B B O   T A R A Z   L N K M F   C B U R R
```

Neither the appearance of the cipher text, nor the I.C. of $\frac{26(1646)}{200\times199}=1.075$, shows much promise for immediate attack. But look what happens when we take a delta of the cipher text, using the normal A–Z sequence as the modulus:

```
D L H G I   H X N Y T   V P I Z M   B I F C S   H T K X M   T H K C S
  H V Y B   Y P P K U   B T S Q M   O G W W P   O L Q M O   G N C R P

L C C X Z   X D S O X   G F I O J   L F Y A X   Z M U X T   H J Z T B
S Q Z U B   X F O V I   I Y C F U   B T S B W   B M H C V   N B P T H

U M Z E L   C L K S I   T O Q K T   R X M I A   X N C L L   G T Z D L
S R M E G   Q I Y H P   K U B T I   X F O V R   W P O I Z   U M F D H

M B J A C   T A G I F   R I X W V   G B A S F   F F Y O I   A Z Z C C
A O H Q B   Q G F B W   L Q O Y Y   K U Y R M   Z Z S P T   R Y Z C Z →

X F X H C   W F E H N   I K E X Z   W Y Y A F   S X O P A   V U M Z Z
← U H R J U   T I Y C F   U B T S B   W B Z B E   M E Q A K   U Y R M Z →

Z S I C U   J S T P I   O M E U R   N T H H H   W E D G E   Q U S N Z
← Z S P T R   O I A V S   F X R P W   V F N Z Z   O H Y C X   L D X U L

F N B B O   T A R A Z   L N K M F   C B U R R
F H N Z M   E G Q I Y   L B W B S   W Y S W Z
```

A plethora of repetitions is manifested, proving conclusively (1) that, unbeknown to us, the normal sequence was somehow involved, and (2) that it was a lucky thing indeed that we had decided to take a delta stream of the cipher text. The many latent isomorphs in the text were too short to permit their recognition, except for the two sets, now in retrospect, of the longer isomorphs.

(2) The appearance of the polygraphic repetitions in the first line of the delta stream and their "interruption" by one letter suggests (1) that spelled-out numbers are probably in the underlying plain text, and (2) that the encipherment probably involves word lengths. If this latter premise is true, the 11-letter delta repetition indicated in the fourth pair of rows must reflect a 12-letter plaintext word. In trying various possible cribs, notice what happens when we test HEADQUARTERS in this position:

```
                        H E A D Q U A R T E R S
                        V G B A S F F F Y O I A
Equivalents of E_p:       G(h i j k l m n)O
Equivalents of A_p:         B(c d e)F
Equivalents of R_p:                   F(g h)I
```

Not only is there corroboration for the assumption of HEADQUARTERS, but the spacing of the cipher equivalents for the repeated plaintext letters along a distance measured on the normal sequence proves that there is a constant motion of one step of the cipher component during the encryption of a word. This leads easily to the recognition of the numbers SEVEN, EIGHT, and THREE in the text; the threefold occurrence of the word STOP; and, in the second pair of lines, the cipher text XGFIOJLF is identified as the word DECEMBER from the pattern of the possible equivalents for a repeated letter (E_p):

```
D E C E M B E R
X G F I O J L F
  G(h)I(j k)L
```

The message is now on the way to solution: when solved, the beginning is found to read "REURAD NUMBER SEVEN FIVE SEVEN DATED ONE EIGHT DECEMBER STOP . . ."; the plain component is found to be the HYDRAULIC . . . XZ keyword-mixed sequence; and the cryptosystem is determined to be a modified progressive alphabet system in which the normal-sequence cipher component moves one step after the encipherment of each letter, with a double step at the end of each word.

(3) For the second example of the delta technique, let us assume that we are faced with traffic in an unknown cryptosystem, but that historical precedent indicates the possibility of cyclic repeating keys. The messages are transmitted in groupings of five dinomes. A typical message, with its associated frequency distribution, is the following:

```
41 70 19 05 97   76 72 58 07 21   11 07 54 46 74   07 40 57 35 56
20 13 07 81 18   44 07 02 56 91   38 22 17 43 57   41 98 34 79 29
76 44 97 96 03   81 06 86 71 68   88 13 16 47 46   13 51 78 29 13
34 05 42 65 56   55 14 72 46 25   81 21 44 11 61   85 16 16 42 65
52 44 41 08 59   26 14 00 90 07   30 16 28 09 43   51 30 89 12 23
82 06 32 39 09   49 25 98 15 22   88 33 26 65 64   42 46 76 53 53
28 28 33 17 32   41 36 96 20 01   41 87 72 60 16   31 65 28 39 74
49 68 51 99 72   24 55 63 16 86   33 37 42 82 27   58 28 70 98 10
14 48 31 90 72   78 61 19 67 22   92 26 44 37 29
```

00 1	10 1	20 2	30 2	40 1	50	60 1	70 2	80	09 2
01 1	11 2	21 2	31 2	41 5	51 3	61 2	71 1	81 3	91 1
02 1	12 1	22 3	32 2	42 4	52 1	62	72 5	82 2	92 1
03 1	13 4	23 1	33 3	43 2	53 2	62 1	73	83	93
04	14 3	24 1	34 2	44 5	54 1	64 1	74 2	84	94
05 2	15 1	25 2	35 1	45	55 2	65 4	75	85 1	95
06 2	16 6	26 3	36 1	46 4	56 3	66	76 3	86 2	96 2
07 6	17 2	27 1	37 2	47 1	57 2	67 1	77	87 1	97 2
08 1	18 1	28 5	38 1	48 1	58 2	68 2	78 2	88 2	98 3
09 2	19 2	29 3	39 2	49 2	59 1	69	79 1	89 1	99 1

The I.C. of $\frac{100(312)}{175\times174}=1.025$ is not informative; there are no polygraphic repetitions of significance; and writing the text on widths has produced nothing of consequence. We shall now try deriving delta streams; first, a delta at an interval of one dinome (with mod 10 arithmetic):

```
C:  41 70 19 05 97   76 72 58 07 21   11 07 54 46 74   07 40 57 35 56 . . .
Δ:     39 49 96 92   89 06 86 59 24   90 96 57 92 38   33 43 17 88 21
```

The entire delta stream at this interval reveals nothing of interest, so we next take a delta stream at an interval of two dinomes, deriving the following Δ_2 stream:

```
C:   41 70 19 05 97   76 72 58 07 21   11 07 54 46 74   07 40 57 35 56 . . .
Δ2:        78 35 88   71 85 82 35 73   14 86 43 49 20   61 76 50 95 09
```


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This stream too shows nothing; but when we finally take a Δ_{12}, we obtain the following:

C:	41	70	19	05	97	76	72	58	07	21	11	07	54	46	74	07	40	57	35	56
Δ_{12}:													13	76	65	02	53	81	63	08 →
C:	20	13	07	81	18	44	07	02	56	91	38	22	17	43	57	41	98	34	79	29
Δ_{12}:	← 23	92	96	84	64	08	33	05	16	44	03	76	97	30	50	60	80	90	72	27
C:	76	44	97	96	03	81	06	86	71	68	88	13	16	47	46	13	51	78	29	13
Δ_{12}:	20	53	69	74	96	48	59	45	83	34	19	94	40	03	59	27	58	97	23	37
C:	34	05	42	65	56	55	14	72	46	25	81	21	44	11	61	85	16	16	42	65
Δ_{12}:	63	47	64	52	40	18	78	69	95	57	62	18	10	16	29	20	60	61	38	93
C:	52	44	41	08	59	26	14	00	90	07	30	16	28	09	43	51	30	89	12	23
Δ_{12}:	16	29	60	87	15	15	53	25	84	91	98	51	76	65	02	53	81	63	08	23 →
C:	82	06	32	39	09	49	25	98	15	22	88	33	26	65	64	42	46	76	53	53
Δ_{12}:	← 92	09	02	23	81	40	82	47	85	43	76	10	44	69	32	13	47	37	38	65
C:	28	28	33	17	32	41	36	96	20	01	41	87	72	60	16	31	65	28	39	74
Δ_{12}:	13	06	55	84	16	86	72	54	84	35	98	34	54	42	83	24	33	87	03	88
C:	49	68	51	99	72	24	55	63	16	86	33	37	42	82	27	58	28	70	98	10
Δ_{12}:	29	67	10	12	00	64	49	32	51	68	04	63	03	24	76	69	56	56	43	57
C:	14	48	31	90	72	78	61	19	67	22	92	26	44	37	29					
Δ_{12}:	08	62	08	63	30	96	44	61	49	52	04	16	30	99	98					

The 9-dinome repetition revealed in the Δ_{12} stream is proof (1) that there is a 21-dinome identical underlying plaintext passage at the positions indicated by the two occurrences of the repetition, and (2) that, if this be a system involving a cyclic repeating numerical key, the length of this key is 12 dinomes.[26]

[26] Note that the period here could not have been revealed by any other means, say writing the cipher text on this width—the underlying text is not rough enough to yield a significant I.C. on the correct width.

The solution from here on is straightforward,[27] based on the reduction of the periodic cryptogram to monoalphabetic terms. For example, if in the fragment of the worksheet below, the cipher dinome 46 at position B2 is assumed to be (arbitrarily) 00_p, then the cipher dinome 28 at position H9 must also represent 00_p, since the delta repetition has already proved the location of the identical underlying plain text. The deriviation of the additives from these decipherments enables the recovery of other plaintext dinomes, finally culminating in the recovery of the entire 12-dinome keying sequence (to a relative base) and the reduction of the cipher text to monoalphabetic terms. (The completion of the solution of this problem, including the recovery of the plain text, is left to the interested reader.)

		1	2	3	4	5	6	7	8	9	10	11	12
	K:		46							28			
A.	(C)	41	70	19	05	97	76	72	58	07	21	11	07
	(P)												
B.	(C)	54	46	74	07	40	57	35	56	20	13	07	81
	(Δ)	13	76	65	02	53	81	63	08	23	92	96	84
	(P)		00										
						*	*	*	*	*			
G.	(C)	44	11	61	85	16	16	42	65	52	44	41	08
	(Δ)	10	16	29	20	60	61	38	93	16	29	60	87
	(P)												
H.	(C)	59	26	14	00	90	07	30	16	28	09	43	51
	(Δ)	15	15	53	25	84	91	98	51	76	65	02	53 →
	(P)									00			
I.	(C)	30	89	12	23	82	06	32	39	09	49	25	98
	(Δ)	← 81	63	08	23	92	09	02	23	81	40	82	47
	(P)												
						*	*	*	*	*			
O.	(C)	67	22	92	26	44	37	29					
	(Δ)	49	52	04	16	30	99	98					
	(P)												

(4) Related to the process of the derivation of delta streams is the process of *autokeying,* which may uncover phenomena when this principle has been incorporated in the cryptosystem. Taking as an illustration the message beginning below,

3667 8406 8424 5012 4749 1868 9463 3122 2118 9456 . . .,

if we perform a *ciphertext autokeying* decipherment, the key is actually at hand, since it is the cipher text itself, but it must be displaced against itself at an interval corresponding to the length of the introductory key. Under the assumption of an introductory key of one digit, the matched cipher, key, and resultant plain text are shown below:

K:	366	7840	6842	4501	2474	9186	8946	3312	2211	8945	
C:	3667	8406	8424	5012	4749	1868	9463	3122	2118	9456	. . .
P:	-301	1666	2682	1511	2375	2782	1527	0810	0907	1511	

It is obvious that the unknown intermediate text is 4-digit code, with a limitation of 0, 1, and 2 in the first position of code groups. Although the foregoing example with an introductory key of one digit is a trival case of ciphertext autokey, an initial key of unknown length would not pose any problems. Taking the case of the message beginning

3442 7900 4372 1948 4469 0128 0539 8863 9783 4489 . . . ,

[27] Cf. subpars. 83*g–j* on pp. 230–235 of *Military Cryptanalytics, Part II.*

all that would be required is to try various offsets of the cipher text against itself, and one of the decipherments would be the actual plain text (to true base, of course); this is shown in the diagram below:

K(5):		344	2790	0437	2194	8446	9012	8053	9886	3978
K(4):		3442	7900	4372	1948	4469	0128	0539	8863	9783
K(3):	3	4427	9004	3721	9484	4690	1280	5398	8639	7834
K(2):	34	4279	0043	7219	4844	6901	2805	3988	6397	8344
K(1):	344	2790	0437	2194	8446	9012	8053	9886	3978	3448
C:	3442	7900	4372	1948	4469	0128	0539	8863	9783	4489 . . .
P(1):	–108	5210	4945	9854	6023	1116	2586	9087	6815	1041
P(2):	––18	3731	4339	4739	0625	4227	8734	5985	3496	6145
P(3):	–––9	3583	5378	8227	5085	6538	9359	3575	1154	7655
P(4):	––––	4568	7472	7676	3521	6769	0411	8334	1920	5706
P(5):	––––	–666	2682	1511	2375	2782	1527	0810	0907	1511

On the trial of an offset of 5, the underlying 4-digit plain code is revealed. (The missing first 5 digits would have to be determined by context after a volume of code text has been solved, or by analysis of the beginnings of messages.)

(5) In deriving a stream by the *plaintext autokeying* method, the "plain text" resulting from a decipherment at the correct offset will either be the true plain, or one of 9 equivalent versions. For example, in the message beginning below,

0331 2722 8840 3662 3502 7950 3679 7891 0997 8662 . . .,

the decipherment at a one-digit offset is predicated on the supposed identity of the first plaintext digit. If we arbitrarily assume that the first plaintext digit is 3, we obtain the following decipherment:

K:	303	8439	3531	9424	8500	2541	9425	4354	7363	4424
C:	0331	2722	8840	3662	3502	7950	3679	7891	0997	8662 . . .
P:	3038	4393	5319	4248	5002	5419	4254	3547	3634	4248

(Note the trinomic repetition in the original cipher text, which becomes a tetranomic repetition when treated by the plaintext autokey method.) If we "complete the plain-component sequence" from this decipherment by going down the columns in alternate directions (down the normal numerical sequence in the odd columns, and down the reversed normal sequence in even columns), as shown in Fig. 169, below, we would recognize in the asterisked row the properties of code text having the limitation of 0, 1, and 2 in the *a* position of the code groups. The foregoing case of plaintext autokey with a one-digit introductory key was quite simple; if the initial key were much longer, and the nature of the intermediate text unknown, the problem becomes exceedingly difficult—in fact, it is only fairly recently that a method of solution has been discovered.[28]

"P":	3038	4393	5319	4248	5002	5419	4254	3547	3634	4248
	3038	4393	5319	4248	5002	5419	4254	3547	3634	4248
	4947	5202	6228	5157	6911	6328	5163	4456	4543	5157
	5856	6111	7137	6066	7820	7237	6072	5365	5452	6066
	6765	7020	8046	7975	8739	8146	7981	6274	6361	7975
	7674	8939	9955	8884	9648	9055	8890	7183	7270	8884
	8583	9848	0864	9793	0557	0964	9709	8092	8189	9793
	9492	0757	1773	0602	1466	1873	0618	9901	9098	0602
	*0301	1666	2682	1511	2375	2782	1527	0810	0907	1511
	1210	2575	3591	2420	3284	3691	2436	1729	1816	2420
	2129	3484	4400	3339	4193	4500	3345	2638	2725	3339

FIGURE 169

[28] See Lambros D. Callimahos, "The Analysis of Digital Plaintext Autokey Systems," appearing in the *NSA Technical Journal*, Vol. XIV, No. 2, Spring 1969.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

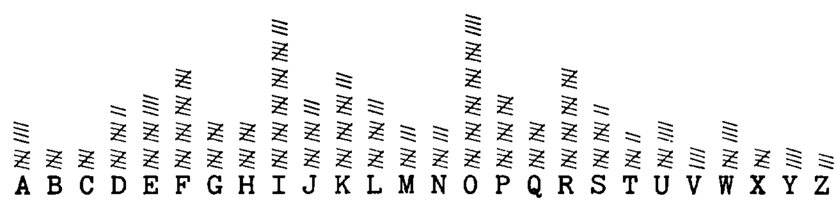
77. **Second step: recognizing the phenomena.**—*a*. The manipulative processes described in the preceding paragraph are undertaken in order to disclose nonrandom characteristics or manifestations. The second step in diagnosis is the *recognition* of the nonrandom characteristics or manifestations when disclosed. When, for example, we make a uniliteral frequency distribution of cipher text we are really hoping that some roughness or other property will be manifested—otherwise we wouldn't even have bothered to make the distribution. And after making the distribution, what's the use of it if we don't recognize the fact that it is rough? The recognition of phenomena, then, implies some sort of measurement, either by eye (relying on past experience and visual memory), or by statistical means, which takes it out of the realm of the subjective. After finding a phenomenon—or what is thought to be a phenomenon—an evaluation should follow. It's not enough to say "I think the doublets are high." How many are there? *Count them.* And, even more important, *how many doublets are expected at random* in a sample of this size?

[32] Note the repeating key, HEAVY WATER, manifested in one of the columns of Fig. 171.

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

b. Some phenomena cannot really be measured (or measured accurately) but permit of their recognition because of either past experience or intuitive feeling of something that's "not quite right." For example, let us examine the following intercepted message and its frequency distribution:

```
FOROF JIEOF KGMVQ IFORR IFRWM  ASEFL QPOIE XCZIF ASDOF GHIRF
QWERP PIFRI FORFO ASDIK JDGHJ  ASPQE POIRP MNCCO ASDFK QPOIR
OIKOP ASDEW UIOKP KOIJL POIEQ  MNBVX IKFRI ROEWJ KIOLK LPOIU
RETYU UINMH GHTYU IOKLP REDFS  QAWSE DRFTG GYHUJ JIKOL ZSERD
XDRTF QAWSZ RDXCF VGYUH NJIOK  MLOKN LPOOP LKJHB FGJHK CVBNM
QPWOL EORIK TIORI TUGJI TUHDG  NJKHF LKOIL SEWDX QLASW NBJMB
```



The δ I.C. here is $\frac{26(4332)}{300\times 299}=1.256$, which represents a sigmage of 10.85σ,[33] highly significant indeed. In subjecting the cryptogram to close scrutiny, we find many points of considerable interest. For example, (1) we observe only 7 doublets and 6 A–A doublets when we expect 14.4 of each; (2) there are 7 tetragraphic repetitions (IFOR, LPOI, OASD, POIR, QAWS, QPOI, UIOK) when we expect not quite a tenth (0.096) of one; (3) there are 23 trigraphic repetitions when we expect only 2.48; (4) on a width of 5, the average I.C. is 1.44, a deviation of 3.0σ, and on a width of 15 the average I.C. is 1.52, a deviation of 2.5σ. We could find other statistical phenomena with which to build up an impressive case for some esoteric cryptosystem, the likes of which had not been seen by man before; but an experienced cryptanalyst

[33] The standard deviation of the δ I.C. is given by the formula $\sigma=\frac{\sqrt{2(c-1)}}{\sqrt{N(N-1)}}$ where c is the number of categories, and N is the sample size. Thus, in this example, $\sigma=\frac{\sqrt{2(26-1)}}{\sqrt{300\times 299}}=.0236$. The sigmage (S) is obtained by dividing the standard deviation into the "bulge" of the I.C., with the formula $S=\frac{\delta-1.00}{\sigma}$, so in this case $S=\frac{0.256}{.0236}=10.85\sigma$. The sigmage of the δ I.C. is interpreted with the chi-square distribution; in this case, 10.85σ may be found to represent about 1 chance in 20,000,000,000 of having occurred at random. The table below gives the interpretation of sigmages from 1 to 5, according to the chi-square and Normal distributions, whichever is applicable in the particular case; the sigmage in the first column will be equaled or exceeded by pure chance, on the average, once every χ times as shown in the table.

Values of x

S	Chi-square distribution		Normal distribution
	c=10	c=26	
1	6. 6	6. 4	6. 3
1½	12	13	15
2	24	28	44
2½	49	66	161
3	102	164	740
3½	220	435	4, 298
4	483	1, 210	31, 500
4½	1080	3, 520	294, 000
5	2470	10, 650	3, 488, 000

(or a younger one who is destined to make his mark in the art) could, without too much difficulty, identify the cryptogram for what it is.[34]

c. The importance of evaluating distributions and repetitive phenomena cannot be overstressed. Let us take as an example the case of 40 messages in a certain unknown system that was under study. The text groups, after the discriminants and indicators were deleted, totalled 9717 letters with the following distribution:

A 356	G 403	L 361	Q 334	V 372
B 347	H 398	M 381	R 395	W 306
C 318	I 347	N 421	S 484	X 372
D 369	J 325	O 401	T 409	Y 418
E 386	K 379	P 313	U 359	Z 375
F 388				

We have also observed 91 repeated tetragraphs and 5 repeated pentagraphs in the traffic, which strikes us as being probably considerably above the random expectations; there are no other longer repetitions. The δ I.C. here is $\frac{26(3{,}659{,}986)}{9717\times9716}=1.0079$, certainly "flat-looking." But we must not forget that the significance of the δ I.C. is tied to the sample size, so we should go ahead and compute the sigmage. The standard deviation for this example is $\frac{\sqrt{2(26-1)}}{\sqrt{9717\times9716}}$, which for all practical purposes becomes $\frac{7.0711}{9717}=$ .00073. Therefore the sigmage is $\frac{.0079}{.00073}=10.8\sigma$—and we revise our estimate of the relative flatness of the distribution. As for the polygraphic repetitions, it would be wise to take a closer look at them. In arriving at the Poisson probability, we first compute $a=Np$ (where N is the sample size and p is the probability of the item under investigation), so for tetragraphs $a=\frac{9717}{26^4}=.0213$, rounded off to .02 for looking up in the tables, and for pentagraphs $a=\frac{9717}{26^5}=.00082$, rounded off to .001. Under the entry of $a=.02$ we find the probability of .0001960 for a two-fold occurrence (i.e., a repetition) of a tetragraph, and we multiply this probability by 26^4 to obtain the number of tetragraphs expected at random in a sample of this size; under the entry of $a=.001$ we find the probability of .0000005 for a repeated pentagraph, and we multiply this probability by 26^5 to obtain the number of pentagraphs expected at random. Our findings are summarized below:

Tetragraphs: $a=\frac{9717}{26^4}=.0213\ (\approx.02)$; Poisson probability $=.0001960$

Expected at random $=.0001960(26^4)=89.6$; observed $=91$

Pentagraphs: $a=\frac{9717}{26^5}=.00082\ (\approx.001)$; Poisson probability $=.0000005$

Expected at random $=.0000005(26^5)=5.5$; observed $=5$

Random on the nose. Statistics *are* useful. This means that, if the system is to be eventually solved, it will not be solved on the basis of its polygraphic repetitions.

d. We have already stated, not in so many words, that the distribution of the δ I.C. is asymptotic to the chi-square distribution with c–1 degrees of freedom; in plain English, this means that the distribution of the δ I.C. is approximated (and thus may be interpreted) by the χ^2 distribution. The ξ I.C.,

[34] A practice message, consisting of random typing of 5-letter groups on a typewriter. Note groups of the text, typical of typewriter random, composed of keyboard sequences and patterns (such as ASDFK, LPOIU, and QWERP, alternating hand patterns (such as FOROF, GHTYU, and QPWOL), and one-hand patterns (such as DRFTG, KIOLK, and REDFS).

however, is approximately Normally distributed, and thus has different formulas for standard deviation and sigmage, as shown below:[35]

(1) The ξ I.C. has the formula $\xi=\frac{c\Sigma f_1f_2}{N_1N_2}$, where c is the number of categories, Σf_1f_2 the sum of the cross products of the corresponding frequencies, and N_1 and N_2 the sizes of the two distributions being compared. If we recall the formula for the γ I.C. (see footnote 22 on p. 345), $\gamma=\frac{c\Sigma f^2}{N^2}$. Now if we compute the γ I.C. for each of the two distributions being matched, the standard deviation of the ξ I.C. is given by the formula $\sigma=\frac{\sqrt{(\gamma_1-1)(\gamma_2-1)}}{\sqrt{c-1}}$. The sigmage of the ξ I.C. is then given by the formula $S=\frac{\xi-1}{\sigma}$ or, in another form, $S=\frac{(\xi-1)\sqrt{c-1}}{\sqrt{(\gamma_1-1)(\gamma_2-1)}}$.

(2) As an example of the use of these formulas, let us take the ξ I.C. of the following two distributions

1. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z N=57

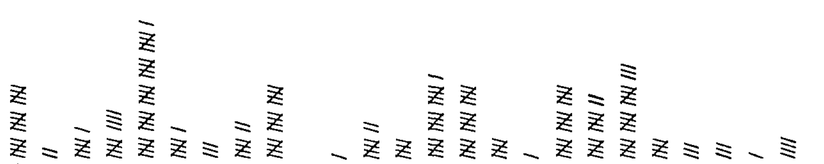
2. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z N=55

Σ 9 1 4 10 28 0 4 16 0 0 35 3 1 1 3 12 12 4 10 42 1 =196

The ξ I.C. here is $\frac{26(196)}{57\cdot 55}=1.63$. The γ I.C. of the first distribution (γ_1) is $\frac{26(219)}{57^2}=1.75$; that of the second ($\gamma_2$) is $\frac{26(239)}{55^2}=2.05$; therefore the standard deviation of our ξ I.C. is $\frac{\sqrt{(1.75-1.00)(2.05-1.00)}}{\sqrt{26-1}}=\frac{\sqrt{(0.75)(1.05)}}{5}=\frac{0.8874}{5}=.178$. The sigmage of the ξ I.C. is $\frac{1.63-1.00}{0.178}=3.5\sigma$. The evaluation of this sigmage follows the Normal distribution, so that 3.5σ represents 1 chance in 4298 of having occurred at random.

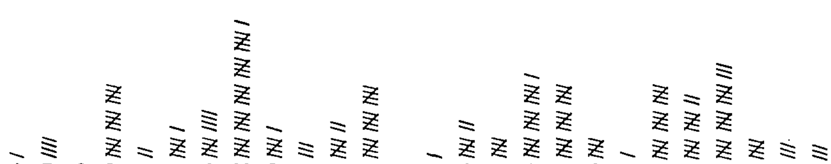
e. Certain cryptosystems have characteristic uniliteral frequency distributions that make them readily identifiable from their distributions. A few examples will serve to stress the need for the cryptanalyst's broad experience and contact with many systems, and the need for tucking away in one's mind the phenomena associated with the systems he has encountered, either personally or vicariously.

(1) The simplest case of all, transposition of plain text, is easily recognizable by its frequency distribution profile, which matches that of the particular language involved. Thus, for English plain text, the theoretical profile for 200 letters is the following:



A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

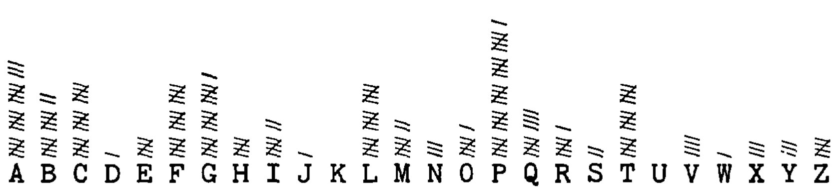
The quick earmarks of a transposition cipher are (1) the high vowel count (approximately 40%) as compared with that of random text (23%), and (2) the low frequency of certain letters of the language involved (in English, the combined frequency of JKQXZ=1.4%). If we obtain the following distributional profile,



A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

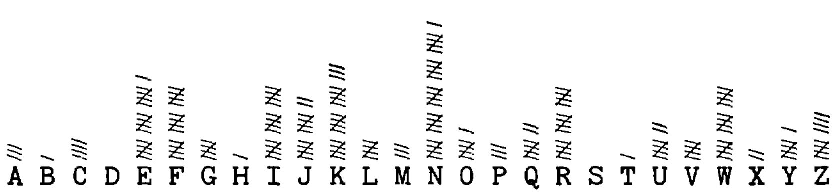
[35] Adapted from Howard H. Campaigne, *The Index of Coincidence*, National Security Agency, 1965.

we diagnose it immediately as that of a direct standard alphabet encipherment, since what we have is the normal profile displaced by three positions ($A_p=D_c$). Likewise, the profile

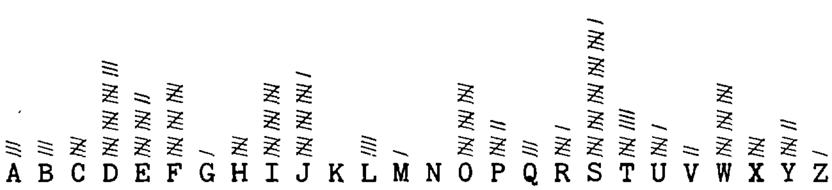


reveals itself as that of a reversed standard alphabet encipherment, since it can be recognized that it is the reversed normal profile, at a slide of $-6(A_p=T_c)$.

(2) Although Porta systems[36] are rarely encountered, nevertheless we should still know enough to recognize immediately distributions arising from encipherment by a Porta table. In the normal Porta, if we draw a vertical line between the M and N of the distribution, we can fit a cyclic permutation of the first half (A–M) of the normal sequence in the *right* half of the distribution, and we can fit a cyclic permutation of the last half (N–Z) of the normal sequence in the *left* half of the distribution; moreover, since Porta encipherment is reciprocal, establishing an identity in one half of the distribution automatically establishes an identity in the other half. For example, in the theoretical distribution below,

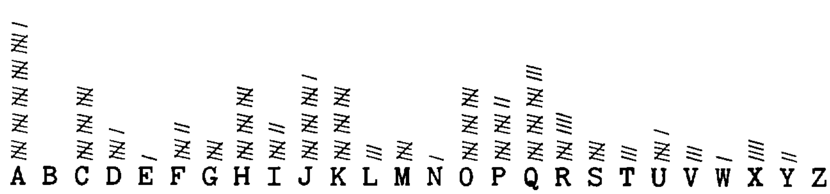


it is clear that the plaintext sequence A . . . E . . . I is at cipher W . . . N . . . R, and that the plaintext sequence RST is at cipher IJK; furthermore, establishing one letter, say $N_c=E_p$, then the reciprocal value $N_p=E_c$ must obtain. In complementary Porta (in which the A–M sequence of the table runs in the opposite direction from the N–Z sequence) the direction of fitting the plaintext sequences to the cipher must be reversed, as in the following example:



Here the plaintext sequence A . . . E . . . I is at cipher W . . . S . . . O, and the plaintext sequence RST is at cipher FED.

(3) A monoalphabetic distribution is recognized by its roughness, which will be that of the underlying text. For English, the expected δ I.C. is 1.73, and a 200-letter distribution involving a mixed alphabet might look as follows:



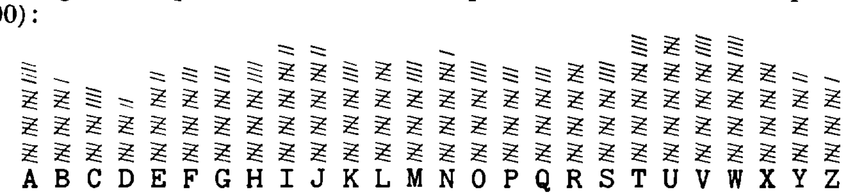
[36] Cf. par. 23 (on pp. 45–47) of *Military Cryptanalytics, Part II.*

The monographic roughness will prevail not only in monoalphabetic substitution, but also in polyalphabetic substitution after the ciphertext letters have been properly allocated into the respective cipher alphabets to which they belong, since this has the effect of reducing the polyalphabetic text to monoalphabetic terms. If this step has not been correctly performed, the resultant distributions will be a merger of two or more monoalphabetic distributions; and if the distributions which were merged are of equal size, the expected δ I.C. is 1.00 plus $1/m$ of the bulge of the I.C. of the underlying text, where m is the number of distributions that were merged. Thus, if a 300-letter periodic polyalphabetic cipher of a period of 30 were incorrectly assumed to have a period of 15, the expected I.C. of the distributions (and therefore also that of the over-all width write-out) would be $1.00+\frac{.73}{2}=1.37$; or if the period were mistakenly assumed to be 10, the expected I.C. would be $1.00+\frac{.73}{3}=1.24$. Such indices can therefore be a guide to what is happening, because if we had written out our cipher text on a width of 10 and obtained an I.C. of 1.24, on observing that this has one-third the bulge expected for English plain text we might have suspected that the true period is 30 and not 10 as assumed.

(4) The Gronsfeld system,[37] in which each plaintext letter has as its cipher equivalent either itself or one of the nine letters following it in the normal alphabetical sequence, has a characteristic frequency distribution even (or especially so) if the source of key is a large book of random digits. The theoretical relative frequencies are shown below, for English underlying text:

A 3.84	G 3.56	L 3.96	Q 3.58	V 4.80
B 3.18	H 3.85	M 3.90	R 4.00	W 4.71
C 2.88	I 4.40	N 4.28	S 3.88	X 3.96
D 2.38	J 4.40	O 3.73	T 4.78	Y 3.40
E 3.42	K 3.70	P 3.71	U 5.01	Z 3.14
F 3.55				100.00

The γ I.C. of this distribution (multiplying the entries above by 100) is $\frac{26(3,940,534)}{(10,000)^2}=1.0245$, which is the characteristic roughness expected of a Gronsfeld cipher.[38] The distributional profile is the following (on a base of 500):



[37] Cf. par. 24 (on pp. 47–48) of *Military Cryptanalytics, Part II.*

[38] We use the γ I.C. here because we are assuming the composition of the universe or total population.

A quick test for identifying a Gronsfeld cipher is as follows: the four highest cipher letters (T, U, V, W) have a combined theoretical frequency of 19.3%, and the two lowest cipher letters (C and D) have a combined frequency of 5.26%, so the ratio $\frac{\text{TUVW}}{\text{CD}}$ of the cipher letters is $\frac{19.3}{5.26}=3.67$, as compared with the ratio $\frac{4/26}{2/26}=2$ for a random (i.e., equiprobable) population. This index, 3.67, may be used as an expected value in a suspected case of Gronsfeld encipherment. As an example, let us consider the following distribution derived from an intercepted 1100-letter message:

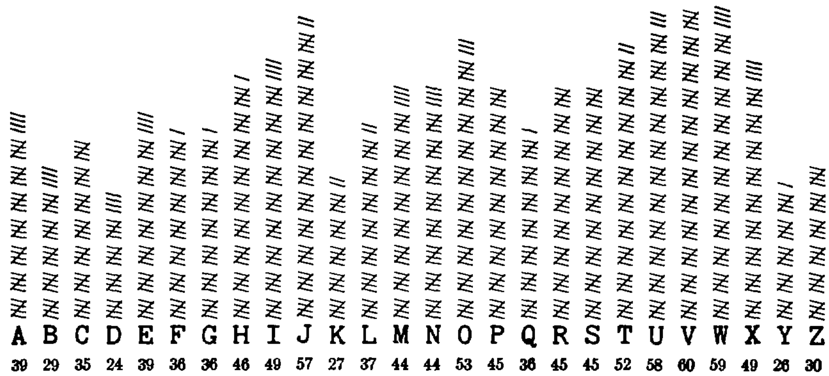


FIGURE 172

The δ I.C. is $\frac{26(48{,}218)}{1100(1099)}=1.037$; the standard deviation is $\frac{7.0711}{1100}=.0064$; and the sigmage is $\frac{.037}{.0064}=5.8\sigma$. The Gronsfeld index of $\frac{\text{TUVW}}{\text{CD}}$ is $\frac{229}{59}=3.88$, supporting the hypothesis that this is a case of Gronsfeld encipherment.

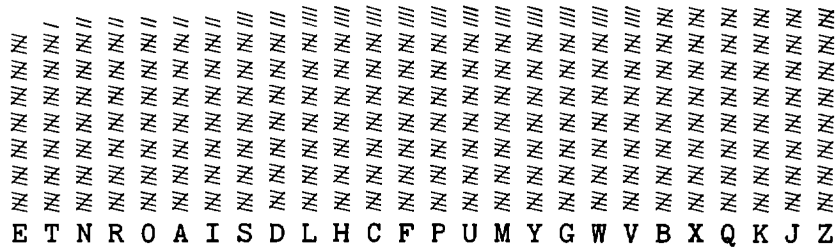
(5) In complex noncrashing systems (e.g., Enigma machine systems) wherein a plaintext letter cannot be enciphered by itself, the 25 cipher equivalents of any plaintext letter are equiprobable, leading to the following theoretical relative ciphertext frequencies for English underlying text:

A	3.70	G	3.94	L	3.86	Q	3.99	V	3.94
B	3.96	H	3.86	M	3.90	R	3.70	W	3.94
C	3.88	I	3.70	N	3.68	S	3.76	X	3.98
D	3.83	J	3.99	O	3.70	T	3.63	Y	3.92
E	3.48	K	3.99	P	3.89	U	3.90	Z	4.00
F	3.88								100.00

The γ I.C. of the cipher text is most easily computed by the formula $\gamma_c=1+\frac{\beta_p}{(c-1)^2}$, where β_p is the bulge of the I.C. of the plain (i.e., $\gamma_p-1.00$) and c is the number of categories; therefore $\gamma=1+\frac{.73}{(26-1)^2}=1.0012$. A property of such systems is that, if the distribution of the cipher letters is made in the descending frequency order of the *plaintext* letters of the language, the slope of frequencies of the ciphertext letters will be that of a gradual ascent; thus:

E	3.48	I	3.70	C	3.88	Y	3.92	X	3.98
T	3.63	S	3.76	F	3.88	G	3.94	Q	3.99
N	3.68	D	3.83	P	3.89	W	3.94	K	3.99
R	3.70	L	3.86	U	3.90	V	3.94	J	3.99
O	3.70	H	3.86	M	3.90	B	3.96	Z	4.00
A	3.70								100.00

Or, in tally-mark form (to a base of 1000 letters):



The slope is so gradual that usually about 8000 letters are required for a positive determination. As an example, let us study the following combined frequency distribution of a number of messages, totalling 4306 letters:

167	171	189	152	141	159	194	152	146	162	154	167	165	137	150	182	189	138	172	187	163	172	159	179	174	185
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The δ I.C. is $\frac{26(715{,}628)}{4306\times4305}=1.0037$; the standard deviation is $\frac{7.0711}{4306}=.00164$; and the sigmage is $\frac{.0037}{.0016}$ $=2.3\sigma$. On the hypothesis that this might possibly be a noncrashing encipherment of English plain text, we construct the distribution in the descending frequency order of English plaintext letters; this is shown by the bar graph in Fig. 173, below. Since the distributional sample is small, we try to smooth the rough bar graph by averaging every pair of letters; this is shown in the figure by the line composed of dashes. Then, in order further to improve matters, we average pairs of averages—we're trying very hard to make things come out the way we wish—and this plotting (shown by the line composed of dots) now displays a gradual ascent from left to right, so we have proved our point.

(6) In plaintext autokey systems, and also in other systems in which the key is literal plain text derived from a book or other source, the expected I.C. will show a measurable departure from random. The bulge of the I.C. (i.e., the excess over random) may be calculated by the formula $\beta_c=\frac{\beta_k\times\beta_p}{c-1}$, where β_c is the bulge of the cipher text, β_p the bulge of the plain text (for English, .73), β_k the bulge of the key (which, since the key text is plain text, is also .73), and c is the number of categories (in this case, 26). Thus the expected bulge of this type of traffic, without regard to the identity of the components involved, is $\frac{.73\times.73}{25}=.0213$, so the expected I.C. is 1.02 in the general case. When the components are known sequences, the theoretical distribution for the ciphertext letters may be computed, as well as the I.C. of the specific case. For example, where the components are direct standard alphabets, the relative frequencies are the following (reduced to a base of 1000 letters):

A 40.6	G 45.9	L 41.5	Q 30.9	V 51.7
B 35.3	H 41.8	M 43.5	R 47.2	W 47.3
C 31.6	I 50.5	N 30.1	S 42.2	X 40.9
D 25.0	J 30.1	O 28.0	T 41.5	Y 31.5
E 47.5	K 35.9	P 34.3	U 30.4	Z 35.5
F 39.3				1000.0

The kappa or repeat rate (i.e., the sum of the squares of the probabilities of the cipher letters) is found to be .0398; therefore the theoretical I.C. for direct standard alphabets is 26(.0398)=1.0348. Where the components are reversed standard alphabets, the relative frequencies are the following (to a base of 1000 letters):

A 66.9	G 34.8	L 45.4	Q 37.7	V 34.4
B 39.9	H 33.9	M 37.4	R 38.1	W 44.1
C 34.1	I 31.3	N 46.7	S 31.3	X 32.1
D 32.1	J 38.1	O 37.4	T 33.9	Y 34.1
E 44.1	K 37.7	P 45.4	U 34.8	Z 39.9
F 34.4				1000.0

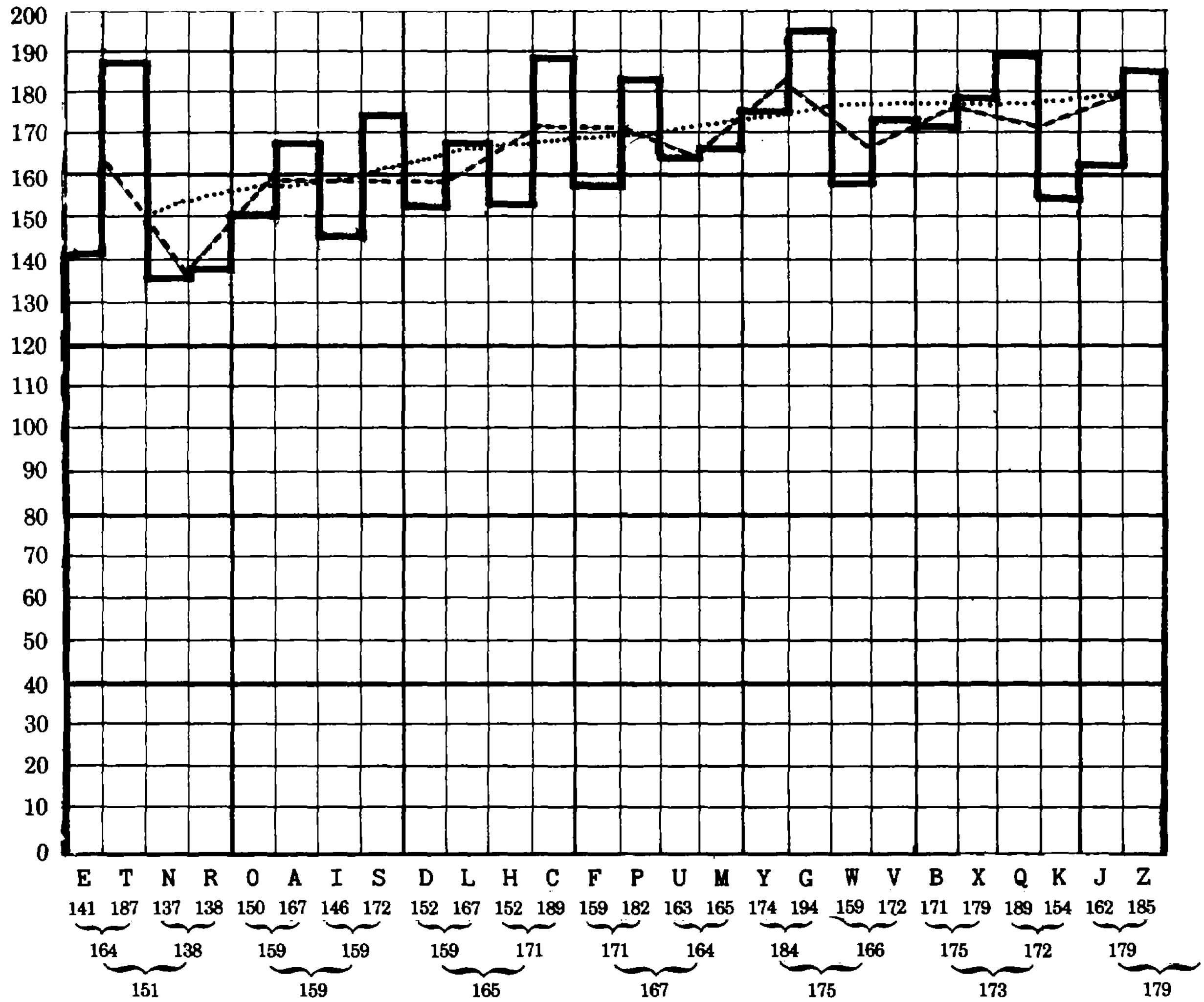


FIGURE 173

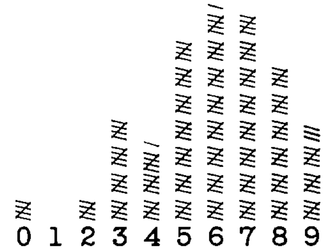
The kappa here too is .0398, so the theoretical I.C. for reversed standard alphabets happens to be the same as that for direct standard alphabets, namely, 1.0348. (Note in particular a striking property of this distribution: not only is the frequency of A_c outstanding, about twice that of the average of the other letters, but the distribution is symmetrical about A_c, so that the frequency of $B_c=Z_c$, $C_c=Y_c$, $D_c=X_c$, etc.) For a quick test to determine whether direct standard alphabets have been used, under the assumption of plaintext keying, we take the ratio $\frac{\text{VIER}}{\text{JNOD}}=\frac{197}{113}=1.7$; and, for testing for reversed standard alphabets, we take the ratio $\frac{\text{ANLP}}{\text{DXIS}}=\frac{204}{127}=1.6$. (The expected value for random in both cases is of course 1.0.) More precise measurement would involve the use of the chi-square test of a sample against the theoretical distribution of the assumed population.

(7) In numerical cryptosystems, one of the most striking distributional profiles is that of the classic Nihilist system,[39] which involves encipherment (usually periodic) by an additive key which is derived from plain text, applied to intermediate text produced by a 5×5 bipartite square with the row- and column coordinates consisting of the digits 1–5 in normal order, the interior of the square containing the

[39] Cf. par. 81 (on pp. 208–218) of *Military Cryptanalytics, Part II.*

206-687 O - 77 - 24

letters A–Z inscribed in straight horizontals (with I and J in the same cell). The following distribution of a 210-digit cryptogram is typical:



The virtual absence of the digit 1, the low frequency of 0 and 2, and the characteristic curve of the digits 3–9 are earmarks of this system, without regard to the length of the additive key. Certain other numerical systems, especially those used without superencipherment, also have distinctive profiles; among the most common of these are monome-dinome systems,[40] particularly those having two or three medium- or low-frequency letters in the top row of the enciphering matrix. Two typical distributions are shown in Figs. 174*a* and *b*, below:

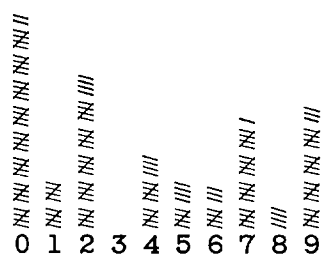


FIGURE 174*a*

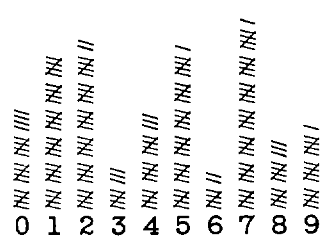


FIGURE 174*b*

The row coordinates of monome-dinome matrices will be found among the digits of highest frequency: in Fig. 174*a*, they are 0 and 2; in Fig. 174*b* they happen to be 1, 5, and 7. The I.C.'s of the matrices will vary, from the lowest possible I.C. of 1.002 to those above 2.00, but in operational cases encountered wherein the top row consists of a key word, the I.C.'s of matrices with two numbered rows are usually in the vicinity of 1.30 to 1.50.[41]

(8) The criterion of 40% vowels should not be the sole basis for the identification of a transposition cipher, because there exist substitution systems in which the vowels are abnormally high in the cipher text. For example, (a) 5-letter codes in which two of the letters in each group are vowels, and (b) certain manual and machine cryptosystems (e.g., the "transpositeur à permutations secrètes," a French commercial cipher device[42]) in which vowels are enciphered by vowels, consonants by consonants. In the former case, the 5-letter code with 2 vowels in each group would be instantly recognized for what it is; if, however, the code groups were further enciphered by a transposition system the phenomena of the individual code groups would be lost, but the over-all text would still contain exactly 40% vowels—in fact, the figure of *exactly* 40%, if consistent in several messages, would be sufficient cause for suspicion.

(9) There have been instances of combined substitution-transposition systems involving standard-alphabet monoalphabetic substitution and single columnar transposition. The frequency distribution would reveal the standard-alphabet encipherment, which when removed would leave as a residue the transposition which can then be easily solved. In those cases involving transposition followed by periodic polyalphabetic substitution with standard alphabets, the period may be determined by writing out the text on various widths, with confirmation by the average δ I.C. of the columns of the correct width.

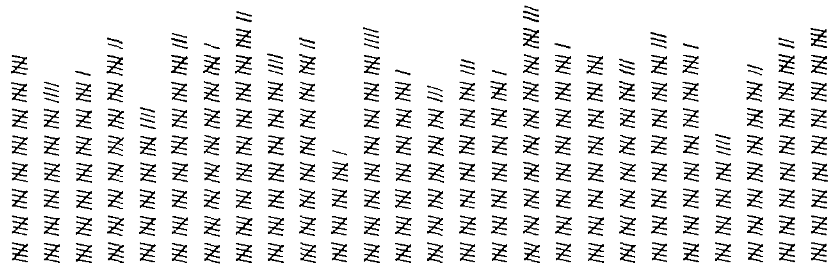
[40] Cf. Chapter X, *Military Cryptanalytics, Part I.*

[41] For a discussion of the I.C.'s of different matrices, see par. 89 (on pp. 261–263) of *Military Cryptanalytics, Part II.*

[42] Cf. par. 76 (on pp. 190–197) of *Military Cryptanalytics, Part II.*

(In favorable cases, there may even be polygraphic repetitions at intervals whose factors give a clue to the period.) The substitution can then be easily removed by fitting the distributions to the normal;[43] what is left, the transposition, can be attacked without complications. If periodic polyalphabetic substitution is *followed* by transposition, the transposition must be removed first—usually a very difficult task—before the residue, the polyalphabetic cipher, can be attacked. Incidentally, it is to be noted that in combined simple substitution and transposition the monographic δ I.C. will not reveal the fact that a transposition is also involved. If, however, we take a *digraphic* I.C., the lack of cohesion in the cipher-text digraphs will give an indication that a transposition is also present; furthermore, the expected polygraphic repetitions typical of monoalphabetic substitution will be destroyed by the transposition.

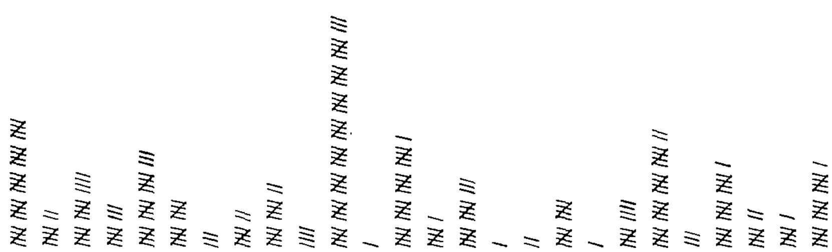
(10) In connection with frequency distributions, it would be well to point out some subterfuges that have been encountered in certain systems. The first example is a 1000-letter composite frequency distribution, taken from several messages in what might be a complex cipher:



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
40	34	36	42	29	43	41	47	39	42	21	44	36	33	38	36	48	41	40	38	43	41	24	37	42	45

It is evident that, no matter what else is involved, the cryptosystem incorporates a 25-letter alphabet (perhaps without a K), thinly disguised here by making about half of the W's into K's. In the second example, shown by the cryptogram below and its accompanying frequency distribution,

```
KABZW WTTMJ KUNDT XCKEA FICOH  QEKEF ZOUAZ CWKOZ TMKIK AEYMR
CTZZE IVKUE OKIAK UCKEW DCKAB  KIAGK ICKIM EZMKO MENDY MRKUZ
EWZKU ECKAU HRKOM UUHDK OCWQA  JKUMK IHUKU RGIGA RCAFU HYMEW
DFHUW KAYOT RKUJA DBVMZ KOXDW  FAZNC KEAFU HWKAW KATSK AKUYM
ZNNZO KIAKU WMAFA FKURT MFEWK  ARTFO MUXMZ RBVCR XOCKA BKIMY
DWKAW EZMKO EMNBM XMZLK EAKUB  WICKU JPEXX
```



A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

it is clear that what we have is an *expanded* alphabet of 30 characters represented by 26 letters: K is always followed by one of five vowels, and these K-digraphs are to be treated as single characters.[44]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

1st letter		2d letter
B	A E I O U Y	
C	. A E I O U Y	
D	. . A E I O U Y	
⋯	⋯	
W	. . . . A E I O U Y .	
X	. . . . A E I O U Y	
Z	Y . . . . A E I O U	

3d letter	4th letter
B C D E F G H I K L M N O P Q R S T U V W X Y Z	A
C D E F G H I K L M N O P Q R S T U V W X Y Z A	B
D E F G H I K L M N O P Q R S T U V W X Y Z A B	C
⋯	
Y Z A B C D E F G H I K L M N O P Q R S T U V W	X
Z A B C D E F G H I K L M N O P Q R S T U V W X	Y
A B C D E F G H I K L M N O P Q R S T U V W X Y	Z

Only a few messages in this system were intercepted and the code was never broken; the only observation of any importance was that every message contained the group COVQ at least once. It now looks as if Message "A" is a monoalphabetic substitution of a message in this code, and Message "B" a transposition of the basic code text.

(2) In each of the two distributions the frequencies of 10, 9, 7, and 5 occur only once; therefore in Message "A" we can make the decipherments $D_c=C_p$, $C_c=I_p$, $W_c=X_p$, and $F_c=L_p$, as shown in the work sheet below:

	1	2	3	4	5	6	7
A	D A W S	D C F L	M S U M	W X Z H	F A A S	P S O W	T X I F
	C X	C I L		X	L	X	L
B	B S K D	J C F W	B A S G	<u>I C K U</u>	W H R E	F X P R	<u>I C K U</u>
	C	I L X		I	X	L	I
C	J A M D	Q A X L	D K D W	E C Q C	R C X Q	Q C D E	I X Z K
	C		C C X	I I	I	I C	
D	J H K D	D K T N	D A E L	W A C K			
	C	C	C	X I			

The crib COVQ has only two possible placements, at D2 or D3; the frequencies of the cipher letters K, T, and N match exactly the frequencies of the plaintext letters O, V, and Q in Message "B", so these decipherments may be made. Furthermore, in the six vowel equivalents (A, C, H, K, S, X) of Message "A" we can identify $H_c=A_p$ by comparing the frequency distributions of the two messages; and since K_c (which has a frequency of 8) has already been identified as O_p from the COVQ crib, then the only other remaining cipher letter with a frequency of 8, A_c, must therefore represent E_p. Our work sheet will now look as follows:

```
      1        2        3        4        5        6        7
A|D A W S  D C F L  M S U M  X W Z H  F A A S  P S O W  T X I F
 |C E X    C I               X Y   A    E E          X  V Y

B|B S K D  J C F W  B A S G  I C K U  W H R E  F X P R  I C K U
 |    O C    I   X    E        I O    X A        Y        I O

C|J A M D  Q A X L  D K D W  E C Q C  R C X Q  Q C D E  I X Z K
 |  E   C    E Y    C O C X    I   I    I Y      I C      Y   O

D|J H K D  D K T N  D A E L  W A C K
 |  A O C  C O V Q  C E      X E I O
```

(3) From here on the solution of Message "A" is automatic. The partially recovered groups CEX–, XY–A, and –AOC can now be completed as CEXU, XYZA, and NAOC by means of the permutation table, and these decipherments will snowball into a complete reconstruction of the plain code message:

```
C E X U  C I L G  P U F P  X Y Z A  L E E U  S U R X  V Y H L
J U O C  N I L X  J E U M  H I O F  X A D K  L Y S D  H I O F
N E P C  T E Y G  C O C X  K I T I  D I Y T  T I C K  H Y Z O
N A O C  C O V Q  C E K G  X E I O
```

The substitution alphabet, *which should have been compiled concurrently with the reconstruction of the plain text*,[47] is found to be as follows:

```
P:  A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
C:  H Y D R A U L I C B E F G J K M N O P Q S T V W X Z
```

Having reconstructed the underlying plain code of Messages "A" and "B", we can now recover the transposition rectangle used to encipher Message "B", together with the numerical key:[48]

21	13	10	18	6	5	16	19	1	20	7	17	14	9	2	12	8	15	11	4	3
C	E	X	U	C	I	L	G	■	P	U	F	P	X	■	Y	Z	A	L	E	■
E	U	S	U	■	■	R	X	V	Y	H	L	J	U	O	C	N	I	L	■	X
J	E	U	M	H	I	O	F	X	A	■	D	K	■	L	Y	■	S	D	H	I
O	F	■	N	E	P	C	T	E	Y	G	C	O	C	X	■	K	I	■	T	I
D	■	I	Y	T	T	I	C	K	H	Y	Z	■	O	N	A	O	■	C	C	O
V	Q	C	■	E	K	■	G	X	E	I	■	O								

It will be observed that the blank cells in the matrix are systematically placed: in the first row, at key positions 1, 2, and 3; in the second row, at 4, 5, and 6; and so forth, with perhaps blanks in a possible seventh row at positions 19, 20, and 21 for this particular key. The blanks in the matrix vitiate any attempts at anagramming the columns, which would have been possible with the COVQ crib had there been no blanks. Thus neither message is solvable individually, but they are vulnerable when taken together.

[47] This sin of omission was purposely committed in this example so that the more general technique of solution, which would not rest on the systematic construction of the enciphering alphabet, could be demonstrated.

[48] The reader might like to recover the literal key from which this numerical key was derived. See in this connection pp. 427–429 of *Military Cryptanalytics, Part II.*

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

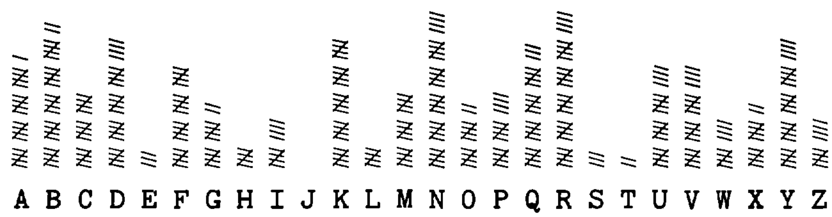
(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(6) Distributional properties, limitations, and associations provide valuable insights into an unknown cryptosystem. Let us study the following message and its accompanying uniliteral distribution:

```
B D A U B   U A R A D   L D Z R B   C K Q F N   F M Q V B   R Z D N B
Y Q Z U R   B M W Z C   C E N A E   U D A L G   B N N Z L   U F P E M
R X Q A K   C N K V D   K G K D U   F W U B Q   D Z R Z P   Y R K G Y
C Y W G Z   N B P B N   A M Y N D   W T C K P   A Q R V A   N A G G K
X N F Q N   B D B M F   I M D K X   D Q Y A M   D Y K C P   S F R Y U
Q V V R R   I U W Q Y   M S F U Q   X K P Y D   V N M A R   Y B Q W N
G W C K O   Y Y R X U   K R U A C   B G L A D   R B N Y O   B P B N V
O V K O K   R F U M H   A Q K N I   Q N A M V   A U N I B   D O K G B
Z R F M D   X B R F O   X R C B D   V R F I G   B P F N Q   I A R V D
V Q V R P   X A N F D   I N Y P K   N Y C O W   Q K Y O Q   B V M K M
V N V Q H   Q R X Y C   Y Q F R V   U X N U V   U L Y G U   T I P W D
K D B C F   D N K V O   N F Q F C   K P B R A   P K O H Y   R B G R Y
H O R K N   X H M A C   S U I R F   Q Y P Y N   D F B O X
```

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36



An interesting profile, no J, a δ I.C. of 1.28 representing a deviation of 15σ. Digraphic distributions are now made, on and off the cut; these are shown in Figs. 176*a* and *b* on the next page. The digraphic I.C. of the on-cut distribution (fig. 176*a*) is $\frac{625(92)}{192\times 191}=1.57$, while that of the off-cut distribution (Fig. 176*b*) is $\frac{625(80)}{192\times 191}=1.36$. Although $\frac{192}{25}=7.7$ doublets are expected if the 25 letters present were equiprobable, and even more in the case at hand with such a rough uniliteral distribution, Fig. 176*b* has only 5 doublets whereas Fig. 176*a* has none whatsoever. Furthermore, Fig. 176*a* shows some striking properties in the physical appearance of the distribution, displaying affinities of some letters for others, and apparent matching propensities among the rows and columns of the distribution; moreover, since there is an apparent symmetry about the main diagonal (from upper left to lower right), this means that the matching of rows also corresponds to identical matching of the columns. Since this now bespeaks a biliteral system with a commutative matrix, we divide the text into digraphs

BD AU BU AR AD LD ZR BX KQ FN FM QV BR ZD NB . . .

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	–	–	1	2	–	–	1	–	–	–	–	–	2	2	–	–	2	2	–	–	2	–	–	–	–	–
B	–	–	2	2	–	–	1	–	–	–	–	–	–	2	1	2	2	2	–	–	1	–	–	–	–	–
C	–	2	–	–	1	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–
D	1	1	–	–	–	1	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	1	1	1	1	1
E	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	1	–	–	–	–	–
F	–	–	–	2	–	–	–	–	–	–	–	–	2	2	1	1	2	2	–	–	2	–	–	–	–	–
G	–	1	–	–	–	–	–	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	1	–	1	–
H	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	1	–	–	–	–	–	–	–	–	–
I	–	–	–	–	–	–	1	–	–	–	–	–	1	1	–	1	1	1	–	–	–	–	–	–	–	–
J	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
K	–	–	2	2	–	–	1	–	–	–	–	–	1	2	1	2	1	2	–	–	–	–	–	–	–	–
L	–	–	–	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–
M	1	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	1	–	–	1	1	–	–	–
N	2	2	–	–	–	1	–	–	1	–	2	–	–	–	–	–	–	–	–	–	–	1	–	1	1	1
O	–	1	–	–	–	–	–	1	–	–	1	–	–	–	–	–	–	–	–	–	–	1	1	–	1	–
P	–	2	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	1	–	–	–	–	1	1	–
Q	1	1	–	–	–	1	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	2	–	1	2	–
R	1	2	–	–	–	1	–	–	1	–	2	–	–	–	–	–	–	–	–	–	–	1	–	2	2	1
S	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–
T	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
U	1	–	–	–	–	1	–	–	–	–	–	1	–	–	–	–	–	–	–	1	–	1	1	–	–	–
V	–	–	–	2	–	–	–	–	–	–	–	–	1	2	1	–	2	2	–	–	1	–	–	–	–	–
W	–	–	–	1	–	–	1	–	–	–	–	–	–	1	–	–	–	–	–	–	1	–	–	–	–	–
X	–	–	–	1	–	–	–	–	–	–	–	–	–	2	–	–	–	1	–	–	1	–	–	–	–	–
Y	–	–	2	1	–	–	1	–	–	–	–	–	–	2	1	2	2	2	–	–	1	–	–	–	–	–
Z	–	–	1	1	–	–	–	–	–	–	–	–	–	1	–	–	–	2	–	–	1	–	–	–	–	–

FIGURE 176*a*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	–	–	1	–	1	–	–	–	–	–	1	1	1	–	–	1	–	1	–	–	–	–	–	–	–	–
B	–	–	–	2	–	–	1	–	–	–	–	–	2	2	–	1	–	1	–	–	–	1	–	–	1	1
C	–	–	1	–	–	1	–	–	–	–	2	–	–	1	1	1	–	–	1	–	–	–	–	–	1	–
D	1	1	–	–	–	–	–	–	1	–	2	1	–	2	1	–	1	1	–	–	1	2	–	–	–	1
E	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–
F	–	1	1	–	–	–	–	–	2	–	–	–	–	–	–	–	1	–	–	–	–	–	1	–	–	–
G	–	2	–	–	–	–	1	–	–	–	1	–	–	–	–	–	–	1	–	–	1	–	–	–	–	1
H	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–
I	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–
J	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
K	–	–	–	–	–	–	2	–	–	–	–	–	–	1	2	1	–	–	–	–	–	2	–	2	1	–
L	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–
M	1	–	–	3	–	–	–	–	–	–	1	–	–	–	–	–	1	1	–	–	–	1	–	–	1	–
N	2	1	–	2	–	3	1	–	1	–	–	–	1	1	–	–	1	–	–	–	1	1	–	–	2	–
O	–	–	–	–	–	–	–	–	–	–	1	–	–	1	–	–	1	1	–	–	–	–	–	2	–	–
P	1	1	–	–	1	1	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	1	–	2	–
Q	–	–	–	1	–	2	–	1	–	–	1	–	–	2	–	–	–	2	–	–	–	1	1	–	1	1
R	1	2	1	–	–	4	–	–	–	–	–	–	–	–	–	1	–	1	–	–	1	2	–	1	1	1
S	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
T	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
U	1	2	–	1	–	1	–	–	1	–	1	–	1	1	–	–	2	1	–	–	–	–	–	1	–	–
V	2	1	–	–	–	–	–	–	–	–	1	–	–	–	1	–	–	1	–	–	1	1	–	–	–	–
W	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	1	–	–	–	–	–	1
X	1	1	–	–	–	–	–	1	–	–	1	–	–	–	–	–	1	–	–	–	–	–	–	–	1	–
Y	1	1	1	–	–	–	–	1	–	–	1	–	1	–	1	–	–	1	–	–	–	–	1	–	1	–
Z	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	1	–	1	–	–	–	–	–	–	–	–

FIGURE 176*b*

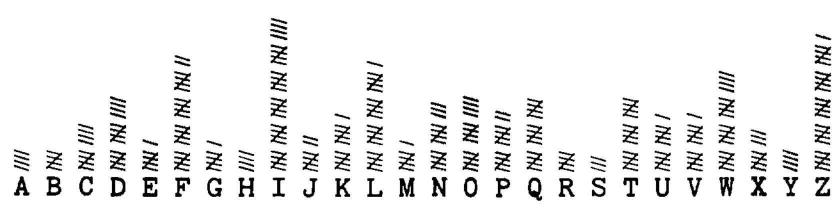
and proceed to establish the two families of letters in the manner illustrated in subpar. *f*(10), above, arriving at the following groupings of 15 and 10 letters:

A B E F H I K L S T V W X Y Z | C D G M N O P Q R U

From the distribution in Fig. 176*a* we see that the A, F, and V rows match (and also these columns), and we may continue the matching process until we reconstruct the 15 variant row coordinates and the 10 variant column coordinates of what becomes a 5 x 5 matrix. Then, using an arbitrary A–Z sequence within the square, we may reduce the biliteral cipher text to monoalphabetic terms. If slight errors are made in the matching, these can be caught and corrected during the recovery of the plain text.[61]

(7) In the next case to be studied, we have the following cryptogram and its accompanying distribution:

```
C W I E L   X G Q Z B   W E L V A   E D N L Q   O I F P Z   O K W Z M
W I F V F   Q I F T B   X E G Q L   P Z I K·W   C P E J V   I D V I D
Q L W Z K   Q U L T F   W C W I E   L V F Q Z   U J T Z C   P F N I I
D Q F R O   K N D T I   B T Z J N   U K T D N   U F S O K   W C V Z O
F P Z O L   W I K P O   M R I B R   Z L T I D   X U L N F   W G T Z F
V O M P Z   M X I F N   U Z C S O   K N D T I   H R G Y Z   L Q L Y U
Z I O C V   C W F Q Z   C N O F X   I Z L S U   A D Q H X   I B X M Y
Z L W J Q   F T I I Z   G X I L T   U Z G R I   L T U Z D   P O L V F
P O D W J   V H N Z A   F N J P I   K Q L W D   T M W Z F   N I D Y A
I L V L W   Z L N I O   D Q J T U   F T H W I   K P I K W   F Q F P Z
```



Another interesting profile, with an I.C. of 1.28 representing a deviation of 12σ. The one hexagraphic repetition present is at an interval of 70, and the two pentagraphic repetitions are at intervals of 71 and 7, so no common factor is involved. Digraphic distributions are now made, on and off the cut, and since these show considerable similarity it means that the cut makes no difference; the two distributions are therefore combined, as shown in Fig. 177, below. There seems to be a remarkable affinity for the letters A–M to contact letters in the last half of the alphabet and vice versa, with the exception of the vowels A, E, I, O, and U; the letter Z also seems not to fit the pattern (note the appearance of the last column of the distribution). In examining the cipher text more closely, it is seen that consonants are clustered in pairs (if we exclude Z), and that these pairs always consist of a letter from the first half of the alphabet followed by a letter from the last half (Y here behaves as a member of the consonant family). No property is discernible among the vowels, other than the fact that they are reluctant to contact each other—there are only 10 such contacts among the 64 vowels AEIOU present; furthermore, the only doublets in the cipher text are the two occurrences of the digraph II. We now conclude that what we have is a uniliteral-biliteral system. We note that the message contains 64 vowels and 105 consonant digraph elements, making a total of 169 underlying plaintext elements. If the cipher vowels actually represent plaintext vowels (say some arrangement of A, E, I, O, and U), and the consonant cipher digraphs represent plaintext consonants (with variants), the ratio of 64:169 is a percentage of 37.9%, which would be acceptable under this hypothesis. Furthermore, the Z_c which might have been assumed to be a null may now be considered, not only from its spacing throughout the cipher text but also from its frequency of 26 out of the 169 plaintext elements (=15.4%) to be a word separator. (The long stretch of 23 cipher letters between the Z's in the last line of the cipher represents only 14 plaintext letters, so that the assumption of a word separator is quite plausible.) The completion of the solution is again left to the reader as an instructive exercise.

[61] Completion of the solution is left to the reader as an exercise.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	-	-	-	1	1	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	-	1	2	-	-
C	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	2	-	-	1	-	-	2	3	-	-	-
D	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	1	4	-	-	3	-	1	1	1	1	-
E	-	-	-	1	-	-	1	-	-	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	4	4	1	1	3	-	2	2	1	-	-
G	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	1	-	1	-	-	-	1	1	-
H	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1	-	-	-	-	1	1	-	-
I	-	3	-	5	2	4	-	1	2	-	5	3	-	-	2	-	-	-	-	-	-	-	-	-	-	2
J	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	1	-	-	2	-	2	-	-	-	-
K	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	2	2	-	-	1	-	-	4	-	-	-
L	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	1	2	-	1	4	-	4	5	1	1	-
M	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	-	-	-	2	1	1	-
N	-	-	-	2	-	1	-	-	3	1	-	1	-	-	1	-	-	-	-	-	3	-	-	-	-	1
O	-	-	1	2	-	2	-	-	1	-	4	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	-	1	1	-	-	2	-	-	-	-	-	3	-	-	-	-	-	-	-	-	-	-	5
Q	-	-	-	-	-	3	-	1	1	1	-	4	-	-	1	-	-	-	-	-	1	-	-	-	-	3
R	-	-	-	-	-	-	1	-	2	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	1
S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	1	-	-	-	-	-
T	-	1	-	1	-	1	-	1	4	-	-	-	1	-	-	-	-	-	-	-	3	-	-	-	-	3
U	1	-	-	-	-	2	-	-	-	1	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	4
V	1	-	1	-	-	3	-	1	2	-	-	1	-	-	1	-	-	-	-	-	-	-	-	-	-	1
W	-	-	3	1	1	2	1	-	5	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
X	-	-	-	-	1	-	1	-	4	-	-	-	1	-	-	-	-	-	-	-	1	-	-	-	-	-
Y	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	2
Z	1	1	3	1	-	2	2	-	2	1	1	5	2	-	3	-	-	-	-	-	1	-	-	-	-	-

FIGURE 177

(8) As a final admonition on the recognition of variant phenomena, let us consider an aspect that might escape our notice. We have at hand the following four message beginnings:

```
(a)  77790  26062  54876  06579  10093  44420  90978  84898  08236 . . .
(b)  95743  42558  03993  11928  74140  29556  30307  92815  00311 . . .
(c)  88521  84625  51975  71358  47204  07600  39699  59667  68331 . . .
(d)  69674  03906  14939  22790  85401  91774  89689  43339  45359 . . .
```

Nothing out of the ordinary is seen, until we divide the texts into trinomes:

```
(a)  777 902 606 254 876 065 791 009 344 420 909 788 489 808 236 . . .
(b)  957 434 255 803 993 119 287 414 029 556 303 079 281 500 311 . . .
(c)  885 218 462 551 975 713 584 720 407 600 396 995 966 768 331 . . .
(d)  696 740 390 614 939 227 908 540 191 774 896 894 333 945 359 . . .
```

Just when we are about to let this go by, too, we suddenly note that the trinomes in the first column all sum to 21, and that all those in the second column sum to 11; in fact, each of the first 9 columns has trinomes summing to a fixed number. The system is now identified as a summing-trinome system, in which each plaintext letter is assigned a unique value of 1 to 26; this value is then expressed as a trinome, the digits of which sum to the designated value of the letter. The idiomorphic pattern of the stereotyped initial 9-letter word reveals itself as REFERENCE, and the message beginnings are now read with ease and all keys recovered.

(9) Repetitive phenomena associated with certain general types of cryptosystems are easily recognized for what they are. For example, if the polygraphic repetitions present in a cryptogram are predominantly of even length, and if the intervals between these repetitions are all even, this may indicate a digraphic system, or for that matter a biliteral system such as a 2-letter code; if the majority of the repetitions in digit traffic are of lengths divisible by 3, and if the intervals between them are also divisible by 3, obviously either a 3-digit code is involved, or a trinome cipher system of some sort. If the majority of polygraphic repetitions are not only pentagraphic, *but also across the entire group* (i.e., begining in the *a* position and ending in the *e* position of the 5-letter groups), 5-letter code is indicated, or some other type of pentagraphic encipherment such as the periodic fractionation system shown below, in which the vertical dinome encipherments within each group are recombined horizontally and converted back to letters, using the same fractionating square:

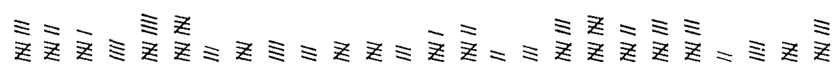
	1	2	3	4	5
1	A	B	C	D	E
2	F	G	H	I	K
3	L	M	N	O	P
4	Q	R	S	T	U
5	V	W	X	Y	Z

```
THREE OFFIC ERSAN DFOUR THREE ENLIS . . .
42411 32221 14413 12344 42411 13324
43255 41143 52313 41452 43255 53143
RQDMZ MGDAS DQPHC BOTDW RQDMZ CMULS
```

The intervals between all causal repetitions here will be factorable by 5, and in the main these repetitions will extend across one or more full groups. If, on the other hand, traffic were enciphered by a 5-alphabet periodic polyalphabetic substitution system, even though the intervals between repetitions would still be factorable by 5, the repetitions would be of varying lengths, and furthermore, distributions of the columns of the text when it is written on a width of 5 would show unmistakable evidence of monoalphabeticity.

(10) In periodic polyalphabetic substitution systems the intervals between the longer (and therefore less likely to be random) polygraphic repetitions are expected to correspond to the length of the period or multiples of the period. That this is not invariably true is shown by the following case of periodic polyalphabetic encipherment containing a repeated pentagraph at an interval of 26, which in a sample of this size represents odds of more than 1000-to-1 against having arisen by pure chance:[62]

```
EEYMF RNNSP UUEUD SLIUR AUFJS ETSFV
KNVVD SXDSA FTHXH JOZTW OCRZR AGLCA
RSXDS AEYZZ YTRLL EGIFR OEBVU FEYHK
UYBCQ OKOOA VZTBW BNTBG XECIQ AMQCA
SOKBV RFZFK NCBUH NZIFZ LMTVS HVPFJ
```



A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

[62] An easy way to calculate this is as follows. Since in this sample of 150 letters there are 146 pentagraphs possible, the expected number of pentagraphic repetitions (R) is given by the formula $R=\frac{N(N-1)}{2c}=\frac{146\cdot 145}{2\cdot 26^5}=0.00089$. Where the expectation is small, the expected number is an indication of the probability and therefore may be translated quite accurately into odds. In this case, then, the odds against a random pentagraphic repetition are $\frac{100{,}000-89}{89}$ or 1123 to 1.

The over-all δ I.C. of 1.003 supports the possibility of a large number of alphabets, so the text is written on a width of 26:

```
        5         10        15        20      25 26
E E Y M F R N N S P U U E U D S L I U R A U F J S E
T S F V K N V V D S X D S A F T H X H J O Z T W O C
R Z R A G L C A R S X D S A E Y Z Z Y T R L L E G I
F R O E B V U F E Y H K U Y B C Q O K O O A V Z T B
W B N T B G X E C I Q A M Q C A S O K B V R F Z F K
N C B U H N Z I F Z L M T V S H V P F J
Σ       2 2       2 2 2 2 2       2 2 2 2     2 2   =26
```

The I.C. of this array is $\frac{26(26)}{20(6\cdot5)+6(5\cdot4)}=0.94$, showing that it is *not* periodic encipherment with 26 alphabets. When, however, the cipher text is written on a width of 20, the I.C. of the array is $\frac{26(76)}{10(8\cdot7)+10(7\cdot6)}=2.02$, proving this period and indicating to us that what must have happened is that the keying sequence must have contained a repeated segment: the polygraph of key elements 2–6 is identical to that of key elements 16–20. A pentagraphic repetition in a cipher message of 150 letters must be causal (i.e., it must have been produced by the application of identical keys to identical plain text) but it does not mean that it is the result of periodic repetition of the total key.[63]

```
        5         10        15        20
E E Y M F R N N S P U U E U D S L I U R
A U F J S E T S F V K N V V D S X D S A
F T H X H J O Z T W O C R Z R A G L C A
R S X D S A E Y Z Z Y T R L L E G I F R
O E B V U F E Y H K U Y B C Q O K O O A
V Z T B W B N T B G X E C I Q A M Q C A
S O K B V R F Z F K N C B U H N Z I F Z
L M T V S H V P F J
Σ 2 2 4 6 2 4 4 6 2 2 2 4 2 4 4 2 6 4 14 =76
```

(11) When long, interrupted polygraphic repetitions (i.e., with occasional nonhitting portions) are found, these are telltale earmarks of variant systems. Note the following ciphertext passages:

```
(a) . . . FI CM TD VG PM IA ML TH PE VR AY DF BD US . . .
(b) . . . FI CM TD VG DS IA ML TH DY VR FR DF BD US . . .
```

Care must be taken, of course, to discount the possibility of the nonhitting portions' being nulls; the finding of other similar interrupted polygraphic repetitions should confirm the variant hypothesis, and the analyst should be on his way to establishing the variant values for the as yet unknown plaintext letters. That near-repetitions are not always the by-product of variant systems may be illustrated by the following example of five selected pairs of message beginnings based on a situation once encountered operationally:

```
(a) 37314 65277 36706 19472 64052 93404 . . .
    37314 65277 36706 19472 64052 86930 . . .

(b) 54991 65277 36706 19472 16328 39726 . . .
    42311 65277 36706 19472 14035 81436 . . .

(c) 60234 65277 36706 19472 69867 94130 . . .
    32231 65277 36706 19472 18832 63597 . . .

(d) 92874 65377 36706 19472 62890 42398 . . .
    35834 65277 36706 19472 69801 02876 . . .

(e) 82381 65277 36706 19472 11034 92179 . . .
    42311 38297 36706 10481 14035 21625 . . .
```

[63] This example involves direct standard alphabets. The reader might find it profitable to solve it by the method of completing the plain-component sequence and to discover the repeating key in the process.

From the appearance of these repetitions, after no doubt a considerable amount of study, we may conjecture (a) that the method of encryption encompasses five groups at a time; (b) that similar beginnings are present, involving in the main the first four groups; and (c) that, as a result of observing the predominantly symmetrically placed hits with the A3 group as the center, the manner of encryption involves dinomic substitution starting with elements of the central group, fanning outward until elements of the first and last groups (i.e., the A1 and A5) are combined. This will be clarified by studying the following example of the encryption of a message beginning, using the dinome substitution table shown below:

```
P:  0 1 2 5 8   2 7 0 3 2   1 8 7 7 3   1 4 3 2 0   0 3 6 6 5 . . .
C:  3 7                 7   3 6 7 0 6   1                 5 2
```

	0	1	2	3	4	5	6	7	8	9
0	31	67	58	24	90	32	18	84	45	97
1	42	82	50	36	53	29	75	23	06	73
2	62	71	88	17	03	93	30	28	27	15
3	26	00	05	91	79	54	13	41	59	56
4	99	37	10	65	86	08	80	94	61	77
5	12	98	81	14	35	96	39	72	02	38
6	87	48	20	83	92	43	69	01	74	66
7	76	21	57	19	68	16	70	49	95	34
8	46	52	11	63	07	89	44	60	22	85
9	04	33	64	25	47	78	55	09	51	40

The central digit of the A3 is left unenciphered, the *b* and *d* digits (8,7) are enciphered dinomically as (6,0) and replaced in the corresponding positions, and the *a* and *e* digits (1,3) are enciphered as (3,6). This process is continued in gradually expanding dinomic treatment until the *b* digit (1) of the A1 is combined with the *d* digit (6) of the A5 group to yield the dinome (7,5), and finally the *a* digit (0) of A1 is combined with the *e* digit (5) of A5 to yield the dinome (3,2). The feature of reciprocal dinomic substitution incorporated in the table avoids the possible cipher clerk's error of using the deciphering table for encipherment. As an anticlimatic revelation, the system on which this example is based was not solved cryptanalytically, nor for that matter even diagnosed, but instead was read as a result of adroit tachydactylurgy: light-fingered techniques applied to the enemy's wastebasket. The cryptanalyst is never too proud to accept help from whatever quarter—and even Faust turned the tables on Mephistopheles in the end.

h. We have already shown a number of examples of the application of mathematics in recognizing phenomena. One of the useful mathematical tools for evaluating distributions, i.e., testing for "goodness of fit" against an assumed statistical population, is the χ^2 (chi square) test. The value of χ^2 is calculated by the formula $\chi^2=\Sigma\frac{(f_i-a_i)^2}{a_i}$, where f_i is the observed frequency of each category and a_i is the expected frequency of each category in a distribution. The χ^2 value and the appropriate number of "degrees of freedom" (symbolized by the Greek letter ν, nu) are then looked up in a table and translated into a probability statement. The number of degrees of freedom is usually $c-1$: thus for digital text $\nu=9$; for 26-letter text, $\nu=25$; and for teleprinter text, $\nu=31$. In Fig. 178 is illustrated an abridged chi-square table that will be found generally useful in estimating the orders of magnitudes of probabilities.

(1) As an example of the application of the χ^2 test, let us assume that we are to evaluate a distribution of 180 digits of supposedly random text, i.e., from an equiprobable population;[64] this distribution is shown in the column labeled f_i of the diagram below:

	f_i	a_i	$f_i - a_i$	$(f_i - a_i)^2$
0	22	18	4	16
1	14	18	−4	16
2	13	18	−5	25
3	18	18	0	0
4	18	18	0	0
5	17	18	−1	1
6	16	18	−2	4
7	18	18	0	0
8	17	18	−1	1
9	27	18	9	81
	180	180	0	144

The sum (144) of the squares of the differences, divided by the expected number (18), gives a χ^2 value of 8; this figure, looked up in the table on the row for 9 degrees of freedom, is found to represent a probability of something greater than 0.5. This means that if we were to examine 100 samples of 10-category random text (the sample size is for all practical purposes immaterial), slightly more than half of them would be as rough as or rougher than the distribution just studied—in other words, this distribution is indistinguishable from a random one. Had the χ^2 value for a distribution of digits turned out to be, say, 28, looking this up in the row for 9 degrees of freedom gives us a probability of .001; i.e., in 1000 samples of random text, only one of them would be expected to be as rough as or rougher than the case at hand.

[64] The sample is taken from the four message beginnings given in subpar. *g*(8), above.

CHI-SQUARE TABLE

Probability

ν	0. 5	0. 1	. 05	. 01	. 005	. 001	. 0001	. 00001	. 000001	ν
1	0. 5	2. 7	3. 8	6. 6	7. 9	10. 8	15. 1	19. 5	23. 9	1
2	1. 4	4. 6	6. 0	9. 2	10. 6	13. 8	18. 4	23. 0	27. 6	2
3	2. 4	6. 2	7. 8	11. 3	12. 8	16. 3	21. 1	25. 9	30. 7	3
4	3. 4	7. 8	9. 5	13. 3	14. 9	18. 5	23. 5	28. 5	33. 4	4
5	4. 4	9. 2	11. 1	15. 1	16. 8	20. 5	25. 7	30. 9	35. 9	5
6	5. 3	10. 6	12. 6	16. 8	18. 5	22. 5	27. 9	33. 1	38. 3	6
7	6. 3	12. 0	14. 1	18. 5	20. 3	24. 3	29. 9	35. 3	40. 5	7
8	7. 3	13. 4	15. 5	20. 1	22. 0	26. 1	31. 8	37. 3	42. 7	8
9	8. 3	14. 7	16. 9	21. 7	23. 6	27. 9	33. 7	39. 3	44. 8	9
10	9. 3	16. 0	18. 3	23. 2	25. 2	29. 6	35. 6	41. 3	46. 9	10
11	10. 3	17. 3	19. 7	24. 7	26. 8	31. 3	37. 4	43. 2	48. 9	11
12	11. 3	18. 6	21. 0	26. 2	28. 3	32. 9	39. 1	45. 1	50. 8	12
13	12. 3	19. 8	22. 4	27. 7	29. 8	34. 5	40. 9	46. 9	52. 8	13
14	13. 3	21. 1	23. 7	29. 1	31. 3	36. 1	42. 6	48. 7	54. 6	14
15	14. 3	22. 3	25. 0	30. 6	32. 8	37. 7	44. 3	50. 5	56. 5	15
16	15. 3	23. 5	26. 3	32. 0	34. 3	39. 3	45. 9	52. 2	58. 3	16
17	16. 3	24. 8	27. 6	33. 4	35. 7	40. 8	47. 6	54. 0	60. 1	17
18	17. 3	26. 0	28. 9	34. 8	37. 2	42. 3	49. 2	55. 7	61. 9	18
19	18. 3	27. 2	30. 1	36. 2	38. 6	43. 8	50. 8	57. 4	63. 7	19
20	19. 3	28. 4	31. 4	37. 6	40. 0	45. 3	52. 4	59. 0	65. 4	20
21	20. 3	29. 6	32. 7	38. 9	41. 4	46. 8	54. 0	60. 7	67. 1	21
22	21. 3	30. 8	33. 9	40. 3	42. 8	48. 3	55. 5	62. 3	68. 9	22
23	22. 3	32. 0	35. 2	41. 6	44. 2	49. 7	57. 1	64. 0	70. 6	23
24	23. 3	33. 2	36. 4	43. 0	45. 6	51. 2	58. 6	65. 6	72. 2	24
25	24. 3	34. 4	37. 7	44. 3	46. 9	52. 6	60. 1	67. 2	73. 9	25
26	25. 3	35. 6	38. 9	45. 6	48. 3	54. 1	61. 7	68. 8	75. 6	26
27	26. 3	36. 7	40. 1	47. 0	49. 6	55. 5	63. 2	70. 4	77. 2	27
28	27. 3	37. 9	41. 3	48. 3	51. 0	56. 9	64. 7	71. 9	78. 8	28
29	28. 3	39. 1	42. 6	49. 6	52. 3	58. 3	66. 2	73. 5	80. 4	29
30	29. 3	40. 3	43. 8	50. 9	53. 7	59. 7	67. 6	75. 0	82. 0	30
31	30. 3	41. 4	45. 0	52. 2	55. 0	61. 1	69. 1	76. 6	83. 6	31
32	31. 3	42. 6	46. 2	53. 5	56. 3	62. 5	70. 6	78. 1	85. 2	32
33	32. 3	43. 8	47. 4	54. 8	57. 7	63. 9	72. 0	79. 6	86. 8	33
34	33. 3	44. 9	48. 6	56. 1	59. 0	65. 3	73. 5	81. 1	88. 4	34
35	34. 3	46. 1	49. 8	57. 3	60. 3	66. 6	74. 9	82. 6	90. 0	35
48	47. 3	60. 9	65. 2	73. 7	77. 0	84. 0	93. 2	101. 7	109. 7	48
49	48. 3	62. 0	66. 3	74. 9	78. 2	85. 4	94. 6	103. 1	111. 1	49
63	62. 3	77. 8	82. 5	92. 0	95. 7	103. 4	113. 5	122. 7	131. 4	63
80	79. 3	96. 6	101. 9	112. 3	116. 3	124. 8	135. 8	145. 8	155. 1	80
99	98. 3	117. 4	123. 2	134. 6	139. 0	148. 2	160. 1	170. 8	180. 8	99

FIGURE 178

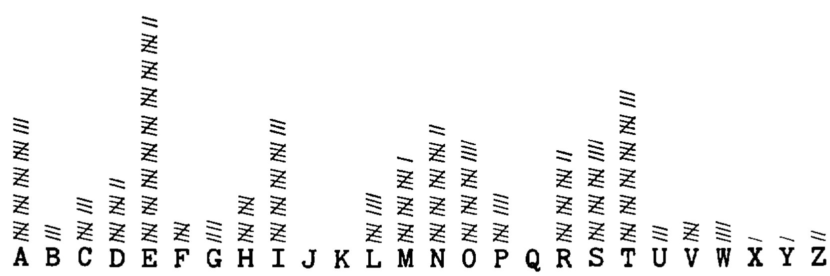
(2) Where the expected frequencies in each category are equal, it is preferable to use an equivalent formula for χ^2. This formula, especially advantageous when a desk calculator is handy, is $\chi^2=\frac{c}{N}\Sigma f_i^2-N$; thus, in the previous example wherein the sum of the squares of the frequencies is 3384, $\chi^2=\frac{10}{180}(3384)-180=8$, as before.[65] In a similar situation involving 26-letter text, we have the following distribution to be tested on the hypothesis of an equiprobable universe:

A	167	G	194	L	167	Q	189	V	172
B	171	H	152	M	165	R	138	W	159
C	189	I	146	N	137	S	172	X	179
D	152	J	162	O	150	T	187	Y	174
E	141	K	154	P	182	U	163	Z	185
F	159								4306

The expected (average) frequency for each letter is $\frac{4306}{26}$ or 165.62, rounded off to the nearest hundredth, and the squaring of the (f_i-a_i) differences would be a little clumsy. Instead, we calculate the sum (719,934) of the squares of the frequencies, and compute $\chi^2=\frac{26}{4306}(719{,}934)-4306=41.0$ which, as may be seen in the table (in the row of *25* degrees of freedom), roughly interpolating by eye between 37.7 and 44.3, means a probability of about .02, i.e., only one random sample in 50 will have a score as large as or larger than 41.0.

(3) We shall now treat a case wherein the expected frequencies are not equal. Let us assume that the following cryptogram has been recognized as being a transposition cipher, but that the language of the underlying text is unknown, perhaps either English or German:

```
ITANM IETNA ICTGV IRCHN PESMS  LECNE EDAST LDEIM ROSWD RAMEM
SNPEO ETTEP VENAE WHNNI TEONO  TESWD SAAED EMEMT MPOTA RRTOT
EXEDE WNEAF GMAEN NRBEA EIFTR  RDNFA ELEAB HAOIE URIOF DPDSO
PLMHR IAODM MIPFM THIZA ERRTC  AERLT EAISS IATAN SSNTY SEOET
CTEIH TSALO ONUVI EDIRV NIIIL  THNDO CHTCN ZOLEE NTTIR BEPAE
ERTIS MNCTS LRHVG GMSSE IMNOE  OOSOA EUTHP
```



The expected frequencies for a given language are obtained by multiplying the probability of each letter by the sample size; thus, if in English the letter A_p has a probability of .0737, then we should expect $.0737\times285$ or 21.00 A's in our sample, and so on. In Fig. 179, below, are listed the computational steps in testing the distribution under the hypothesis that the cryptogram is transposed English plain text, and in Fig. 180 are these steps under the hypothesis that it is transposed German plain text; the "Sq"

[65] The most convenient sequence of operations for obtaining the values for χ^2 and δ I.C., when a desk calculator is available, is to derive the γ I.C. first, as follows:

(1) $\gamma=\frac{c\Sigma f_i^2}{N^2}$

(2) $\chi^2=N(\gamma-1)$

(3) $\delta=1+\frac{\chi^2+1-c}{N-1}$, or $\frac{\chi^2+N-c}{N-1}$

in the column headings means of course $(f_i - a_i)^2$. The χ^2 of 34.69 with 25 degrees of freedom for the English hypothesis, representing a probability of 0.1 is not a bad fit, while the value of 105.54 shows that German plain text is astronomically out of the question. We must, however, take another look at our computations. The test is prone to error when the expected frequencies in some categories are too small (look in particular at the contribution of 10.57 for Z in Fig. 179). What we should do is lump together these categories which fall below a preselected level, adjusting the number of degrees of freedom accordingly; here we shall combine the probabilities and observed frequencies of those letters whose expected frequencies are less than 2.00:

	P(E)	f_i	a_i
J	.0016	—	(0.46)
K	.0030	—	(0.86)
Q	.0035	—	(1.00)
X	.0046	1	(1.31)
Z	.0010	2	(0.29)
	.0137	3	

	P(G)	f_i	a_i
J	.0032	—	(0.91)
Q	.0001	—	(0.03)
X	.0002	1	(0.06)
Y	.0004	1	(0.11)
	.0039	2	

These combined entries are now substituted in the calculations, as shown in Figs. 179*a* and 180*a* in the lines labelled "&". It is seen that the value of 22.43 (now with *21* degrees of freedom) for the hypothesis of English plain text is much improved (probability of 0.5), and it can be shown (with more extensive χ^2 tables) that even the χ^2 of 83.49 with *22* degrees of freedom has been improved by a factor of 1000 to a still useless 0.00000001. (Note, by the way, the high score contributed by the letter M: there happens to be an inordinate number of M's in the plain text, many more than would be expected in an average sample of the language.)

	P(E)	f_i	a_i	$f_i - a_i$	Sq	$\frac{Sq}{a_i}$
A	.0737	23	21.00	2.00	4.00	0.19
B	.0097	3	2.76	0.24	0.06	0.02
C	.0307	8	8.75	−0.75	0.56	0.06
D	.0424	12	12.08	−0.08	0.01	0.00
E	.1300	42	37.05	4.95	24.50	0.66
F	.0283	5	8.07	−3.07	9.42	1.17
G	.0164	4	4.67	−0.67	0.45	0.10
H	.0339	10	9.66	0.34	0.12	0.01
I	.0735	23	20.95	2.05	4.20	0.20
J	.0016	—	0.46	−0.46	0.21	0.46
K	.0030	—	0.86	−0.86	0.74	0.86
L	.0364	9	10.37	−1.37	1.88	0.18
M	.0247	16	7.04	8.96	80.28	11.40
N	.0795	22	22.66	−0.66	0.44	0.02
O	.0753	19	21.46	−2.46	6.05	0.28
P	.0267	9	7.61	1.39	1.93	0.25
Q	.0035	—	1.00	−1.00	1.00	1.00
R	.0758	17	21.60	−4.60	21.16	0.98
S	.0612	19	17.44	1.56	2.43	0.14
T	.0919	28	26.19	1.81	3.28	0.13
U	.0260	3	7.41	−4.41	19.45	2.62
V	.0153	5	4.36	0.64	0.41	0.09
W	.0156	4	4.45	−0.45	0.20	0.04
X	.0046	1	1.31	−0.31	0.10	0.08
Y	.0193	1	5.50	−4.50	20.25	3.68
Z	.0010	2	0.29	1.71	2.92	10.07
	1.0000	285	285.00	0.00		34.69

FIGURE 179

	P(G)	f_i	a_i	$f_i - a_i$	Sq	$\frac{Sq}{a_i}$
A	.0600	23	17.10	5.90	34.81	2.04
B	.0170	3	4.85	−1.85	3.42	0.71
C	.0270	8	7.70	0.30	0.09	0.01
D	.0541	12	15.42	−3.42	11.70	0.76
E	.1795	42	51.16	−9.16	83.91	1.64
F	.0160	5	4.56	0.44	0.19	0.04
G	.0320	4	9.12	−5.12	26.21	2.87
H	.0413	10	11.77	−1.77	3.13	0.27
I	.0813	23	23.17	−0.17	0.03	0.00
J	.0032	—	0.91	−0.91	0.83	0.91
K	.0124	—	3.53	−3.53	12.46	3.53
L	.0331	9	9.43	−0.43	0.18	0.02
M	.0226	16	6.44	9.56	91.39	14.19
N	.1055	22	30.07	−8.07	65.12	2.17
O	.0272	19	7.75	11.25	126.56	16.33
P	.0083	9	2.37	6.63	43.96	18.55
Q	.0001	—	0.03	−0.03	0.00	0.00
R	.0723	17	20.61	−3.61	13.03	0.63
S	.0687	19	19.58	−0.58	0.34	0.02
T	.0574	28	16.36	11.64	135.49	8.28
U	.0458	3	13.05	−10.05	101.00	7.74
V	.0087	5	2.48	2.52	6.35	2.56
W	.0150	4	4.28	−0.28	0.08	0.02
X	.0002	1	0.06	0.94	0.88	14.67
Y	.0004	1	0.11	0.89	0.79	7.18
Z	.0109	2	3.11	−1.11	1.23	0.40
	1.0000	285	285.02	0.02		105.54

FIGURE 180

	P(E)	f_i	a_i	f_i-a_i	Sq	$\frac{Sq}{a_i}$
A	.0737	23	21.00	2.00	4.00	0.19
B	.0097	3	2.76	0.24	0.06	0.02
C	.0307	8	8.75	−0.75	0.56	0.06
D	.0424	12	12.08	−0.08	0.01	0.00
E	.1300	42	37.05	4.95	24.50	0.66
F	.0283	5	8.07	−3.07	9.42	1.17
G	.0164	4	4.67	−0.67	0.45	0.10
H	.0339	10	9.66	0.34	0.12	0.01
I	.0735	23	20.95	2.05	4.20	0.20
J	.0016	—				
K	.0030	—				
L	.0364	9	10.37	−1.37	1.88	0.18
M	.0247	16	7.04	8.96	80.28	11.40
N	.0795	22	22.66	−0.66	0.44	0.02
O	.0753	19	21.46	−2.46	6.05	0.28
P	.0267	9	7.61	1.39	1.93	0.25
Q	.0035	—				
R	.0758	17	21.60	−4.60	21.16	0.98
S	.0612	19	17.44	1.56	2.43	0.14
F	.0919	28	26.19	1.81	3.28	0.13
U	.0260	3	7.41	−4.41	19.45	2.62
V	.0153	5	4.36	0.64	0.41	0.09
W	.0156	4	4.45	−0.45	0.20	0.04
X	.0046	(1)				
Y	.0193	1	5.50	−4.50	20.25	3.68
Z	.0010	(2)				
&	.0137	3	3.90	−0.90	0.81	0.21
	1.0000	285	284.98	0.02		22.43

FIGURE 179*a*

	P(G)	f_i	a_i	f_i-a_i	Sq	$\frac{Sq}{a_i}$
A	.0600	23	17.10	5.90	34.81	2.04
B	.0170	3	4.85	−1.85	3.42	0.71
C	.0270	8	7.70	0.30	0.09	0.01
D	.0541	12	15.42	−3.42	11.70	0.76
E	.1795	42	51.16	−9.16	83.91	1.64
F	.0160	5	4.56	0.44	0.19	0.04
G	.0320	4	9.12	−5.12	26.21	2.87
H	.0413	10	11.77	−1.77	3.13	0.27
I	.0813	23	23.17	−0.17	0.03	0.00
J	.0032	—				
K	.0124	—	3.53	−3.53	12.46	3.53
L	.0331	9	9.43	−0.43	0.18	0.02
M	.0226	16	6.44	9.56	91.39	14.19
N	.1055	22	30.07	−8.07	65.12	2.17
O	.0272	19	7.75	11.25	126.56	16.33
P	.0083	9	2.37	6.63	43.96	18.55
Q	.0001	—				
R	.0723	17	20.61	−3.61	13.03	0.63
S	.0687	19	19.58	−0.58	0.34	0.02
T	.0574	28	16.36	11.64	135.49	8.28
U	.0458	3	13.05	−10.05	101.00	7.74
V	.0087	5	2.48	2.52	6.35	2.56
W	.0150	4	4.28	−0.28	0.08	0.02
X	.0002	(1)				
Y	.0004	(1)				
Z	.0109	2	3.11	−1.11	1.23	0.40
&	.0039	2	1.11	0.89	0.79	0.71
	1.0000	285	285.02	−0.02		83.49

FIGURE 180*a*

(4) One more example might be apropos at this point. If we wished to test the distribution of the 4306 letters given in subpar. (2), above, for the possibility of a complex noncrashing encipherment, we will have the following computational steps:[66]

	P	f_i	a_i	$f_i - a_i$	$(f_i - a_i)^2$	$\frac{(f_i - a_i)^2}{a_i}$
A	.0370	167	159.32	7.68	58.98	0.37
B	.0396	171	170.52	0.48	0.23	0.00
C	.0388	189	167.07	21.93	480.92	2.88
D	.0383	152	164.92	−12.92	166.93	1.01
E	.0348	141	149.85	−8.85	78.32	0.52
F	.0388	159	167.07	−8.07	65.12	0.39
G	.0394	194	169.66	24.34	592.44	3.49
H	.0386	152	166.21	−14.21	201.92	1.21
I	.0370	146	159.32	−13.32	177.42	1.11
J	.0399	162	171.81	−9.81	96.24	0.56
K	.0399	154	171.81	−17.81	317.20	1.85
L	.0386	167	166.21	0.79	0.62	0.00
M	.0390	165	167.93	−2.93	8.58	0.05
N	.0368	137	158.46	−21.46	460.53	2.91
O	.0370	150	159.32	−9.32	86.86	0.55
P	.0389	182	167.50	14.50	210.25	1.26
Q	.0399	189	171.81	17.19	295.50	1.72
R	.0370	138	159.32	−21.32	454.54	2.85
S	.0376	172	161.91	10.09	101.81	0.63
T	.0363	187	156.31	30.69	941.88	6.03
U	.0390	163	167.93	−4.93	24.30	0.14
V	.0394	172	169.66	2.34	5.48	0.03
W	.0394	159	169.66	−10.66	113.64	0.67
X	.0398	179	171.38	7.62	58.06	0.34
Y	.0392	174	168.80	5.20	27.04	0.16
Z	.0400	185	172.24	12.76	162.82	0.95
	1.0000	4306	4306.00	0.00		31.68

Interpolating in the χ^2 table for 25 degrees of freedom, we find that the value 31.68 represents a probability of about 0.2; in other words, this is a good fit for the hypothesis of noncrashing encipherment.

(5) We cannot leave this discussion without an admonishment on the application of mathematical tools. High scores in significance tests are regarded by the experienced cryptanalyst as indicative of a probability of a particular hypothesis, and not proof of the matter. Take for example the following 440-character cryptogram sent by an enemy known to be using cyclic additive keys of lengths between 20 and 40 digits:

```
90723 78168 94849 15771 16844  17454 66220 88312 87103 45436
51844 80725 95351 25207 71062  43897 67340 60921 05986 85348
28147 15733 58293 45515 05206  88337 34666 19895 67818 09150
66954 13321 61791 75797 51414  31979 86210 85627 71095 58825
20894 16966 09087 59634 80149  82880 81862 77470 01320 13674
11794 57837 24849 06800 00520  19613 90147 16045 20150 35129
06260 72364 91991 17821 62194  36516 06329 85610 90458 05395
55013 35800 92354 04365 92886  96225 20858 05926 00264 38137
81653 45991 26864 97686 06029  44327 74662 65027
```

[66] The probabilities here are those given on p. 363, and have been calculated by the formula $P_i = \frac{1-p_i}{c-1}$, where P_i is the probability of the cipher letter, p_i the probability of the plaintext letter in the language, and c the number of categories. Thus, since E_p in English has a probability of .1300, the probability of E in the cipher text is $\frac{1-.1300}{25} = .0348$.

The frequency distribution of the cipher text written out on a width of 20 is the following:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
0	4	6	1	2	4	5	1	1	2	3	5	3	4		3		1	1	1	3	
1	1	3	5	2	2	5		2	1	2	1	7	1	2	1	5	1	2	4	3	
2	4	2	3	3		1	2	1	4		2	1	3	3	3			1	7	1	
3			4		3	1	1	5	3	1	1		1		2	4	2	3	3	1	
4		1	1	6	4	3	2	2	1	3		3	1	2	6	2		1	1	1	
5		2		5			8	1	2	3	4	2	1	2	1	3	7	1		5	
6	4	4	1	1	1	1	4	2	4	4	2	4	2	6	3	1	1	4		1	
7		3	2		2	3	1	2	1	5	3		1		1	1	4	2	3	4	
8	4	1	5		3	3	2	2	1	1	1	1	5	3		4	3	6	1	1	
9	5			3	3		1	4	3		3	1	3	4	2	2	3	1	2	2	
	68	58	60	66	46	58	74	42	40	52	48	68	46	60	52	54	68	52	68	46	=1126

The δ I.C. here is $\frac{10(1126)}{20(22\cdot 21)}=1.22$, and its sigmage is $\frac{440(1.22-1.00)}{\sqrt{2(20\cdot 9)}}=5.1\sigma$.

This sigmage, which can be interpreted by the χ^2 distribution with 20×9 or 180 degrees of freedom, turns out to have a probability of 0.000005 of having occurred by chance in a random sample of this size.[67] This shows that we are *certainly on the right track, but it doesn't prove that we necessarily have the right answer*. If we should write the cipher text on a width of 30, however, we will have the following distributional diagram:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
0	9	3	2	1	5	1			1	1		3	2	1		1		2		1		3	1		2	3	2		2	4	
1		1		1	1	3		2	3	1	1	6	4		2	5				3	1	3	2	3		2	1	2	2	1	
2		1	5	6		1	1	1	3	1	1	2			2		1	1	4		5	1	1		1				4		
3			4		1	1	2	7	3	1	1				2		1	1	3	1			1		2	4					
4					3	1	2		1	3		2	1	4	1	3		1	1			1	1	4	6	1		2		1	
5		3	1	3		1	4			1				2	1	2	7			5	4	1		2			4	2	2	2	
6	2	6		2	2			5	3		1		3	3	2	1	2			1	3	2		2		1	3	1	1	4	
7			1		1	2			1	5	3	1	2		2	1	2	1	1	4		2				1	3	1	2		
8	1			2		5	4			2	3	1	2	1	1	1	1	6	1		1	1	8		2	1		2	1		
9	3	1	2		2		2				5		1	4	2	1	1	3	5					3	1	1	1	4		2	
	80	42	36	40	30	28	30	64	24	28	32	40	24	32	12	28	24	38	38	38	38	16	58	28	36	20	26	20	20	28	=1020

The δ I.C. here is $\frac{10(1020)}{20(15\cdot 14)+10(14\cdot 13)}=1.69$, while the sigmage is $\frac{440(1.69-1.00)}{\sqrt{2(30\cdot 9)}}=13.06\sigma$. This sigmage, on the χ^2 distribution with 30×9 or 270 degrees of freedom, can be interpreted and shown to have a probability of less than 10^{-23} of occurring by chance. The explanation is that a cyclic 30-digit additive sequence was actually used with underlying monome-dinome text of a theoretical I.C. of approximately 1.69; on a width of 20, however, each column of the distribution diagram is composed not of a random sampling of ciphertext digits, but instead represents a merger of three monoalphabetic distributions, as is in fact suggested by the average I.C. of 1.22 for this array—almost exactly one-third the bulge of the underlying plain text.

[67] The manner of this computation, a bit complicated, is outlined here for the interested reader. Since the distribution of the δ I.C. is asymptotically related to the χ^2 distribution, the latter may be used to evaluate the δ I.C. The mean of any χ^2 distribution is equal to ν, and the variance (σ^2) is equal to 2ν, which in this case is 360; the standard deviation (σ), being equal to the square root of the variance, is in this case $\sqrt{360}$ or 19.0. The sum of the mean (180) plus the product 5.1×19.0 gives the equivalent χ^2 score of 276.9. Now the probability that a χ^2 score equal to or greater than k will occur is given by 1 minus the Poisson cumulative entry for $\frac{\nu}{2}$, where $\frac{a}{2}$ is expected. Thus, in this particular case, we look up $a=\frac{276.9}{2}$ (rounded off to 138) in the Poisson cumulative tables, and opposite the expected value $\frac{180}{2}$ or 90 we find the entry .999995, which when subtracted from 1 gives the answer 0.000005.

(6) A typical example of the use of log weights in substitution ciphers has been treated in subpar. 76*g*(4) in connection with the selection of generatrices. Log weights are also useful in the solution of transposition ciphers, as will be shown by the subsequent discussion. Let us suppose that we have at hand the transposition cipher given in subpar. *h*(3), above, and that it has been recognized that the underlying language is English. The message length, 285 letters, suggests the possibility of a completely filled rectangular matrix, either 15×19 or 19×15. If this assumption is correct and if the usual form of keyed columnar transposition is involved, we can determine the size of the matrix actually used. This we do by inscribing the cipher text into the matrices by columns from left to right, counting the vowels in the rows and noting the deviations from the expected. The correct matrix size will exhibit a consistency of smaller deviations from the expected 40% vowels; the sum of the deviations, divided by the column lengths, gives a convenient index pointing to the correct matrix from among several possibilities—the correct matrix will be the one with the smallest index. In Fig. 181, below, the expected number of vowels in each row is $.40\times 15=6.0$ and the index is $\frac{27.0}{19}=1.4$, whereas in Fig. 182 the expected number is $.40\times 19=7.6$ and the index is $\frac{15.0}{15}=1.0$; it is obvious that Fig. 182 is the correct matrix.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	V	D
I	N	I	T	E	R	N	A	M	R	S	O	D	P	M	5	1.0
T	P	M	E	S	R	N	B	H	R	S	O	O	A	S	4	2.0
A	E	R	P	W	T	R	H	R	T	N	N	C	E	S	3	3.0
N	S	O	V	D	O	B	A	I	C	T	U	H	E	E	7	1.0
M	M	S	E	S	T	E	O	A	A	Y	V	T	R	I	7	1.0
I	S	W	N	A	E	A	I	O	E	S	I	C	T	M	8	2.0
E	L	D	A	A	X	E	E	D	R	E	E	N	I	N	8	2.0
T	E	R	E	E	E	I	U	M	L	O	D	Z	S	O	8	2.0
N	C	A	W	D	D	F	R	M	T	E	I	O	M	E	5	1.0
A	N	M	H	E	E	T	I	I	E	T	R	L	N	O	7	1.0
I	E	E	N	M	W	R	O	P	A	C	V	E	C	O	7	1.0
C	E	M	N	E	N	R	F	F	I	T	N	E	T	S	4	2.0
T	D	S	I	M	E	D	D	M	S	E	I	N	S	O	5	1.0
G	A	N	T	T	A	N	P	T	S	I	I	T	L	A	5	1.0
V	S	P	E	M	F	F	D	H	I	H	I	T	R	E	4	2.0
I	T	E	O	P	G	A	S	I	A	T	L	I	H	U	8	2.0
R	L	O	N	O	M	E	O	Z	T	S	T	R	V	T	4	2.0
C	D	E	O	T	A	L	P	A	A	A	H	B	G	H	6	0.0
H	E	T	T	A	E	E	L	E	N	L	N	E	G	P	6	0.0
															111	27.0

FIGURE 181

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	V	D
I	I	E	R	V	T	M	W	E	H	P	T	E	S	O	T	N	M	I	7	.6
T	R	D	A	E	E	P	N	I	A	L	H	A	E	N	H	T	N	M	7	.6
A	C	A	M	N	S	O	E	F	O	M	I	I	O	U	N	T	C	N	9	1.4
N	H	S	E	A	W	T	A	T	I	H	Z	S	E	V	D	I	T	O	7	.6
M	N	T	M	E	D	A	F	R	E	R	A	S	T	I	O	R	S	E	7	.6
I	P	L	S	W	S	R	G	R	U	I	E	I	C	E	C	B	L	O	7	.6
E	E	D	N	H	A	R	M	D	R	A	R	A	T	D	H	E	R	O	7	.6
T	S	E	P	N	A	T	A	N	I	O	R	T	E	I	T	P	H	S	7	.6
N	M	I	E	N	E	O	E	F	O	D	T	A	I	R	C	A	V	O	10	2.4
A	S	M	O	I	D	T	N	A	F	M	C	N	H	V	N	E	G	A	6	1.6
I	L	R	E	T	E	E	N	E	D	M	A	S	T	N	Z	E	G	E	8	.4
C	E	O	T	E	M	X	R	L	P	I	E	S	S	I	O	R	M	U	8	.4
T	C	S	T	O	E	E	B	E	D	P	R	N	A	I	L	T	S	T	6	1.6
G	N	W	E	N	M	D	E	A	S	F	L	T	L	I	E	I	S	H	6	1.6
V	E	D	P	O	T	E	A	B	O	M	T	Y	O	L	E	S	E	P	9	1.4
																			111	15.0

FIGURE 182

The columns of the matrix of Fig. 182 are correct; only their order remains to be determined. Unless col. 1 is the last column of the matrix, it must be followed by one of the other columns, which may be determined by considering the sum of the log weights of the digraphs formed by the 18 pairings shown below:[68]

1 2		1 3		1 4		1 5		1 6		1 7		1 8		1 9		1 10	
II	--	IE	59	IR	73	IV	72	IT	73	IM	53	IW	—	IE	59	IH	--
TR	64	TD	45	TA	74	TE	91	TE	91	TP	25	TN	48	TI	82	TA	74
AC	61	AA	33	AM	61	AN	89	AS	80	AO	25	AE	13	AF	38	AO	25
NH	38	NS	71	NE	87	NA	72	NW	33	NT	93	NA	72	NT	93	NI	51
MN	—	MT	25	MM	59	ME	72	MD	13	MA	78	MF	13	MR	25	ME	72
IP	48	IL	70	IS	78	IW	--	IS	78	IR	73	IG	67	IR	73	IU	--
EE	81	ED	88	EN	99	EH	48	EA	78	ER	94	EM	61	ED	88	ER	94
TS	67	TE	91	TP	25	TN	48	TA	74	TT	67	TA	74	TN	48	TI	82
NM	42	NI	75	NE	87	NN	51	NE	87	NO	66	NE	87	NF	53	NO	66
AS	80	AM	61	AO	25	AI	64	AD	73	AT	83	AN	89	AA	33	AF	38
IL	70	IR	73	IE	59	IT	73	IE	59	IE	59	IN	92	IE	59	ID	45
CE	76	CO	80	CT	61	CE	76	CM	13	CX	--	CR	38	CL	42	CP	—
TC	45	TS	67	TT	67	TO	84	TE	91	TE	91	TB	33	TE	91	TD	45
GN	33	GW	13	GE	61	GN	33	GM	13	GD	13	GE	61	GA	48	GS	33
VE	87	VD	—	VP	—	VO	13	VT	13	VE	87	VA	45	VB	—	VO	13
	792		851		916		886		869		907		793		832		638

[68] The weights used here are the digraphic weights (to the base 224) given in Table 15 of Appendix 2, *Military Cryptanalytics, Part I.*

1 11		1 12		1 13		1 14		1 15		1 16		1 17		1 18		1 19	
IP	48	IT	73	IE	59	IS	78	IO	80	IT	73	IN	92	IM	53	II	--
TL	42	TH	92	TA	74	TE	91	TN	48	TH	92	TT	67	TN	48	TM	45
AM	61	AI	64	AI	64	AO	25	AU	59	AN	89	AT	83	AC	61	AN	89
NH	38	NZ	--	NS	71	NE	87	NV	33	ND	85	NI	75	NT	93	NO	66
MR	25	MA	78	MS	38	MT	25	MI	53	MO	55	MR	25	MS	38	ME	72
II	--	IE	59	II	--	IC	69	IE	59	IC	69	IB	25	IL	70	IO	80
EA	78	ER	94	EA	78	ET	79	ED	88	EH	48	EE	81	ER	94	EO	58
TO	84	TR	64	TT	67	TE	91	TI	82	TT	67	TP	25	TH	92	TS	67
ND	85	NT	51	NA	72	NI	75	NR	38	NC	67	NA	72	NV	33	NO	66
AM	61	AC	61	AN	89	AH	25	AV	48	AN	89	AE	13	AG	45	AA	33
IM	53	IA	51	IS	78	IT	73	IN	92	IZ	25	IE	59	IG	67	IE	59
CI	48	CE	76	CS	13	CS	13	CI	48	CO	80	CR	38	CM	13	CU	38
TP	25	TR	64	TN	48	TA	74	TI	82	TL	42	TT	67	TS	67	TT	67
GF	25	GL	25	GT	38	GL	25	GI	42	GE	61	GI	42	GS	33	GH	67
VM	--	VT	13	VY	--	VO	13	VL	--	VE	87	VS	--	VE	87	VP	--
	673		865		789		843		852		1029		764		894		807

The high score of 1029 for the pairing of cols. 1–16 shows that these columns are correctly juxtaposed. We could now continue the process, finding which column should be to the left of col. 1, and which column is to the right of col. 16, and in the end we would establish the order of all the columns, even if we had but a minimum background in the language.[69]

i. As a final word on the recognition of nonrandom phenomena, it must be stressed that diagnostic examination in the initial phases should rely heavily on powers of *observation*, rather than calculation; in particular, the spotting of bust messages and other errors, which has often been the crux of the first entry into a new cryptosystem, is more readily done by eye than by statistical analysis or machine methods. Nevertheless, machine aids can and do make valuable contributions to diagnosis, especially in performing much of the clerical and statistical work incidental to the study of a new cryptosystem. One of the basic exploratory (or so-called "diagnostic") machine programs is called the STETHOSCOPE program, for obvious reasons. The particular computer on which it is run may change as time goes on, but in essence the program presents a variety of fundamental information about a single message (or a single stretch of key) under study, as follows:

a. The uniliteral frequency distribution, the monographic I.C., and its sigmage.

b. The over-all digraphic I.C., as well as the digraphic I.C.'s on cut "A" and cut "B" and their sigmages.

c. The over-all trigraphic I.C., and the trigraphic I.C.'s on cuts "A", "B", and "C" and their sigmages.

d. The local roughness (in terms of the observed and expected number of hits, and sigmage), when the message is offset against itself at offsets of 1 to 33.

e. Width tests, giving average columnar I.C.'s and sigmages of the message if it were written out on widths from 2 to 51.

f. The observed and expected number of tetragraphic and pentagraphic repetitions, and their I.C.'s and sigmages.

g. A listing of polygraphic repetitions of length 4 or longer.

h. If desired, the same categories of statistical information on the delta stream.

The STETHOSCOPE program exists in several versions, including one in which several messages may be treated together; its general purpose remains the same, as in a doctor's first examination of a patient.[70]

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[69] A modification of this general procedure is adapted in machine methods for the solution of transposition ciphers with incompletely filled matrices, whether or not the number of columns of the matrix is known.

[70] The reader might be interested in examining a work entitled "Cryptanalytic Diagnosis with the Aid of a Computer—a Collection of 147 STETHOSCOPE Listings" which I prepared for use in my *Intensive Study Program in General Cryptanalysis*. This anthology is derived from cryptographic examples embracing a wide variety of manual and machine cryptosystems, many of which are diagnosable from the STETHOSCOPE listing.

78. Third step: interpreting the phenomena.—*a.* The third step in diagnosis, that of explaining the nonrandom characteristics or manifestations once they are recognized, is the most crucial step in the diagnostic process; as indicated in subpar. 72*b*, this takes experience and imagination, and, in addition, *intelligence.* In some cases interpretation will be practically synonymous with recognition: a striking roughness in a literal cryptogram, together with the presence of a plethora of polygraphic repetitions whose lengths look not unlike those expected for plain text, identify the message as simple substitution. Even in a more complicated system, interpretation may follow directly on the heels of recognition. In the following example from a pre-World War II course in cryptanalysis, it is given that the plain text is a message famous in English history:

253 269 863 261 471 958 220 370 4 21 20 25

Although very short, the message can be easily and quickly solved by inspection, and the nature of the cryptosystem determined.

b. Specific guidelines for the interpretation of nonrandom phenomena cannot be formulated: the cerebral gifts with which the analyst has been endowed will have to suffice—and the pooling of such gifts, especially in a particularly difficult problem, is to be encouraged and indeed may be absolutely necessary. Nevertheless, there are some generalities which bear restating. Certain distributional profiles can be easily identified for what they are (cf. subpar. 77*e* on pp. 360–367), and their interpretative exploitation may follow. Even when a rough distributional profile itself cannot be interpreted, nevertheless when there are changes in the distributional profile this signifies a key change, i.e., it denotes the beginning of a new cryptoperiod. Phenomena connected with certain bust or isolog situations may likewise be easily interpretable. The presence of depths shows that the key is not influenced by the plain or cipher text during the course of encipherment, and the presence of polygraphic repetitions in excess of those expected at random automatically excludes certain major classes of systems.[71] The presence of isomorphs which exhibit properties of indirect symmetry of position and which can be chained out to yield a complete sequence, identifies the system as one involving sliding components and Vigenère properties.

c. The absence of certain letters may yield interpretable information. An absent J may be indicative of a small-matrix digraphic system, or it may be another 25-letter system such as a 5×5 fractionating system, or a code in which there is no J in the code groups. If a missing letter is K, Q, W, X, or Z, this may point to a code (especially a commercial code) since permutation tables for codes have been constructed with one of these letters missing. On the other hand, messages containing, say, only the letters A through O merely mean that 15 letters have been employed, just that: the system might be a code system, or some type of multiliteral cipher. If the cipher text consisted only of consonants, without the 6 vowels, the system could be either a code or a multiliteral cipher—but even a digraphic system could not be excluded (cf. the system illustrated on pp. 387–391 of *Military Cryptanalytics, Part I.*)

d. Certain types of aberrations, isologs, and other special situations in transposition ciphers lend themselves to facile interpretation, as will be shown by the examples below.

(1) For the first case, let us study the following message:

```
A R A R U  L O S E F  E S O O T  U L O C O  M E N A T  F M T V Z→
←N V D W L  H A R W C  L M E I E  O P I N O  E S N D G  O T T E I
F W R O F  R T N O W  T P T I T  C G A M H  E E J I C  N E T O S
L L M T V  Z N V D E  L R T E F
```

The presence of a 7-letter ciphertext repetition, and the *location* of these segments in the message, point to an error involving a *repeated column* of the matrix in a keyed columnar transposition cipher. The column lengths involved are 7 and 6: there are 7 letters in the repeated column; there are 6 letters following the second occurrence of the repetition; and there are 26 letters ($26=2\times7+2\times6$) preceding the first occurrence of the repetition. Furthermore, since the interval between the two repetitions is 59, the only sum of integral multiples of 7 and 6 possible is $5\times7+4\times6$; thus there must be a total of 9 long columns and 7 short columns in the matrix of 16 columns.

[71] Careful judgment must be exercised in arriving at conclusions: as an illustration, the presence of long polygraphic repetitions in the cipher text of messages would not exclude, for example, one-time key based upon running plain text from a book.

(2) For the next example, the following two messages are to be studied:

Message "A"

```
HINDR WOMSM ACPEG EABIR ELTAN ILWFB
SEAPO CTRUH NOTLF IPNEA TALFD OTVCS
NIEET LEBLL LITEO
```

Message "B"

```
HINDR WOMSM ACPEG EABIR ELTAN ILWFB
SEAPO CTRUH NOTLF IPNEA TALFD O(GFEE
ENTMY N)TVCS NIEET LEBLL LITEO
```

The two messages are identical except for the insertion of 10 letters in Message "B" starting at the 57th position. Had these 10 letters been in two complete groups, it could have merely signified that these groups were inadvertently dropped in Message "A" by the transmitting operator; but in this context, it shows an *omitted column* of the transposition diagram, and that one column length must be 10. The 19 letters following the interpolated segment in Message "B" shows that there must be one long column of 10 and one short column of 9 letters there; and the 56 letters ($56=2\times10+4\times9$) prior to the interpolated segment must consist of two long columns and four short columns, making a matrix of 9 columns, four of them long and five of them short.

(3) For the third example we have the following pair of messages isolated for study:

Message "A"

```
TTOTE OTPLG FSDRR RUSNH AJWLA EUBFB
TOMTH NMTER AGARO OIYEH EHMEY SNRER
UHNOS IREAR
```

Message "B"

```
OMTHN MTPLG FSDRR RUSNH AJWLA EUBFB
TTTOT EOTER AGARO OIYEH EHMEY SNRER
UHNOS IREAR
```

This is clearly a case of a pair of *transposed columns* in a keyed columnar transposition cipher, and the number of letters between the two underlined segments, 24, points either to four columns of 6 letters each, or three columns of 8 letters each; but the number of letters after the second underlined column, 32, points to the latter case; thus in the matrix of 9 columns there are seven long columns of 8 letters, and two short columns of 7 letters.

(4) The next case deals with the repetitive phenomena present in the following two messages:

Message "A"

```
NOSMO BOOOI TEARS PRDEO EPOOL GNTIF
EUUNS ILATT ATODO LYIBR ARLIM CTIMC
TTRSE EMEAA YENCO FBPIO NYIIC NDSRY
CCFAD TUGSE
```

Message "B"

```
NOSEA CLITC OITFD NLEAI NDEOI LLWEC
SDGNT ILALN EKNSI TETST EFETO DOMLS
BSGOA RLCTT ERTOR MCTFI BTPRE JMEAE
AAIOA SUOFB PMNUE EORIC NVYEI DAICF
AEEOA RPRI
```

These manifestations are the result of *similar beginnings* in a columnar transposition system. There are 12 segments initiated by the underlined polygraphs, indicating 12 columns of the matrix; in Message "A" the column lengths are 9 and 8, while in Message "B" the column lengths are 11 and 10. Since the column breaks are known, anagramming of the texts is a trivial matter.

(5) In this next example the manifestations admittedly would be hard to detect in a volume of traffic, but on detailed examination it is found that there is a large number of polygraphic repetitions in common between the two messages, spaced at a more-or-less uniform distance apart, but not preserving the same order of these repetitions in the two messages:

Message "A"

```
C Y M R S   T U H I T   O O A N B   E O D H E   D Y T S D   E T E O V
S B E R H   E A V U E   E O I E D   D D C R E   I A L E N   S R L O G
M S R F H   R E N A S   U R A P N   N A G I E   S F E H N   R I C N C
```

Message "B"

```
M T I U S   R L R O M   I H E A E   N P J S U   R Y S N S   R E I E E
O N A G R   B T T O I   E E M D N   R F H H E   E O C N C   S C V I F
E H F F G   S T S D E   R O T O V   S N E U S   O O A I I   P O D H N
N O R S T
```

These are the manifestations of a pair of messages in columnar transposition with *similar endings*, relatively offset in the bottom rows of the two matrices. Although the segments delineated by the repetitions could be anagrammed as in the case of similar beginnings, we are able by a special method of solution *to derive the transposition key directly*, without recourse to anagramming (see pp. 425–427 of *Military Cryptanalytics, Part II*, for the solution of this example).

(6) In another example of the interpretation of phenomena in transposition ciphers, let us study the following pair of messages:

Message "A"

```
C N S D O   M E D U T   D M F L T   F A L S E   A D O S I   N A L E O
N U T V C   E T I G O   A F I L S   H W O A N   E D A A T   P A S N T
L A U O I   M F L E E   T L E R L   C I I Y V   N T N O I   R R T M C
N F S R G   I P E L P   R O I U C   E E S N L   T E B S E   A S I C F
```

Message "B"

```
C N S D O   L E I T M   F L T A P   I T G D O   S W O C H   N U T E T
F I O A L   E T A I O   A N P B A   E N A S N   A I E D M   F L E N U
O M H C I   K N F N O   I R R T I   P T T G I   P L R V O   I U B T N
I T T E B   S R O E G
```

Although the repetitive phenomena present are reminiscent of similar beginnings in columnar transposition ciphers, the lengths (2, 3, 4, 5, and 6) of the polygraphic repetitions in common are puzzling at first glance, since we expect them to be of lengths n and $(n+1)$, barring accidental contacts which might cause an occasional fluke longer repetition. Furthermore, the column lengths of Message "A" appear to be from 7 through 10 letters, inclusive, and those of Message "B" appear to be from 5 through 10 letters. A little thought on the subject, however, will lead us to the hypothesis that what is involved is a matrix *with blank cells* incorporated, and that these blank cells are placed near the *beginnings* of the columns (thus giving rise to the different-length repetitions). Solution of this case, then, would proceed by anagramming the *ends* of the 13 columns delineated by the repetitions.

(7) For the last example, let us consider a case which, although rarely encountered, serves to illustrate certain interpretative considerations. The following message is being studied:

```
TOROE REVST ENFOS RAIOU OSADY  ODIYP ESDTE IXPOS MOTOH BRCGA
FGOVC RRSSI REFOI GPSIN NSPLP  OESOA LANTL EVERE NSHEI LATST
EAWIS ENDIN IVASL BEDOP TOLLA  SASFO EATOE MATFI NEGIU RHTWI
OFNOI ETLEV RRSTN ESSST ENENV  IRWOA NIRZU RNSEE OOONI OUPDE
INDTO LMEMI SELLF ERTSW OCENI  VONYO IOIVT RDIIT PSEWT NTEPW
VEFCA KESIN REEDO STRXF ITSFU  RVIOS TOONG HECTO WPROO PTHEM
LLARD SSEER
```

From its uniliteral frequency distribution, it has been determined that the message is an English-language transposition cipher, but initial efforts to anagram the text on the basis of single columnar transposition have proved fruitless. In further study of the cryptogram, we make a digraphic distribution on the cut and find the digraphic ϕ to be 178, so the digraphic δ I.C. is $\frac{676(178)}{155 \cdot 154}=5.04$; the digraphic ϕ off the cut, however is 110, so the digraphic δ I.C. for this latter case is $\frac{676(110)}{154 \cdot 153}=312$. The situation is now clear: what we have is a case of *digraphic transposition*, in which each cell of the matrix contains a digraph instead of a single letter. The digraphic kappa plain constant for English is .0069, so the expected digraphic I.C. is $676(.0069)=4.66$; on the other hand, the I.C. of digraphs composed of *disconnected* letters from an English-language population, thus destroying the digraphic cohesion of the language, is $676(.0667)^2=3.01$. On cut, then, this message displays the characteristics of English plaintext digraphs. Once the nature of the system is deduced, anagramming is much simpler than in the case of monographic transposition since we are dealing with larger units.

(8) The foregoing discussion has been confined to manifestations in cases of monophase (i.e., single) transposition. The phenomena in double transposition are much obscured, and each case presents a very special case. In spite of casual references to the alleged existence of a test to be able to distinguish single columnar transposition from double, the truth of the matter is that no practical test exists.[72] The best way of distinguishing single columnar transposition from double is this: if it's single, you solve it with ease.

e. Proforma systems consisting of stereotyped messages in which the order and nature of the successive elements are determined by prearrangement (as for example, weather messages) are identified as such by their very appearance. The specific type of proforma system may have to be determined through patent characteristics associated with certain types of traffic or through collateral information. Sometimes the problem of interpretation is simple enough to be solved without any extraneous information, as in

[72] This test is based on sliding the first several letters of a cipher message (i.e., those presumed to come from col. 1, the first numbered column of the matrix) and the last several letters of the message (i.e., those presumed to come from the last column of the matrix) throughout the remainder of the cipher text, and evaluating the digraphs thus formed, using log weights in a procedure similar to that illustrated in subpar. 77*h*(6); if the cryptogram is a single transposition, a certain mean score of four sets of log weights should be attained—but by this time we would already have *solved* the cryptogram.

the case of the following 14 messages given in their entirety, together with the intercept information enclosed in parentheses:

1.	(MIH DE FSE 1030 16 DEC)	04941	45401	02124	44464	44149	42401
2.	(MIH DE FSE 1044 16 DEC)	04942	48421	53625	46424	94244	46411
3.	(MIH DE FSE 1055 16 DEC)	04944	49421	53625	46424	94244	47411
4.	(MIH DE FSE 1119 16 DEC)	04040	40401	02124	44464	44149	41401
5.	(MIH DE FSE 1135 16 DEC)	04042	40401	02124	44464	44149	45411
6.	(MIH DE FSE 1154 16 DEC)	04043	46471	14724	44464	44047	42401
7.	(MIH DE FSE 1205 16 DEC)	04044	49421	53625	46424	94244	47411
8.	(MIH DE FSE 1229 16 DEC)	04141	42401	02124	44464	44149	42401
9.	(MIH DE FSE 1252 16 DEC)	04143	45471	14724	44464	44047	44401
10.	(MIH DE FSE 1307 16 DEC)	04249	40471	14724	44464	44047	43401
11.	(MIH DE FSE 1314 16 DEC)	04249	47401	02124	44464	44149	41401
12.	(MIH DE FSE 1323 16 DEC)	04240	46421	53625	46424	94949	43421
13.	(MIH DE FSE 1342 16 DEC)	04242	45492	61215	46424	94241	42401
14.	(MIH DE FSE 1355 16 DEC)	04243	47471	14724	44464	44048	49421

This example, based upon a situation encountered during World War II in the Pacific Theater of Operations, is capable of being solved in its entirety, in spite of the brevity of the messages and the small amount of traffic.

f. We shall now take up the interpretation of phenomena in connection with indicators, and we shall use as an example one which may be considered typical of the approaches in such problems.

(1) Let us study the following collection of A1 and Z0 groups (known to be the indicator and indicator-check groups) of 40 messages in an unknown cryptosystem:

	A1	Z0		A1	Z0
1.	T X N E A	X G F K H	21.	D P G I M	T H P C K
2.	R F C W P	G N G L V	22.	R C Y F W	N O Z V D
3.	A K P L M	I Z V R K	23.	G S T B C	P Y I Z E
4.	S G W F T	I N N E Z	24.	K Z H U X	D O Q B A
5.	Z E R O M	H W X G S	25.	P A G S V	E L P K G
6.	Q I F W E	G O O H M	26.	M I Y E Q	H C Z H T
7.	C O K R Z	L A R U E	27.	E A T M R	A I I X G
8.	L M D K G	P A A I Z	28.	M W Q Z G	M A M Z W
9.	A Q X B I	T L D M D	29.	G E B O D	X U L C P
10.	T G Z V E	C E B P M	30.	N B F H Z	S M O K B
11.	W M I U R	D F H W Z	31.	Y S K L Q	I Y R Y Z
12.	B W U G E	C P W N Z	32.	C O L T S	B H S U E
13.	Y A V L E	V S E D Z	33.	H B X S N	O B D S T
14.	R Y K Z E	G S R L D	34.	S H L D O	U X S O X
15.	C R E B X	K S C D V	35.	U N A Z O	W S K F W
16.	L X U Y P	C B W R S	36.	F D M X U	O D T R U
17.	K Y M C A	D G T B D	37.	D T B W C	G U L Q K
18.	V W E Y M	Q Z C F G	38.	X U R C A	D G X O D
19.	O M T R D	A X I H W	39.	V N O D U	Y C U F P
20.	E L D F T	I N A S V	40.	O V S H L	S I Y E F

(2) The following information has been gleaned from an initial study of the traffic:

a. The messages have flat counts, and polygraphic repetitions within messages are no more than those expected for random; but a high hit rate between certain pairs of messages proves that the cryptosystem is one in which depths are possible.

b. The texts of message nos. 16 and 18 are in flush depth.

c. Message no. 38 is offset 30 positions to the right of message no. 15; and message no. 28 is offset 30 positions to the right of message no. 14.

d. Message no. 37 is offset 60 positions to the right of message no. 31; and message no. 30 is offset 60 positions to the right of message no. 33.

e. Message no. 6 is offset 90 positions to the right of message no. 3; message no. 16 is offset

90 positions to the right of message no. 39; and message no. 5 is offset 90 positions to the right of message no. 25.

f. Message nos. 6, 13, 28, 30, 32, 34, and 35 have been determined from traffic analysis to be high-precedence messages.

(3) It has been observed that there are no J's present in any of the indicator groups, although there are J's in the cipher texts of the messages. The A1 group always consists of a set of five different letters; the over-all I.C.'s of the A1 groups (0.97) and the Z0 groups (1.01) are flat, as are the ten individual columnar counts except for the Z0*e* which has an I.C. of 1.47. The only doublets present in the Z0 groups are in the *bc* position. Several instances of apparent digraphic relationships are seen between the A1 and Z0 groups (e.g., note the LM's in the A1 groups of message nos. 3 and 8 associated with IZ's in the Z0 groups), and it is observed that these relationships are reversible (e.g., note the CR in message no. 15 and the RC in message no. 22 associated with DV and VD). It is further noted that these digraphic relationships are confined to the first and last digraphs of the two indicator groups; it is therefore conjectured that the last digraph of Z0 represents an encipherment of the first digraph of A1, and that the first digraph of Z0 represents an encipherment of the last digraph of A1, leaving the *c* position of Z0 perhaps a monographic encipherment of the *c* position of A1. The absence of the letter J, the lack of doublets in the Z0*ab* and *de* positions, together with the reversibility feature, suggests Playfair; but encipherments such as the $AK_p=RK_c$ in message no. 3 seem to require that the Z0 groups be read *in reverse* before establishing equivalencies with the A1 groups. The Playfair hypothesis is quickly proved with the reconstruction of the square and the derivation of a key word on which it is based, and it is found that the *c* position of A1 is enciphered with the square by taking the letter to its immediate right in the row in which it is located.

(4) The A1*c* position of the high-precedence messages consists of one of the letters A, F, L, Q, or V; these letters are 5 apart on the normal sequence (if J is omitted), and we might even conjecture that these letters come from the first column of a 5×5 square into which has been inscribed the normal sequence, less J. Is it possible that the A1*c* represents the precedence, and that the latter is based on the identity of the particular one of five columns (of such a square) in which the letter is found? Message nos. 16 and 18 which are in flush depth have the A1 groups LXUYP and VWEYM; and since the indicators in a flush depth situation may be presumed to represent identical information, the letters in the *c* position, U and E, *could* come from the fifth column EKPUZ of our putative 5×5 square. These manifestations in the *c* position may either be encipherments, or perhaps even more likely. some type of *variants.*

(5) The existence of depths composed of pairs of messages offset at intervals of 30, 60, and 90 suggests that the keying sequence is in multiples of 30 (i.e., if a key book is involved, the pages contain lines of 30 letters each), and that therefore the indicators designate the starting line used. If this hypothesis is correct, then the *ab* and *de* digraphs of A1 could indicate the page and line, probably in that order. Then from message nos. 16 and 18 in flush depth with A1's of LXUYP and VWEYM, it should follow that LX=VW, and YP=YM. The suggestion of variants is still strong, so perhaps A1*ab* and *de* are digraphs representing *single letters* indicating page and line in a key book: thus the book might contain 25 pages of 25 lines each. If the interior of a 5×5 square is inscribed with the normal sequence, perhaps the row- and column coordinates of the bipartite variant square also consist of the normal sequence following some prearranged route. The urge is very strong to hypothesize the following coordinate configuration, in which the elements LX=VW and YP=YM are shown in ringed capital letters, the lower-case letters being the completion of the assumed order of coordinate letters:

					e	k	P	u	z
					d	i	o	t	y
					c	h	n	s	X
					b	g	M	r	W
					a	f	l	q	v
V	q	L	f	a	A	B	C	D	E
w	r	m	g	b	F	G	H	I	K
x	s	n	h	c	L	M	N	O	P
Y	t	o	i	d	Q	R	S	T	U
z	u	p	k	e	V	W	X	Y	Z

In order to test what may seem to be a bold assumption, we consider a pair of messages, nos. 15 and 38, which are in offset depth 30 intervals apart on the keying sequence, with A1 groups **CREBX** and **XURCA**: sure enough, **CR**=**XU** on the diagram above, and we find that **BX** and **CA** represent sequent positions (=30 letters in the keying sequence). A second test is in order, so we pick messages nos. 31 and 37 which are in offset depth 60 intervals apart, with A1 groups **YSKLQ** and **DTBWC**: again on the diagram above **YS**=**DT**, and **LQ** and **WC** are two positions apart. We shall make one final test to be triply sure, so we examine messages nos. 3 and 6 which are in offset depth 90 intervals apart, with A1 groups **AKPLM** and **QIFWE**: **AK**=**QI** in the diagram, and **LM** is separated by three positions from **WE**. All the messages may now be correctly superimposed with respect to the keying sequence of $25 \times 25 \times 30 = 18{,}750$ letters, keeping in mind the two possibilities that when the bottom of a key page is reached, the key is continued (a) from the top of the same page, or (b) from the top of the next page (perhaps with a proviso in the cryptographic instructions that a message should not be enciphered with keys starting past, let us say, the middle of page Z, the last page).

(6) This example, although artificial, nevertheless contains complexities similar in nature to those of certain complicated indicator systems encountered in actual operations. Without sufficient material for study, and without the element of luck which is all too often essential to success, we might not have been able to complete the solution. Sometimes we are able to solve a problem in spite of errors in thought or procedure, and at other times we are able to reach a solution *because* of such errors.[73] As a demonstration, we assumed in subpar. (4), above, that the indicators **LXUYP** and **VWEYM** must represent identical information, because the messages to which they appertained were in flush depth, and, encouraged by our findings, we went ahead and solved our problem. This is faulty reasoning, because the *c* position of the indicator (which happens to be a *precedence* symbol) bears no relation to the page and line keys used in the encipherment, and thus the two messages in depth would not necessarily have the same precedence. Had the letters in the *c* position of the two groups been, say **U** and **O**, we might have argued ourselves out of the hypothesis being postulated and thus either delayed the solution, or missed it completely. Furthermore, although in the preceding subparagraph it was conjectured that A1*ab* and *de* were digraphs representing *single* letters for the page and line indicators, these digraphs could just as well have represented as many as 25^2 pages and some number greater than 25 (say, 50, 75, or 100, with a variant usage) for the line designator.

(7) As an epilogue to the discussion of interpretation of indicator phenomena, we should keep in mind certain situations which have arisen in operational practice. We must not overlook the possibility of *cleartext indicators* in various contexts with which we may be familiar, and extrapolations from familiar contexts. The situation in subpar. 76*f*(3) may be cited as an example, that of Hagelin C–38 indicators in the clear which were interpretable at once from the five columnar distributions of the *a–e* positions of the indicator groups. If these indicators had been enciphered by five different direct or reversed standard cipher alphabets, recognition and interpretation would hardly have been delayed. And even if the encipherment had been accomplished by five mixed alphabets, the severe limitation in the indicator positions representing the last two wheels would still enable easy recognition and interpretation. In connection with cleartext indicators, we should also be on the lookout for an indicator in the clear every *n* groups of the cipher text (as in a case in which every 40th group was such an indicator for purposes of check on the encipherment). *Tailing indicators* are often easily interpretable under apparent or assumed rules of motion when the rules are not too complex, such as in certain pin-wheel devices or in certain varieties of notch control in wired-wheel systems. Null letters in indicators may be identified as such by finding indicators of messages known to be in flush depth, with a discrepancy in one of the positions; e.g., indicators of the form **LDPCX** and **LDPCY** in a machine cipher system could

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[73] The most striking example that comes to mind of incorrect reasoning making solution possible occurred many years ago during World War II, before the advent of computers, in a very complex teleprinter cipher system about which nothing was known. A long sequence of key was recovered from reading a depth, and, because of a totally unwarranted conjecture of the apparent limitation of two of the indicator positions to 23 and 25 letters, the key levels were written out on a width of the product of these numbers, 575. In this write-out, some long polygraphic repetitions were observed on successive rows, but displaced one position to the left, indicating a significant period of 574, not 575. Since $574 = 14 \times 41$, the key levels were written on the prime width of 41; more polygraphic repetitions were found on this width, proving that the cycle of 41 played a part in the machine. Had it not been for the totally erroneous interpretation of the indicator phenomena, it is possible that the machine would not have been solved.

suggest the possibility of four wheels and a null position.[74] Finally, literal systems have been encountered which have an underlying *numerical base,* such as in the simple scheme of A=1, B=2 . . . J=0, or as in the case of the three pairs of indicators and indicator-check groups shown below:

	A1	A2
1.	M A N U S	C U D K I
2.	B G K H R	V Q A R H
3.	Z U Q M G	P A G W Q

These could have come from the variant values of the following diagram:

1	2	3	4	5	6	7	8	9	0
A	B	C	D	E	F	G	H	I	J
K	L	M	N	O	P	Q	R	S	T
U	V	W	X	Y	Z				

Furthermore, given the order 1–0 for the digits in the top line, it is seen that the A1 and A2 are both sum-checking groups, confirming this order.

79. The treatment: hypothesis formulation and testing.—*a.* In modern practice, cryptodiagnosis may be thought of as embracing hypothesis formulation and hypothesis testing. The establishment of a hypothesis is followed by a particular diagnostic test or set of tests, and their outcome will determine the validity of the hypothesis and the subsequent road to be followed.

b. As an elementary example, in studying the 385-letter cryptogram given in subpar 77*g*(6) with its accompanying on-the-cut digraphic distribution as shown below,

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	-	-	1	2	-	-	1	-	-	-	-	-	2	2	-	-	2	2	-	-	2	-	-	-	-	-
B	-	-	2	2	-	-	1	-	-	-	-	-	-	2	1	2	2	2	-	-	1	-	-	-	-	-
C	-	2	-	-	1	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-
D	1	1	-	-	-	1	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	1	1	1	1	1
E	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	1	-	-	-	-	-
F	-	-	-	2	-	-	-	-	-	-	-	-	2	2	-	-	2	2	-	-	2	-	-	-	-	-
G	-	1	-	-	-	-	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	1	-	1	-
H	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	1	-	-	-	-	-	-	-	-	-
I	-	-	-	-	-	-	1	-	-	-	-	-	1	1	-	1	1	1	-	-	-	-	-	-	-	-
J	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
K	-	-	2	2	-	-	1	-	-	-	-	-	1	2	1	2	1	2	-	-	-	-	-	-	-	-
L	-	-	-	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-
M	1	-	-	-	-	1	-	1	-	-	-	-	-	-	-	-	-	-	1	-	-	1	1	-	-	-
N	2	2	-	-	-	1	-	-	1	-	2	-	-	-	-	-	-	-	-	-	-	1	-	1	1	1
O	-	1	-	-	-	-	-	1	-	-	1	-	-	-	-	-	-	-	-	-	-	1	1	-	1	-
P	-	2	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	1	-	-	-	-	1	1	-
Q	1	1	-	-	-	1	-	-	1	-	1	-	-	-	-	-	-	-	-	-	-	2	-	1	2	-
R	1	2	-	-	-	1	-	-	1	-	2	-	-	-	-	-	-	-	-	-	-	1	-	2	2	1
S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-
T	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
U	1	-	-	-	-	1	-	-	-	-	-	1	-	-	-	-	-	-	-	1	-	1	1	-	-	-
V	-	-	-	2	-	-	-	-	-	-	-	-	1	2	1	-	2	2	-	-	1	-	-	-	-	-
W	-	-	-	1	-	-	1	-	-	-	-	-	-	1	-	-	-	-	-	-	1	-	-	-	-	-
X	-	-	-	1	-	-	-	-	-	-	-	-	-	2	-	-	-	1	-	-	1	-	-	-	-	-
Y	-	-	2	1	-	-	1	-	-	-	-	-	-	2	1	2	2	2	-	-	1	-	-	-	-	-
Z	-	-	1	1	-	-	-	-	-	-	-	-	-	1	-	-	-	2	-	-	1	-	-	-	-	-

[74] The possibility of *variants* should of course not be overlooked; one must also keep in mind that an apparent null or dummy position may not always be what it seems—it may be information extraneous to the keying, as was the case in the example in subpar. (4), above. For that matter, each indicator letter may control more than one keying element, so that each of the first four letters LDPC in the example above might be used to set a pair of wheels in a 8-wheel machine.

we were struck by (a) the absence of the letter J, (b) the lack of doublets, (c) the apparent matching propensities among the rows and columns of the distribution, and (d) the symmetry about the main diagonal. These characteristics lead us to the hypothesis of a bipartite system with a commutative matrix, and under this hypothesis we amalgamate the variant rows and columns of a 5×5 square, reducing the text to uniliteral, monoalphabetic terms with a satisfactory over-all δ I.C., following which the plain text is recovered, proving the hypothesis.

c. In another elementary example, if we have identified a cipher as being digraphic substitution and we wish to test it for the possibility of its being enciphered with an *inverse four-square system*,[75] all we have to do is to convert the cipher text with an arbitrary four-square matrix containing the normal sequence in the *ciphertext* sections and, if the hypothesis is correct, the distributions of the initial letters and of the final letters of the converted digraphs should exhibit the characteristics of monoalphabeticity.[76]

[redacted]

80. Post mortem.—*a*. After a system has been solved (or for that matter, just as important, even if a system has *not* been solved), the diagnostic steps taken and a résumé of the work done on the problem should be properly documented. Technical reporting is all too often inadequate, in spite of the fact that it should be a major facet of cryptanalytic operations if the reports are to be of benefit to technicians, particularly the newer personnel.[77] In those cases wherein a system has not been solved, it is important to record the work which has been done, in order to evaluate the steps already taken, and to avoid unnecessary duplication if the problem is restudied later. In Fig. 183 is illustrated a suggested reporting form for unsolved systems which contains on a single sheet of paper a convenient résumé of the system.

b. As mentioned in subpar. 74*f*, it is important that the cryptanalyst have readily available the salient cryptolinguistic data in the languages with which he is working. Among these data, the indispensable items are the following:[78]

1. The frequencies of single letters, the monographic kappa plain constant, and the monographic I.C.
2. The frequencies of digraphs, the digraphic kappa plain constant, and the digraphic I.C.
3. The frequencies of initial letters of words.
4. The frequencies of initial diagraphs of words.
5. A listing of the most frequent trigraphs, with their frequencies.
6. A listing of the most frequent initial trigraphs of words, with their frequencies.
7. A listing of the most frequent tetragraphs, with their frequencies.
8. The average length of words.

In addition to the foregoing, we should also have lists of frequent words and common phrases expected to be in the traffic, as well as a collection of representative samples of different types of messages.

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[77] For suggestions on technical report writing, and for a model of a technical report, see pp. 386–391 of *Military Cryptanalytics, Part I*.

[78] See in this connection the compilation I prepared in 1956 entitled *Letter Frequency Data, Foreign Languages*, which contains the indispensable cryptolinguistic data, through tetragraphs in the following 20 languages: Albanian, Arabic, Bulgarian, Czech, Dutch, French, German, Greek, Hungarian, Indonesian, Italian, Persian, Polish, Portguese, Romanian, Russian, Serbo-Croat, Slovak, Spanish, and Turkish.

Reporting Form for Unsolved Systems

* * * * *

1. Short Title: ________________ 2. Service Description: ______________________________
3. Time element covered: __________________________ to ______________________________
4. Message volume: ________ per week; ________ per month; __________________ total on hand
5. Msgs sent in ____ -letter groups; ____ -digit groups. Missing chars.: ______________________
6. Discriminant: _______________; Indicator(s): ____
7. Other nontextual groups: ____
8. Possible indicator explanation: ____
9. Evidence of tailing, trailing, or flush starts: ____
10. Remainder test on ___________ msgs: ____
11. Frequency counts:

Msg No.	No. of chars.	Mono I.C.	Dig I.C.	Dig Cut A	Dig Cut B

12. Width tests:
    Msg No. _________; width _________; av. depth _________; av. I.C. _________
    Msg No. _________; width _________; av. depth _________; av. I.C. _________
    Msg No. _________; width _________; av. depth _________; av. I.C. _________
    Msg No. _________; width _________; av. depth _________; av. I.C. _________
13. Evidence of local roughness: ____
14. Positional roughness, ___________ group: 1 ______ 2 ______ 3 ______ 4 ______ 5 ______
15. Phenomena in delta stream: ____
16. Polygraphic repeats within msgs _______; between msgs _______; at beginnings ________; at endings _______; in body of text _______.
17. Evidence of isomorphism: ____
18. No. of flush depths _______; offset depths _______; maximum depth _________ deep.
19. Bust msgs: ____
20. Analysts most familiar with system: ____
21. Additional remarks (coded to item nos. above); related facts, collateral information (e.g., predecessor systems, probable type of system, traffic analysis observations):

Date: ___________ 19____ Reported by: ______________________ Org. ___________________

FIGURE 183

c. There is nothing like live traffic in an unknown cryptosystem to test one's assimilation and understanding of diagnostic principles and techniques. If the traffic is in a low- or medium-grade system, diagnosis and solution should be a matter of course, given a sufficient amount of material for study. But even if the traffic is in a high-grade system, something can often be discovered in fairly short order, even if the magnitude of the discovery is not particularly startling—after all, a jigsaw puzzle consists of many pieces. For example, (a) the identification of buried indicator groups in certain positions of the messages, or (b) the recognition of probable machine-cipher traffic (implied by the level of correspondents and the volume of traffic passed) as Enigma from the over-all uniliteral frequency distribution typical of a noncrashing system, should be made quite early in the cryptanalytic game. In the meantime, or perhaps even as a concurrent exercise, the analyst would derive benefit and gain experience from participation in the Zendian Problem [79] (especially if he worked as a member of a team [80]), which provides considerable

[79] Cf. Appendix 8 ("The Zendian Problem: An Exercise in Communication Intelligence Operations"), *Military Cryptanalytics, Part II.*

[80] For example, as do the students in my *Intensive Study Program in General Cryptanalysis.*

practice both in traffic analysis and in the diagnosis and solution of the low-, medium- and high-grade systems contained therein.

d. This chapter began with a definition; it is therefore fitting that we also end it with a definition:

"diagnostician, n. An experienced cryptanalyst of ability, just before retirement age."

This is but a gentle reminder to the reader of an aphorism of Hippocrates, as translated by Chaucer: "The lyf so short, the craft so long to lerne." [81]

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[81] What Hippocrates actually said was ὁ βίος βραχύς, ἡ δε τέχνη μακρή.

CHAPTER XII

CONCLUDING REMARKS

81. Special cases of aperiodic encipherment.—*a*. This text has treated the principal methods for achieving aperiodic polyalphabetic substitution, which, from the standpoint of the cryptographic mechanics, fall into two categories: (1) systems in which the key elements are not in any way determined or influenced by any elements of the plain or cipher text; and (2) systems in which the key elements are generated or governed by the plain text being enciphered or by the resultant cipher text. Complicated operational examples have been encountered in each of these categories, and sometimes as a merger of the two categories.

b. In addition to the monographic methods already treated, there are theoretically possible aperiodic *digraphic* substitution systems, paralleling the periodic digraphic systems illustrated in the preceding text.[1] For example, in a scheme involving a plurality of Playfair squares generated from a basic square such as the following,[2] in which a permutation is accomplished two columns at a time by inscribing the five initial row digraphs of a square vertically into the two columns to the right of each succeeding generated square,

```
 1   2                       1   2   3
|H Y D R A|H F              H Y|D R A H F|D K
|U L I C B|Y N              U L|I C B Y N|R P
|E F G K M|U O              E F|G K M U O|I Q
|N O P Q S|L T              N O|P Q S L T|C W
|T V W X Z|E V              T V|W X Z E V|G X
```

producing the following band of 25 squares:

```
1   2   3   4   5   6   7   8   9   10  11  12  13
H Y D R A H F D K A U F I K B U N I P B S N T P W S Z T V
U L I C B Y N R P H S D T A W F Z K V U X I E B G N M P O
E F G K M U O I Q B L N C P Y S R T H W D Z A V F X K E U
N O P Q S L T C W Y Z R V H X D E A G F M K O U Q I L B C
T V W X Z E V G X M E O G Q M L O C Q Y L R C H Y D R A H

13  14  15  16  17  18  19  20  21  22  23  24  25      (check)
W S Z T V W X Z E V G X M E O G Q M L O C Q Y L R C|H Y D  R A|
G N M P O S Q T L W C Z Y V R X H E D G A M F O K Q|U L I  C B|
F X K E U G I M B O N Q P L S C T Y W R Z H V D X A|E F G  K M|
Q I L B C N Y P R S H T D W A Z F V K X U E I G B M|N O P  Q S|
Y D R A H F D K A U F I K B U N I P B S N T P W S Z|T V W  X Z|
```

[1] Chapter XIII, *Military Cryptanalytics, Part II.*

[2] Cf. subpar. 91*c*, *Military Cryptanalytics, Part II.*

These squares, used sequentially, would yield periodic polyalphabetic substitution with a period of 25 diagraphs; but the band of squares may be used in various ways to produce aperiodic encipherment. For example, in a plaintext interruptor system with E_p as the interruptor letter, whenever E_p occurs as the *first* letter of a digraph,[3] a double skip in the otherwise normal key progression could take place. In the illustrative encipherment below, we have used the foregoing Playfair band, starting in square no. 1 with E_p as the plaintext interruptor:

Square no.:	1	3	5	6	7	8	9	10	11	12	13	14	16	17	18
Plain:	EN	EM	YP	AT	RO	LS	AC	TI	VE	IN	AR	EA	WE	ST	OF . . .
Cipher:	NT	ZU	WH	WA	EN	UD	CI	NE	AB	DX	HA	BT	LV	HD	GU

Or, in a plaintext autokey variation, the *second* letter of the plaintext digraph could designate the particular square (identified by the letter in the upper left-hand corner of the square) for the next encipherment; e.g., the "A" square is square no. 3, the "B" square is no. 8, and so on. This is illustrated in the following example:

Square no.:	1	9	19	10	12	20	11	23	7	17	9	25	3	17	12
Plain:	EN	EM	YP	AT	RO	LS	AC	TI	VE	IN	AR	EA	WE	ST	OF . . .
Cipher:	NT	AE	PD	OE	CL	OW	OT	PE	HV	FQ	ET	FE	LX	HD	QA

Ciphertext interruption and ciphertext autokey could be accomplished in a similar manner. An added complication would be to predicate the keying, not on a single letter, but on the mod-25 sum (as measured on the normal A–Z sequence, less J) of *both* letters of the keying digraph. Thus, in the example below, the second digraph is enciphered in square no. $(E+N=5+13=18)$, the third digraph is square no. $(E+M=5+12=17)$, the fourth digraph in square no. $(Y+P=24+15\equiv 14)$, and so on:

Square no.:	1	18	17	14	20	6	4	4	3	1	22	18	6	2	12
Plain:	EN	EM	YP	AT	RO	LS	AC	TI	VE	IN	AR	EA	WE	ST	OF . . .
Cipher:	NT	OE	PD	TP	SR	ZL	DY	CO	GV	UP	GZ	OW	SM	LP	QA

This latter scheme could yield ciphers of considerable complexity, as would the use of a very long literal key (with J either being ignored, or combined with I), such as in the following example:

Square:	W	H	E	N	I	N	T	H	E	C	O	U	R	S	E
Plain:	EN	EM	YP	AT	RO	LS	AC	TI	VE	IN	AR	EA	WE	ST	OF . . .
Cipher:	XP	FE	PD	CA	EN	SX	OT	WU	GV	UP	US	QS	TG	NP	NU

c. Many types of aperiodic substitution systems initially appear very complicated or difficult in their cryptanalysis, but, as we have seen from our studies thus far, methods for their solution become readily available or demonstrable.[4] Even in apparently more abstruse cryptographic cases, methods for their solution can readily be postulated. We shall now take up two illustrative special cases.

82. Analysis and solution of a first case.—*a.* For the first example of what at first blush appears to be a complex case of aperiodic encipherment, we shall treat a cryptosystem that once occurred to the author in a dream after a particularly sumptuous meal.[5] In that dream there came to us the principle for

---

[3] For that matter, an E_p in *either* position of a diagraph could be used to govern the key interruption.

[4] In this connection, the following quotation may provide some food for thought:

> "It has been said, with regard to musical problems, that musicians generally give the correct answers supported by illogical argument, but mathematicians arrive at incorrect answers through a process of irrefutable reasoning."
>
> —A. L. Leigh Silver, in *The American Mathematical Monthly*, Vol. 78, No. 4, April 1971.

From this can be extrapolated that the intuitive cryptanalyst may hit upon a solution for reasons that are entirely irrelevant, whereas a mathematician may correctly define the steps in a problem, predicated however on initial assumptions that later prove to be invalid.

[5] This story, including complete details of the meal which gave rise to the dream, is written up in an article entitled "A Cryptanalyst's Nightmare: An Oneirotic Problem and Its Solution," appearing in the *NSA Technical Journal*, Vol. XVI, No. 4, Fall 1971.

a cipher machine, a principle startling in its simplicity and yet apparently devilishly difficult in its cryptanalytic aspects—a machine incorporating sliding components with both plaintext *and* ciphertext autokeying in its cryptography. In this system, the first letter of the message plain text is enciphered by a key letter designated by an indicator; after that, every subsequent plaintext letter is enciphered by a key letter which is the mod-26 sum of the immediately preceding plaintext letter and its ciphertext equivalent. Thus the system would embrace all of the nasty features of plaintext and ciphertext autokeying, and none of their [for the cryptanalyst] welcome attributes: polygraphic repetitions, which are plentiful in plaintext autokey, are here drastically reduced; and isomorphs, which arise in ciphertext autokey, should here be nonexistent. If the introductory key consists of several letters, there is no statistical way of determining its length, and other mathematical techniques applicable in autokey systems are fruitless here. A veritable cryptanalyst's nightmare!

b. But let us approach the subject calmly: after all, there must be *some* solution, albeit for a special situation. Let us consider the trivial case of known components and examine what happens in the following message enciphered with direct standard alphabets:

		6	21	22	9	26	11	8	11	26	9	14	23	6	17	26	1	14	19	22	7	24	11	2	5
K:	M	F	U	V	I	Z	K	H	K	Z	I	N	W	F	Q	Z	A	N	S	V	G	X	K	B	E
P:	W	H	A	T	I	S	Y	O	U	R	P	R	E	S	E	N	T	P	O	S	I	T	I	O	N
C:	I	M	U	O	Q	R	I	V	E	Q	X	E	A	X	U	M	T	C	G	N	O	Q	S	P	R

The first plaintext letter, W_p, enciphered in the initial key of M_k yields I_c as the cipher equivalent. The second plaintext letter, H_p, is enciphered by the mod-26 sum ($\equiv 6 = F_k$) of the preceding W_p and I_c, yielding M_c as the result. The third plaintext letter, A_p, is enciphered by the mod-26 sum ($\equiv 21 = U_k$) of the preceding H_p and M_c, yielding U_c. And so on.

c. Now if all we had were the ciphertext message and a knowledge of the general system, we could make trial decipherments on the basis of direct standard cipher alphabets. In the three diagrams below, we have assumed that the first plaintext letter is A, B, and C, in turn, and we have deciphered the first five letters accordingly:

		10	17	26	5
K:		J	Q	Z	E
C:	I	M	U	O	Q
P:	A	D	E	P	M

	11	16	1	4
	K	P	A	D
I	M	U	O	Q
B	C	F	O	N

	12	15	2	3
	L	O	B	C
I	M	U	O	Q
C	B	G	N	O

Noting the alternation of progression of the 1st, 2d, 3d . . . letters of the decipherments, we construct from the first "decipherment" a generatrix diagram in which the sequence in the even columns runs in the opposite direction from that of the odd columns, as follows:

	I	M	U	O	Q
	A	D	E	P	M
	B	C	F	O	N
	C	B	G	N	O
	D	A	H	M	P
	E	Z	I	L	Q
	F	Y	J	K	R
	G	X	K	J	S
	H	W	L	I	T
	I	V	M	H	U
	J	U	N	G	V
	K	T	O	F	W
	L	S	P	E	X
	M	R	Q	D	Y
	N	Q	R	C	Z
	O	P	S	B	A
	P	O	T	A	B
	Q	N	U	Z	C
	R	M	V	Y	D
	S	L	W	X	E
	T	K	X	W	F
	U	J	Y	V	G
	V	I	Z	U	H
*	W	H	A	T	I
	X	G	B	S	J
	Y	F	C	R	K
	Z	E	D	Q	L

The message plain text comes out on one generatrix, as is indicated by the asterisked row.

d. The foregoing case was patently trivial, the components being known. If, however, the components were *unknown*, it would seem that the only practical solutions must be predicated on either a huge sample of cipher text (correctly assuming—without foundation—the length of the introductory key) for the possible application of Fourier techniques and other palpable mathematical approaches, or a sizeable quantity of matched plain and cipher text—if we are to call these practical solutions. In the next case to be examined, let us assume a knowledge of the general system, and let us further assume that the plain and cipher are unknown mixed sequences but that the sequence for the keying (i.e., for performing the modular arithmetic) is the normal sequence. Therefore if we had available some matched plain and cipher, we would also be privy to the *keys* involved. Let us now study the two message beginnings given below:

```
                  5         10        15        20        25
           21 20 14 10
     K:     U T N J Z B U T Z F G J Z R P Y L M I F B R J X E
1.   P:   R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
     C:   C O H E H W G Q U G R O H E K F S L B A N W W V Z W

           12 7 17 24
     K:     L G Q X H A N P W M S V K L G B G H N J S V T H A
2.   P:   R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
     C:   T B K S P V Z M R N D A S Y B I N G G E E A G F V O
```

If the keying sequence is the normal sequence, then in Message No. 1 the sum $R_p+C_c=21=U$ will be the key for enciphering the second plaintext letter, and the sum $E_p+O_c=20=T$ will be the key for enciphering the third plaintext letter, and so on. The 50 key-plain-cipher relationships derivable from the foregoing diagram are inserted in a reconstruction matrix, as shown in Fig. 184, below.

Plain

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A														Z				O								
B														G					N		W					
C																										
D																										
E																		W								
F														N	R											
G	G					K													I		O					
H					V		G																			
I					A																					
J		V												E				H								
K													Y													
L	L				B																					
M							B								D											
N			M		E																					
O																										
P					R														F							
Q					S																					
R					K								W													
S																					A					
T		F			U	H																				
U			Q		O																					
V													G					S								
W																									N	
X					Z													P								
Y																			S							
Z					W								E												G	

FIGURE 184

Noting in the A column that G_c and L_c are opposite G_k and L_k, we capitalize on this implication and chain our cipher letters to the Ø column outside the matrix. The partial chains are the following:

Cols.								
Ø–B:	JV	TF						
Ø–C:	NM	UQ						
Ø–E:	HV	IA	LB	NE	PRK	QS	TUO	XZW
Ø–F:	GK	TH						
Ø–G:	HG	MB						
Ø–M:	KY	RW	VG	ZE				
Ø–N:	AZ	BG	FN	JE				
Ø–O:	FR	MD						
Ø–R:	AO	EW	JH	VS	XP			
Ø–S:	BN	GI	PF	YS				
Ø–U:	BW	GO	SA					
Ø–Y:	WN	ZG						

From these chains the following equivalent primary cipher component can easily be reconstructed:

X Z W Y T U O Q S P R K L B F J D G N E M C I A H V

This sequence is then decimated at various trial intervals, and when the decimation of −7 is reached, it will be discovered that the original cipher component is based upon the diagram below:

```
6 1 2 4 5 7 3
M A C H I N E
B D F G J K L
O P Q R S T U
V W X Y Z
```

The plain component can now be recovered by juxtaposing a blank strip against the known cipher component, designating an arbitrary cell as the index position against which to set the key letter, and filling in the plaintext values at the various positionings of the sliding cipher component. This yields the following sequence:

A G . S . . B . O U . E F . . . M . N . Y R C . . .

Although there are only 13 different plaintext letters in our component, the recovered sequence can be put together as in the diagram below, revealing the key word DREAM:

```
2 5 3 1 4
. R E A M
B C F G .
. . . . N
O . . S .
U . . . Y
.
```

e. For the third case, we have the following matched plain and cipher beginnings of three messages:

```
            5         10             15             20             25
1.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    O B P X S V P U Z L J A L F A X X E H X V H E F P L
2.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    A T F O B K F C L D E G D M G O O H S O K S H M F D
3.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    D E K L J O K B U C F I C R I L L M Q L O Q M R K C
```

To our utter astonishment, isomorphism has appeared in a system we could have sworn could not possibly yield isomorphs, since *plaintext* autokey is one of the features involved in the encipherment! Nevertheless, doing what comes naturally, we derive the following sets of partial chains from the foregoing beginnings:

Msgs 1–2: XOAG JEHSBT PFM VK UC ZLD
Msgs 1–3: VOD ZUBEM PK XLC SJFR AI HQ
Msgs 2–3: ADCBJ TEFKOLU GI HMR SQ

These partial chains are quickly put together into the following cipher-component sequence, J E H S B T . U C . Z L D I X O A G V K R . P F M Q, and upon trial decimation we discover that at the interval of −5 we can recover the original key word upon which the sequence is based:

```
7 5 3 4 9 6 1 8 2
. I G H T M A R E
B C D F J K L O P
Q S U V . X . Z
```

The plain component is found to be identical to the cipher component, as is also the keying sequence. The isomorphism is a result of identical cipher and keying sequences, the constitution of the plain component being irrelevant.

f(1). For the last case, we shall examine the matched plain and cipher of the following eight message beginnings:

```
                 5         10        15        20        25        30
1.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    F A O N A G C Z M K W V B D K K U U U V B F B F Q D
2.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    H Q K H S S O H V J T Q T A E H T T N H W S T V F X
3.  R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
    A G X X V L N M S T V D D G R P X X B U X Q F E O H
4.  T O C O M M A N D I N G O F F I C E R S A L L U N I T S . . .
    R N H S A I Y Z L L O B J Q Z X U K C A A V X N H I O I
5.  T O C O M M A N D I N G O F F I C E R S A L L U N I T S . . .
    V T G Z J H Q C B D D H E J H Y X W J J J U R G B Q N S
6.  T O C O M M A N D I N G O F F I C E R S A L L U N I T S . . .
    M F C L T T U A S O E J O S O M A C Z L L W C T G J M H
7.  R E C O N N A I S S A N C E R E P O R T S I N D I C A T E . . .
    T K J I E L K E Q A A N M S Y O K A C W K G A S O V L Y C
8.  R E C O N N A I S S A N C E R E P O R T S I N D I C A T E . . .
    J E R J A Y Q I B G G T N G L Y R B A H F D D I E L S N I
```

We begin by indexing the vertical plain-cipher digraphs and, in the hope that commutative keying (i.e., involving only one keying sequence) is present, we index the first four vertical digraphs of Msg No. 1 as **FR**, **AE**, **FO**, and **EN**. From this index we list the key digraphs occuring two or more times together with the plain-cipher equivalencies which directly follow them:

	Key	P-C equivalents				
1.	AA	LV	NN			
2.	AL	LW	TY			
3.	AM	EE	MI			
4.	AN	DS	NY			
5.	AQ	NC	II			
6.	AR	EG	TH			
7.	AS	AA	TN			
8.	AU	GU	NA			
9.	BG	EU	OJ			
10.	BN	UF	IQ			
11.	CH	EV	OS			
12.	CL	UT	AS			
13.	CN	CZ	DB	EG		
14.	CR	SA	TW	OJ		
15.	DI	ND	IE			
16.	DN	GH	DI			
17.	EE	SH	CR			
18.	EG	FX	NC	RL		
19.	EH	RS	NW			
20.	EI	SQ	CL			
21.	EK	SK	RC	CJ		
22.	EN	RA	GJ	NL		
23.	EO	RH	FJ	PK		
24.	EQ	RD	FK			
25.	ES	NO	YT	RY		
26.	EV	NB	YJ			
27.	FO	EN	IM	CC		
28.	FS	FO	ID			
29.	GN	EH	IJ			
30.	GU	EV	NB			
31.	II	TO	SB			
32.	IM	AY	CA			
33.	IO	NE	CV			
34.	JO	FQ	NA			
35.	JR	SJ	EE			
36.	KS	SU	IG			
37.	LR	UG	EY			
38.	MT	BV	MT	AU	SH	OF
39.	NO	CH	GB			
40.	NT	CN	SS	EI		
41.	OT	UQ	SI	CG		
42.	QU	RT	MF			
43.	RT	MA	ON	EK		
44.	SU	AU	MT			
45.	TY	OV	EC			

The foregoing plain-cipher equivalents are now indexed in order to equate *key* equivalents. It is discovered that the plain-cipher equivalents AU, EE, EG, EV, MT, NA, NB, NC, OJ, and SH have occurred twice each, so we are able to arrive at the diagram of equivalencies shown in Fig. 185, below:

	Keys			P-C equivalencies							Keys	P-C equivalencies		
1.		BG	CR	TW	OJ	EU	SA			19.	EN	RA	GJ	NL
2.			CL	AS	UT					20.	EO	RH	FJ	PK
3.	EE	SU	MT	AU	BV	MT	SH	OF	CR	21.	EQ	RD	FK	
4.		JR	AM	EE	MI	SJ				22.	ES	NO	YT	RY
5.		CN	AR	EG	TH	CZ	DB			23.	FO	EN	IM	CC
6.	EV	GU	CH	EV	OS	NB	YJ			24.	FS	FO	ID	
7.		JO	AU	NA	GU	FQ				25.	GN	EH	IJ	
8.		EG	AQ	NC	II	FX	RL			26.	II	TO	SB	
9.			AA	LV	NN					27.	IM	AY	CA	
10.			AL	LW	TY					28.	IO	NE	CV	
11.			AN	DS	NY					29.	KS	SU	IG	
12.			AS	AA	TN					30.	LR	UG	EY	
13.			BN	UF	IQ					31.	NO	CH	GB	
14.			DI	ND	IE					32.	NT	CN	SS	EI
15.			DN	GH	DI					33.	OT	UQ	SI	CG
16.			EH	RS	NW					34.	QU	RT	MF	
17.			EI	SQ	CL					35.	RT	MA	ON	EK
18.			EK	SK	RC	CJ				36.	TY	OV	EC	

FIGURE 185

The values of the plain-cipher equivalencies are inserted into a reconstruction matrix, as shown in Fig. 186, below:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1					U										J				A	W						
2	S																				T					
3	U	V	R										T		F				H							
4					E								I						J							
5			Z	B	G															H						
6					V									B	S										J	
7						Q	U							A												
8						X			I					C				L								
9												V		N												
10												W								Y						
11				S										Y												
12	A																			N						
13									Q												F					
14									E					D												
15				I			H																			
16														W				S								
17			L																Q							
18			J															C	K							
19							J							L				A								
20						J										K		H								
21						K												D								
22														O				Y							T	
23			C		N				M																	
24						O			D																	
25					H				J																	
26																			B	O						
27	Y		A																							
28			V											E												
29									G										U							
30					Y																G					
31			H				B																			
32			N		I														S							
33			G																I		Q					
34													F					T								
35					K								A		N											
36					C										V											

FIGURE 186

The following chains are derived from the foregoing matrix:

1– 3:	JF	AH	4–32:	EI	JS	8–24:	XO	ID
1– 4:	UE	AJ	4–35:	EK	IA	16–19:	WL	SA
1– 5:	UG	WH	5–23:	ZC	GN	16–22:	WO	SY
1– 6:	UV	JS	5–32:	ZN	GI	17–18:	LJ	QK
1–26:	AB	WO	6–22:	BO	JT	17–32:	LN	QS
1–32:	UI	AS	6–35:	VK	SN	17–33:	LG	QI
1–35:	UK	JN	6–36:	SVC		18–32:	JN	KS
1–36:	UC	JV	7– 8:	QX	AC	18–33:	JG	KI
3– 4:	TI	HJ	7–19:	UJ	AL	19–22:	LO	AY
3–17:	RL	HQ	8–14:	IE	CD	20–21:	JK	HD
3–18:	RJ	HK	8–16:	CW	LS	23–25:	NH	MJ
3–27:	UY	RA	8–19:	CLA		23–32:	CNI	
3–32:	RN	HS	8–20:	XJ	LH	32–33:	NG	SI
3–33:	RG	HI	8–21:	XK	LD	35–36:	KC	NV
3–35:	TA	FN	8–22:	CO	LY			

These chains may now be combined into the following cipher-component sequence:

S V C L A X F Q P E O Y G R T B I H Z J U D N K M W

When decimated at an interval of −9, it can be analyzed and found that this sequence is derived from the following diagram:

```
6 1 2 4 5 7 3
M A C H I N E
B D)
F G J)
K L O P Q R S)
T U V W)
X Y Z     )
```

(2) We now place our recovered cipher component in juxtaposition against a blank plain-component strip and, referring to line 1 of Fig. 186, we write an E_p over the U_c and O_p over the J_c, an S_p over the A_c, and a T_p over the W_c. We then slide F_c under the O_p, and, referring to line 3, we write A_p over the U_c, B_p over the V_c, C_p over the R_c, and so forth for all entries of line 3. Continuing in this vein, we recover the entire plain component and find that it is based upon the following diagram:

```
2 5 3 1 4
D R E A M
B C F G)
H)
I J K)
L N O P Q)
S T)
U V W X)
Y)
Z     )
```

(3) The final step is to recover the mixed sequence used for the modular sums of the key. Referring to Fig. 185, since we now know the plain and cipher components, we can arrive at artificial key values as measured on the pair of enciphering components. For example, in line 1 of Fig. 185, the equivalence of $E_p=E_c$ is designated as the key of N (using A_p as an arbitrary index letter), and in line 2 the equivalence $E_p=G_c$ as in the key of W, and so on. In this fashion we can determine the following equivalent modular sums *as measured on the unknown keying sequence:*

```
 1.  JR = AM
 2.  CN = AR
 3.  EI = RT = OS = AN
 4.  EG = AQ
 5.  QS = CR = BG
 6.  MT = EE = DI
 7.  AL = OT
 8.  AS = GN
 9.  CL = EK = DN
10.  EH = LR = NO
11.  EO = TY
12.  ES = FO
13.  JM = LN
14.  JO = NT
```

FIGURE 187

This means that the mod-26 sum of J plus R is the same as the mod-26 sum of A plus M, and so on, within each of the 14 lines of equivalencies. Two approaches are now possible: either a mathematical one involving a rather messy set of simultaneous equations with barely enough material for a solution, or a cryptologic one involving the standard techniques of indirect symmetry of position.

(4) We insert the relationships of the 14 rows of Fig. 187 into a matrix, as shown below in capital letters.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	M									R			a					j								
2	R		N											c				a								
3	N				I				e					a	S			T	o	r						
4	Q				G		e										a									
5		G	R				b										S	c	q							
6				I	E				d				T							m						
7	L											a			T					o						
8	S						N							g					a							
9			L	N	K						e	c		d												
10					H			e				R		O	n			l								
11					O										e					Y					T	
12					S	O									f				e							
13										M		N	j	l												
14										O				T	j					n						

The top row (the Ø row) in conjunction with one of the other rows shows that the expressed equivalencies have identical modular sums: that is, in rows 0–1, the combinations AM and JR sum to the same number, mod 26. Since we have already proved that there is commutative keying in the cryptosystem, the MA and RJ must also sum to the same number with AM and JR; therefore we may insert these reciprocal values into the matrix, as shown by the lower-case letters. Chains are then derived from this matrix, as shown below:

1– 2:	MR	JA		3– 8:	NS	OAG			7–11:	TE	OY	
1– 3:	MN	JT		3– 9:	IK	AD			7–14:	TJ	ON	
1–13:	RM	AJ		3–10:	IH	AO	SN	TL	9–10:	KH	CR	DO
2– 3:	RN	CAT		3–11:	IO	SE	RY		9–13:	CN	DL	
2– 5:	NR	AC		3–12:	ISF	OE			10–11:	HO	NE	
2– 8:	RS	CG		3–14:	AT	SJ	RN		10–12:	HS	NF	
2– 9:	NL	CD		4– 5:	EB	AS			10–13:	RN	OL	
2–10:	CO	AL		4– 8:	QS	EN			10–14:	OT	NJ	
3– 4:	NQ	IG		5– 8:	BN	QA			11–12:	OS	EF	
3– 5:	TC	OQ		6– 9:	IN	EK			11–14:	EJ	YN	
3– 6:	IED	RM		6–11:	EO	MY			13–14:	MO	LT	
3– 7:	NL	ST	RO	7–10:	AR	TN						

These partial chains can now be amalgamated into a single chain with six blanks, as follows:

```
A J Y . O . Q R M I E D T H C S L . . N K F G . . B
```

This sequence is either the original keying sequence or, more likely, a decimation thereof. Referring now to the matched plain and cipher of Message No. 1,

```
R E F E R E N C E Y O U R M E S S A G E N U M B E R . . .
F A O N A G C Z M K W V B D K K U U U V B F B F Q D
```

and by using our recovered plain and cipher components, we can determine that if $E_p = A_c$ at position 2, the key letter must be H_k (under the assumption of A_p as the index letter). If the commutative keying is expressed as a strip sliding against itself, the mod-26 sum of R+F=H at position 1 can be represented on the keying strips as follows:

```
(1)  A J Y . O . Q R M I E D T H C S L . . N K (F) G . . B
(2)  . N K F G . . B A J Y . O . Q (R) M I E D T (H) C S L .
```

which means that the first element (R) of the equation R+F=H is under S as an index. At position 3, $F_p = O_c$ with our sliding strips means a key of F_k, so the alignment of the keying strips for the mod-26 sum of E+A=F must be

```
(1)  (A) J Y . O . Q R M I E D T H C S L . . N K F G . . B
(2)  (F) G . . B A J Y . O . Q R M I (E) D T H C S L . . N K
```

and again the index letter in the upper strip is seen to be S. By using the data in the message beginnings, we can recover the entire keying strip as follows:

```
A J Y Z O U Q R M I E D T H C S L V W N K F G X P B
```

Even though this is a decimation of the original keying sequence, it nevertheless will suffice for the cryptographic operations of the system, which has now been laid completely bare.

g. These solutions teach us a lesson we are sometimes prone to forget. The original cryptoprinciple of the machine appeared at the outset to be extremely complicated in analysis, and as we learned more about it, at each step we were temporarily beset by insurmountable obstacles that we speedily managed to surmount. In the end, all the approaches to solution appeared quite straightforward and obvious.

83. Analysis and solution of a second case.—*a.* For the second example of an apparently complex case of aperiodic encipherment which yielded to facile attack we shall quote verbatim an article by William Lutwiniak which appeared in the *NSA Technical Journal*, Vol. I, No. 3, October 1956:

SOLUTION OF A POLYALPHABETIC MONOME DINOME SYSTEM

By William Lutwiniak

An interesting use of monome-dinome matrix encipherment has been suggested by Mr. L. D. Callimahos. His scheme calls for 10 matrices, used successively, to encipher the plain text. The ten matrices are generated from one basic matrix by systematically displacing the coordinate sequence one position at a time. By way of example, if the basic matrix is the first one shown below,

	4	5	1	7	0	6	2	8	3	9
-	S	E	-	R	A	T	I	O	-	N
1	B	C	D	F	G	H	J	K	L	M
3	P	Q	U	V	W	X	Y	Z	.	,

Matrix 1

	5	1	7	0	6	2	8	3	9	4
-	S	E	-	R	A	T	I	O	-	N
7	B	C	D	F	G	H	J	K	L	M
9	P	Q	U	V	W	X	Y	Z	.	,

Matrix 2

	1	7	0	6	2	8	3	9	4	5
-	S	E	-	R	A	T	I	O	-	N
0	B	C	D	F	G	H	J	K	L	M
4	P	Q	U	V	W	X	Y	Z	.	,

Matrix 3

. . . .

	9	4	5	1	7	0	6	2	8	3
-	S	E	-	R	A	T	I	O	-	N
5	B	C	D	F	G	H	J	K	L	M
8	P	Q	U	V	W	X	Y	Z	.	,

Matrix 10

then the 2d, 3d . . . 10th are generated as indicated. Plain text THE if started in the first matrix, would be enciphered as 6727. Note that the row coordinates are slides of the column coordinates. The first row is successively 1, 7, 0, . . . 5 and the second 3, 9, 4, . . . 8. The distance between the pair of row coordinates for a given matrix, measured on the sequence of the column coordinates, is fixed by the position of the blanks on the monomic row; this distance is the same for all ten matrices.

The system gives rise to some interesting properties. The original plain language has a rigid cycle of 10, but the average cipher cycle equals 10 plus the relative frequency of the letters enciphered by dinomes. In general, these frequencies vary between 33% and 60%, depending on the number and relative frequencies of the monomes composing the top row. In the example given above, the eight monomes are the eight most frequent letters in English, which normally make up 66% of plain text. The cycle of the cipher would thus average 13.3%; this, however, is a purely statistical cycle, and the cipher text would give the appearance of being aperiodic.

One other feature is that the cipher text is guaranteed to be monomically flat. Each digit is used in turn for all columns and rows; hence whatever inherent bias exists monomically in the basic matrix will be systematically and equally distributed over all 10 digits.

Mr. Callimahos submitted for consideration a 925-digit message as a test problem. All that was known was that the encipherment proceeded as indicated above, by systematically displacing the horizontal coordinate one position; the locations of the blanks on the monome row determine the successive row coordinates. The number of the row coordinates is unknown, as is the underlying language. The first 210 digits of the message are given below:

1 0 4 4 9	0 7 9 7 5	1 7 3 8 5	2 2 1 0 5	6 6 9 1 7	5 9 4 3 9
3 4 6 0 9	2 5 0 6 8	4 5 3 9 0	4 1 7 0 8	1 0 8 5 3	8 8 2 4 4
7 7 0 7 5	5 6 8 1 3	6 4 9 0 7	5 3 2 1 9	8 1 5 1 2	1 0 7 4 7
6 7 9 9 5	8 5 5 5 9	4 2 2 3 9	9 7 9 6 0	9 9 5 8 8	9 8 8 1 5
6 7 0 8 1	4 6 0 0 1	3 6 0 6 8	4 6 8 5 8	2 3 9 1 8	4 6 2 5 6
2 9 0 2 2	7 1 3 5 4	9 0 8 6 2	9 3 8 2 3	0 7 7 7 8	0 3 2 1 8
1 1 5 0 1	6 3 7 5 8	4 0 1 1 6	1 8 2 9 2	7 0 3 5 2	2 9 2 5 0 . . .

A STETHOSCOPE run on the material elicited the following facts:

Monomic delta I.C.: 0.999 (As anticipated, flat)
Dinomic delta I.C.: 1.07 (4.6 sigmas)
Trinomic delta I.C.: 1.29 (6 sigmas)

The repeat rates at distances 1 to 10 were not significant. The following polynomic repeats, all occurring twice, were listed:

Size	*Number of pairs*
5	5
6	6
7	4
8	1
10	1
11	1

The distances between these repeats, as expected, gave every indication of being aperiodic; the shorter intervals involved were 24, 51 (twice), 63, 69, 75, and 82. It had been hoped that the statistical cycle could be inferred from the shorter intervals between repeats, but the observed data are insufficient. A count of repeats at distances from 1 to 20 should peak at the statistical cycle; however, an attack was devised that did not require this information.

Referring to the sample matrices at the beginning of this article, the following are the 10 successive encipherments of GETTYSBURG:

```
                      Plain

            G   E T T  Y   S  B   U  R  G
      1    1 0 1 8 3 1 4 6 3 2 6 9 5 5 7
      2    7 6 7 3 9 7 5 2 9 8 2 4 1 1 0
      3    0 2 0 9 4 0 1 8 4 3 8 5 7 7 6
      4    6 8 6 4 5 6 7 3 5 9 3 1 0 0 2
      5    2 3 2 5 1 2 0 9 1 4 9 7 6 6 8
      6    8 9 8 1 7 8 6 4 7 5 4 0 2 2 3
      7    3 4 3 7 0 3 2 5 0 1 5 6 8 8 9
      8    9 5 9 0 6 9 8 1 6 7 1 2 3 3 4
      9    4 1 4 6 2 4 3 7 2 0 7 8 9 9 5
     10    5 7 5 2 8 5 9 0 8 6 0 3 4 4 1
Pattern:   A B A C D A E F D G F H I I J
```

It will be noted that all versions are isomorphic to each other, since all have the pattern ABACDAEFDGFHIIJ. Further, each of the 15 columns is a slide of the actual horizontal coordinates. At first glance, it seems a little strange that the row coordinates behave no differently from the column coordinates: all are slides of the basic sequence. However, it is readily apparent that the designations for each row are determined by the fixed location of the blanks in the monomic portion of the matrix, and hence must be slides of the basic sequence. What is not known is the distance (as measured on the basic sequence) between the blanks, and there is no information derivable from the isomorphs that bears on this question.

Consideration of the encipherment scheme makes the reason for isomorphism quite clear: the location of the plain values in the matrix is invariant; only the coordinate values designating the plain vary, and these values progress regularly one position at a time. Each individual plaintext repeat must be associated with the isomorph caused by invariant plain positions in the matrix, regardless of where the repeat starts in the sequence of 10 matrices. A ciphertext repeat is merely a special case of isomorphism; i.e., a plaintext repeat which starts at the same point in the 10-matrix sequence.

This suggests a simple, straightforward attack based on this phenomenon: locate long isomorphs; then reconstruct the basic coordinate sequence by the principles of indirect symmetry. It is true that this reconstructed coordinate sequence may be a decimation of the true one; but there is a procedure for selecting the correct one from among the possibilities.

In order to avoid a tedious isomorphic search, the longer hits located by STETHOSCOPE were listed and patterned as follows:

Repeat	*Pattern*
2 3 7 9 9 3 5 8 9 7 7	A B C D D B E F D C C
8 1 3 6 4 9 0 7 5 3	A B C D E F G H I C
2 8 5 0 9 5 7 0	A B C D E C F D
6 8 0 4 6 5 5	A B C D A E E
4 7 2 2 3 2 1	A B C C D C E
2 7 8 5 2 3 3	A B C D A E E
8 5 2 2 1 0 5	A B C C D E B

Fortunately, there is an isomorphic repeat with patterns ABCDAEE among these repeats. This pattern was searched for elsewhere in the message; since it already indicated a plaintext group occurring four times, it was expected that there might be other occurrences. The following possibilities, including the original pair, were found:

	A	B	C	D	A	E	E
(1)	6	8	0	4	6	5	5
(2)	2	7	8	5	2	3	3
(3)	3	0	9	2	3	4	4
(4)	9	3	5	8	9	7	7
(5)	1	5	4	0	1	8	8
(6)	7	5	2	3	7	9	9
(7)	7	0	8	4	7	2	2

If the members of the original pair (1) and (2) are taken as correct, (7) cannot be right, since there is a repeated digit (8) in column C of the pattern. The mechanics of the system demands that between a pair of isomorphs, the corresponding digits must either all be different or all be identical. The other conflict remaining, after eliminating (7), is that both (5) and (6) cannot be correct: there is a clash in column B. This clears up quickly when it is noted that (4) is contained in the 11-digit repeat, and that (6) involves the beginning of this repeat—23799. The fact that one of the 7-digit isomorphs was contained in the 11-digit repeat led to testing the 4 digits preceding the remaining 7-digit isomorphs, and the following was built up:

	A	B	C	D	D	B	E	F	D	C	C
(1)	1	8	5	6	6	8	0	4	6	5	5
(2)	9	7	3	2	2	7	8	5	2	3	3
(3)	7	0	4	3	3	0	9	2	3	4	4
(4)	2	3	7	9	9	3	5	8	9	7	7
(5)					1	5	4	0	1	8	8

The principles of indirect symmetry were applied, and the following equivalent sequence reconstructed: 9306827415. This is either the true basic sequence or a decimation of the true sequence at intervals 3, 7, or 9. (Had the true sequence been such that even decimations, or a decimation of 5, produced either 2 sets of 5 or 5 sets of 2, all relationships obtained from the isomorphs would have had these properties and hence an unbroken sequence, as reconstructed automatically, eliminates the even decimations and the one at distance 5.)

Before attempting to select the correct decimation from the four possibilities, let us consider the properties of a monome-dinome matrix. The numbered rows, in the least favorable case (where 8 monomes represent ETNROAIS), must contain 33% of the plain text. This percentage increases with fewer and/or less frequent monomes. Thus, there is often a tendency for dinomes to follow one another sufficiently often to be distinguished from the random case. In the system under consideration, an "A–A" dinome count (in the sequence ABCDE . . ., A–A dinomes are AC, BD, CE, etc.) will cover a situation where a dinome from a particular row is followed by another dinome from that same row. However, in this respect, we cannot distinguish one row from another: each time a dinome from any given row is followed by a dinome from that same row it will be reflected in the A–A dinome count, since each row is a slide of the basic sequence starting at a different point. We do lose the information when a dinome from a given row is followed by a dinome from some other row.

The following is the A–A dinome distribution:

θ_c^1 \ θ_c^2	0	1	2	3	4	5	6	7	8	9
0	4	13	9	9	10	11	9	9	8	10
1	8	10	9	8	4	11	15	5	10	6
2	16	5	6	13	7	3	6	11	11	15
3	10	12	6	4	11	5	4	9	10	5
4	14	5	10	6	4	11	7	11	13	7
5	10	2	9	8	12	9	5	10	13	13
6	6	12	7	1	6	8	8	4	9	14
7	7	7	8	11	6	13	12	9	12	10
8	11	10	13	10	13	10	10	11	8	11
9	5	6	14	8	12	14	4	17	13	9

The four decimations of the reconstructed sequence are tried in turn by noting the frequency of each of the 10 chained dinomes composing them, and summing the set of 10 frequencies for each decimation. Thus:

											Sum
Decimation of +1:	9	3	0	6	8	2	7	4	1	5	
Frequencies:	8	10	9	9	13	11	6	5	11	13	=95
Decimation of +3:	9	6	7	5	0	2	1	3	8	4	
Frequencies:	4	4	13	10	9	5	8	10	13	7	=83
Decimation of +7:	9	4	8	3	1	2	0	5	7	6	
Frequencies:	12	13	10	12	9	16	11	10	12	14	=119
Decimation of +9:	9	5	1	4	7	2	8	6	0	3	
Frequencies:	14	2	4	9	8	11	10	6	9	5	=78

The decimation of +7 yields the highest value and is in all probability the basic sequence; i.e., in the true matrix, a dinome beginning with 9 is followed frequently by one beginning with 4, which in turn is followed by one beginning with 8, etc. The only information now needed to convert the entire text to monoalphabetic terms (i.e., in terms of one matrix) is the number and identity of the row coordinates. One source of data bearing on the identity of the monomes (and hence, on the row coordinates) stems from the situation where a monome intervenes between two dinomes. Representing this situation as AB C DE, it is apparent that what is required is a selection of dinomes AD such that the digits involved will come from matrices once removed from each other. The basic coordinate sequence involved, 9483120576, indicates that the following A--A dinomes should be extracted from the text:

A	B	C	D	E
9	.	.	8	.
4	.	.	3	.
8	.	.	1	.
3	.	.	2	.
1	.	.	0	.
2	.	.	5	.
0	.	.	7	.
5	.	.	6	.
7	.	.	9	.
6	.	.	4	.

Then the frequency of digits appearing in position C would be tallied; these 10 distributions can be combined by equating them according to the basic sequence; i.e., arranging the counts in a 10×10 matrix and summing the left-to-right downward diagonals. The lowest diagonal sums should be the row coordinates sought.

Herewith are the distributions of the C digits for each of the 10 specified dinomes from the sequence 9483120576.

Dinome:	98	43	81	32	10	25	07	56	79	64
9	2	1	1	1	1	–	2	–	3	1
4	1	1	1	–	1	–	3	1	–	1
8	1	–	3	1	–	3	–	1	–	–
3	1	–	2	1	1	–	2	–	1	1
1	–	–	1	1	4	2	1	1	2	–
2	3	2	–	1	2	–	1	–	2	–
0	1	–	1	–	1	2	1	–	–	3
5	5	1	1	1	1	3	4	4	–	–
7	–	–	1	1	1	–	4	1	2	–
6	–	–	1	1	2	–	–	–	1	3
Diagonal Sums:	7	<u>2</u>	19	7	12	9	<u>4</u>	12	16	21

This would indicate that there are two rows, and that the blanks are spaced 5 apart on the basic sequence. Thus the 10 successive pairs of row coordinates are 92, 40, 85, 37, 16, 29, 04, 58, 73, and 61. It only remains to determine where in this sequence the message starts. The easiest way to test the 10 starting points in turn is to complete the plain component sequence, extract the putative monome-dinome stream for each assumption, and evaluate it by the proportion of monomes to dinomes. In the correct case this will approach 1.9 or less.

Herewith are the first 60 digits of the message, with the plain component sequence completed down the columns:

```
1 0 4 4 9 0 7 9 7 5 1 7 3 8 5 2 2 1 0 5 6 6 9 1 7 5 9 4 3 9
2 5 8 8 4 5 6 4 6 7 2 6 1 3 7 0 0 2 5 7 9 9 4 2 6 7 4 8 1 4
0 7 3 3 8 7 9 8 9 6 0 9 2 1 6 5 5 0 7 6 4 4 8 0 9 6 8 3 2 8
5 6 1 1 3 6 4 3 4 9 5 4 0 2 9 7 7 5 6 9 8 8 3 5 4 9 3 1 0 3
7 9 2 2 1 9 8 1 8 4 7 8 5 0 4 6 6 7 9 4 3 3 1 7 8 4 1 2 5 1
6 4 0 0 2 4 3 2 3 8 6 3 7 5 8 9 9 6 4 8 1 1 2 6 3 8 2 0 7 2
9 8 5 5 0 8 1 0 1 3 9 1 6 7 3 4 4 9 8 3 2 2 0 9 1 3 0 5 6 0
4 3 7 7 5 3 2 5 2 1 4 2 9 6 1 8 8 4 3 1 0 0 5 4 2 1 5 7 9 5
8 1 6 6 7 1 0 7 0 2 8 0 4 9 2 3 3 8 1 2 5 5 7 8 0 2 7 6 4 7
3 2 9 9 6 2 5 6 5 0 3 5 8 4 0 1 1 3 2 0 7 7 6 3 5 0 6 9 8 6

3 4 6 0 9 2 5 0 6 8 4 5 3 9 0 4 1 7 0 8 1 0 8 5 3 8 8 2 4 4
1 8 9 5 4 0 7 5 9 3 8 7 1 4 5 8 2 6 5 3 2 5 3 7 1 3 3 0 8 8
2 3 4 7 8 5 6 7 4 1 3 6 2 8 7 3 0 9 7 1 0 7 1 6 2 1 1 5 3 3
0 1 8 6 3 7 9 6 8 2 1 9 0 3 6 1 5 4 6 2 5 6 2 9 0 2 2 7 1 1
5 2 3 9 1 6 4 9 3 0 2 4 5 1 9 2 7 8 9 0 7 9 0 4 5 0 0 6 2 2
7 0 1 4 2 9 8 4 1 5 0 8 7 2 4 0 6 3 4 5 6 4 5 8 7 5 5 9 0 0
6 5 2 8 0 4 3 8 2 7 5 3 6 0 8 5 9 1 8 7 9 8 7 3 6 7 7 4 5 5
9 7 0 3 5 8 1 3 0 6 7 1 9 5 3 7 4 2 3 6 4 3 6 1 9 6 6 8 7 7
4 6 5 1 7 3 2 1 5 9 6 2 4 7 1 6 8 0 1 9 8 1 9 2 4 9 9 3 6 6
8 9 7 2 6 1 0 2 7 4 9 0 8 6 2 9 3 5 2 4 3 2 4 0 8 4 4 1 9 9
```

In testing the ten possible starting points on the completed plain component, the rule is to read on the same generatrix one position for a monome, or two for a dinome, and then ascend one generatrix for the matrix. The reverse would be true had the basic sequence been recovered as a right-to-left progression. In the actual case at hand, it is necessary to test for only five assumptions. The positions of the blanks (five apart) gives 5 sets of 2; thus (2, 9) is identical with (9, 2) insofar as reading the generatrices is concerned.

The five streams (derived from the foregoing diagram) representing the ten possibilities for the first 60 digits of the message follow; all start with the initial digit of cipher text. The fraction following each stream is the monome-to-dinome ratio of the stream. The introductory dinomes are the two row coordinates associated with the particular matrix under consideration.

```
9, 2    1 29  6  5  8  3  1  4  6 26  3  4 23  8 98  8  3  8  8 26
and     5  6  6 95  6  0  3  6  8  0  5 27 96  1  6 24 27  4  7  3
2, 9    1 24 24  6  7 90  2–                                          33/14

4, 0    1  2  6  7 08  3  1 49 09  1  8 01  3 43  3  1  3  3 09  7
and     9  9 47  9  5  1  9  3  5  7 06 49  2  9 08 06  8  6  1  2
0, 4   08 08  9  6 45  0–                                             31/15

8, 5    1  2  6  7  0  4 81  4  6  2  7 84  2 88  9  4  4 88 80  6
and    59  9  4 59 52  4  1  7  6 59 84  0  4 53 59  3  9  2  0 53
5, 8   53  4  9 87  5–                                                29/16

3, 7    1  2  6 75  8 32  8  9  0  6 38  0 33  4  8  8 33 35  9 74
and     4  8 74 70  8  2  6  9 74 38  5  8 71 74  1  4  0  5 71 71
7, 3    8  4 36  7–                                                   27/17

1, 6   10  9 67  3 10  3  4  5  9 13  5 11  8  3  3 11 17  4 68  8
and     3 68 65  3  0  9  4 68 13  7  3 62 68  2  8  5  7 62 62  3
6, 1    8 19  6–                                                      25/18
```

The (1, 6) possibility looks the best from the standpoint of monome-to-dinome ratio, and, in fact, is correct. In the actual solution, 250 digits were examined on the basis of the five possibilities, and the (1, 6) possibility was outstandingly the correct one. The following matrix was recovered quickly:

	8	3	1	2	0	5	7	6	9	4
-	R	E	-	A	C	T	I	-	O	N
1	B	F	H	K	M	Q	U	W	Y	
6	D	G	J	L	P	S	V	X	Z	

The key word is obviously REACTION, and the following is the basic format:

	9	4	8	3	1	2	0	5	7	6
–	O	N	R	E	–	A	C	T	I	–
6	Z		D	G	J	L	P	S	V	X
1	Y		B	F	H	K	M	Q	U	W

The sequence of column coordinates is found to be the numerical key derived from the internal key word expanded to ten letters, i.e., REACTIONRE; the message plain text begins with the words "MOVEMENT OF THREE HUNDRED. . . ."

Thus, what at first glance appears to be a rather complicated system yields to a straightforward attack which derives largely from simple cryptanalytic fundamentals. The attack depends in the main on the principles of indirect symmetry of related alphabets, and the fact that in this instance these principles were applied to digits rather than letters merely underscores the fundamental importance of the principles. This is just one more illustration of the desirability of every cryptanalyst's having a complete grasp of cryptanalytic fundamentals: one never knows where they may be applied next.

The basic system itself was assumed to be known, but the attack would probably have proceeded as above, albeit less directly, had nothing been known about the cryptosystem. Thus, aperiodic, causal ciphertext repeats would have been observed. Sooner or later isomorphism would be noticed among these repeats and the principles of indirect symmetry applied to yield the basic sequence. Once the fact was ascertained that 10 digital related alphabets were involved, an attempt would be made to determine what caused the aperiodicity. Eventually these investigations would lead to the fact that the 10 alphabets were used in order, and a statistical period determined. Among the hypotheses which might account for such a statistical period would be monome-dinome substitution.

Under a long additive for a superencipherment, the system would afford considerable security. It could, however, be solved, if sufficient depth were available to permit entry via the classic repeated differences attack, at which point the stripped text would be treated as above. The selection could be controlled by a key word, expressed as digits through the basic matrix, each digit selecting a matrix in turn. This would render part of the attack outlined above ineffective, but the entry via isomorphs would still obtain. Also, selection other than cyclical would no longer ensure monome flatness, and the roughness thus exhibited might suggest specific attacks. Isologs would be extremely vulnerable; indeed, most of the variations on related alphabet systems and the appropriate attacks relating to them are applicable to the basic system set forth here, with the modifications necessitated by the peculiarities of monome-dinome encipherment. A nasty version of this system would embrace a matrix with coordinate sequences selected from the ten rows of a Latin square, superenciphered by a long additive—we reserve judgment on this one.

b. The final paragraph in the article above mentions some of the "if's, and's, and but's," variations possible in the system. In any cryptosystem if the cryptography is too complicated, cipher clerks' errors are bound to arise, and it is often through these messages and their subsequent correction that entries may be made, particularly in a difficult system.

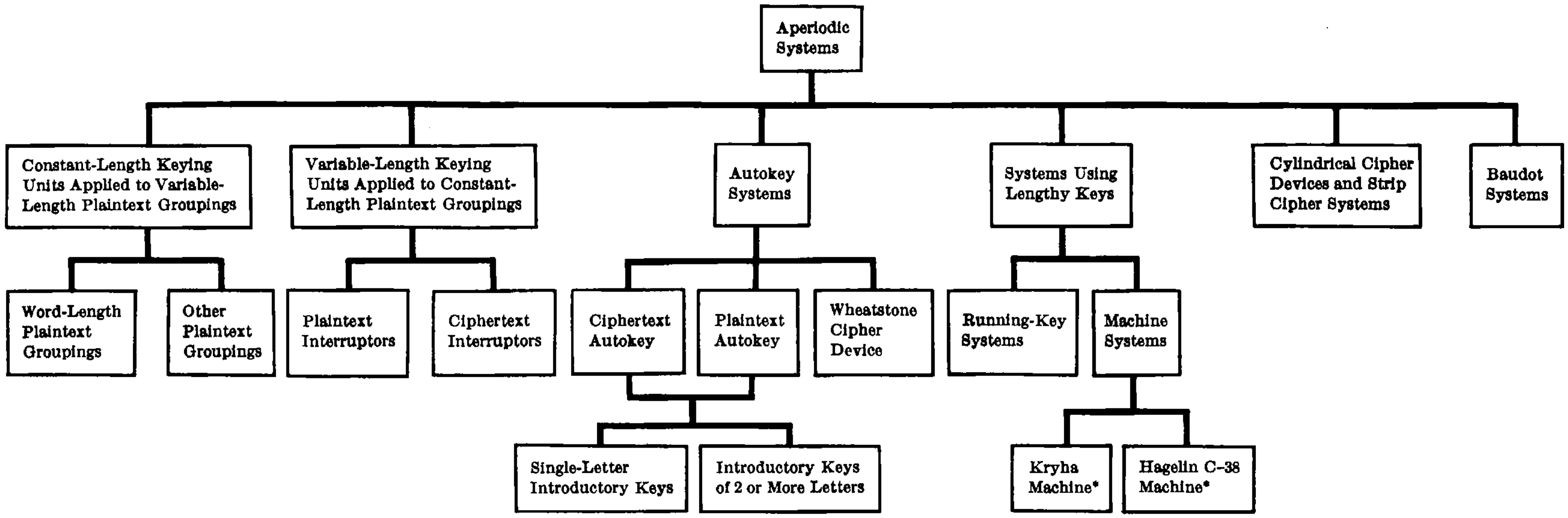
84. Final remarks.—*a*. Unlike the preceding two texts, because of space considerations this volume does not contain a glossary as one of its appendices. It is assumed that the reader has already acquired a basic technical vocabulary from a study of the first two texts; furthermore, new terms in this volume have been defined on their first occurrence and therefore should be absorbed during the course of systematic study. The reader may wish to resort to the NSA *Basic Cryptologic Glossary* (June 1971) to refresh his memory.

b. Four of the appendices have been included in order to enhance the usefulness of this volume as an operational reference manual. Appendix 3, "Tables of the Poisson distribution," and Appendix 4, "Table of the Binomial distribution for p = 1/10," are of service in evaluating probabilities; Appendix 5, "Plain-text and random material for sampling purposes," will be found useful in certain experimenting and testing situations; and Appendix 6, "Basic letter frequency data, 24 foreign languages," shows the cryptolinguistic similarities and differences among monographic and digraphic frequencies of some major languages.

c. As was the case with the previous texts, mere reading of the methods of solution of the various types of cryptosystems covered in this volume is not enough to ensure complete understanding of the principles and techniques presented. It is therefore strongly recommended that the reader solve the problems contained in Appendix 7, "Problems—Military Cryptanalytics, Part III," as a means of acquiring facility and adroitness in the solution of aperiodic ciphers.

d. The formal portion of this text will be closed with a synoptic chart of cryptography found on the next page, continuing the series of charts established in the first volume, showing the relationships among the various cryptosystems treated in *Military Cryptanalytics, Part III.* This chart is a continuation of the chart on p. 318 of *Military Cryptanalytics Part II,* if we consider the present chart as an amplification of the box labelled "Aperiodic" in the previous chart.

FKUPK JYZVB NCJNC RCAVP XIIAI P



*Strictly speaking, these are periodic systems; but this classification is academic, since they are solved as if they were aperiodic systems.

Synoptic chart of cryptography for *Military Cryptanalytics, Part III.*

APPENDICES

APPENDIX 1

DE PROFUNDIS; OR THE ABC OF DEPTH READING

This article, reprinted here with minor emendations, first appeared in the *NSA Technical Journal*, Vol. 2, No. 1. It is included here not only because it is a self-contained, concise explanation of the principles of depth reading in alphabetical, digital, and Baudot systems, but also because it is a particularly instructive example and model of how to write a lucid technical paper calculated to reach a majority of readers, especially including those who may not be versed in a subject under discussion.

DE PROFUNDIS; OR

THE ABC OF DEPTH READING

By Lambros D. Callimahos

(A concise explanation of the principles of depth reading in alphabetical, digital, and Baudot systems.)

The Editor [of the *Technical Journal*] informs us that he is perpetually trying to cajole our engineers to write the ABC of this or that, in a fashion calculated to explain to the *non*engineer the mysteries of black boxes; that is, produce articles intended to be basic introductions to something or other, explaining the obvious for the benefit of those of us to whom it is *not* obvious. Much to the Editor's surprise, one of these engineers buttonholed him, saying: "How about an article explaining what [redacted] Most engineers don't know." The Editor approached the writer; the writer, tit for tatting, borrowed the title from the engineers; what follows is, we hope, an answer to this request.

It is assumed that the reader has some idea of the concepts of letter frequencies of plain text.[1] We know that, for instance, the letters of English text have greatly varying frequencies, ranging from an E of 13% and a T of 9% to the group of letters JKQXZ whose combined individual frequencies sum to 1.4 per cent.[2] If English plain text is subjected to a simple monoalphabetic substitution, the ratios between the frequencies are not changed—only the *identities* of the letters; [redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

[redacted]

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

206-687 O - 77 - 29

(b)(1)
(b)(3)-18 USC 798
(b)(3)-50 USC 3024(i)
(b)(3)-P.L. 86-36

APPENDIX 2

SYNOPTIC TABLES, CIPHER DEVICES M-94

Synoptic Table for A

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
1	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	C	D	K	D	N	P	X	H	D	E	T	N	C	D	B	J	M	D	O	T	N	V	K	J	D
3	E	E	O	C	Q	O	J	P	S	L	M	F	J	W	V	N	Y	M	J	R	K	S	W	P	N
4	I	H	M	B	U	C	E	J	K	B	S	L	I	P	H	U	O	C	Y	Z	H	F	R	X	B
5	G	F	J	I	K	I	Z	O	Q	D	X	H	L	K	I	B	F	N	L	X	R	D	E	M	U
6	D	I	U	F	D	X	B	B	O	F	V	Q	D	J	Y	T	T	E	F	Q	G	L	V	V	H
7	J	J	B	G	O	L	N	W	I	J	Q	G	H	V	K	G	H	Q	X	L	O	I	D	K	Y
8	F	K	G	J	P	U	I	K	V	G	P	C	B	I	S	I	E	B	N	Y	X	E	T	B	F
9	V	T	E	H	I	R	K	C	T	H	N	U	M	U	G	M	U	O	G	I	E	B	U	Q	W
10	U	L	P	L	T	N	P	V	Z	O	O	J	K	Q	U	W	S	Z	W	O	Y	H	F	W	J
11	Y	M	H	K	J	D	V	F	E	N	H	T	G	H	E	Z	Z	P	H	V	B	K	O	U	L
12	M	O	S	M	B	Y	R	Z	F	M	U	B	X	Z	N	R	J	L	V	B	F	N	Y	G	V
13	H	U	C	R	R	Z	O	L	H	T	W	Y	U	C	T	V	X	G	C	P	S	R	H	L	G
14	T	V	Z	U	H	H	G	Q	G	P	D	P	Z	T	C	L	D	V	M	E	J	J	M	O	R
15	Q	Y	I	O	C	W	S	E	Y	R	I	Z	T	X	X	X	P	J	I	S	M	Q	L	S	C
16	K	G	N	Q	Y	B	Y	R	U	Q	Z	K	S	B	O	C	C	R	R	N	U	Z	S	T	Q
17	Z	Z	X	V	S	J	D	Y	N	S	Y	X	W	L	W	S	W	K	B	H	D	G	I	E	M
18	O	N	F	P	L	S	U	N	L	V	C	I	Q	E	F	H	G	Y	S	J	Q	M	Q	C	P
19	L	P	Y	T	W	Q	L	S	P	Z	G	S	Y	G	Q	D	Q	T	E	W	C	X	N	H	S
20	R	Q	Q	N	E	F	C	U	M	U	K	R	V	N	D	E	I	F	K	M	L	P	J	N	O
21	X	X	R	W	M	K	F	M	B	X	R	D	O	Y	R	O	B	U	U	D	Z	U	C	Z	E
22	S	R	T	Y	Z	V	M	G	X	Y	F	V	R	R	L	K	K	I	P	G	W	C	P	F	X
23	P	W	V	X	V	M	Q	T	W	W	B	E	P	S	J	F	L	W	D	F	T	O	G	R	T
24	W	S	W	Z	X	E	T	D	C	I	E	W	F	M	Z	P	N	X	Z	C	I	T	B	I	K
25	N	B	L	S	G	T	W	X	R	C	J	O	E	F	M	Y	V	H	Q	K	P	Y	Z	D	I

Synoptic Table for B

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
1	C	A	G	I	R	J	N	W	X	D	E	Y	M	L	V	T	K	O	S	P	F	H	Z	Q	U
2	E	C	E	F	H	S	I	K	W	F	J	P	K	E	H	G	L	Z	E	E	S	K	A	W	H
3	I	D	P	G	C	Q	K	C	C	J	A	Z	G	G	I	I	N	P	K	S	J	N	X	U	Y
4	G	E	H	J	Y	F	P	V	R	G	L	K	X	N	Y	M	V	L	U	N	M	R	K	G	F
5	D	H	S	H	S	K	V	F	A	H	T	X	U	Y	K	W	A	G	P	H	U	J	W	L	W
6	J	F	C	L	L	V	R	Z	J	O	M	I	Z	R	S	Z	R	V	D	J	D	Q	R	O	J
7	F	I	Z	K	W	M	O	L	D	N	S	S	T	S	G	R	M	J	Z	W	Q	Z	E	S	L
8	V	J	I	M	E	E	G	Q	S	M	X	R	S	M	U	V	Y	R	Q	M	C	G	V	T	V
9	U	K	N	R	M	T	S	E	K	T	V	D	W	F	E	L	O	K	A	D	L	M	D	E	G
10	Y	T	X	U	Z	A	Y	R	Q	P	Q	V	Q	A	N	X	F	Y	T	G	Z	X	T	C	R
11	M	L	F	O	V	G	D	Y	O	R	P	E	Y	O	T	C	T	T	O	F	W	P	U	H	C
12	H	M	Y	Q	X	P	U	N	I	Q	N	W	V	D	C	S	H	F	J	C	T	U	F	N	Q
13	T	O	Q	V	G	O	L	S	V	S	O	O	O	W	X	H	E	U	Y	K	I	C	O	Z	M
14	Q	U	R	P	A	C	C	U	T	V	H	A	R	P	O	D	U	I	L	A	P	O	Y	F	P
15	K	V	T	T	F	I	F	M	Z	Z	U	M	P	K	W	E	S	W	F	U	A	T	H	R	S
16	Z	Y	V	N	N	X	M	G	E	U	W	N	F	J	F	O	Z	X	X	T	V	Y	M	I	O
17	O	G	W	W	Q	L	Q	T	F	X	D	F	E	V	Q	K	J	H	N	R	N	A	L	D	E
18	L	Z	L	Y	U	U	T	D	H	Y	I	L	A	I	D	F	X	A	G	Z	K	W	S	A	X
19	R	N	A	X	K	R	W	X	G	W	Z	H	N	U	R	P	D	S	W	X	H	V	I	Y	T
20	X	P	D	Z	D	N	A	A	Y	I	Y	Q	C	Q	L	Y	P	D	H	Q	R	S	Q	J	K
21	S	Q	K	S	O	D	H	I	U	C	C	G	J	H	J	A	C	M	V	L	G	F	N	P	I
22	P	X	O	A	P	Y	X	H	N	A	G	C	I	Z	Z	Q	W	C	C	Y	O	D	J	X	A
23	W	R	M	E	I	Z	J	P	L	K	K	U	L	C	M	J	G	N	M	I	X	L	C	M	Z
24	N	W	J	D	T	H	E	J	P	E	R	J	D	T	A	N	Q	E	I	O	E	I	P	V	D
25	A	S	U	C	J	W	Z	O	M	L	F	T	H	X	P	U	I	Q	R	V	Y	E	G	K	N

Synoptic Table for C

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1	E	D	Z	B	Y	I	F	V	R	A	G	U	J	T	X	S	W	N	M	K	L	O	P	H	Q
2	I	E	I	I	S	X	M	F	A	K	K	J	I	X	O	H	G	E	I	A	Z	T	G	N	M
3	G	H	N	F	L	L	Q	Z	J	E	R	T	L	B	W	D	Q	Q	R	U	W	Y	B	Z	P
4	D	F	X	G	W	U	T	L	D	L	F	B	D	L	F	E	I	B	B	T	T	A	Z	F	S
5	J	I	F	J	E	R	W	Q	S	B	B	Y	H	E	Q	O	B	O	S	R	I	W	A	R	O
6	F	J	Y	H	M	N	A	E	K	D	E	P	B	G	D	K	K	Z	E	Z	P	V	X	I	E
7	V	K	Q	L	Z	D	H	R	Q	F	J	Z	M	N	R	F	L	P	K	X	A	S	K	D	X
8	U	T	R	K	V	Y	X	Y	O	J	A	K	K	Y	L	P	N	L	U	Q	V	F	W	A	T
9	Y	L	T	M	X	Z	J	N	I	G	L	X	G	R	J	Y	V	G	P	L	N	D	R	Y	K
10	M	M	V	R	G	H	E	S	V	H	T	I	X	S	Z	A	A	V	D	Y	K	L	E	J	I
11	H	O	W	U	A	W	Z	U	T	O	M	S	U	M	M	Q	R	J	Z	I	H	I	V	P	A
12	T	U	L	O	F	B	B	M	Z	N	S	R	Z	F	A	J	M	R	Q	O	R	E	D	X	Z
13	Q	V	A	Q	N	J	N	G	E	M	X	D	T	A	P	N	Y	K	A	V	G	B	T	M	D
14	K	Y	D	V	Q	S	I	T	F	T	V	V	S	O	B	U	O	Y	T	B	O	H	U	V	N
15	Z	G	K	P	U	Q	K	D	H	P	Q	E	W	D	V	B	F	T	O	P	X	K	F	K	B
16	O	Z	O	T	K	F	P	X	G	R	P	W	Q	W	H	T	T	F	J	E	E	N	O	B	U
17	L	N	M	N	D	K	V	A	Y	Q	N	O	Y	P	I	G	H	U	Y	S	Y	R	Y	Q	H
18	R	P	J	W	O	V	R	I	U	S	O	A	V	K	Y	I	E	I	L	N	B	J	H	W	Y
19	X	Q	U	Y	P	M	O	H	N	V	H	M	O	J	K	M	U	W	F	H	F	Q	M	U	F
20	S	X	B	X	I	E	G	P	L	Z	U	N	R	V	S	W	S	X	X	J	S	Z	L	G	W
21	P	R	G	Z	T	T	S	J	P	U	W	F	P	I	G	Z	Z	H	N	W	J	G	S	L	J
22	W	W	E	S	J	A	Y	O	M	X	D	L	F	U	U	R	J	A	G	M	M	M	I	O	L
23	N	S	P	A	B	G	D	B	B	Y	I	H	E	Q	E	V	X	S	W	D	U	X	Q	S	V
24	A	B	H	E	R	P	U	W	X	W	Z	Q	A	H	N	L	D	D	H	G	D	P	N	T	G
25	B	A	S	D	H	O	L	K	W	I	Y	G	N	Z	T	X	P	M	V	F	Q	U	J	E	R

Synoptic Table for D

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
1	J	E	K	C	O	Y	U	X	S	F	I	V	H	W	R	E	P	M	Z	G	Q	L	T	A	N
2	F	H	O	B	P	Z	L	A	K	J	Z	E	B	P	L	O	C	C	Q	F	C	I	U	Y	B
3	V	F	M	I	I	H	C	I	Q	G	Y	W	M	K	J	K	W	N	A	C	L	E	F	J	U
4	U	I	J	F	T	W	F	H	O	H	C	O	K	J	Z	F	G	E	T	K	Z	B	O	P	H
5	Y	J	U	G	J	B	M	P	I	O	G	A	G	V	M	P	Q	Q	O	A	W	H	Y	X	Y
6	M	K	B	J	B	J	Q	J	V	N	K	M	X	I	A	Y	I	B	J	U	T	K	H	M	F
7	H	T	G	H	R	S	T	O	T	M	R	N	U	U	P	A	B	O	Y	T	I	N	M	V	W
8	T	L	E	L	H	Q	W	B	Z	T	F	F	Z	Q	B	Q	K	Z	L	R	P	R	L	K	J
9	Q	M	P	K	C	F	A	W	E	P	B	L	T	H	V	J	L	P	F	Z	A	J	S	B	L
10	K	O	H	M	Y	K	H	K	F	R	E	H	S	Z	H	N	N	L	X	X	V	Q	I	Q	V
11	Z	U	S	R	S	V	X	C	H	Q	J	Q	W	C	I	U	V	G	N	Q	N	Z	Q	W	G
12	O	V	C	U	L	M	J	V	G	S	A	G	Q	T	Y	B	A	V	G	L	K	G	N	U	R
13	L	Y	Z	O	W	E	E	F	Y	V	L	C	Y	X	K	T	R	J	W	Y	H	M	J	G	C
14	R	G	I	Q	E	T	Z	Z	U	Z	T	U	V	B	S	G	M	R	H	I	R	X	C	L	Q
15	X	Z	N	V	M	A	B	L	N	U	M	J	O	L	G	I	Y	K	V	O	G	P	P	O	M
16	S	N	X	P	Z	G	N	Q	L	X	S	T	R	E	U	M	O	Y	C	V	O	U	G	S	P
17	P	P	F	T	V	P	I	E	P	Y	X	B	P	G	E	W	F	T	M	B	X	C	B	T	S
18	W	Q	Y	N	X	O	K	R	M	W	V	Y	F	N	N	Z	T	F	I	P	E	O	Z	E	O
19	N	X	Q	W	G	C	P	Y	B	I	Q	P	E	Y	T	R	H	U	R	E	Y	T	A	C	E
20	A	R	R	Y	A	I	V	N	X	C	P	Z	A	R	C	V	E	I	B	S	B	Y	X	H	X
21	B	W	T	X	F	X	R	S	W	A	N	K	N	S	X	L	U	W	S	N	F	A	K	N	T
22	C	S	V	Z	N	L	O	U	C	K	O	X	C	M	O	X	S	X	E	H	S	W	W	Z	K
23	E	B	W	S	Q	U	G	M	R	E	H	I	J	F	W	C	Z	H	K	J	J	V	R	F	I
24	I	A	L	A	U	R	S	G	A	L	U	S	I	A	F	S	J	A	U	W	M	S	E	R	A
25	G	C	A	E	K	N	Y	T	J	B	W	R	L	O	Q	H	X	S	P	M	U	F	V	I	Z

Synoptic Table for E

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
1	I	H	P	D	M	T	Z	R	F	L	J	W	A	G	N	O	U	Q	K	S	Y	B	V	C	X
2	G	F	H	C	Z	A	B	Y	H	B	A	O	N	N	T	K	S	B	U	N	B	H	D	H	T
3	D	I	S	B	V	G	N	N	G	D	L	A	C	Y	C	F	Z	O	P	H	F	K	T	N	K
4	J	J	C	I	X	P	I	S	Y	F	T	M	J	R	X	P	J	Z	D	J	S	N	U	Z	I
5	F	K	Z	F	G	O	K	U	U	J	M	N	I	S	O	Y	X	P	Z	W	J	R	F	F	A
6	V	T	I	G	A	C	P	M	N	G	S	F	L	M	W	A	D	L	Q	M	M	J	O	R	Z
7	U	L	N	J	F	I	V	G	L	H	X	L	D	F	F	Q	P	G	A	D	U	Q	Y	I	D
8	Y	M	X	H	N	X	R	T	P	O	V	H	H	A	Q	J	C	V	T	G	D	Z	H	D	N
9	M	O	F	L	Q	L	O	D	M	N	Q	Q	B	O	D	N	W	J	O	F	Q	G	M	A	B
10	H	U	Y	K	U	U	G	X	B	M	P	G	M	D	R	U	G	R	J	C	C	M	L	Y	U
11	T	V	Q	M	K	R	S	A	X	T	N	C	K	W	L	B	Q	K	Y	K	L	X	S	J	H
12	Q	Y	R	R	D	N	Y	I	W	P	O	U	G	P	J	T	I	Y	L	A	Z	P	I	P	Y
13	K	G	T	U	O	D	D	H	C	R	H	J	X	K	Z	G	B	T	F	U	W	U	Q	X	F
14	Z	Z	V	O	P	Y	U	P	R	Q	U	T	U	J	M	I	K	F	X	T	T	C	N	M	W
15	O	N	W	Q	I	Z	L	J	A	S	W	B	Z	V	A	M	L	U	N	R	I	O	J	V	J
16	L	P	L	V	T	H	C	O	J	V	D	Y	T	I	P	W	N	I	G	Z	P	T	C	K	L
17	R	Q	A	P	J	W	F	B	D	Z	I	P	S	U	B	Z	V	W	W	X	A	Y	P	B	V
18	X	X	D	T	B	B	M	W	S	U	Z	Z	W	Q	V	R	A	X	H	Q	V	A	G	Q	G
19	S	R	K	N	R	J	Q	K	K	X	Y	K	Q	H	H	V	R	H	V	L	N	W	B	W	R
20	P	W	O	W	H	S	T	C	Q	Y	C	X	Y	Z	I	L	M	A	C	Y	K	V	Z	U	C
21	W	S	M	Y	C	Q	W	V	O	W	G	I	V	C	Y	X	Y	S	M	I	H	S	A	G	Q
22	N	B	J	X	Y	F	A	F	I	I	K	S	O	T	K	C	O	D	I	O	R	F	X	L	M
23	A	A	U	Z	S	K	H	Z	V	C	R	R	R	X	S	S	F	M	R	V	G	D	K	O	P
24	B	C	B	S	L	V	X	L	T	A	F	D	P	B	G	H	T	C	B	B	O	L	W	S	S
25	C	D	G	A	W	M	J	Q	Z	K	B	V	F	L	U	D	H	N	S	P	X	I	R	T	O

Synoptic Table for F

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
1	V	I	Y	G	N	K	M	Z	H	J	B	L	E	A	Q	P	T	U	X	C	S	D	O	R	W
2	U	J	Q	J	Q	V	Q	L	G	G	E	H	A	O	D	Y	H	I	N	K	J	L	Y	I	J
3	Y	K	R	H	U	M	T	Q	Y	H	J	Q	N	D	R	A	E	W	G	A	M	I	H	D	L
4	M	T	T	L	K	E	W	E	U	O	A	G	C	W	L	Q	U	X	W	U	U	E	M	A	V
5	H	L	V	K	D	T	A	R	N	N	L	C	J	P	J	J	S	H	H	T	D	B	L	Y	G
6	T	M	W	M	O	A	H	Y	L	M	T	U	I	K	Z	N	Z	A	V	R	Q	H	S	J	R
7	Q	O	L	R	P	G	X	N	P	T	M	J	L	J	M	U	J	S	C	Z	C	K	I	P	C
8	K	U	A	U	I	P	J	S	M	P	S	T	D	V	A	B	X	D	M	X	L	N	Q	X	Q
9	Z	V	D	O	T	O	E	U	B	R	X	B	H	I	P	T	D	M	I	Q	Z	R	N	M	M
10	O	Y	K	Q	J	C	Z	M	X	Q	V	Y	B	U	B	G	P	C	R	L	W	J	J	V	P
11	L	G	O	V	B	I	B	G	W	S	Q	P	M	Q	V	I	C	N	B	Y	T	Q	C	K	S
12	R	Z	M	P	R	X	N	T	C	V	P	Z	K	H	H	M	W	E	S	I	I	Z	P	B	O
13	X	N	J	T	H	L	I	D	R	Z	N	K	G	Z	I	W	G	Q	E	O	P	G	G	Q	E
14	S	P	U	N	C	U	K	X	A	U	O	X	X	C	Y	Z	Q	B	K	V	A	M	B	W	X
15	P	Q	B	W	Y	R	P	A	J	X	H	I	U	T	K	R	I	O	U	B	V	X	Z	U	T
16	W	X	G	Y	S	N	V	I	D	Y	U	S	Z	X	S	V	B	Z	P	P	N	P	A	G	K
17	N	R	E	X	L	D	R	H	S	W	W	R	T	B	G	L	K	P	D	E	K	U	X	L	I
18	A	W	P	Z	W	Y	O	P	K	I	D	D	S	L	U	X	L	L	Z	S	H	C	K	O	A
19	B	S	H	S	E	Z	G	J	Q	C	I	V	W	E	E	C	N	G	Q	N	R	O	W	S	Z
20	C	B	S	A	M	H	S	O	O	A	Z	E	Q	G	N	S	V	V	A	H	G	T	R	T	D
21	E	A	C	E	Z	W	Y	B	I	K	Y	W	Y	N	T	H	A	J	T	J	O	Y	E	E	N
22	I	C	Z	D	V	B	D	W	V	E	C	O	V	Y	C	D	R	R	O	W	X	A	V	C	B
23	G	D	I	C	X	J	U	K	T	L	G	A	O	R	X	E	M	K	J	M	E	W	D	H	U
24	D	E	N	B	G	S	L	C	Z	B	K	M	R	S	O	O	Y	Y	Y	D	Y	V	T	N	H
25	J	H	X	I	A	Q	C	V	E	D	R	N	P	M	W	K	O	T	L	G	B	S	U	Z	Y

Synoptic Table for G

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G
1	D	Z	E	J	A	P	S	T	Y	H	K	C	X	N	U	I	Q	V	W	F	O	M	B	L	R
2	J	N	P	H	F	O	Y	D	U	O	R	U	U	Y	E	M	I	J	H	C	X	X	Z	O	C
3	F	P	H	L	N	C	D	X	N	N	F	J	Z	R	N	W	B	R	V	K	E	P	A	S	Q
4	V	Q	S	K	Q	I	U	A	L	M	B	T	T	S	T	Z	K	K	C	A	Y	U	X	T	M
5	U	X	C	M	U	X	L	I	P	T	E	B	S	M	C	R	L	Y	M	U	B	C	K	E	P
6	Y	R	Z	R	K	L	C	H	M	P	J	Y	W	F	X	V	N	T	I	T	F	O	W	C	S
7	M	W	I	U	D	U	F	P	B	R	A	P	Q	A	O	L	V	F	R	R	S	T	R	H	O
8	H	S	N	O	O	R	M	J	X	Q	L	Z	Y	O	W	X	A	U	B	Z	J	Y	E	N	E
9	T	B	X	Q	P	N	Q	O	W	S	T	K	V	D	F	C	R	I	S	X	M	A	V	Z	X
10	Q	A	F	V	I	D	T	B	C	V	M	X	O	W	Q	S	M	W	E	Q	U	W	D	F	T
11	K	C	Y	P	T	Y	W	W	R	Z	S	I	R	P	D	H	Y	X	K	L	D	V	T	R	K
12	Z	D	Q	T	J	Z	A	K	A	U	X	S	P	K	R	D	O	H	U	Y	Q	S	U	I	I
13	O	E	R	N	B	H	H	C	J	X	V	R	F	J	L	E	F	A	P	I	C	F	F	D	A
14	L	H	T	W	R	W	X	V	D	Y	Q	D	E	V	J	O	T	S	D	O	L	D	O	A	Z
15	R	F	V	Y	H	B	J	F	S	W	P	V	A	I	Z	K	H	D	Z	V	Z	L	Y	Y	D
16	X	I	W	X	C	J	E	Z	K	I	N	E	N	U	M	F	E	M	Q	B	W	I	H	J	N
17	S	J	L	Z	Y	S	Z	L	Q	C	O	W	C	Q	A	P	U	C	A	P	T	E	M	P	B
18	P	K	A	S	S	Q	B	Q	O	A	H	O	J	H	P	Y	S	N	T	E	I	B	L	X	U
19	W	T	D	A	L	F	N	E	I	K	U	A	I	Z	B	A	Z	E	O	S	P	H	S	M	H
20	N	L	K	E	W	K	I	R	V	E	W	M	L	C	V	Q	J	Q	J	N	A	K	I	V	Y
21	A	M	O	D	E	V	K	Y	T	L	D	N	D	T	H	J	X	B	Y	H	V	N	Q	K	F
22	B	O	M	C	M	M	P	N	Z	B	I	F	H	X	I	N	D	O	L	J	N	R	N	B	W
23	C	U	J	B	Z	E	V	S	E	D	Z	L	B	B	Y	U	P	Z	F	W	K	J	J	Q	J
24	E	V	U	I	V	T	R	U	F	F	Y	H	M	L	K	B	C	P	X	M	H	Q	C	W	L
25	I	Y	B	F	X	A	O	M	H	J	C	Q	K	E	S	T	W	L	N	D	R	Z	P	U	V

Synoptic Table for H

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
1	T	F	S	L	C	W	X	P	G	O	U	Q	B	Z	I	D	E	A	V	J	R	K	M	N	Y
2	Q	I	C	K	Y	B	J	J	Y	N	W	G	M	C	Y	E	U	S	C	W	G	N	L	Z	F
3	K	J	Z	M	S	J	E	O	U	M	D	C	K	T	K	O	S	D	M	M	O	R	S	F	W
4	Z	K	I	R	L	S	Z	B	N	T	I	U	G	X	S	K	Z	M	I	D	X	J	I	R	J
5	O	T	N	U	W	Q	B	W	L	P	Z	J	X	B	G	F	J	C	R	G	E	Q	Q	I	L
6	L	L	X	O	E	F	N	K	P	R	Y	T	U	L	U	P	X	N	B	F	Y	Z	N	D	V
7	R	M	F	Q	M	K	I	C	M	Q	C	B	Z	E	E	Y	D	E	S	C	B	G	J	A	G
8	X	O	Y	V	Z	V	K	V	B	S	G	Y	T	G	N	A	P	Q	E	K	F	M	C	Y	R
9	S	U	Q	P	V	M	P	F	X	V	K	P	S	N	T	Q	C	B	K	A	S	X	P	J	C
10	P	V	R	T	X	E	V	Z	W	Z	R	Z	W	Y	C	J	W	O	U	U	J	P	G	P	Q
11	W	Y	T	N	G	T	R	L	C	U	F	K	Q	R	X	N	G	Z	P	T	M	U	B	X	M
12	N	G	V	W	A	A	O	Q	R	X	B	X	Y	S	O	U	Q	P	D	R	U	C	Z	M	P
13	A	Z	W	Y	F	G	G	E	A	Y	E	I	V	M	W	B	I	L	Z	Z	D	O	A	V	S
14	B	N	L	X	N	P	S	R	J	W	J	S	O	F	F	T	B	G	Q	X	Q	T	X	K	O
15	C	P	A	Z	Q	O	Y	Y	D	I	A	R	R	A	Q	G	K	V	A	Q	C	Y	K	B	E
16	E	Q	D	S	U	C	D	N	S	C	L	D	P	O	D	I	L	J	T	L	L	A	W	Q	X
17	I	X	K	A	K	I	U	S	K	A	T	V	F	D	R	M	N	R	O	Y	Z	W	R	W	T
18	G	R	O	E	D	X	L	U	Q	K	M	E	E	W	L	W	V	K	J	I	W	V	E	U	K
19	D	W	M	D	O	L	C	M	O	E	S	W	A	P	J	Z	A	Y	Y	O	T	S	V	G	I
20	J	S	J	C	P	U	F	G	I	L	X	O	N	K	Z	R	R	T	L	V	I	F	D	L	A
21	F	B	U	B	I	R	M	T	V	B	V	A	C	J	M	V	M	F	F	B	P	D	T	O	Z
22	V	A	B	I	T	N	Q	D	T	D	Q	M	J	V	A	L	Y	U	X	P	A	L	U	S	D
23	U	C	G	F	J	D	T	X	Z	F	P	N	I	I	P	X	O	I	N	E	V	I	F	T	N
24	Y	D	E	G	B	Y	W	A	E	J	N	F	L	U	B	C	F	W	G	S	N	E	O	E	B
25	M	E	P	J	R	Z	A	I	F	G	O	L	D	Q	V	S	T	X	W	N	K	B	Y	C	U

Synoptic Table for I

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
1	G	J	N	F	T	X	K	H	V	C	Z	S	L	U	Y	M	B	W	R	O	P	E	Q	D	A
2	D	K	X	G	J	L	P	P	T	A	Y	R	D	Q	K	W	K	X	B	V	A	B	N	A	Z
3	J	T	F	J	B	U	V	J	Z	K	C	D	H	H	S	Z	L	H	S	B	V	H	J	Y	D
4	F	L	Y	H	R	R	R	O	E	E	G	V	B	Z	G	R	N	A	E	P	N	K	C	J	N
5	V	M	Q	L	H	N	O	B	F	L	K	E	M	C	U	V	V	S	K	E	K	N	P	P	B
6	U	O	R	K	C	D	G	W	H	B	R	W	K	T	E	L	A	D	U	S	H	R	G	X	U
7	Y	U	T	M	Y	Y	S	K	G	D	F	O	G	X	N	X	R	M	P	N	R	J	B	M	H
8	M	V	V	R	S	Z	Y	C	Y	F	B	A	X	B	T	C	M	C	D	H	G	Q	Z	V	Y
9	H	Y	W	U	L	H	D	V	U	J	E	M	U	L	C	S	Y	N	Z	J	O	Z	A	K	F
10	T	G	L	O	W	W	U	F	N	G	J	N	Z	E	X	H	O	E	Q	W	X	G	X	B	W
11	Q	Z	A	Q	E	B	L	Z	L	H	A	F	T	G	O	D	F	Q	A	M	E	M	K	Q	J
12	K	N	D	V	M	J	C	L	P	O	L	L	S	N	W	E	T	B	T	D	Y	X	W	W	L
13	Z	P	K	P	Z	S	F	Q	M	N	T	H	W	Y	F	O	H	O	O	G	B	P	R	U	V
14	O	Q	O	T	V	Q	M	E	B	M	M	Q	Q	R	Q	K	E	Z	J	F	F	U	E	G	G
15	L	X	M	N	X	F	Q	R	X	T	S	G	Y	S	D	F	U	P	Y	C	S	C	V	L	R
16	R	R	J	W	G	K	T	Y	W	P	X	C	V	M	R	P	S	L	L	K	J	O	D	O	C
17	X	W	U	Y	A	V	W	N	C	R	V	U	O	F	L	Y	Z	G	F	A	M	T	T	S	Q
18	S	S	B	X	F	M	A	S	R	Q	Q	J	R	A	J	A	J	V	X	U	U	Y	U	T	M
19	P	B	G	Z	N	E	H	U	A	S	P	T	P	O	Z	Q	X	J	N	T	D	A	F	E	P
20	W	A	E	S	Q	T	X	M	J	V	N	B	F	D	M	J	D	R	G	R	Q	W	O	C	S
21	N	C	P	A	U	A	J	G	D	Z	O	Y	E	W	A	N	P	K	W	Z	C	V	Y	H	O
22	A	D	H	E	K	G	E	T	S	U	H	P	A	P	P	U	C	Y	H	X	L	S	H	N	E
23	B	E	S	D	D	P	Z	D	K	X	U	Z	N	K	B	B	W	T	V	Q	Z	F	M	Z	X
24	C	H	C	C	O	O	B	X	Q	Y	W	K	C	J	V	T	G	F	C	L	W	D	L	F	T
25	E	F	Z	B	P	C	N	A	O	W	D	X	J	V	H	G	Q	U	M	Y	T	L	S	R	K

Synoptic Table for J

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J
1	F	K	U	H	B	S	E	O	D	G	A	T	I	V	Z	N	X	R	Y	W	M	Q	C	P	L
2	V	T	B	L	R	Q	Z	B	S	H	L	B	L	I	M	U	D	K	L	M	U	Z	P	X	V
3	U	L	G	K	H	F	B	W	K	O	T	Y	D	U	A	B	P	Y	F	D	D	G	G	M	G
4	Y	M	E	M	C	K	N	K	Q	N	M	P	H	Q	P	T	C	T	X	G	Q	M	B	V	R
5	M	O	P	R	Y	V	I	C	O	M	S	Z	B	H	B	G	W	F	N	F	C	X	Z	K	C
6	H	U	H	U	S	M	K	V	I	T	X	K	M	Z	V	I	G	U	G	C	L	P	A	B	Q
7	T	V	S	O	L	E	P	F	V	P	V	X	K	C	H	M	Q	I	W	K	Z	U	X	Q	M
8	Q	Y	C	Q	W	T	V	Z	T	R	Q	I	G	T	I	W	I	W	H	A	W	C	K	W	P
9	K	G	Z	V	E	A	R	L	Z	Q	P	S	X	X	Y	Z	B	X	V	U	T	O	W	U	S
10	Z	Z	I	P	M	G	O	Q	E	S	N	R	U	B	K	R	K	H	C	T	I	T	R	G	O
11	O	N	N	T	Z	P	G	E	F	V	O	D	Z	L	S	V	L	A	M	R	P	Y	E	L	E
12	L	P	X	N	V	O	S	R	H	Z	H	V	T	E	G	L	N	S	I	Z	A	A	V	O	X
13	R	Q	F	W	X	C	Y	Y	G	U	U	E	S	G	U	X	V	D	R	X	V	W	D	S	T
14	X	X	Y	Y	G	I	D	N	Y	X	W	W	W	N	E	C	A	M	B	Q	N	V	T	T	K
15	S	R	Q	X	A	X	U	S	U	Y	D	O	Q	Y	N	S	R	C	S	L	K	S	U	E	I
16	P	W	R	Z	F	L	L	U	N	W	I	A	Y	R	T	H	M	N	E	Y	H	F	F	C	A
17	W	S	T	S	N	U	C	M	L	I	Z	M	V	S	C	D	Y	E	K	I	R	D	O	H	Z
18	N	B	V	A	Q	R	F	G	P	C	Y	N	O	M	X	E	O	Q	U	O	G	L	Y	N	D
19	A	A	W	E	U	N	M	T	M	A	C	F	R	F	O	O	F	B	P	V	O	I	H	Z	N
20	B	C	L	D	K	D	Q	D	B	K	G	L	P	A	W	K	T	O	D	B	X	E	M	F	B
21	C	D	A	C	D	Y	T	X	X	E	K	H	F	O	F	F	H	Z	Z	P	E	B	L	R	U
22	E	E	D	B	O	Z	W	A	W	L	R	Q	E	D	Q	P	E	P	Q	E	Y	H	S	I	H
23	I	H	K	I	P	H	A	I	C	B	F	G	A	W	D	Y	U	L	A	S	B	K	I	D	Y
24	G	F	O	F	I	W	H	H	R	D	B	C	N	P	R	A	S	G	T	N	F	N	Q	A	F
25	D	I	M	G	T	B	X	P	A	F	E	U	C	K	L	Q	Z	V	O	H	S	R	N	Y	W

Synoptic Table for K

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K
1	Z	T	O	M	D	V	P	C	Q	E	R	X	G	J	S	F	L	Y	U	A	H	N	W	B	I
2	O	L	M	R	O	M	V	V	O	L	F	I	X	V	G	P	N	T	P	U	R	R	R	Q	A
3	L	M	J	U	P	E	R	F	I	B	B	S	U	I	U	Y	V	F	D	T	G	J	E	W	Z
4	R	O	U	O	I	T	O	Z	V	D	E	R	Z	U	E	A	A	U	Z	R	O	Q	V	U	D
5	X	U	B	Q	T	A	G	L	T	F	J	D	T	Q	N	Q	R	I	Q	Z	X	Z	D	G	N
6	S	V	G	V	J	G	S	Q	Z	J	A	V	S	H	T	J	M	W	A	X	E	G	T	L	B
7	P	Y	E	P	B	P	Y	E	E	G	L	E	W	Z	C	N	Y	X	T	Q	Y	M	U	O	U
8	W	G	P	T	R	O	D	R	F	H	T	W	Q	C	X	U	O	H	O	L	B	X	F	S	H
9	N	Z	H	N	H	C	U	Y	H	O	M	O	Y	T	O	B	F	A	J	Y	F	P	O	T	Y
10	A	N	S	W	C	I	L	N	G	N	S	A	V	X	W	T	T	S	Y	I	S	U	Y	E	F
11	B	P	C	Y	Y	X	C	S	Y	M	X	M	O	B	F	G	H	D	L	O	J	C	H	C	W
12	C	Q	Z	X	S	L	F	U	U	T	V	N	R	L	Q	I	E	M	F	V	M	O	M	H	J
13	E	X	I	Z	L	U	M	M	N	P	Q	F	P	E	D	M	U	C	X	B	U	T	L	N	L
14	I	R	N	S	W	R	Q	G	L	R	P	L	F	G	R	W	S	N	N	P	D	Y	S	Z	V
15	G	W	X	A	E	N	T	T	P	Q	N	H	E	N	L	Z	Z	E	G	E	Q	A	I	F	G
16	D	S	F	E	M	D	W	D	M	S	O	Q	A	Y	J	R	J	Q	W	S	C	W	Q	R	R
17	J	B	Y	D	Z	Y	A	X	B	V	H	G	N	R	Z	V	X	B	H	N	L	V	N	I	C
18	F	A	Q	C	V	Z	H	A	X	Z	U	C	C	S	M	L	D	O	V	H	Z	S	J	D	Q
19	V	C	R	B	X	H	X	I	W	U	W	U	J	M	A	X	P	Z	C	J	W	F	C	A	M
20	U	D	T	I	G	W	J	H	C	X	D	J	I	F	P	C	C	P	M	W	T	D	P	Y	P
21	Y	E	V	F	A	B	E	P	R	Y	I	T	L	A	B	S	W	L	I	M	I	L	G	J	S
22	M	H	W	G	F	J	Z	J	A	W	Z	B	D	O	V	H	G	G	R	D	P	I	B	P	O
23	H	F	L	J	N	S	B	O	J	I	Y	Y	H	D	H	D	Q	V	B	G	A	E	Z	X	E
24	T	I	A	H	Q	Q	N	B	D	C	C	P	B	W	I	E	I	J	S	F	V	B	A	M	X
25	Q	J	D	L	U	F	I	W	S	A	G	Z	M	P	Y	O	B	R	E	C	N	H	X	V	T

Synoptic Table for L

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L	L
1	R	M	A	K	W	U	C	Q	P	B	T	H	D	E	J	X	N	G	F	Y	Z	I	S	O	V
2	X	O	D	M	E	R	F	E	M	D	M	Q	H	G	Z	C	V	V	X	I	W	E	I	S	G
3	S	U	K	R	M	N	M	R	B	F	S	G	B	N	M	S	A	J	N	O	T	B	Q	T	R
4	P	V	O	U	Z	D	Q	Y	X	J	X	C	M	Y	A	H	R	R	G	V	I	H	N	E	C
5	W	Y	M	O	V	Y	T	N	W	G	V	U	K	R	P	D	M	K	W	B	P	K	J	C	Q
6	N	G	J	Q	X	Z	W	S	C	H	Q	J	G	S	B	E	Y	Y	H	P	A	N	C	H	M
7	A	Z	U	V	G	H	A	U	R	O	P	T	X	M	V	O	O	T	V	E	V	R	P	N	P
8	B	N	B	P	A	W	H	M	A	N	N	B	U	F	H	K	F	F	C	S	N	J	G	Z	S
9	C	P	G	T	F	B	X	G	J	M	O	Y	Z	A	I	F	T	U	M	N	K	Q	B	F	O
10	E	Q	E	N	N	J	J	T	D	T	H	P	T	O	Y	P	H	I	I	H	H	Z	Z	R	E
11	I	X	P	W	Q	S	E	D	S	P	U	Z	S	D	K	Y	E	W	R	J	R	G	A	I	X
12	G	R	H	Y	U	Q	Z	X	K	R	W	K	W	W	S	A	U	X	B	W	G	M	X	D	T
13	D	W	S	X	K	F	B	A	Q	Q	D	X	Q	P	G	Q	S	H	S	M	O	X	K	A	K
14	J	S	C	Z	D	K	N	I	O	S	I	I	Y	K	U	J	Z	A	E	D	X	P	W	Y	I
15	F	B	Z	S	O	V	I	H	I	V	Z	S	V	J	E	N	J	S	K	G	E	U	R	J	A
16	V	A	I	A	P	M	K	P	V	Z	Y	R	O	V	N	U	X	D	U	F	Y	C	E	P	Z
17	U	C	N	E	I	E	P	J	T	U	C	D	R	I	T	B	D	M	P	C	B	O	V	X	D
18	Y	D	X	D	T	T	V	O	Z	X	G	V	P	U	C	T	P	C	D	K	F	T	D	M	N
19	M	E	F	C	J	A	R	B	E	Y	K	E	F	Q	X	G	C	N	Z	A	S	Y	T	V	B
20	H	H	Y	B	B	G	O	W	F	W	R	W	E	H	O	I	W	E	Q	U	J	A	U	K	U
21	T	F	Q	I	R	P	G	K	H	I	F	O	A	Z	W	M	G	Q	A	T	M	W	F	B	H
22	Q	I	R	F	H	O	S	C	G	C	B	A	N	C	F	W	Q	B	T	R	U	V	O	Q	Y
23	K	J	T	G	C	C	Y	V	Y	A	E	M	C	T	Q	Z	I	O	O	Z	D	S	Y	W	F
24	Z	K	V	J	Y	I	D	F	U	K	J	N	J	X	D	R	B	Z	J	X	Q	F	H	U	W
25	O	T	W	H	S	X	U	Z	N	E	A	F	I	B	R	V	K	P	Y	Q	C	D	M	G	J

Synoptic Table for M

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
1	H	O	J	R	Z	E	Q	G	B	T	S	N	K	F	A	W	Y	C	I	D	U	X	L	V	P
2	T	U	U	U	V	T	T	T	X	P	X	F	G	A	P	Z	O	N	R	G	D	P	S	K	S
3	Q	V	B	O	X	A	W	D	W	R	V	L	X	O	B	R	F	E	B	F	Q	U	I	B	O
4	K	Y	G	Q	G	G	A	X	C	Q	Q	H	U	D	V	V	T	Q	S	C	C	C	Q	Q	E
5	Z	G	E	V	A	P	H	A	R	S	P	Q	Z	W	H	L	H	B	E	K	L	O	N	W	X
6	O	Z	P	P	F	O	X	I	A	V	N	G	T	P	I	X	E	O	K	A	Z	T	J	U	T
7	L	N	H	T	N	C	J	H	J	Z	O	C	S	K	Y	C	U	Z	U	U	W	Y	C	G	K
8	R	P	S	N	Q	I	E	P	D	U	H	U	W	J	K	S	S	P	P	T	T	A	P	L	I
9	X	Q	C	W	U	X	Z	J	S	X	U	J	Q	V	S	H	Z	L	D	R	I	W	G	O	A
10	S	X	Z	Y	K	L	B	O	K	Y	W	T	Y	I	G	D	J	G	Z	Z	P	V	B	S	Z
11	P	R	I	X	D	U	N	B	Q	W	D	B	V	U	U	E	X	V	Q	X	A	S	Z	T	D
12	W	W	N	Z	O	R	I	W	O	I	I	Y	O	Q	E	O	D	J	A	Q	V	F	A	E	N
13	N	S	X	S	P	N	K	K	I	C	Z	P	R	H	N	K	P	R	T	L	N	D	X	C	B
14	A	B	F	A	I	D	P	C	V	A	Y	Z	P	Z	T	F	C	K	O	Y	K	L	K	H	U
15	B	A	Y	E	T	Y	V	V	T	K	C	K	F	C	C	P	W	Y	J	I	H	I	W	N	H
16	C	C	Q	D	J	Z	R	F	Z	E	G	X	E	T	X	Y	G	T	Y	O	R	E	R	Z	Y
17	E	D	R	C	B	H	O	Z	E	L	K	I	A	X	O	A	Q	F	L	V	G	B	E	F	F
18	I	E	T	B	R	W	G	L	F	B	R	S	N	B	W	Q	I	U	F	B	O	H	V	R	W
19	G	H	V	I	H	B	S	Q	H	D	F	R	C	L	F	J	B	I	X	P	X	K	D	I	J
20	D	F	W	F	C	J	Y	E	G	F	B	D	J	E	Q	N	K	W	N	E	E	N	T	D	L
21	J	I	L	G	Y	S	D	R	Y	J	E	V	I	G	D	U	L	X	G	S	Y	R	U	A	V
22	F	J	A	J	S	Q	U	Y	U	G	J	E	L	N	R	B	N	H	W	N	B	J	F	Y	G
23	V	K	D	H	L	F	L	N	N	H	A	W	D	Y	L	T	V	A	H	H	F	Q	O	J	R
24	U	T	K	L	W	K	C	S	L	O	L	O	H	R	J	G	A	S	V	J	S	Z	Y	P	C
25	Y	L	O	K	E	V	F	U	P	N	T	A	B	S	Z	I	R	D	C	W	J	G	H	X	Q

Synoptic Table for N

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
1	A	P	X	W	Q	D	I	S	L	M	O	F	C	Y	T	U	V	E	G	H	K	R	J	Z	B
2	B	Q	F	Y	U	Y	K	U	P	T	H	L	J	R	C	B	A	Q	W	J	H	J	C	F	U
3	C	X	Y	X	K	Z	P	M	M	P	U	H	I	S	X	T	R	B	H	W	R	Q	P	R	H
4	E	R	Q	Z	D	H	V	G	B	R	W	Q	L	M	O	G	M	O	V	M	G	Z	G	I	Y
5	I	W	R	S	O	W	R	T	X	Q	D	G	D	F	W	I	Y	Z	C	D	O	G	B	D	F
6	G	S	T	A	P	B	O	D	W	S	I	C	H	A	F	M	O	P	M	G	X	M	Z	A	W
7	D	B	V	E	I	J	G	X	C	V	Z	U	B	O	Q	W	F	L	I	F	E	X	A	Y	J
8	J	A	W	D	T	S	S	A	R	Z	Y	J	M	D	D	Z	T	G	R	C	Y	P	X	J	L
9	F	C	L	C	J	Q	Y	I	A	U	C	T	K	W	R	R	H	V	B	K	B	U	K	P	V
10	V	D	A	B	B	F	D	H	J	X	G	B	G	P	L	V	E	J	S	A	F	C	W	X	G
11	U	E	D	I	R	K	U	P	D	Y	K	Y	X	K	J	L	U	R	E	U	S	O	R	M	R
12	Y	H	K	F	H	V	L	J	S	W	R	P	U	J	Z	X	S	K	K	T	J	T	E	V	C
13	M	F	O	G	C	M	C	O	K	I	F	Z	Z	V	M	C	Z	Y	U	R	M	Y	V	K	Q
14	H	I	M	J	Y	E	F	B	Q	C	B	K	T	I	A	S	J	T	P	Z	U	A	D	B	M
15	T	J	J	H	S	T	M	W	O	A	E	X	S	U	P	H	X	F	D	X	D	W	T	Q	P
16	Q	K	U	L	L	A	Q	K	I	K	J	I	W	Q	B	D	D	U	Z	Q	Q	V	U	W	S
17	K	T	B	K	W	G	T	C	V	E	A	S	Q	H	V	E	P	I	Q	L	C	S	F	U	O
18	Z	L	G	M	E	P	W	V	T	L	L	R	Y	Z	H	O	C	W	A	Y	L	F	O	G	E
19	O	M	E	R	M	O	A	F	Z	B	T	D	V	C	I	K	W	X	T	I	Z	D	Y	L	X
20	L	O	P	U	Z	C	H	Z	E	D	M	V	O	T	Y	F	G	H	O	O	W	L	H	O	T
21	R	U	H	O	V	I	X	L	F	F	S	E	R	X	K	P	Q	A	J	V	T	I	M	S	K
22	X	V	S	Q	X	X	J	Q	H	J	X	W	P	B	S	Y	I	S	Y	B	I	E	L	T	I
23	S	Y	C	V	G	L	E	E	G	G	V	O	F	L	G	A	B	D	L	P	P	B	S	E	A
24	P	G	Z	P	A	U	Z	R	Y	H	Q	A	E	E	U	Q	K	M	F	E	A	H	I	C	Z
25	W	Z	I	T	F	R	B	Y	U	O	P	M	A	G	E	J	L	C	X	S	V	K	Q	H	D

Synoptic Table for O

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
1	L	U	M	Q	P	C	G	B	I	N	H	A	R	D	W	K	F	Z	J	V	X	T	Y	S	E
2	R	V	J	V	I	I	S	W	V	M	U	M	P	W	F	F	T	P	Y	B	E	Y	H	T	X
3	X	Y	U	P	T	X	Y	K	T	T	W	N	F	P	Q	P	H	L	L	P	Y	A	M	E	T
4	S	G	B	T	J	L	D	C	Z	P	D	F	E	K	D	Y	E	G	F	E	B	W	L	C	K
5	P	Z	G	N	B	U	U	V	E	R	I	L	A	J	R	A	U	V	X	S	F	V	S	H	I
6	W	N	E	W	R	R	L	F	F	Q	Z	H	N	V	L	Q	S	J	N	N	S	S	I	N	A
7	N	P	P	Y	H	N	C	Z	H	S	Y	Q	C	I	J	J	Z	R	G	H	J	F	Q	Z	Z
8	A	Q	H	X	C	D	F	L	G	V	C	G	J	U	Z	N	J	K	W	J	M	D	N	F	D
9	B	X	S	Z	Y	Y	M	Q	Y	Z	G	C	I	Q	M	U	X	Y	H	W	U	L	J	R	N
10	C	R	C	S	S	Z	Q	E	U	U	K	U	L	H	A	B	D	T	V	M	D	I	C	I	B
11	E	W	Z	A	L	H	T	R	N	X	R	J	D	Z	P	T	P	F	C	D	Q	E	P	D	U
12	I	S	I	E	W	W	W	Y	L	Y	F	T	H	C	B	G	C	U	M	G	C	B	G	A	H
13	G	B	N	D	E	B	A	N	P	W	B	B	B	T	V	I	W	I	I	F	L	H	B	Y	Y
14	D	A	X	C	M	J	H	S	M	I	E	Y	M	X	H	M	G	W	R	C	Z	K	Z	J	F
15	J	C	F	B	Z	S	X	U	B	C	J	P	K	B	I	W	Q	X	B	K	W	N	A	P	W
16	F	D	Y	I	V	Q	J	M	X	A	A	Z	G	L	Y	Z	I	H	S	A	T	R	X	X	J
17	V	E	Q	F	X	F	E	G	W	K	L	K	X	E	K	R	B	A	E	U	I	J	K	M	L
18	U	H	R	G	G	K	Z	T	C	E	T	X	U	G	S	V	K	S	K	T	P	Q	W	V	V
19	Y	F	T	J	A	V	B	D	R	L	M	I	Z	N	G	L	L	D	U	R	A	Z	R	K	G
20	M	I	V	H	F	M	N	X	A	B	S	S	T	Y	U	X	N	M	P	Z	V	G	E	B	R
21	H	J	W	L	N	E	I	A	J	D	X	R	S	R	E	C	V	C	D	X	N	M	V	Q	C
22	T	K	L	K	Q	T	K	I	D	F	V	D	W	S	N	S	A	N	Z	Q	K	X	D	W	Q
23	Q	T	A	M	U	A	P	H	S	J	Q	V	Q	M	T	H	R	E	Q	L	H	P	T	U	M
24	K	L	D	R	K	G	V	P	K	G	P	E	Y	F	C	D	M	Q	A	Y	R	U	U	G	P
25	Z	M	K	U	D	P	R	J	Q	H	N	W	V	A	X	E	Y	B	T	I	G	C	F	L	S

Synoptic Table for P

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
1	W	Q	H	T	I	O	V	J	M	R	N	Z	F	K	B	Y	C	L	D	E	A	U	G	X	S
2	N	X	S	N	T	C	R	O	B	Q	O	K	E	J	V	A	W	G	Z	S	V	C	B	M	O
3	A	R	C	W	J	I	O	B	X	S	H	X	A	V	H	Q	G	V	Q	N	N	O	Z	V	E
4	B	W	Z	Y	B	X	G	W	W	V	U	I	N	I	I	J	Q	J	A	H	K	T	A	K	X
5	C	S	I	X	R	L	S	K	C	Z	W	S	C	U	Y	N	I	R	T	J	H	Y	X	B	T
6	E	B	N	Z	H	U	Y	C	R	U	D	R	J	Q	K	U	B	K	O	W	R	A	K	Q	K
7	I	A	X	S	C	R	D	V	A	X	I	D	I	H	S	B	K	Y	J	M	G	W	W	W	I
8	G	C	F	A	Y	N	U	F	J	Y	Z	V	L	Z	G	T	L	T	Y	D	O	V	R	U	A
9	D	D	Y	E	S	D	L	Z	D	W	Y	E	D	C	U	G	N	F	L	G	X	S	E	G	Z
10	J	E	Q	D	L	Y	C	L	S	I	C	W	H	T	E	I	V	U	F	F	E	F	V	L	D
11	F	H	R	C	W	Z	F	Q	K	C	G	O	B	X	N	M	A	I	X	C	Y	D	D	O	N
12	V	F	T	B	E	H	M	E	Q	A	K	A	M	B	T	W	R	W	N	K	B	L	T	S	B
13	U	I	V	I	M	W	Q	R	O	K	R	M	K	L	C	Z	M	X	G	A	F	I	U	T	U
14	Y	J	W	F	Z	B	T	Y	I	E	F	N	G	E	X	R	Y	H	W	U	S	E	F	E	H
15	M	K	L	G	V	J	W	N	V	L	B	F	X	G	O	V	O	A	H	T	J	B	O	C	Y
16	H	T	A	J	X	S	A	S	T	B	E	L	U	N	W	L	F	S	V	R	M	H	Y	H	F
17	T	L	D	H	G	Q	H	U	Z	D	J	H	Z	Y	F	X	T	D	C	Z	U	K	H	N	W
18	Q	M	K	L	A	F	X	M	E	F	A	Q	T	R	Q	C	H	M	M	X	D	N	M	Z	J
19	K	O	O	K	F	K	J	G	F	J	L	G	S	S	D	S	E	C	I	Q	Q	R	L	F	L
20	Z	U	M	M	N	V	E	T	H	G	T	C	W	M	R	H	U	N	R	L	C	J	S	R	V
21	O	V	J	R	Q	M	Z	D	G	H	M	U	Q	F	L	D	S	E	B	Y	L	Q	I	I	G
22	L	Y	U	U	U	E	B	X	Y	O	S	J	Y	A	J	E	Z	Q	S	I	Z	Z	Q	D	R
23	R	G	B	O	K	T	N	A	U	N	X	T	V	O	Z	O	J	B	E	O	W	G	N	A	C
24	X	Z	G	Q	D	A	I	I	N	M	V	B	O	D	M	K	X	O	K	V	T	M	J	Y	Q
25	S	N	E	V	O	G	K	H	L	T	Q	Y	R	W	A	F	D	Z	U	B	I	X	C	J	M

Synoptic Table for Q

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
1	K	X	R	V	U	F	T	E	O	S	P	G	Y	H	D	J	I	B	A	L	C	Z	N	W	M
2	Z	R	T	P	K	K	W	R	I	V	N	C	V	Z	R	N	B	O	T	Y	L	G	J	U	P
3	O	W	V	T	D	V	A	Y	V	Z	O	U	O	C	L	U	K	Z	O	I	Z	M	C	G	S
4	L	S	W	N	O	M	H	N	T	U	H	J	R	T	J	B	L	P	J	O	W	X	P	L	O
5	R	B	L	W	P	E	X	S	Z	X	U	T	P	X	Z	T	N	L	Y	V	T	P	G	O	E
6	X	A	A	Y	I	T	J	U	E	Y	W	B	F	B	M	G	V	G	L	B	I	U	B	S	X
7	S	C	D	X	T	A	E	M	F	W	D	Y	E	L	A	I	A	V	F	P	P	C	Z	T	T
8	P	D	K	Z	J	G	Z	G	H	I	I	P	A	E	P	M	R	J	X	E	A	O	A	E	K
9	W	E	O	S	B	P	B	T	G	C	Z	Z	N	G	B	W	M	R	N	S	V	T	X	C	I
10	N	H	M	A	R	O	N	D	Y	A	Y	K	C	N	V	Z	Y	K	G	N	N	Y	K	H	A
11	A	F	J	E	H	C	I	X	U	K	C	X	J	Y	H	R	O	Y	W	H	K	A	W	N	Z
12	B	I	U	D	C	I	K	A	N	E	G	I	I	R	I	V	F	T	H	J	H	W	R	Z	D
13	C	J	B	C	Y	X	P	I	L	L	K	S	L	S	Y	L	T	F	V	W	R	V	E	F	N
14	E	K	G	B	S	L	V	H	P	B	R	R	D	M	K	X	H	U	C	M	G	S	V	R	B
15	I	T	E	I	L	U	R	P	M	D	F	D	H	F	S	C	E	I	M	D	O	F	D	I	U
16	G	L	P	F	W	R	O	J	B	F	B	V	B	A	G	S	U	W	I	G	X	D	T	D	H
17	D	M	H	G	E	N	G	O	X	J	E	E	M	O	U	H	S	X	R	F	E	L	U	A	Y
18	J	O	S	J	M	D	S	B	W	G	J	W	K	D	E	D	Z	H	B	C	Y	I	F	Y	F
19	F	U	C	H	Z	Y	Y	W	C	H	A	O	G	W	N	E	J	A	S	K	B	E	O	J	W
20	V	V	Z	L	V	Z	D	K	R	O	L	A	X	P	T	O	X	S	E	A	F	B	Y	P	J
21	U	Y	I	K	X	H	U	C	A	N	T	M	U	K	C	K	D	D	K	U	S	H	H	X	L
22	Y	G	N	M	G	W	L	V	J	M	M	N	Z	J	X	F	P	M	U	T	J	K	M	M	V
23	M	Z	X	R	A	B	C	F	D	T	S	F	T	V	O	P	C	C	P	R	M	N	L	V	G
24	H	N	F	U	F	J	F	Z	S	P	X	L	S	I	W	Y	W	N	D	Z	U	R	S	K	R
25	T	P	Y	O	N	S	M	L	K	R	V	H	W	U	F	A	G	E	Z	X	D	J	I	B	C

Synoptic Table for R

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
1	X	W	T	U	H	N	O	Y	A	Q	F	D	P	S	L	V	M	K	B	Z	G	J	E	I	C
2	S	S	V	O	C	D	G	N	J	S	B	V	F	M	J	L	Y	Y	S	X	O	Q	V	D	Q
3	P	B	W	Q	Y	Y	S	S	D	V	E	E	E	F	Z	X	O	T	E	Q	X	Z	D	A	M
4	W	A	L	V	S	Z	Y	U	S	Z	J	W	A	A	M	C	F	F	K	L	E	G	T	Y	P
5	N	C	A	P	L	H	D	M	K	U	A	O	N	O	A	S	T	U	U	Y	Y	M	U	J	S
6	A	D	D	T	W	W	U	G	Q	X	L	A	C	D	P	H	H	I	P	I	B	X	F	P	O
7	B	E	K	N	E	B	L	T	O	Y	T	M	J	W	B	D	E	W	D	O	F	P	O	X	E
8	C	H	O	W	M	J	C	D	I	W	M	N	I	P	V	E	U	X	Z	V	S	U	Y	M	X
9	E	F	M	Y	Z	S	F	X	V	I	S	F	L	K	H	O	S	H	Q	B	J	C	H	V	T
10	I	I	J	X	V	Q	M	A	T	C	X	L	D	J	I	K	Z	A	A	P	M	O	M	K	K
11	G	J	U	Z	X	F	Q	I	Z	A	V	H	H	V	Y	F	J	S	T	E	U	T	L	B	I
12	D	K	B	S	G	K	T	H	E	K	Q	Q	B	I	K	P	X	D	O	S	D	Y	S	Q	A
13	J	T	G	A	A	V	W	P	F	E	P	G	M	U	S	Y	D	M	J	N	Q	A	I	W	Z
14	F	L	E	E	F	M	A	J	H	L	N	C	K	Q	G	A	P	C	Y	H	C	W	Q	U	D
15	V	M	P	D	N	E	H	O	G	B	O	U	G	H	U	Q	C	N	L	J	L	V	N	G	N
16	U	O	H	C	Q	T	X	B	Y	D	H	J	X	Z	E	J	W	E	F	W	Z	S	J	L	B
17	Y	U	S	B	U	A	J	W	U	F	U	T	U	C	N	N	G	Q	X	M	W	F	C	O	U
18	M	V	C	I	K	G	E	K	N	J	W	B	Z	T	T	U	Q	B	N	D	T	D	P	S	H
19	H	Y	Z	F	D	P	Z	C	L	G	D	Y	T	X	C	B	I	O	G	G	I	L	G	T	Y
20	T	G	I	G	O	O	B	V	P	H	I	P	S	B	X	T	B	Z	W	F	P	I	B	E	F
21	Q	Z	N	J	P	C	N	F	M	O	Z	Z	W	L	O	G	K	P	H	C	A	E	Z	C	W
22	K	N	X	H	I	I	I	Z	B	N	Y	K	Q	E	W	I	L	L	V	K	V	B	A	H	J
23	Z	P	F	L	T	X	K	L	X	M	C	X	Y	G	F	M	N	G	C	A	N	H	X	N	L
24	O	Q	Y	K	J	L	P	Q	W	T	G	I	V	N	Q	W	V	V	M	U	K	K	K	Z	V
25	L	X	Q	M	B	U	V	E	C	P	K	S	O	Y	D	Z	A	J	I	T	H	N	W	F	G

Synoptic Table for S

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
1	P	B	C	A	L	Q	Y	U	K	V	X	R	W	M	G	H	Z	D	E	N	J	F	I	T	O
2	W	A	Z	E	W	F	D	M	Q	Z	V	D	Q	F	U	D	J	M	K	H	M	D	Q	E	E
3	N	C	I	D	E	K	U	G	O	U	Q	V	Y	A	E	E	X	C	U	J	U	L	N	C	X
4	A	D	N	C	M	V	L	T	I	X	P	E	V	O	N	O	D	N	P	W	D	I	J	H	T
5	B	E	X	B	Z	M	C	D	V	Y	N	W	O	D	T	K	P	E	D	M	Q	E	C	N	K
6	C	H	F	I	V	E	F	X	T	W	O	O	R	W	C	F	C	Q	Z	D	C	B	P	Z	I
7	E	F	Y	F	X	T	M	A	Z	I	H	A	P	P	X	P	W	B	Q	G	L	H	G	F	A
8	I	I	Q	G	G	A	Q	I	E	C	U	M	F	K	O	Y	G	O	A	F	Z	K	B	R	Z
9	G	J	R	J	A	G	T	H	F	A	W	N	E	J	W	A	Q	Z	T	C	W	N	Z	I	D
10	D	K	T	H	F	P	W	P	H	K	D	F	A	V	F	Q	I	P	O	K	T	R	A	D	N
11	J	T	V	L	N	O	A	J	G	E	I	L	N	I	Q	J	B	L	J	A	I	J	X	A	B
12	F	L	W	K	Q	C	H	O	Y	L	Z	H	C	U	D	N	K	G	Y	U	P	Q	K	Y	U
13	V	M	L	M	U	I	X	B	U	B	Y	Q	J	Q	R	U	L	V	L	T	A	Z	W	J	H
14	U	O	A	R	K	X	J	W	N	D	C	G	I	H	L	B	N	J	F	R	V	G	R	P	Y
15	Y	U	D	U	D	L	E	K	L	F	G	C	L	Z	J	T	V	R	X	Z	N	M	E	X	F
16	M	V	K	O	O	U	Z	C	P	J	K	U	D	C	Z	G	A	K	N	X	K	X	V	M	W
17	H	Y	O	Q	P	R	B	V	M	G	R	J	H	T	M	I	R	Y	G	Q	H	P	D	V	J
18	T	G	M	V	I	N	N	F	B	H	F	T	B	X	A	M	M	T	W	L	R	U	T	K	L
19	Q	Z	J	P	T	D	I	Z	X	O	B	B	M	B	P	W	Y	F	H	Y	G	C	U	B	V
20	K	N	U	T	J	Y	K	L	W	N	E	Y	K	L	B	Z	O	U	V	I	O	O	F	Q	G
21	Z	P	B	N	B	Z	P	Q	C	M	J	P	G	E	V	R	F	I	C	O	X	T	O	W	R
22	O	Q	G	W	R	H	V	E	R	T	A	Z	X	G	H	V	T	W	M	V	E	Y	Y	U	C
23	L	X	E	Y	H	W	R	R	A	P	L	K	U	N	I	L	H	X	I	B	Y	A	H	G	Q
24	R	R	P	X	C	B	O	Y	J	R	T	X	Z	Y	Y	X	E	H	R	P	B	W	M	L	M
25	X	W	H	Z	Y	J	G	N	D	Q	M	I	T	R	K	C	U	A	B	E	F	V	L	O	P

Synoptic Table for T

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1	Q	L	V	N	J	A	W	D	Z	P	M	B	S	X	C	G	H	F	O	R	I	Y	U	E	K
2	K	M	W	W	B	G	A	X	E	R	S	Y	W	B	X	I	E	U	J	Z	P	A	F	C	I
3	Z	O	L	Y	R	P	H	A	F	Q	X	P	Q	L	O	M	U	I	Y	X	A	W	O	H	A
4	O	U	A	X	H	O	X	I	H	S	V	Z	Y	E	W	W	S	W	L	Q	V	V	Y	N	Z
5	L	V	D	Z	C	C	J	H	G	V	Q	K	V	G	F	Z	Z	X	F	L	N	S	H	Z	D
6	R	Y	K	S	Y	I	E	P	Y	Z	P	X	O	N	Q	R	J	H	X	Y	K	F	M	F	N
7	X	G	O	A	S	X	Z	J	U	U	N	I	R	Y	D	V	X	A	N	I	H	D	L	R	B
8	S	Z	M	E	L	L	B	O	N	X	O	S	P	R	R	L	D	S	G	O	R	L	S	I	U
9	P	N	J	D	W	U	N	B	L	Y	H	R	F	S	L	X	P	D	W	V	G	I	I	D	H
10	W	P	U	C	E	R	I	W	P	W	U	D	E	M	J	C	C	M	H	B	O	E	Q	A	Y
11	N	Q	B	B	M	N	K	K	M	I	W	V	A	F	Z	S	W	C	V	P	X	B	N	Y	F
12	A	X	G	I	Z	D	P	C	B	C	D	E	N	A	M	H	G	N	C	E	E	H	J	J	W
13	B	R	E	F	V	Y	V	V	X	A	I	W	C	O	A	D	Q	E	M	S	Y	K	C	P	J
14	C	W	P	G	X	Z	R	F	W	K	Z	O	J	D	P	E	I	Q	I	N	B	N	P	X	L
15	E	S	H	J	G	H	O	Z	C	E	Y	A	I	W	B	O	B	B	R	H	F	R	G	M	V
16	I	B	S	H	A	W	G	L	R	L	C	M	L	P	V	K	K	O	B	J	S	J	B	V	G
17	G	A	C	L	F	B	S	Q	A	B	G	N	D	K	H	F	L	Z	S	W	J	Q	Z	K	R
18	D	C	Z	K	N	J	Y	E	J	D	K	F	H	J	I	P	N	P	E	M	M	Z	A	B	C
19	J	D	I	M	Q	S	D	R	D	F	R	L	B	V	Y	Y	V	L	K	D	U	G	X	Q	Q
20	F	E	N	R	U	Q	U	Y	S	J	F	H	M	I	K	A	A	G	U	G	D	M	K	W	M
21	V	H	X	U	K	F	L	N	K	G	B	Q	K	U	S	Q	R	V	P	F	Q	X	W	U	P
22	U	F	F	O	D	K	C	S	Q	H	E	G	G	Q	G	J	M	J	D	C	C	P	R	G	S
23	Y	I	Y	Q	O	V	F	U	O	O	J	C	X	H	U	N	Y	R	Z	K	L	U	E	L	O
24	M	J	Q	V	P	M	M	M	I	N	A	U	U	Z	E	U	O	K	Q	A	Z	C	V	O	E
25	H	K	R	P	I	E	Q	G	V	M	L	J	Z	C	N	B	F	Y	A	U	W	O	D	S	X

Synoptic Table for U

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
1	Y	V	B	O	K	R	L	M	N	X	W	J	Z	Q	E	B	S	I	P	T	D	C	F	G	H
2	M	Y	G	Q	D	N	C	G	L	Y	D	T	T	H	N	T	Z	W	D	R	Q	O	O	L	Y
3	H	G	E	V	O	D	F	T	P	W	I	B	S	Z	T	G	J	X	Z	Z	C	T	Y	O	F
4	T	Z	P	P	P	Y	M	D	M	I	Z	Y	W	C	C	I	X	H	Q	X	L	Y	H	S	W
5	Q	N	H	T	I	Z	Q	X	B	C	Y	P	Q	T	X	M	D	A	A	Q	Z	A	M	T	J
6	K	P	S	N	T	H	T	A	X	A	C	Z	Y	X	O	W	P	S	T	L	W	W	L	E	L
7	Z	Q	C	W	J	W	W	I	W	K	G	K	V	B	W	Z	C	D	O	Y	T	V	S	C	V
8	O	X	Z	Y	B	B	A	H	C	E	K	X	O	L	F	R	W	M	J	I	I	S	I	H	G
9	L	R	I	X	R	J	H	P	R	L	R	I	R	E	Q	V	G	C	Y	O	P	F	Q	N	R
10	R	W	N	Z	H	S	X	J	A	B	F	S	P	G	D	L	Q	N	L	V	A	D	N	Z	C
11	X	S	X	S	C	Q	J	O	J	D	B	R	F	N	R	X	I	E	F	B	V	L	J	F	Q
12	S	B	F	A	Y	F	E	B	D	F	E	D	E	Y	L	C	B	Q	X	P	N	I	C	R	M
13	P	A	Y	E	S	K	Z	W	S	J	J	V	A	R	J	S	K	B	N	E	K	E	P	I	P
14	W	C	Q	D	L	V	B	K	K	G	A	E	N	S	Z	H	L	O	G	S	H	B	G	D	S
15	N	D	R	C	W	M	N	C	Q	H	L	W	C	M	M	D	N	Z	W	N	R	H	B	A	O
16	A	E	T	B	E	E	I	V	O	O	T	O	J	F	A	E	V	P	H	H	G	K	Z	Y	E
17	B	H	V	I	M	T	K	F	I	N	M	A	I	A	P	O	A	L	V	J	O	N	A	J	X
18	C	F	W	F	Z	A	P	Z	V	M	S	M	L	O	B	K	R	G	C	W	X	R	X	P	T
19	E	I	L	G	V	G	V	L	T	T	X	N	D	D	V	F	M	V	M	M	E	J	K	X	K
20	I	J	A	J	X	P	R	Q	Z	P	V	F	H	W	H	P	Y	J	I	D	Y	Q	W	M	I
21	G	K	D	H	G	O	O	E	E	R	Q	L	B	P	I	Y	O	R	R	G	B	Z	R	V	A
22	D	T	K	L	A	C	G	R	F	Q	P	H	M	K	Y	A	F	K	B	F	F	G	E	K	Z
23	J	L	O	K	F	I	S	Y	H	S	N	Q	K	J	K	Q	T	Y	S	C	S	M	V	B	D
24	F	M	M	M	N	X	Y	N	G	V	O	G	G	V	S	J	H	T	E	K	J	X	D	Q	N
25	V	O	J	R	Q	L	D	S	Y	Z	H	C	X	I	G	N	E	F	K	A	M	P	T	W	B

Synoptic Table for V

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V	V
1	U	Y	W	P	X	M	R	F	T	Z	Q	E	O	I	H	L	A	J	C	B	N	S	D	K	G
2	Y	G	L	T	G	E	O	Z	Z	U	P	W	R	U	I	X	R	R	M	P	K	F	T	B	R
3	M	Z	A	N	A	T	G	L	E	X	N	O	P	Q	Y	C	M	K	I	E	H	D	U	Q	C
4	H	N	D	W	F	A	S	Q	F	Y	O	A	F	H	K	S	Y	Y	R	S	R	L	F	W	Q
5	T	P	K	Y	N	G	Y	E	H	W	H	M	E	Z	S	H	O	T	B	N	G	I	O	U	M
6	Q	Q	O	X	Q	P	D	R	G	I	U	N	A	C	G	D	F	F	S	H	O	E	Y	G	P
7	K	X	M	Z	U	O	U	Y	Y	C	W	F	N	T	U	E	T	U	E	J	X	B	H	L	S
8	Z	R	J	S	K	C	L	N	U	A	D	L	C	X	E	O	H	I	K	W	E	H	M	O	O
9	O	W	U	A	D	I	C	S	N	K	I	H	J	B	N	K	E	W	U	M	Y	K	L	S	E
10	L	S	B	E	O	X	F	U	L	E	Z	Q	I	L	T	F	U	X	P	D	B	N	S	T	X
11	R	B	G	D	P	L	M	M	P	L	Y	G	L	E	C	P	S	H	D	G	F	R	I	E	T
12	X	A	E	C	I	U	Q	G	M	B	C	C	D	G	X	Y	Z	A	Z	F	S	J	Q	C	K
13	S	C	P	B	T	R	T	T	B	D	G	U	H	N	O	A	J	S	Q	C	J	Q	N	H	I
14	P	D	H	I	J	N	W	D	X	F	K	J	B	Y	W	Q	X	D	A	K	M	Z	J	N	A
15	W	E	S	F	B	D	A	X	W	J	R	T	M	R	F	J	D	M	T	A	U	G	C	Z	Z
16	N	H	C	G	R	Y	H	A	C	G	F	B	K	S	Q	N	P	C	O	U	D	M	P	F	D
17	A	F	Z	J	H	Z	X	I	R	H	B	Y	G	M	D	U	C	N	J	T	Q	X	G	R	N
18	B	I	I	H	C	H	J	H	A	O	E	P	X	F	R	B	W	E	Y	R	C	P	B	I	B
19	C	J	N	L	Y	W	E	P	J	N	J	Z	U	A	L	T	G	Q	L	Z	L	U	Z	D	U
20	E	K	X	K	S	B	Z	J	D	M	A	K	Z	O	J	G	Q	B	F	X	Z	C	A	A	H
21	I	T	F	M	L	J	B	O	S	T	L	X	T	D	Z	I	I	O	X	Q	W	O	X	Y	Y
22	G	L	Y	R	W	S	N	B	K	P	T	I	S	W	M	M	B	Z	N	L	T	T	K	J	F
23	D	M	Q	U	E	Q	I	W	Q	R	M	S	W	P	A	W	K	P	G	Y	I	Y	W	P	W
24	J	O	R	O	M	F	K	K	O	Q	S	R	Q	K	P	R	L	L	W	I	P	A	R	X	J
25	F	U	T	Q	Z	K	P	C	I	S	X	D	Y	J	B	R	N	G	H	O	A	W	E	M	L

Synoptic Table for W

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
1	N	S	L	Y	E	B	A	K	C	I	D	O	Q	P	F	Z	G	X	H	M	T	V	R	U	J
2	A	B	A	X	M	J	H	C	R	C	I	A	Y	K	Q	R	Q	H	V	D	I	S	E	G	L
3	B	A	D	Z	Z	S	X	V	A	A	Z	M	V	J	D	V	I	A	C	G	P	F	V	L	V
4	C	C	K	S	V	Q	J	F	J	K	Y	N	O	V	R	L	B	S	M	F	A	D	D	O	G
5	E	D	O	A	X	F	E	Z	D	E	C	F	R	I	L	X	K	D	I	C	V	L	T	S	R
6	I	E	M	E	G	K	Z	L	S	L	G	L	P	U	J	C	L	M	R	K	N	I	U	T	C
7	G	H	J	D	A	V	B	Q	K	B	K	H	F	Q	Z	S	N	C	B	A	K	E	F	E	Q
8	D	F	U	C	F	M	N	E	Q	D	R	Q	E	H	M	H	V	N	S	U	H	B	O	C	M
9	J	I	B	B	N	E	I	R	O	F	F	G	A	Z	A	D	A	E	E	T	R	H	Y	H	P
10	F	J	G	I	Q	T	K	Y	I	J	B	C	N	C	P	E	R	Q	K	R	G	K	H	N	S
11	V	K	E	F	U	A	P	N	V	G	E	U	C	T	B	O	M	B	U	Z	O	N	M	Z	O
12	U	T	P	G	K	G	V	S	T	H	J	J	J	X	V	K	Y	O	P	X	X	R	L	F	E
13	Y	L	H	J	D	P	R	U	Z	O	A	T	I	B	H	F	O	Z	D	Q	E	J	S	R	X
14	M	M	S	H	O	O	O	M	E	N	L	B	L	L	I	P	F	P	Z	L	Y	Q	I	I	T
15	H	O	C	L	P	C	G	G	F	M	T	Y	D	E	Y	Y	T	L	Q	Y	B	Z	Q	D	K
16	T	U	Z	K	I	I	S	T	H	T	M	P	H	G	K	A	H	G	A	I	F	G	N	A	I
17	Q	V	I	M	T	X	Y	D	G	P	S	Z	B	N	S	Q	E	V	T	O	S	M	J	Y	A
18	K	Y	N	R	J	L	D	X	Y	R	X	K	M	Y	G	J	U	J	O	V	J	X	C	J	Z
19	Z	G	X	U	B	U	U	A	U	Q	V	X	K	R	U	N	S	R	J	B	M	P	P	P	D
20	O	Z	F	O	R	R	L	I	N	S	Q	I	G	S	E	U	Z	K	Y	P	U	U	G	X	N
21	L	N	Y	Q	H	N	C	H	L	V	P	S	X	M	N	B	J	Y	L	E	D	C	B	M	B
22	R	P	Q	V	C	D	F	P	P	Z	N	R	U	F	T	T	X	T	F	S	Q	O	Z	V	U
23	X	Q	R	P	Y	Y	M	J	M	U	O	D	Z	A	C	G	D	F	X	N	C	T	A	K	H
24	S	X	T	T	S	Z	Q	O	B	X	H	V	T	O	X	I	P	U	N	H	L	Y	X	B	Y
25	P	R	V	N	L	H	T	B	X	Y	U	E	S	D	O	M	C	I	G	J	Z	A	K	Q	F

Synoptic Table for X

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1	S	R	F	Z	G	L	J	A	W	Y	V	I	U	B	O	C	D	H	N	Q	E	P	K	M	T
2	P	W	Y	S	A	U	E	I	C	W	Q	S	Z	L	W	S	P	A	G	L	Y	U	W	V	K
3	W	S	Q	A	F	R	Z	H	R	I	P	R	T	E	F	H	C	S	W	Y	B	C	R	K	I
4	N	B	R	E	N	N	B	P	A	C	N	D	S	G	Q	D	W	D	H	I	F	O	E	B	A
5	A	A	T	D	Q	D	N	J	J	A	O	V	W	N	D	E	G	M	V	O	S	T	V	Q	Z
6	B	C	V	C	U	Y	I	O	D	K	H	E	Q	Y	R	O	Q	C	C	V	J	Y	D	W	D
7	C	D	W	B	K	Z	K	B	S	E	U	W	Y	R	L	K	I	N	M	B	M	A	T	U	N
8	E	E	L	I	D	H	P	W	K	L	W	O	V	S	J	F	B	E	I	P	U	W	U	G	B
9	I	H	A	F	O	W	V	K	Q	B	D	A	O	M	Z	P	K	Q	R	E	D	V	F	L	U
10	G	F	D	G	P	B	R	C	O	D	I	M	R	F	M	Y	L	B	B	S	Q	S	O	O	H
11	D	I	K	J	I	J	O	V	I	F	Z	N	P	A	A	A	N	O	S	N	C	F	Y	S	Y
12	J	J	O	H	T	S	G	F	V	J	Y	F	F	O	P	Q	V	Z	E	H	L	D	H	T	F
13	F	K	M	L	J	Q	S	Z	T	G	C	L	E	D	B	J	A	P	K	J	Z	L	M	E	W
14	V	T	J	K	B	F	Y	L	Z	H	G	H	A	W	V	N	R	L	U	W	W	I	L	C	J
15	U	L	U	M	R	K	D	Q	E	O	K	Q	N	P	H	U	M	G	P	M	T	E	S	H	L
16	Y	M	B	R	H	V	U	E	F	N	R	G	C	K	I	B	Y	V	D	D	I	B	I	N	V
17	M	O	G	U	C	M	L	R	H	M	F	C	J	J	Y	T	O	J	Z	G	P	H	Q	Z	G
18	H	U	E	O	Y	E	C	Y	G	T	B	U	I	V	K	G	F	R	Q	F	A	K	N	F	R
19	T	V	P	Q	S	T	F	N	Y	P	E	J	L	I	S	I	T	K	A	C	V	N	J	R	C
20	Q	Y	H	V	L	A	M	S	U	R	J	T	D	U	G	M	H	Y	T	K	N	R	C	I	Q
21	K	G	S	P	W	G	Q	U	N	Q	A	B	H	Q	U	W	E	T	O	A	K	J	P	D	M
22	Z	Z	C	T	E	P	T	M	L	S	L	Y	B	H	E	Z	U	F	J	U	H	Q	G	A	P
23	O	N	Z	N	M	O	W	G	P	V	T	P	M	Z	N	R	S	U	Y	T	R	Z	B	Y	S
24	L	P	I	W	Z	C	A	T	M	Z	M	Z	K	C	T	V	Z	I	L	R	G	G	Z	J	O
25	R	Q	N	Y	V	I	H	D	B	U	S	K	G	T	C	L	J	W	F	Z	O	M	A	P	E

Synoptic Table for Y

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
1	M	G	Q	X	S	Z	D	N	U	W	C	P	V	R	K	A	O	T	L	I	B	A	H	J	F
2	H	Z	R	Z	L	H	U	S	N	I	G	Z	O	S	S	Q	F	F	F	O	F	W	M	P	W
3	T	N	T	S	W	W	L	U	L	C	K	K	R	M	G	J	T	U	X	V	S	V	L	X	J
4	Q	P	V	A	E	B	C	M	P	A	R	X	P	F	U	N	H	I	N	B	J	S	S	M	L
5	K	Q	W	E	M	J	F	G	M	K	F	I	F	A	E	U	E	W	G	P	M	F	I	V	V
6	Z	X	L	D	Z	S	M	T	B	E	B	S	E	O	N	B	U	X	W	E	U	D	Q	K	G
7	O	R	A	C	V	Q	Q	D	X	L	E	R	A	D	T	T	S	H	H	S	D	L	N	B	R
8	L	W	D	B	X	F	T	X	W	B	J	D	N	W	C	G	Z	A	V	N	Q	I	J	Q	C
9	R	S	K	I	G	K	W	A	C	D	A	V	C	P	X	I	J	S	C	H	C	E	C	W	Q
10	X	B	O	F	A	V	A	I	R	F	L	E	J	K	O	M	X	D	M	J	L	B	P	U	M
11	S	A	M	G	F	M	H	H	A	J	T	W	I	J	W	W	D	M	I	W	Z	H	G	G	P
12	P	C	J	J	N	E	X	P	J	G	M	O	L	V	F	Z	P	C	R	M	W	K	B	L	S
13	W	D	U	H	Q	T	J	J	D	H	S	A	D	I	Q	R	C	N	B	D	T	N	Z	O	O
14	N	E	B	L	U	A	E	O	S	O	X	M	H	U	D	V	W	E	S	G	I	R	A	S	E
15	A	H	G	K	K	G	Z	B	K	N	V	N	B	Q	R	L	G	Q	E	F	P	J	X	T	X
16	B	F	E	M	D	P	B	W	Q	M	Q	F	M	H	L	X	Q	B	K	C	A	Q	K	E	T
17	C	I	P	R	O	O	N	K	O	T	P	L	K	Z	J	C	I	O	U	K	V	Z	W	C	K
18	E	J	H	U	P	C	I	C	I	P	N	H	G	C	Z	S	B	Z	P	A	N	G	R	H	I
19	I	K	S	O	I	I	K	V	V	R	O	Q	X	T	M	H	K	P	D	U	K	M	E	N	A
20	G	T	C	Q	T	X	P	F	T	Q	H	G	U	X	A	D	L	L	Z	T	H	X	V	Z	Z
21	D	L	Z	V	J	L	V	Z	Z	S	U	C	Z	B	P	E	N	G	Q	R	R	P	D	F	D
22	J	M	I	P	B	U	R	L	E	V	W	U	T	L	B	O	V	V	A	Z	G	U	T	R	N
23	F	O	N	T	R	R	O	Q	F	Z	D	J	S	E	V	K	A	J	T	X	O	C	U	I	B
24	V	U	X	N	H	N	G	E	H	U	I	T	W	G	H	F	R	R	O	Q	X	O	F	D	U
25	U	V	F	W	C	D	S	R	G	X	Z	B	Q	N	I	P	M	K	J	L	E	T	O	A	H

Synoptic Table for Z

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z
1	O	N	I	S	V	H	B	L	E	U	Y	K	T	C	M	R	J	P	Q	X	W	G	A	F	D
2	L	P	N	A	X	W	N	Q	F	X	C	X	S	T	A	V	X	L	A	Q	T	M	X	R	N
3	R	Q	X	E	G	B	I	E	H	Y	G	I	W	X	P	L	D	G	T	L	I	X	K	I	B
4	X	X	F	D	A	J	K	R	G	W	K	S	Q	B	B	X	P	V	O	Y	P	P	W	D	U
5	S	R	Y	C	F	S	P	Y	Y	I	R	R	Y	L	V	C	C	J	J	I	A	U	R	A	H
6	P	W	Q	B	N	Q	V	N	U	C	F	D	V	E	H	S	W	R	Y	O	V	C	E	Y	Y
7	W	S	R	I	Q	F	R	S	N	A	B	V	O	G	I	H	G	K	L	V	N	O	B	J	F
8	N	B	T	F	U	K	O	U	L	K	E	E	R	N	Y	D	Q	Y	F	B	K	T	D	P	W
9	A	A	V	G	K	V	G	M	P	E	J	W	P	Y	K	E	I	T	X	P	H	Y	T	X	J
10	B	C	W	J	D	M	S	G	M	L	A	O	F	R	S	O	B	F	N	E	R	A	U	M	L
11	C	D	L	H	O	E	Y	T	B	B	L	A	E	S	G	K	K	U	G	S	G	W	F	V	V
12	E	E	A	L	P	T	D	D	X	D	T	M	A	M	U	F	L	I	W	N	O	V	O	K	G
13	I	H	D	K	I	A	U	X	W	F	M	N	N	F	E	P	N	W	H	H	X	S	Y	B	R
14	G	F	K	M	T	G	L	A	C	J	S	F	C	A	N	Y	V	X	V	J	E	F	H	Q	C
15	D	I	O	R	J	P	C	I	R	G	X	L	J	O	T	A	A	H	C	W	Y	D	M	W	Q
16	J	J	M	U	B	O	F	H	A	H	V	H	I	D	C	Q	R	A	M	M	B	L	L	U	M
17	F	K	J	O	R	C	M	P	J	O	Q	Q	L	W	X	J	M	S	I	D	F	I	S	G	P
18	V	T	U	Q	H	I	Q	J	D	N	P	G	D	P	O	N	Y	D	R	G	S	E	I	L	S
19	U	L	B	V	C	X	T	O	S	M	N	C	H	K	W	U	O	M	B	F	J	B	Q	O	O
20	Y	M	G	P	Y	L	W	B	K	T	O	U	B	J	F	B	F	C	S	C	M	H	N	S	E
21	M	O	E	T	S	U	A	W	Q	P	H	J	M	V	Q	T	T	N	E	K	U	K	J	T	X
22	H	U	P	N	L	R	H	K	O	R	U	T	K	I	D	G	H	E	K	A	D	N	C	E	T
23	T	V	H	W	W	N	X	C	I	Q	W	B	G	U	R	I	E	Q	U	U	Q	R	P	C	K
24	Q	Y	S	Y	E	D	J	V	V	S	D	Y	X	Q	L	M	U	B	P	T	C	J	G	H	I
25	K	G	C	X	M	Y	E	F	T	V	I	P	U	H	J	W	S	O	D	R	L	Q	B	N	A

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Appendix 3

TABLES OF THE POISSON DISTRIBUTION

(Individual and cumulative terms)

TABLES OF THE POISSON DISTRIBUTION

(Individual and cumulative terms)

I. INTRODUCTION

1. The importance of the phenomena of repetitions in cryptanalysis is very well known. When the cryptanalyst has at hand a sample of encrypted text, it is often necessary or desirable to determine whether or not the observed repetitions are *causal* repetitions (e.g., produced by the action of identical keying sequences upon identical portions of plain text), or *random* repetitions, (i.e., produced purely by "accident" or "chance"). The number of random repetitions to be expected in a sample of text of a given length may be calculated, and this calculation serves as a criterion for the evaluation of the phenomena present in the sample.

2. If each of the *n* possible elements (where n=10, 26, 100, 676, etc., for single digits, single letters, dinomes, digraphs, etc.) has the same chance of occurrence, the probability, P, that an element will occur *exactly x times* (where $x=0, 1, 2 \ldots N$) in a sample of size N is given by the formula

$$P_x = C_x^N (1-p)^{N-x} p^x$$

where C_x^N has its usual meaning (i.e., the number of combinations of N things taken x at a time), and $p=\frac{1}{n}$. The Poisson Tables, so called because they are derived from the Poisson exponential formula $\frac{a^x e^{-a}}{x!}$ where $a = Np$, give an approximation to this probability that is sufficiently accurate for values of *n* when *n* is equal to or greater than 26.[1]

3. In order to use the Poisson Tables it is necessary to determine $a=Np$, i.e., the average or mean number [2] of occurrences of each of the elements in a given sample of size N. Table I (Individual Terms) gives for values of *a* the probabilities for x-fold repetitions (i.e., *exactly* 0, 1, 2 . . . N times). The expected number of x-fold repetitions is computed from the Table by multiplying the entries in the Table by *n*. This number, *n*P, is then compared with the sample under study. Table II (Cumulative Terms) gives the probabilities of repetitions of multiplicity *c or greater*, where *c* is the sum of all entries in Table I for $x \geqq c$. The expected number of repetitions occurring *c* or more times is here also computed by multiplying the entries in Table II by *n*.

4. Samples of random text may exhibit repetitions which vary considerably from the expected number. The percentage deviation may be quite large in cases where the number of available elements (letters or digits) is small. Only if the deviation is too great, in a given sample under investigation, to be due to fluctuations from sample to sample should this circumstance be attributed to causal factors. In using Table I, a practical guide that has been found useful for measuring excessive deviation is to compare the deviation from the expected with 2½ times the square root of the expected number. For example, if 72 3-fold repetitions are expected in a certain sample, then the deviation in random samples may be $\pm 2\frac{1}{2}\sqrt{72} = \pm 2\frac{1}{2}(8.5) = \pm 21$, and therefore the observed number will usually lie between 51 and 93. Only if the observed number is outside these limits should the repetitions be considered as manifestations causal to the nature of the cryptographic system.

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[1] For cases where n is considerably less than 26, the Poisson distribution becomes increasingly inaccurate and therefore the Binomial distribution should be used. Appendix 4, "Table of the Binomial distribution for p=1/10," for ranges of N between 10 and 1200, gives accurate probability statements for digital cases.

[2] The average number is usually represented by a small *a* but, for convenience in printing, the tables have this element denoted by a capital A.

5. The Table below will assist the analyst in interpreting the deviations from the expected. The statistical quantity called "standard deviation" (represented by σ) is in this case approximated by the square root of the expected number of x-fold repetitions.

Expected number plus 1 σ will be equalled or exceeded once every	6.3	times
Expected number plus $1\frac{1}{2}\sigma$ will be equalled or exceeded once every	15	times
Expected number plus 2 σ will be equalled or exceeded once every	44	times
Expected number plus $2\frac{1}{2}\sigma$ will be equalled or exceeded once every	161	times
Expected number plus 3 σ will be equalled or exceeded once every	740	times
Expected number plus $3\frac{1}{2}\sigma$ will be equalled or exceeded once every	4, 310	times
Expected number plus 4 σ will be equalled or exceeded once every	31, 500	times
Expected number plus $4\frac{1}{2}\sigma$ will be equalled or exceeded once every	294, 000	times
Expected number plus 5 σ will be equalled or exceeded once every	3, 480, 000	times

II. EXAMPLES OF USE OF THE TABLES

1. Two samples of 40 letters each are under study, and the x-fold repetitions of single letters are recorded under A and B respectively in the diagram below. The expected numbers of x-fold repetitions have been entered in the last column, and were obtained thus: since $n=26$ and $N=40$, then $a=\frac{40}{26}=1.54$. The entries under 1.5 (the value for a in the Table closest to 1.54) in Table I were then multiplied by 26.

x	Sample A	Sample B	Expected
0	5	9	5. 8
1	8	8	8. 7
2	7	2	6. 5
3	6	5	3. 3
4	–	–	1. 2
5	–	–	. 37
6	–	1	. 09
7	–	1	. 02

The interpretation of the meaning of the decimal fractions in the last column may be given as follows. Out of 100 samples of random text of size N=40, 37 of them would have one 5-fold repetition, 9 would have one 6-fold repetition, and 2 would have one 7-fold repetition.

Variation from the expected is usual even for random text. In sample A six letters occur 3 times each where only three letters are expected to occur 3 times each. However, there are no occurrences of 4-fold or 5-fold repetitions, although there is a slight expectancy of their occurrence in these cases; this deviation therefore is not sufficient to imply that the repetitions in sample A are other than accidental. In sample B the variations from the expected number are considerable. Note the comparatively great number of blanks (0-fold repetitions) which would rarely occur in a sample of random text; but of much greater significance is the occurrence of 6- and 7-fold repetitions, which have a combined expectancy of only .11. From these phenomena it may be safely inferred that sample B is definitely not a random assortment of letters. Actually, sample A was enciphered with 25 non-related alphabets, whereas sample B is a monoalphabetic substitution cipher.

The individual variations of the samples from the expected are "smoothed" by a consideration of the distribution of the number of letters occurring *x or more times* as is given in the diagram below. Samples A and B are the cumulative data of the two samples, while the last column is calculated from Table II by multiplying the entries under $a=1.5$ by 26.

c	Sample A	Sample B	Expected Cumulative Data
0	26	26	26
1	21	17	20
2	13	9	11.5
3	6	7	4.97
4		2	1.71
5		2	.48
6		2	.12
7		1	.02

2. In another sample of cipher text consisting of 116 digraphs the observed and calculated data for individual and cumulative distributions are as follows:

	Table I			Table II	
x	Observed	Calculated	c	Observed	Calculated
0	606	570	0	676	676
1	34	97	1	70	106
2	27	8.2	2	36	8.7
3	8	.47	3	9	.49
4	1	.02	4	1	.02

Since $n=676$ and $N=116$, then $a=\frac{116}{676}=.17$; and the values under .17 in the tables are multiplied by 676 to derive the calculated expectancies. A few moments' study of the foregoing data shows that the manifestations in this sample must be due to causal factors, since the respective numbers of 2-fold, 3-fold and 4-fold repetitions in the sample far exceed those which could reasonably be attributed to chance. The system used in this cryptogram was actually a Playfair cipher.

3. A cipher message of 209 dinomes has the following distribution of x-fold repetitions. Given also is the calculated number of x-fold random repetitions to be expected in cases where all dinomes have an equal probability of occurrence. The calculated numbers were obtained by multiplying by 100 the entries in Table I and Table II for the value of $a=\frac{209}{100}$=approximately 2.1.

	Table I			Table II	
x	Observed	Calculated	c	Observed	Calculated
0	30	12	0	100	100
1	24	26	1	70	88
2	12	27	2	46	62
3	11	19	3	34	35
4	9	10	4	23	16
5	2	4	5	14	6
6	7	1	6	12	2
7	3	0.4	7	5	0.6
8	–	.1	8	2	.1
9	1	.03	9	2	.03
10	1	.006	10	1	.007

A comparison of the observed data with the expected data for non-causal repetitions leaves little doubt that the repetitions are a causal by-product of the nature of the cryptographic system involved. The cryptogram was actually produced by a variant system in which each plaintext letter has four dinome cipher equivalents.

4. The following tabulation gives the distributions of x-fold repetitions found in three samples, A, B, and C, of four-letter code text, each containing 275,000 groups. The calculated expectancies shown in the second column were derived from Table I, using the value $a=\frac{275,000}{26^4}=.6$

x	Expected	Sample A	Sample B	Sample C
0	250, 793	249, 992	249, 481	250, 122
1	150, 476	151, 348	152, 344	152, 343
2	45, 143	44, 987	44, 685	44, 312
3	9, 028	9, 121	8, 825	8, 131
4	1, 354	1, 342	1, 427	1, 305
5	163	171	188	395
6	16. 5	13	21	201
7	1. 4	2	3	123
8			2	26
9				12
10				4
11				2

The deviation of the entries in sample A from the expected could be due to chance, and therefore there is nothing in the number of repetitions to confirm a hypothesis that the sample is other than a sample of random 4-letter groups. It is also quite clear from an inspection of the foregoing data that sample C could not have occurred by chance, hence sample C may easily be vulnerable to cryptanalytic attack. However, in the case of sample B, although there are small deviations from the expected, it should be noted that these deviations, some plus and some minus, reflect the characteristics of deviations usual for samples of non-random text (cf. sample C). Furthermore, these deviations are less than our estimated norm of $2\frac{1}{2}\sigma$, except for the first two entries.

x	Deviations from the Expected in Sample B	$2\frac{1}{2}\sigma$
0	−1, 312	1, 252
1	1, 868	970
2	−458	531
3	−203	237
4	73	92
5	25	32
6	4. 4	10. 2
7	1. 6	2. 9

However, when examined critically, by other mathematical tests[3] it may be shown that the set of such deviations could have come from a random sample in less than 1 chance in 10,000; therefore sample B should undergo further scrutiny to reveal possible weaknesses in other than repetitive phenomena. Actually, sample A is an extremely complex form of enciphered code and sample B is code text subjected to a fairly complex encipherment; whereas sample C is typical of a simple encipherment superimposed upon a code of rather large proportions and condensing power.

[3] The chi-square test.

TABLE I

X	A=.001	A=.002	A=.003	A=.004	A=.005	A=.006	A=.007	A=.008	X
0	.9990005	.9980020	.9970045	.9960080	.9950125	.9940180	.9930244	.9920319	0
1	.0009990	.0019960	.0029910	.0039840	.0049751	.0059641	.0069512	.0079363	1
2	.0000005	.0000020	.0000045	.0000080	.0000124	.0000179	.0000243	.0000317	2
3							.0000001	.0000001	3

X	A=.009	A=.010	A=.02	A=.03	A=.04	A=.05	A=.06	A=.07	X
0	.9910404	.9900498	.9801987	.9704455	.9607894	.9512294	.9417645	.9323938	0
1	.0089194	.0099005	.0196040	.0291134	.0384316	.0475615	.0565059	.0652676	1
2	.0000401	.0000495	.0001960	.0004367	.0007686	.0011890	.0016952	.0022844	2
3	.0000001	.0000002	.0000013	.0000044	.0000102	.0000198	.0000339	.0000533	3
4					.0000001	.0000002	.0000005	.0000009	4

X	A=.08	A=.09	A=.10	A=.11	A=.12	A=.13	A=.14	A=.15	X
0	.9231163	.9139312	.9048374	.8958341	.8869204	.8780954	.8693582	.8607080	0
1	.0738493	.0822538	.0904837	.0985418	.1064304	.1141524	.1217102	.1291062	1
2	.0029540	.0037014	.0045242	.0054198	.0063858	.0074199	.0085197	.0096830	2
3	.0000788	.0001110	.0001508	.0001987	.0002554	.0003215	.0003976	.0004841	3
4	.0000016	.0000025	.0000038	.0000055	.0000077	.0000104	.0000139	.0000182	4
5			.0000001	.0000001	.0000002	.0000003	.0000004	.0000005	5

X	A=.16	A=.17	A=.18	A=.19	A=.20	A=.21	A=.22	A=.23	X
0	.8521438	.8436648	.8352702	.8269591	.8187308	.8105842	.8025188	.7945336	0
1	.1363430	.1434230	.1503486	.1571222	.1637462	.1702227	.1765541	.1827427	1
2	.0109074	.0121910	.0135314	.0149266	.0163746	.0178734	.0194210	.0210154	2
3	.0005817	.0006908	.0008119	.0009454	.0010916	.0012511	.0014242	.0016112	3
4	.0000233	.0000294	.0000365	.0000449	.0000546	.0000657	.0000783	.0000926	4
5	.0000007	.0000010	.0000013	.0000017	.0000022	.0000028	.0000034	.0000043	5
6				.0000001	.0000001	.0000001	.0000001	.0000002	6

X	A=.24	A=.25	A=.26	A=.27	A=.28	A=.29	A=.30	A=.40	X
0	.7866279	.7788008	.7710516	.7633795	.7557837	.7482636	.7408182	.6703200	0
1	.1887907	.1947002	.2004734	.2061125	.2116194	.2169964	.2222455	.2681280	1
2	.0226549	.0243375	.0260615	.0278252	.0296267	.0314645	.0333368	.0536256	2
3	.0018124	.0020281	.0022587	.0025043	.0027652	.0030416	.0033337	.0071501	3
4	.0001087	.0001268	.0001468	.0001690	.0001936	.0002205	.0002500	.0007150	4
5	.0000052	.0000063	.0000076	.0000091	.0000108	.0000128	.0000150	.0000572	5
6	.0000002	.0000003	.0000003	.0000004	.0000005	.0000006	.0000008	.0000038	6
7								.0000002	7

X	A=.5	A=.6	A=.7	A=.8	A=.9	A=1.0	A=1.1	A=1.2	X
0	.606531	.548812	.496585	.449329	.406570	.367879	.332871	.301194	0
1	.303265	.329287	.347610	.359463	.365913	.367879	.366158	.361433	1
2	.075816	.098786	.121663	.143785	.164661	.183940	.201387	.216860	2
3	.012636	.019757	.028388	.038343	.049398	.061313	.073842	.086744	3
4	.001580	.002964	.004968	.007669	.011115	.015328	.020307	.026023	4
5	.000158	.000356	.000696	.001227	.002001	.003066	.004467	.006246	5
6	.000013	.000036	.000081	.000164	.000300	.000511	.000819	.001249	6
7	.000001	.000003	.000008	.000019	.000039	.000073	.000129	.000214	7
8			.000001	.000002	.000004	.000009	.000018	.000032	8
9						.000001	.000002	.000004	9
10								.000001	10

X	A=1.3	A=1.4	A=1.5	A=1.6	A=1.7	A=1.8	A=1.9	A=2.0	X
0	.272532	.246597	.223130	.201897	.182684	.165299	.149569	.135335	0
1	.354291	.345236	.334695	.323034	.310562	.297538	.284180	.270671	1
2	.230289	.241665	.251021	.258428	.263978	.267784	.269971	.270671	2
3	.099792	.112777	.125511	.137828	.149587	.160671	.170982	.180447	3
4	.032432	.039472	.047067	.055131	.063575	.072302	.081216	.090224	4
5	.008432	.011052	.014120	.017642	.021615	.026029	.030862	.036089	5
6	.001827	.002579	.003530	.004705	.006124	.007809	.009773	.012030	6
7	.000339	.000516	.000756	.001075	.001487	.002008	.002653	.003437	7
8	.000055	.000090	.000142	.000215	.000316	.000452	.000630	.000859	8
9	.000008	.000014	.000024	.000038	.000060	.000090	.000133	.000191	9

TABLE I

X	A=1.3	A=1.4	A=1.5	A=1.6	A=1.7	A=1.8	A=1.9	A=2.0	X
10	.000001	.000002	.000004	.000006	.000010	.000016	.000025	.000038	10
11				.000001	.000002	.000003	.000004	.000007	11
12							.000001	.000001	12

X	A=2.1	A=2.2	A=2.3	A=2.4	A=2.5	A=2.6	A=2.7	A=2.8	X
0	.122456	.110803	.100259	.090718	.082085	.074274	.067206	.060810	0
1	.257159	.243767	.230595	.217723	.205212	.193111	.181455	.170268	1
2	.270016	.268144	.265185	.261268	.256516	.251045	.244964	.238375	2
3	.189012	.196639	.203308	.209014	.213763	.217572	.220468	.222484	3
4	.099231	.108151	.116902	.125409	.133602	.141422	.148816	.155739	4
5	.041677	.047587	.053775	.060196	.066801	.073539	.080360	.087214	5
6	.014587	.017448	.020614	.024078	.027834	.031867	.036162	.040700	6
7	.004376	.005484	.006773	.008255	.009941	.011836	.013948	.016280	7
8	.001149	.001508	.001947	.002477	.003106	.003847	.004708	.005698	8
9	.000268	.000369	.000498	.000660	.000863	.001111	.001412	.001773	9
10	.000056	.000081	.000114	.000159	.000216	.000289	.000381	.000496	10
11	.000011	.000016	.000024	.000035	.000049	.000068	.000094	.000126	11
12	.000002	.000003	.000005	.000007	.000010	.000015	.000021	.000029	12
13		.000001	.000001	.000001	.000002	.000003	.000004	.000006	13
14						.000001	.000001	.000001	14

X	A=2.9	A=3.0	A=3.1	A=3.2	A=3.3	A=3.4	A=3.5	A=3.6	X
0	.055023	.049787	.045049	.040762	.036883	.033373	.030197	.027324	0
1	.159567	.149361	.139653	.130439	.121714	.113469	.105691	.098365	1
2	.231373	.224042	.216461	.208702	.200829	.192898	.184959	.177058	2
3	.223660	.224042	.223677	.222616	.220912	.218617	.215785	.212469	3
4	.162154	.168031	.173350	.178093	.182252	.185825	.188812	.191222	4
5	.094049	.100819	.107477	.113979	.120286	.126361	.132169	.137680	5
6	.045457	.050409	.055530	.060789	.066158	.071604	.077098	.082608	6
7	.018832	.021604	.024592	.027789	.031189	.034779	.038549	.042484	7
8	.006827	.008102	.009529	.011116	.012865	.014781	.016865	.019118	8
9	.002200	.002701	.003282	.003952	.004717	.005584	.006559	.007647	9
10	.000638	.000810	.001018	.001265	.001557	.001899	.002296	.002753	10
11	.000168	.000221	.000287	.000368	.000467	.000587	.000730	.000901	11
12	.000041	.000055	.000074	.000098	.000128	.000166	.000213	.000270	12
13	.000009	.000013	.000018	.000024	.000033	.000043	.000057	.000075	13
14	.000002	.000003	.000004	.000006	.000008	.000011	.000014	.000019	14
15		.000001	.000001	.000001	.000002	.000002	.000003	.000005	15
16						.000001	.000001	.000001	16

X	A=3.7	A=3.8	A=3.9	A=4.0	A=4.1	A=4.2	A=4.3	A=4.4	X
0	.024724	.022371	.020242	.018316	.016573	.014996	.013569	.012277	0
1	.091477	.085009	.078943	.073263	.067948	.062981	.058345	.054020	1
2	.169233	.161517	.153940	.146525	.139293	.132261	.125441	.118845	2
3	.208720	.204588	.200122	.195367	.190368	.185165	.179799	.174305	3
4	.193066	.194359	.195119	.195367	.195127	.194424	.193284	.191736	4
5	.142869	.147713	.152193	.156293	.160004	.163316	.166224	.168728	5
6	.088103	.093551	.098925	.104196	.109336	.114321	.119127	.123734	6
7	.046568	.050785	.055115	.059540	.064040	.068593	.073178	.077775	7
8	.021538	.024123	.026869	.029770	.032820	.036011	.039333	.042776	8
9	.008854	.010185	.011643	.013231	.014951	.016805	.018793	.020913	9
10	.003276	.003870	.004541	.005292	.006130	.007058	.008081	.009202	10
11	.001102	.001337	.001610	.001925	.002285	.002695	.003159	.003681	11
12	.000340	.000423	.000523	.000642	.000781	.000943	.001132	.001350	12
13	.000097	.000124	.000157	.000197	.000246	.000305	.000374	.000457	13
14	.000026	.000034	.000044	.000056	.000072	.000091	.000115	.000144	14
15	.000006	.000009	.000011	.000015	.000020	.000026	.000033	.000042	15
16	.000001	.000002	.000003	.000004	.000005	.000007	.000009	.000012	16
17			.000001	.000001	.000001	.000002	.000002	.000003	17
18							.000001	.000001	18

X	A=4.5	A=4.6	A=4.7	A=4.8	A=4.9	A=5.0	A=5.1	A=5.2	X
0	.011109	.010052	.009095	.008230	.007447	.006738	.006097	.005517	0
1	.049990	.046238	.042748	.039503	.036488	.033690	.031093	.028686	1
2	.112479	.106348	.100457	.094807	.089396	.084224	.079288	.074584	2

TABLE I

X	A=4.5	A=4.6	A=4.7	A=4.8	A=4.9	A=5.0	A=5.1	A=5.2	X
3	.168718	.163068	.157383	.151691	.146014	.140374	.134790	.129279	3
4	.189808	.187528	.184925	.182029	.178867	.175467	.171857	.168063	4
5	.170827	.172526	.173830	.174748	.175290	.175467	.175294	.174785	5
6	.128120	.132270	.136167	.139798	.143153	.146223	.149000	.151480	6
7	.082363	.086920	.091426	.095862	.100207	.104445	.108557	.112528	7
8	.046329	.049979	.053713	.057517	.061377	.065278	.069205	.073143	8
9	.023165	.025545	.028050	.030676	.033416	.036266	.039216	.042261	9
10	.010424	.011751	.013184	.014724	.016374	.018133	.020000	.021976	10
11	.004264	.004914	.005633	.006425	.007294	.008242	.009273	.010388	11
12	.001599	.001884	.002206	.002570	.002978	.003434	.003941	.004502	12
13	.000554	.000667	.000798	.000949	.001123	.001321	.001546	.001801	13
14	.000178	.000219	.000268	.000325	.000393	.000472	.000563	.000669	14
15	.000053	.000067	.000084	.000104	.000128	.000157	.000191	.000232	15
16	.000015	.000019	.000025	.000031	.000039	.000049	.000061	.000075	16
17	.000004	.000005	.000007	.000009	.000011	.000014	.000018	.000023	17
18	.000001	.000001	.000002	.000002	.000003	.000004	.000005	.000007	18
19				.000001	.000001	.000001	.000001	.000002	19

X	A=5.3	A=5.4	A=5.5	A=5.6	A=5.7	A=5.8	A=5.9	A=6.0	X
0	.004992	.004517	.004087	.003698	.003346	.003028	.002739	.002479	0
1	.026455	.024390	.022477	.020708	.019072	.017560	.016163	.014873	1
2	.070107	.065852	.061812	.057983	.054355	.050923	.047680	.044618	2
3	.123856	.118533	.113323	.108234	.103275	.098452	.093771	.089235	3
4	.164109	.160020	.155819	.151528	.147167	.142755	.138312	.133853	4
5	.173955	.172821	.171401	.169711	.167770	.165596	.163208	.160623	5
6	.153660	.155539	.157117	.158397	.159382	.160076	.160488	.160623	6
7	.116343	.119987	.123449	.126717	.129782	.132635	.135268	.137677	7
8	.077077	.080991	.084871	.088702	.092470	.096160	.099760	.103258	8
9	.045390	.048595	.051866	.055193	.058564	.061970	.065398	.068838	9
10	.024057	.026241	.028526	.030908	.033382	.035943	.038585	.041303	10
11	.011591	.012882	.014263	.015735	.017298	.018952	.020696	.022529	11
12	.005119	.005797	.006537	.007343	.008216	.009160	.010175	.011264	12
13	.002087	.002408	.002766	.003163	.003603	.004087	.004618	.005199	13
14	.000790	.000929	.001087	.001265	.001467	.001693	.001946	.002228	14
15	.000279	.000334	.000398	.000472	.000557	.000655	.000765	.000891	15
16	.000092	.000113	.000137	.000165	.000199	.000237	.000282	.000334	16
17	.000029	.000036	.000044	.000054	.000067	.000081	.000098	.000118	17
18	.000008	.000011	.000014	.000017	.000021	.000026	.000032	.000039	18
19	.000002	.000003	.000004	.000005	.000006	.000008	.000010	.000012	19
20	.000001	.000001	.000001	.000001	.000002	.000002	.000003	.000004	20
21						.000001	.000001	.000001	21

X	A=6.1	A=6.2	A=6.3	A=6.4	A=6.5	A=6.6	A=6.7	A=6.8	X
0	.002243	.002029	.001836	.001662	.001503	.001360	.001231	.001114	0
1	.013681	.012582	.011569	.010634	.009772	.008978	.008247	.007574	1
2	.041729	.039006	.036441	.034029	.031760	.029629	.027628	.025750	2
3	.084848	.080612	.076527	.072595	.068814	.065183	.061702	.058368	3
4	.129393	.124948	.120530	.116151	.111822	.107553	.103351	.099225	4
5	.157860	.154936	.151868	.148674	.145369	.141969	.138490	.134946	5
6	.160491	.160100	.159461	.158585	.157483	.156166	.154648	.152939	6
7	.139856	.141803	.143515	.144992	.146234	.147243	.148020	.148569	7
8	.106640	.109897	.113018	.115994	.118815	.121475	.123967	.126284	8
9	.072279	.075707	.079113	.082484	.085811	.089082	.092286	.095415	9
10	.044090	.046938	.049841	.052790	.055777	.058794	.061832	.064882	10
11	.024450	.026456	.028545	.030714	.032959	.035276	.037661	.040109	11
12	.012429	.013669	.014986	.016381	.017853	.019402	.021028	.022728	12
13	.005832	.006519	.007263	.008064	.008926	.009850	.010837	.011889	13
14	.002541	.002887	.003268	.003687	.004144	.004644	.005186	.005774	14
15	.001033	.001193	.001373	.001573	.001796	.002043	.002317	.002618	15
16	.000394	.000462	.000540	.000629	.000730	.000843	.000970	.001113	16
17	.000141	.000169	.000200	.000237	.000279	.000327	.000382	.000445	17
18	.000048	.000058	.000070	.000084	.000101	.000120	.000142	.000168	18
19	.000015	.000019	.000023	.000028	.000034	.000042	.000050	.000060	19
20	.000005	.000006	.000007	.000009	.000011	.000014	.000017	.000020	20
21	.000001	.000002	.000002	.000003	.000003	.000004	.000005	.000007	21
22			.000001	.000001	.000001	.000001	.000002	.000002	22

TABLE I

X	A=6.1	A=6.2	A=6.3	A=6.4	A=6.5	A=6.6	A=6.7	A=6.8	X
23								.000001	23

X	A=6.9	A=7.0	A=7.1	A=7.2	A=7.3	A=7.4	A=7.5	A=7.6	X
0	.001008	.000912	.000825	.000747	.000676	.000611	.000553	.000500	0
1	.006954	.006383	.005858	.005375	.004931	.004523	.004148	.003803	1
2	.023990	.022341	.020797	.019352	.018000	.016736	.015555	.014453	2
3	.055178	.052129	.049219	.046444	.043799	.041282	.038889	.036614	3
4	.095182	.091226	.087364	.083598	.079934	.076372	.072916	.069567	4
5	.131351	.127717	.124057	.120382	.116703	.113031	.109375	.105742	5
6	.151053	.149003	.146800	.144458	.141989	.139405	.136718	.133940	6
7	.148895	.149003	.148897	.148586	.148074	.147371	.146484	.145421	7
8	.128422	.130377	.132146	.133727	.135118	.136318	.137329	.138150	8
9	.098457	.101405	.104249	.106982	.109596	.112084	.114440	.116660	9
10	.067935	.070983	.074017	.077027	.080005	.082942	.085830	.088661	10
11	.042614	.045171	.047774	.050418	.053094	.055797	.058521	.061257	11
12	.024503	.026350	.028267	.030251	.032299	.034408	.036575	.038796	12
13	.013005	.014188	.015438	.016754	.018137	.019586	.021101	.022681	13
14	.006410	.007094	.007829	.008616	.009457	.010353	.011304	.012312	14
15	.002949	.003311	.003706	.004136	.004602	.005107	.005652	.006238	15
16	.001272	.001448	.001644	.001861	.002100	.002362	.002649	.002963	16
17	.000516	.000596	.000687	.000788	.000902	.001028	.001169	.001325	17
18	.000198	.000232	.000271	.000315	.000366	.000423	.000487	.000559	18
19	.000072	.000085	.000101	.000119	.000141	.000165	.000192	.000224	19
20	.000025	.000030	.000036	.000043	.000051	.000061	.000072	.000085	20
21	.000008	.000010	.000012	.000015	.000018	.000021	.000026	.000031	21
22	.000003	.000003	.000004	.000005	.000006	.000007	.000009	.000011	22
23	.000001	.000001	.000001	.000002	.000002	.000002	.000003	.000004	23
24					.000001	.000001	.000001	.000001	24

X	A=7.7	A=7.8	A=7.9	A=8.0	A=8.1	A=8.2	A=8.3	A=8.4	X
0	.000453	.000410	.000371	.000335	.000304	.000275	.000249	.000225	0
1	.003487	.003196	.002929	.002684	.002459	.002252	.002063	.001889	1
2	.013424	.012464	.011569	.010735	.009958	.009234	.008560	.007933	2
3	.034455	.032407	.030465	.028626	.026886	.025239	.023683	.022213	3
4	.066326	.063193	.060169	.057252	.054443	.051740	.049142	.046648	4
5	.102142	.098581	.095067	.091604	.088198	.084854	.081576	.078369	5
6	.131082	.128156	.125171	.122138	.119067	.115967	.112847	.109716	6
7	.144191	.142802	.141264	.139587	.137778	.135848	.133805	.131659	7
8	.138783	.139232	.139499	.139587	.139500	.139244	.138823	.138242	8
9	.118737	.120668	.122449	.124077	.125550	.126866	.128025	.129026	9
10	.091427	.094121	.096735	.099262	.101696	.104031	.106261	.108382	10
11	.063999	.066740	.069473	.072190	.074885	.077550	.080179	.082764	11
12	.041066	.043381	.045736	.048127	.050547	.052993	.055457	.058035	12
13	.024324	.026029	.027794	.029616	.031495	.033426	.035407	.037435	13
14	.013378	.014502	.015684	.016924	.018222	.019578	.020991	.022461	14
15	.006867	.007541	.008260	.009026	.009840	.010703	.011615	.012578	15
16	.003305	.003676	.004078	.004513	.004981	.005485	.006025	.006604	16
17	.001497	.001687	.001895	.002124	.002374	.002646	.002942	.003263	17
18	.000640	.000731	.000832	.000944	.001068	.001205	.001357	.001523	18
19	.000260	.000300	.000346	.000397	.000455	.000520	.000593	.000673	19
20	.000100	.000117	.000137	.000159	.000184	.000213	.000246	.000283	20
21	.000037	.000043	.000051	.000061	.000071	.000083	.000097	.000113	21
22	.000013	.000015	.000018	.000022	.000026	.000031	.000037	.000043	22
23	.000004	.000005	.000006	.000008	.000009	.000011	.000013	.000016	23
24	.000001	.000002	.000002	.000003	.000003	.000004	.000005	.000006	24
25		.000001	.000001	.000001	.000001	.000001	.000002	.000002	25
26								.000001	26

X	A=8.5	A=8.6	A=8.7	A=8.8	A=8.9	A=9.0	A=9.1	A=9.2	X
0	.000203	.000184	.000167	.000151	.000136	.000123	.000112	.000101	0
1	.001729	.001583	.001449	.001326	.001214	.001111	.001016	.000930	1
2	.007350	.006808	.006304	.005836	.005402	.004998	.004624	.004276	2
3	.020826	.019517	.018283	.017120	.016025	.014994	.014025	.013113	3
4	.044255	.041961	.039765	.037664	.035656	.033737	.031906	.030160	4
5	.075233	.072174	.069192	.066289	.063467	.060727	.058069	.055494	5

TABLE I

X	A=8.5	A=8.6	A=8.7	A=8.8	A=8.9	A=9.0	A=9.1	A=9.2	X
6	.106581	.103449	.100328	.097224	.094143	.091090	.088072	.085091	6
7	.129419	.127094	.124693	.122224	.119696	.117116	.114493	.111834	7
8	.137508	.136626	.135604	.134446	.133161	.131756	.130236	.128609	8
9	.129869	.130554	.131084	.131459	.131682	.131756	.131683	.131467	9
10	.110388	.112277	.114043	.115684	.117197	.118580	.119832	.120950	10
11	.085300	.087780	.090197	.092547	.094823	.097020	.099133	.101158	11
12	.060421	.062909	.065393	.067868	.070327	.072765	.075176	.077555	12
13	.039506	.041617	.043763	.045941	.048147	.050376	.052623	.054885	13
14	.023986	.025565	.027196	.028877	.030608	.032384	.034205	.036067	14
15	.013592	.014657	.015773	.016941	.018161	.019431	.020751	.022121	15
16	.007221	.007878	.008577	.009318	.010102	.010930	.011802	.012720	16
17	.003610	.003985	.004389	.004823	.005289	.005786	.006318	.006984	17
18	.001705	.001904	.002122	.002358	.002615	.002893	.003194	.003518	18
19	.000763	.000862	.000971	.001092	.001225	.001370	.001530	.001704	19
20	.000324	.000371	.000423	.000481	.000545	.000617	.000696	.000784	20
21	.000131	.000152	.000175	.000201	.000231	.000264	.000302	.000343	21
22	.000051	.000059	.000069	.000081	.000093	.000108	.000125	.000144	22
23	.000019	.000022	.000026	.000031	.000036	.000042	.000049	.000057	23
24	.000007	.000008	.000009	.000011	.000013	.000016	.000019	.000022	24
25	.000002	.000003	.000003	.000004	.000005	.000006	.000007	.000008	25
26	.000001	.000001	.000001	.000001	.000002	.000002	.000002	.000003	26
27					.000001	.000001	.000001	.000001	27

X	A=9.3	A=9.4	A=9.5	A=9.6	A=9.7	A=9.8	A=9.9	A=10.0	X
0	.000091	.000083	.000075	.000068	.000061	.000055	.000050	.000045	0
1	.000850	.000778	.000711	.000650	.000594	.000543	.000497	.000454	1
2	.003954	.003655	.003378	.003121	.002883	.002663	.002459	.002270	2
3	.012256	.011452	.010696	.009987	.009322	.008698	.008114	.007567	3
4	.028496	.026911	.025403	.023969	.022606	.021311	.020082	.018917	4
5	.053002	.050593	.048266	.046020	.043855	.041770	.039763	.037833	5
6	.082154	.079262	.076421	.073632	.070899	.068224	.065609	.063055	6
7	.109147	.106438	.103714	.100981	.098246	.095514	.092790	.090079	7
8	.126883	.125065	.123160	.121178	.119123	.117004	.114827	.112599	8
9	.131113	.130623	.130003	.129256	.128388	.127405	.126310	.125110	9
10	.121935	.122786	.123502	.124086	.124537	.124857	.125047	.125110	10
11	.103090	.104926	.106661	.108293	.109819	.111236	.112542	.113736	11
12	.079895	.082192	.084440	.086634	.088770	.090843	.092847	.094780	12
13	.057156	.059431	.061706	.063976	.066236	.068481	.070707	.072908	13
14	.037968	.039904	.041872	.043869	.045892	.047937	.050000	.052077	14
15	.023540	.025006	.026519	.028076	.029677	.031319	.033000	.034718	15
16	.013683	.014691	.015746	.016846	.017992	.019183	.020410	.021699	16
17	.007485	.008123	.008799	.009513	.010266	.011058	.011891	.012764	17
18	.003867	.004242	.004644	.005074	.005532	.006021	.006540	.007091	18
19	.001893	.002099	.002322	.002563	.002824	.003105	.003408	.003732	19
20	.000880	.000986	.001103	.001230	.001370	.001522	.001687	.001866	20
21	.000390	.000442	.000499	.000563	.000633	.000710	.000795	.000889	21
22	.000165	.000189	.000215	.000245	.000279	.000316	.000358	.000404	22
23	.000067	.000077	.000089	.000102	.000118	.000135	.000154	.000176	23
24	.000026	.000030	.000035	.000041	.000048	.000055	.000064	.000073	24
25	.000010	.000011	.000013	.000016	.000018	.000022	.000025	.000029	25
26	.000003	.000004	.000005	.000006	.000007	.000008	.000010	.000011	26
27	.000001	.000001	.000002	.000002	.000002	.000003	.000004	.000004	27
28			.000001	.000001	.000001	.000001	.000001	.000001	28
29								.000001	29

X	A=10.1	A=10.2	A=10.3	A=10.4	A=10.5	A=10.6	A=10.7	A=10.8	X
0	.000041	.000037	.000034	.000030	.000028	.000025	.000023	.000020	0
1	.000415	.000379	.000346	.000317	.000289	.000264	.000241	.000220	1
2	.002095	.001934	.001784	.001646	.001518	.001400	.001291	.001190	2
3	.007054	.006574	.006125	.005705	.005313	.004946	.004603	.004283	3
4	.017811	.016764	.015773	.014834	.013946	.013107	.012313	.011564	4
5	.035979	.034199	.032492	.030855	.029287	.027786	.026350	.024978	5
6	.060565	.058139	.055777	.053482	.051252	.049089	.046991	.044960	6
7	.087387	.084716	.082072	.079458	.076878	.074334	.071830	.069367	7
8	.110326	.108013	.105668	.103296	.100902	.098493	.096072	.093646	8
9	.123810	.122415	.120931	.119364	.117720	.116003	.114219	.112375	9
10	.125048	.124863	.124559	.124139	.123606	.122963	.122215	.121365	10

TABLE I

X	A=10.1	A=10.2	A=10.3	A=10.4	A=10.5	A=10.6	A=10.7	A=10.8	X
11	.114817	.115782	.116633	.117368	.117987	.118492	.118882	.119159	11
12	.096637	.098415	.100110	.101719	.103239	.104668	.106003	.107243	12
13	.075080	.077218	.079318	.081375	.083385	.085344	.087248	.089094	13
14	.054165	.056259	.058355	.060450	.062539	.064618	.066683	.068730	14
15	.036471	.038256	.040071	.041912	.043777	.045663	.047567	.049485	15
16	.023022	.024388	.025795	.027243	.028729	.030252	.031810	.033403	16
17	.013678	.014633	.015629	.016666	.017744	.018863	.020022	.021220	17
18	.007675	.008292	.008943	.009629	.010351	.011108	.011902	.012732	18
19	.004080	.004451	.004848	.005271	.005720	.006197	.006703	.007237	19
20	.002060	.002270	.002497	.002741	.003003	.003285	.003586	.003908	20
21	.000991	.001103	.001225	.001357	.001502	.001658	.001827	.002010	21
22	.000455	.000511	.000573	.000642	.000717	.000799	.000889	.000987	22
23	.000200	.000227	.000257	.000290	.000327	.000368	.000413	.000463	23
24	.000084	.000096	.000110	.000126	.000143	.000163	.000184	.000208	24
25	.000034	.000039	.000045	.000052	.000060	.000069	.000079	.000090	25
26	.000013	.000015	.000018	.000021	.000024	.000028	.000032	.000037	26
27	.000005	.000006	.000007	.000008	.000009	.000011	.000013	.000015	27
28	.000002	.000002	.000003	.000003	.000004	.000004	.000005	.000006	28
29	.000001	.000001	.000001	.000001	.000001	.000002	.000002	.000002	29
30						.000001	.000001	.000001	30

X	A=10.9	A=11.0	A=11.1	A=11.2	A=11.3	A=11.4	A=11.5	A=11.6	X
0	.000018	.000017	.000015	.000014	.000012	.000011	.000010	.000009	0
1	.000201	.000184	.000168	.000153	.000140	.000128	.000117	.000106	1
2	.001097	.001010	.000931	.000858	.000790	.000727	.000670	.000617	2
3	.003984	.003705	.003445	.003202	.002975	.002764	.002568	.002385	3
4	.010856	.010189	.009559	.008965	.008406	.007879	.007382	.006915	4
5	.023667	.022415	.021221	.020082	.018997	.017963	.016979	.016043	5
6	.042995	.041095	.039259	.037487	.035778	.034130	.032544	.031017	6
7	.066949	.064577	.062253	.059979	.057755	.055584	.053465	.051400	7
8	.091218	.088794	.086376	.083970	.081579	.079207	.076856	.074529	8
9	.110475	.108526	.106531	.104496	.102427	.100328	.098204	.096060	9
10	.120418	.119378	.118249	.117036	.115743	.114374	.112935	.111430	10
11	.119323	.119378	.119324	.119164	.118899	.118533	.118068	.117508	11
12	.108385	.109430	.110375	.111220	.111964	.112607	.113149	.113591	12
13	.090877	.092595	.094243	.095820	.097322	.098747	.100093	.101358	13
14	.070754	.072753	.074721	.076656	.078553	.080409	.082220	.083982	14
15	.051415	.053352	.055294	.057236	.059177	.061111	.063035	.064946	15
16	.035026	.036680	.038360	.040065	.041793	.043541	.045306	.047086	16
17	.022458	.023734	.025047	.026396	.027780	.029198	.030648	.032129	17
18	.013600	.014504	.015446	.016424	.017440	.018492	.019581	.020706	18
19	.007802	.008397	.009023	.009682	.010372	.011095	.011852	.012641	19
20	.004252	.004618	.005008	.005422	.005860	.006324	.006815	.007332	20
21	.002207	.002419	.002647	.002892	.003153	.003433	.003732	.004050	21
22	.001093	.001210	.001336	.001472	.001620	.001779	.001951	.002135	22
23	.000518	.000578	.000645	.000717	.000796	.000882	.000975	.001077	23
24	.000235	.000265	.000298	.000335	.000375	.000419	.000467	.000521	24
25	.000103	.000117	.000132	.000150	.000169	.000191	.000215	.000242	25
26	.000043	.000049	.000057	.000065	.000074	.000084	.000095	.000108	26
27	.000017	.000020	.000023	.000027	.000031	.000035	.000041	.000046	27
28	.000007	.000008	.000009	.000011	.000012	.000014	.000017	.000019	28
29	.000003	.000003	.000004	.000004	.000005	.000006	.000007	.000008	29
30	.000001	.000001	.000001	.000002	.000002	.000002	.000003	.000003	30
31				.000001	.000001	.000001	.000001	.000001	31

X	A=11.7	A=11.8	A=11.9	A=12.0	A=12.1	A=12.2	A=12.3	A=12.4	X
0	.000008	.000008	.000007	.000006	.000006	.000005	.000005	.000004	0
1	.000097	.000089	.000081	.000074	.000067	.000061	.000056	.000051	1
2	.000568	.000522	.000481	.000442	.000407	.000374	.000344	.000317	2
3	.002214	.002055	.001907	.001770	.001642	.001522	.001412	.001309	3
4	.006476	.006062	.005674	.005309	.004966	.004643	.004341	.004057	4
5	.015153	.014307	.013504	.012741	.012017	.011330	.010679	.010062	5
6	.029549	.028137	.026782	.025481	.024234	.023037	.021892	.020794	6
7	.049388	.047432	.045530	.043682	.041889	.040151	.038467	.036836	7
8	.072231	.069962	.067725	.065523	.063358	.061230	.059142	.057095	8
9	.093900	.054776	.089548	.087364	.085181	.083001	.080828	.078665	9

TABLE I

X	A=11.7	A=11.8	A=11.9	A=12.0	A=12.1	A=12.2	A=12.3	A=12.4	X
10	.109863	.108239	.106562	.104837	.103069	.101261	.099418	.097544	10
11	.116854	.116110	.115281	.114368	.113376	.112308	.111168	.109959	11
12	.113933	.114175	.114320	.114368	.114321	.114180	.113947	.113624	12
13	.102539	.103636	.104647	.105570	.106406	.107153	.107811	.108380	13
14	.085694	.087350	.088950	.090489	.091965	.093376	.094720	.095994	14
15	.066841	.068716	.070567	.072391	.074185	.075946	.077670	.079355	15
16	.048877	.050678	.052484	.054293	.056103	.057909	.059709	.061500	16
17	.033639	.035176	.036739	.038325	.039932	.041558	.043201	.044859	17
18	.021865	.023060	.024288	.025550	.026843	.028167	.029521	.030903	18
19	.013465	.014322	.015212	.016137	.017095	.018086	.019111	.020168	19
20	.007877	.008450	.009051	.009682	.010342	.011033	.011753	.012504	20
21	.004388	.004748	.005129	.005533	.005959	.006409	.006884	.007383	21
22	.002334	.002547	.002774	.003018	.003278	.003554	.003849	.004162	22
23	.001187	.001307	.001435	.001574	.001724	.001885	.002058	.002244	23
24	.000579	.000642	.000712	.000787	.000869	.000958	.001055	.001159	24
25	.000271	.000303	.000339	.000378	.000421	.000468	.000019	.000575	25
26	.000122	.000138	.000155	.000174	.000196	.000219	.000246	.000274	26
27	.000053	.000060	.000068	.000078	.000088	.000099	.000112	.000126	27
28	.000022	.000025	.000029	.000033	.000038	.000043	.000049	.000056	28
29	.000009	.000010	.000012	.000014	.000016	.000018	.000021	.000024	29
30	.000003	.000004	.000005	.000005	.000006	.000007	.000009	.000010	30
31	.000001	.000002	.000002	.000002	.000002	.000003	.000003	.000004	31
32		.000001	.000001	.000001	.000001	.000001	.000001	.000002	32
33								.000001	33

X	A=12.5	A=12.6	A=12.7	A=12.8	A=12.9	A=13.0	A=13.1	A=13.2	X
0	.000004	.000003	.000003	.000003	.000002	.000002	.000002	.000002	0
1	.000047	.000042	.000039	.000035	.000032	.000029	.000027	.000024	1
2	.000291	.000268	.000246	.000226	.000208	.000191	.000175	.000161	2
3	.001213	.001124	.001042	.000965	.000894	.000828	.000766	.000709	3
4	.003791	.003541	.003307	.003088	.002882	.002690	.002510	.002341	4
5	.009477	.008924	.008400	.007905	.007436	.006994	.006575	.006180	5
6	.019745	.018740	.017781	.016864	.015988	.015153	.014356	.013596	6
7	.035258	.033733	.032259	.030837	.029464	.028141	.026866	.025639	7
8	.055091	.053129	.051212	.049339	.047511	.045730	.043994	.042304	8
9	.076515	.074381	.072265	.070171	.068100	.066054	.064036	.062046	9
10	.095644	.093720	.091777	.089819	.087849	.085870	.083887	.081901	10
11	.108686	.107352	.105961	.104516	.103023	.101483	.099901	.098281	11
12	.113215	.112720	.112142	.111484	.110749	.109940	.109059	.108109	12
13	.108860	.109251	.109554	.109769	.109897	.109940	.109898	.109773	13
14	.097197	.098326	.099381	.100360	.101263	.102087	.102833	.103500	14
15	.080997	.082594	.084143	.085641	.087086	.088475	.089807	.091080	15
16	.063279	.065043	.066788	.068513	.070213	.071886	.073530	.075141	16
17	.046529	.048208	.049895	.051586	.053279	.054972	.056661	.058345	17
18	.032312	.033746	.035204	.036683	.038183	.039702	.041237	.042786	18
19	.021258	.022379	.023531	.024713	.025925	.027164	.028432	.029725	19
20	.013286	.014099	.014942	.015816	.016721	.017657	.018623	.019619	20
21	.007908	.008459	.009036	.009640	.010272	.010930	.011617	.012332	21
22	.004493	.004845	.005216	.005609	.006023	.006459	.006917	.007399	22
23	.002442	.002654	.002880	.003122	.003378	.003651	.003940	.004246	23
24	.001272	.001393	.001524	.001665	.001816	.001977	.002151	.002336	24
25	.000636	.000702	.000774	.000852	.000937	.001028	.001127	.001233	25
26	.000306	.000340	.000378	.000420	.000465	.000514	.000568	.000626	26
27	.000142	.000159	.000178	.000199	.000222	.000248	.000275	.000306	27
28	.000063	.000071	.000081	.000091	.000102	.000115	.000129	.000144	28
29	.000027	.000031	.000035	.000040	.000046	.000052	.000058	.000066	29
30	.000011	.000013	.000015	.000017	.000020	.000022	.000025	.000029	30
31	.000005	.000005	.000006	.000007	.000008	.000009	.000011	.000012	31
32	.000002	.000002	.000002	.000003	.000003	.000004	.000004	.000005	32
33	.000001	.000001	.000001	.000001	.000001	.000001	.000002	.000002	33
34						.000001	.000001	.000001	34

X	A=13.3	A=13.4	A=13.5	A=13.6	A=13.7	A=13.8	A=13.9	A=14.0	X
0	.000002	.000002	.000001	.000001	.000001	.000001	.000001	.000001	0
1	.000022	.000020	.000019	.000017	.000015	.000014	.000013	.000001	1
2	.000148	.000136	.000125	.000115	.000105	.000097	.000089	.000012	2
3	.000657	.000608	.000562	.000520	.000481	.000445	.000411	.000081	3

TABLE I

X	A=13.3	A=13.4	A=13.5	A=13.6	A=13.7	A=13.8	A=13.9	A=14.0	X
4	.002183	.002035	.001897	.001768	.001648	.001535	.001429	.000380	4
5	.005807	.005455	.005123	.004810	.004514	.004236	.003974	.001331	5
6	.012872	.012183	.011526	.010902	.010308	.009743	.009206	.003727	6
7	.024458	.023322	.022230	.021181	.020173	.019207	.018280	.008696	7
8	.040661	.039064	.037512	.036007	.034547	.033132	.031762	.017392	8
9	.060088	.058161	.056269	.054410	.052588	.050802	.049054	.030436	9
10	.079917	.077936	.075962	.073998	.072046	.070107	.068185	.047344	10
11	.096626	.094940	.093227	.091489	.089730	.087953	.086162	.066282	11
12	.107094	.106017	.104880	.103687	.102441	.101146	.099804	.084359	12
13	.109566	.109279	.108914	.108473	.107957	.107370	.106713	.098418	13
14	.104087	.104595	.105024	.105374	.105644	.105836	.105951	.105989	14
15	.092291	.093439	.094522	.095539	.096488	.097369	.098181	.105989	15
16	.076717	.078255	.079753	.081208	.082618	.083981	.085295	.098923	16
17	.060020	.061683	.063333	.064966	.066580	.068173	.069741	.086558	17
18	.044348	.045920	.047500	.049086	.050675	.052266	.053856	.071283	18
19	.031043	.032385	.033750	.035135	.036539	.037962	.039400	.055442	19
20	.020644	.021698	.022781	.023892	.025030	.026193	.027383	.040852	20
21	.013074	.013846	.014645	.015473	.016329	.017213	.018125	.028597	21
22	.007904	.008433	.008987	.009565	.010168	.010797	.011452	.019064	22
23	.004571	.004913	.005275	.005656	.006057	.006478	.006921	.012132	23
24	.002533	.002743	.002967	.003205	.003457	.003725	.004008	.007385	24
25	.001347	.001470	.001602	.001744	.001895	.002056	.002229	.004308	25
26	.000689	.000758	.000832	.000912	.000998	.001091	.001191	.002412	26
27	.000340	.000376	.000416	.000459	.000507	.000558	.000613	.001299	27
28	.000161	.000180	.000201	.000223	.000248	.000275	.000304	.000674	28
29	.000074	.000083	.000093	.000105	.000117	.000131	.000146	.000337	29
30	.000033	.000037	.000042	.000047	.000053	.000060	.000068	.000163	30
31	.000014	.000016	.000018	.000021	.000024	.000027	.000030	.000076	31
32	.000006	.000007	.000008	.000009	.000010	.000012	.000013	.000034	32
33	.000002	.000003	.000003	.000004	.000004	.000005	.000006	.000015	33
34	.000001	.000001	.000001	.000001	.000002	.000002	.000002	.000006	34
35				.000001	.000001	.000001	.000001	.000003	35
36								.000001	36

X	A=14.1	A=14.2	A=14.3	A=14.4	A=14.5	A=14.6	A=14.7	A=14.8	X
0	.000001	.000001	.000001	.000001	.000001				0
1	.000011	.000010	.000009	.000008	.000007	.000007	.000006	.000006	1
2	.000075	.000069	.000063	.000058	.000053	.000049	.000045	.000041	2
3	.000352	.000325	.000300	.000277	.000256	.000237	.000219	.000202	3
4	.001239	.001153	.001073	.000999	.000929	.000864	.000803	.000747	4
5	.003494	.003276	.003070	.002876	.002694	.002523	.002362	.002211	5
6	.008212	.007752	.007316	.006902	.006510	.006139	.005787	.005454	6
7	.016541	.015726	.014946	.014199	.013486	.012804	.012152	.011530	7
8	.029153	.027913	.026715	.025559	.024443	.023367	.022330	.021331	8
9	.045673	.044040	.042447	.040894	.039380	.037906	.036472	.035078	9
10	.064399	.062537	.060700	.058887	.057101	.055343	.053614	.051915	10
11	.082547	.080730	.078910	.077089	.075270	.073456	.071648	.069850	11
12	.096993	.095530	.094034	.092507	.090951	.089371	.087769	.086148	12
13	.105200	.104349	.103437	.102469	.101446	.100371	.099247	.098076	13
14	.105951	.105839	.105654	.105396	.105069	.104672	.104209	.103681	14
15	.099594	.100195	.100723	.101181	.101566	.101881	.102125	.102298	15
16	.087768	.088923	.090021	.091063	.092045	.092967	.093827	.094626	16
17	.072795	.074277	.075724	.077135	.078509	.079842	.081133	.082380	17
18	.057023	.058596	.060158	.061708	.063243	.064761	.066259	.067735	18
19	.042317	.043793	.045277	.046768	.048264	.049763	.051263	.052762	19
20	.029834	.031093	.032373	.033673	.034992	.036327	.037678	.039044	20
21	.020031	.021025	.022045	.023090	.024161	.025256	.026375	.027517	21
22	.012838	.013570	.014329	.015114	.015924	.016761	.017623	.018511	22
23	.007870	.008378	.008909	.009462	.010039	.010640	.011264	.011912	23
24	.004624	.004957	.005308	.005677	.006065	.006472	.006899	.007345	24
25	.002608	.002816	.003036	.003270	.003518	.003780	.004057	.004348	25
26	.001414	.001538	.001670	.001811	.001962	.002123	.002294	.002475	26
27	.000739	.000809	.000884	.000966	.001054	.001148	.001249	.001357	27
28	.000372	.000410	.000452	.000497	.000546	.000598	.000656	.000717	28

TABLE I

X	A=14.1	A=14.2	A=14.3	A=14.4	A=14.5	A=14.6	A=14.7	A=14.8	X
29	.000181	.000201	.000223	.000247	.000273	.000301	.000332	.000366	29
30	.000085	.000095	.000106	.000118	.000132	.000147	.000163	.000181	30
31	.000039	.000044	.000049	.000055	.000062	.000069	.000077	.000086	31
32	.000017	.000019	.000022	.000025	.000028	.000032	.000035	.000040	32
33	.000007	.000008	.000009	.000011	.000012	.000014	.000016	.000018	33
34	.000003	.000003	.000004	.000005	.000005	.000006	.000007	.000008	34
35	.000001	.000001	.000002	.000002	.000002	.000002	.000003	.000003	35
36		.000001	.000001	.000001	.000001	.000001	.000001	.000001	36
37								.000001	37

X	A=14.9	A=15.0	A=16	A=17	A=18	A=19	A=20	A=21	X
1	.000005	.000005	.000002	.000001					1
2	.000038	.000034	.000014	.000006	.000002	.000001			2
3	.000186	.000172	.000077	.000034	.000015	.000006	.000003	.000001	3
4	.000694	.000645	.000307	.000144	.000067	.000030	.000014	.000006	4
5	.002069	.001936	.000983	.000490	.000240	.000116	.000055	.000026	5
6	.005138	.004839	.002622	.001388	.000719	.000366	.000183	.000090	6
7	.010937	.010370	.005994	.003371	.001850	.000994	.000523	.000271	7
8	.020370	.019444	.011987	.007163	.004163	.002360	.001309	.000711	8
9	.033723	.032407	.021311	.013529	.008325	.004982	.002908	.001660	9
10	.050247	.048611	.034098	.023000	.014985	.009466	.005816	.003485	10
11	.068062	.066287	.049597	.035545	.024521	.016351	.010575	.006654	11
12	.084510	.082859	.066129	.050355	.036782	.025889	.017625	.011644	12
13	.096862	.095607	.081389	.065849	.050929	.037837	.027116	.018810	13
14	.103089	.102436	.093016	.079960	.065480	.051351	.038737	.028215	14
15	.102402	.102436	.099218	.090621	.078576	.065044	.051649	.039501	15
16	.095361	.096034	.099218	.096285	.088397	.077240	.064561	.051845	16
17	.083582	.084736	.093381	.096285	.093597	.086327	.075954	.064044	17
18	.069187	.070613	.083006	.090935	.093597	.091123	.084394	.074717	18
19	.054257	.055747	.069899	.081363	.088671	.091123	.088835	.082582	19
20	.040422	.041810	.055920	.069159	.079804	.086567	.088835	.086712	20
21	.028680	.029865	.042605	.055986	.068403	.078328	.084605	.086712	21
22	.019424	.020362	.030986	.043262	.055966	.067642	.076914	.082770	22
23	.012584	.013280	.021555	.031976	.043800	.055878	.066881	.075573	23
24	.007812	.008300	.014370	.022650	.032850	.044237	.055735	.066126	24
25	.004656	.004980	.009197	.015402	.023652	.033620	.044588	.055546	25
26	.002668	.002873	.005660	.010070	.016374	.024569	.034298	.044864	26
27	.001473	.001596	.003354	.006341	.010916	.017289	.025406	.034894	27
28	.000784	.000855	.001916	.003850	.007018	.011732	.018147	.026171	28
29	.000403	.000442	.001057	.002257	.004356	.007686	.012515	.018951	29
30	.000200	.000221	.000564	.001279	.002613	.004868	.008344	.013266	30
31	.000096	.000107	.000291	.000701	.001517	.002984	.005383	.008987	31
32	.000045	.000050	.000146	.000373	.000854	.001772	.003364	.005897	32
33	.000020	.000023	.000071	.000192	.000466	.001020	.002039	.003753	33
34	.000009	.000010	.000033	.000096	.000246	.000570	.001199	.002318	34
35	.000004	.000004	.000015	.000047	.000127	.000309	.000685	.001391	35
36	.000002	.000002	.000007	.000022	.000063	.000163	.000381	.000811	36
37	.000001	.000001	.000003	.000010	.000031	.000084	.000206	.000460	37
38			.000001	.000005	.000015	.000042	.000108	.000254	38
39			.000001	.000002	.000007	.000020	.000056	.000137	39
40				.000001	.000003	.000010	.000028	.000072	40
41					.000001	.000004	.000014	.000037	41
42					.000001	.000002	.000006	.000018	42
43						.000001	.000003	.000009	43
44							.000001	.000004	44
45							.000001	.000002	45
46								.000001	46

X	A=22	A=23	A=24	A=25	A=26	A=27	A=28	A=29	X
4	.000003	.000001	.000001						4
5	.000012	.000006	.000003	.000001					5
6	.000044	.000021	.000010	.000005	.000002	.000001			6
7	.000138	.000069	.000034	.000017	.000008	.000004	.000002	.000001	7
8	.000380	.000199	.000103	.000053	.000026	.000013	.000006	.000003	8
9	.000928	.000509	.000275	.000146	.000076	.000039	.000020	.000010	9

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TABLE I

X	A=22	A=23	A=24	A=25	A=26	A=27	A=28	A=29	X
10	.002042	.001171	.000660	.000365	.000199	.000107	.000056	.000029	10
11	.004083	.002449	.001439	.000830	.000470	.000262	.000144	.000078	11
12	.007486	.004695	.002878	.001728	.001018	.000589	.000335	.000188	12
13	.012669	.008306	.005314	.003323	.002036	.001223	.000722	.000419	13
14	.019908	.013646	.009109	.005935	.003781	.002359	.001444	.000868	14
15	.029199	.020924	.014575	.009891	.006553	.004246	.002695	.001679	15
16	.040148	.030078	.021862	.015455	.010649	.007166	.004717	.003042	16
17	.051956	.040694	.030864	.022727	.016286	.011381	.007769	.005190	17
18	.063502	.051998	.041152	.031566	.023525	.017071	.012085	.008361	18
19	.073529	.062945	.051982	.041534	.032192	.024259	.017810	.012762	19
20	.080882	.072387	.062378	.051917	.041849	.032749	.024934	.018505	20
21	.084733	.079281	.071289	.061807	.051813	.042106	.033245	.025555	21
22	.084733	.082884	.077770	.070235	.061234	.051676	.042312	.033686	22
23	.081049	.082884	.081152	.076342	.069221	.060663	.051510	.042474	23
24	.074295	.079431	.081152	.079523	.074989	.068245	.060095	.051322	24
25	.065380	.073076	.077905	.079523	.077989	.073705	.067307	.059534	25
26	.055321	.064645	.071913	.076464	.077989	.076540	.072484	.066403	26
27	.045077	.055068	.063922	.070800	.075100	.076540	.075169	.071322	27
28	.035417	.045234	.054791	.063215	.069736	.073806	.075169	.073869	28
29	.026868	.035875	.045344	.054495	.062522	.068716	.072577	.073869	29
30	.019703	.027504	.036275	.045413	.054186	.061845	.067738	.071407	30
31	.013983	.020406	.028084	.036623	.045446	.053865	.061183	.066800	31
32	.009613	.014667	.021063	.028612	.036925	.045448	.053535	.060537	32
33	.006409	.010223	.015319	.021676	.029092	.037185	.045424	.053200	33
34	.004147	.006915	.010813	.015938	.022247	.029529	.037408	.045376	34
35	.002607	.004544	.007415	.011384	.016526	.022780	.029926	.037597	35
36	.001593	.002903	.004943	.007906	.011936	.017085	.023276	.030287	36
37	.000947	.001805	.003206	.005342	.008387	.012467	.017614	.023738	37
38	.000548	.001092	.002025	.003514	.005739	.008858	.012979	.018116	38
39	.000309	.000644	.001246	.002253	.003826	.006133	.009318	.013471	39
40	.000170	.000370	.000748	.001408	.002487	.004140	.006523	.009766	40
41	.000091	.000208	.000438	.000859	.001577	.002726	.004455	.006908	41
42	.000048	.000114	.000250	.000511	.000976	.001752	.002970	.004770	42
43	.000024	.000061	.000140	.000297	.000590	.001100	.001934	.003217	43
44	.000012	.000032	.000076	.000169	.000349	.000675	.001231	.002120	44
45	.000006	.000016	.000041	.000094	.000202	.000405	.000766	.001366	45
46	.000003	.000008	.000021	.000051	.000114	.000238	.000466	.000861	46
47	.000001	.000004	.000011	.000027	.000063	.000137	.000278	.000531	47
48	.000001	.000002	.000005	.000014	.000034	.000077	.000162	.000321	48
49		.000001	.000003	.000007	.000018	.000042	.000093	.000190	49
50			.000001	.000004	.000009	.000023	.000052	.000110	50
51			.000001	.000002	.000005	.000012	.000028	.000063	51
52				.000001	.000002	.000006	.000015	.000035	52
53					.000001	.000003	.000008	.000019	53
54					.000001	.000002	.000004	.000010	54
55						.000001	.000002	.000005	55
56							.000001	.000003	56
57							.000001	.000001	57
58								.000001	58

X	A=30	A=31	A=32	A=33	A=34	A=35	A=36	A=37	X
8	.000002	.000001							8
9	.000005	.000003	.000001	.000001					9
10	.000015	.000008	.000004	.000002	.000001				10
11	.000042	.000022	.000011	.000006	.000003	.000002	.000001		11
12	.000104	.000057	.000030	.000016	.000009	.000004	.000002	.000001	12
13	.000240	.000135	.000075	.000041	.000022	.000012	.000006	.000003	13
14	.000513	.000299	.000172	.000097	.000054	.000030	.000016	.000009	14
15	.001027	.000618	.000366	.000214	.000123	.000070	.000039	.000022	15
16	.001925	.001197	.000732	.000440	.000261	.000153	.000088	.000050	16
17	.003397	.002182	.001377	.000855	.000522	.000315	.000187	.000110	17
18	.005662	.003759	.002449	.001567	.000987	.000612	.000374	.000225	18
19	.008941	.006133	.004124	.002722	.001766	.001127	.000708	.000438	19
20	.013411	.009506	.006599	.004492	.003002	.001972	.001274	.000811	20
21	.019159	.014032	.010055	.007059	.004861	.003287	.002185	.001429	21

TABLE I

X	A=30	A=31	A=32	A=33	A=34	A=35	A=36	A=37	X
22	.026126	.019773	.014625	.010588	.007512	.005229	.003575	.002403	22
23	.034077	.026650	.020348	.015192	.011105	.007957	.005596	.003866	23
24	.042596	.034423	.027131	.020889	.015732	.011604	.008394	.005960	24
25	.051115	.042684	.034728	.027573	.021395	.016246	.012087	.008821	25
26	.058979	.050893	.042742	.034997	.027978	.021870	.016736	.012552	26
27	.065532	.058432	.050657	.042774	.035232	.028350	.022314	.017202	27
28	.070213	.064693	.057894	.050412	.042782	.035437	.028690	.022731	28
29	.072635	.069155	.063883	.057365	.050158	.042769	.035615	.029001	29
30	.072635	.071460	.068142	.063102	.056845	.049897	.042738	.035768	30
31	.070291	.071460	.070340	.067173	.062347	.056335	.049631	.042691	31
32	.065898	.069227	.070340	.069272	.066243	.061617	.055835	.049361	32
33	.059908	.065031	.068209	.069272	.068251	.065351	.060911	.055345	33
34	.052860	.059293	.064196	.067234	.068251	.067273	.064494	.060228	34
35	.045308	.052517	.058694	.063392	.066301	.067273	.066337	.063670	35
36	.037757	.045223	.052172	.058110	.062617	.065404	.066337	.065438	36
37	.030614	.037889	.045122	.051828	.057540	.061869	.064544	.065438	37
38	.024169	.030910	.037998	.045008	.051483	.056985	.061147	.063716	38
39	.018591	.024569	.031177	.038084	.044883	.051140	.056443	.060449	39
40	.013943	.019041	.024942	.031419	.038150	.044748	.050799	.055915	40
41	.010203	.014397	.019467	.025289	.031637	.038199	.044604	.050460	41
42	.007288	.010626	.014832	.019870	.025611	.031833	.038232	.044453	42
43	.005084	.007661	.011038	.015249	.020250	.025910	.032008	.038250	43
44	.003467	.005397	.008027	.011437	.015648	.020610	.026188	.032165	44
45	.002311	.003718	.005708	.008387	.011823	.016030	.020951	.026447	45
46	.001507	.002506	.003971	.006017	.008739	.012197	.016396	.021272	46
47	.000962	.001653	.002704	.004224	.006322	.009083	.012559	.016746	47
48	.000601	.001067	.001802	.002904	.004478	.006623	.009419	.012909	48
49	.000368	.000675	.001177	.001956	.003107	.004731	.006920	.009747	49
50	.000221	.000419	.000753	.001291	.002113	.003311	.004983	.007213	50
51	.000130	.000254	.000473	.000835	.001409	.002273	.003517	.005233	51
52	.000075	.000152	.000291	.000530	.000921	.001530	.002435	.003723	52
53	.000042	.000089	.000176	.000330	.000591	.001010	.001654	.002599	53
54	.000024	.000051	.000104	.000202	.000372	.000655	.001103	.001781	54
55	.000013	.000029	.000061	.000121	.000230	.000417	.000722	.001198	55
56	.000007	.000016	.000035	.000071	.000140	.000260	.000464	.000792	56
57	.000004	.000009	.000019	.000041	.000083	.000160	.000293	.000514	57
58	.000002	.000005	.000011	.000023	.000049	.000096	.000182	.000328	58
59	.000001	.000002	.000006	.000013	.000028	.000057	.000111	.000206	59
60		.000001	.000003	.000007	.000016	.000033	.000067	.000127	60
61		.000001	.000002	.000004	.000009	.000019	.000039	.000077	61
62			.000001	.000002	.000005	.000011	.000023	.000046	62
63				.000001	.000003	.000006	.000013	.000027	63
64				.000001	.000001	.000003	.000007	.000016	64
65					.000001	.000002	.000004	.000009	65
66						.000001	.000002	.000005	66
67							.000001	.000003	67
68							.000001	.000001	68
69								.000001	69

X	A=38	A=39	A=40	A=41	A=42	A=43	A=44	A=45	X
12	.000001								12
13	.000002	.000001							13
14	.000005	.000002	.000001	.000001					14
15	.000012	.000006	.000003	.000002	.000001	.000001			15
16	.000028	.000016	.000009	.000005	.000003	.000001	.000001		16
17	.000063	.000036	.000021	.000011	.000006	.000003	.000002	.000001	17
18	.000134	.000079	.000046	.000026	.000015	.000008	.000005	.000003	18
19	.000268	.000161	.000096	.000056	.000033	.000019	.000011	.000006	19
20	.000509	.000315	.000192	.000116	.000069	.000041	.000024	.000014	20
21	.000920	.000584	.000366	.000226	.000138	.000083	.000050	.000029	21
22	.001590	.001036	.000665	.000421	.000263	.000163	.000099	.000060	22
23	.002626	.001756	.001156	.000751	.000481	.000304	.000190	.000117	23
24	.004158	.002853	.001927	.001282	.000841	.000544	.000348	.000219	24
25	.006321	.004451	.003084	.002103	.001413	.000936	.000612	.000395	25
26	.009238	.006677	.004744	.003317	.002283	.001548	.001036	.000683	26
27	.013001	.009644	.007028	.005036	.003551	.002466	.001688	.001139	27
28	.017645	.013430	.010041	.007375	.005327	.003787	.002652	.001830	28

TABLE I

X	A=38	A=39	A=40	A=41	A=42	A=43	A=44	A=45	X
29	.023120	.018065	.013849	.010426	.007715	.005615	.004024	.002840	29
30	.029286	.023485	.018465	.014249	.010801	.008049	.005902	.004261	30
31	.035899	.029546	.023826	.018845	.014633	.011164	.008376	.006185	31
32	.042630	.036009	.029783	.024146	.019206	.015002	.011517	.008697	32
33	.049089	.042556	.036101	.029999	.024444	.019548	.015357	.011860	33
34	.054864	.048814	.042471	.036175	.030196	.024723	.019873	.015697	34
35	.059567	.054393	.048539	.042377	.036235	.030374	.024984	.020182	35
36	.062876	.058926	.053932	.048263	.042274	.036280	.030535	.025227	36
37	.064575	.062111	.058305	.053480	.047986	.042163	.036312	.030681	37
38	.064575	.063746	.061373	.057702	.053038	.047711	.042046	.036333	38
39	.062919	.063746	.062947	.060661	.057117	.052604	.047436	.041923	39
40	.059773	.062152	.062947	.062178	.059973	.056550	.052180	.047163	40
41	.055400	.059120	.061412	.062178	.061436	.059308	.055998	.051765	41
42	.050124	.054897	.058487	.060697	.061436	.060720	.058665	.055462	42
43	.044295	.049791	.054407	.057874	.060007	.060720	.060029	.058042	43
44	.038255	.044133	.049461	.053928	.057280	.059340	.060029	.059361	44
45	.032304	.038248	.043965	.049135	.053461	.056703	.058695	.059361	45
46	.026686	.032428	.038231	.043794	.048812	.053005	.056143	.058070	46
47	.021576	.026908	.032537	.038203	.043620	.048494	.052559	.055599	47
48	.017081	.021863	.027114	.032632	.038167	.043442	.048180	.052124	48
49	.013246	.017401	.022134	.027304	.032715	.038123	.043263	.047869	49
50	.010067	.013573	.017707	.022389	.027480	.032786	.038072	.043082	50
51	.007501	.010379	.013888	.017999	.022631	.027643	.032846	.038014	51
52	.005482	.007784	.010683	.014192	.018279	.022859	.027793	.032897	52
53	.003930	.005728	.008063	.010979	.014485	.018546	.023073	.027931	53
54	.002766	.004137	.005972	.008336	.011266	.014768	.018800	.023276	54
55	.001911	.002934	.004343	.006214	.008603	.011546	.015040	.019044	55
56	.001297	.002043	.003102	.004549	.006452	.008865	.011817	.015303	56
57	.000864	.001398	.002177	.003272	.004754	.006688	.009122	.012081	57
58	.000566	.000940	.001502	.002313	.003443	.004958	.006920	.009374	58
59	.000365	.000621	.001018	.001607	.002451	.003614	.005161	.007149	59
60	.000231	.000404	.000679	.001098	.001716	.002590	.003785	.005362	60
61	.000144	.000258	.000445	.000738	.001181	.001826	.002730	.003956	61
62	.000088	.000162	.000287	.000488	.000800	.001266	.001937	.002871	62
63	.000053	.000101	.000182	.000318	.000533	.000864	.001353	.002051	63
64	.000032	.000061	.000114	.000204	.000350	.000581	.000930	.001442	64
65	.000018	.000037	.000070	.000128	.000226	.000384	.000630	.000998	65
66	.000011	.000022	.000042	.000080	.000144	.000250	.000420	.000681	66
67	.000006	.000013	.000025	.000049	.000090	.000161	.000276	.000457	67
68	.000003	.000007	.000015	.000029	.000056	.000102	.000178	.000303	68
69	.000002	.000004	.000009	.000017	.000034	.000063	.000114	.000197	69
70	.000001	.000002	.000005	.000010	.000020	.000039	.000072	.000127	70
71	.000001	.000001	.000003	.000006	.000012	.000024	.000044	.000080	71
72		.000001	.000002	.000003	.000007	.000014	.000027	.000050	72
73			.000001	.000002	.000004	.000008	.000016	.000031	73
74				.000001	.000002	.000005	.000010	.000019	74
75				.000001	.000001	.000003	.000006	.000011	75
76					.000001	.000002	.000003	.000007	76
77						.000001	.000002	.000004	77
78							.000001	.000002	78
79							.000001	.000001	79
80								.000001	80

X	A=46	A=47	A=48	A=49	A=50	A=51	A=52	A=53	X
17	.000001								17
18	.000001	.000001							18
19	.000003	.000002	.000001	.000001					19
20	.000008	.000004	.000002	.000001	.000001				20
21	.000017	.000010	.000006	.000003	.000002	.000001	.000001		21
22	.000036	.000021	.000012	.000007	.000004	.000002	.000001	.000001	22
23	.000071	.000043	.000026	.000015	.000009	.000005	.000003	.000002	23
24	.000137	.000084	.000051	.000031	.000019	.000011	.000006	.000004	24
25	.000252	.000158	.000099	.000061	.000037	.000022	.000013	.000008	25
26	.000445	.000286	.000182	.000115	.000071	.000044	.000027	.000016	26
27	.000758	.000499	.000324	.000208	.000132	.000083	.000051	.000032	27
28	.001246	.000837	.000555	.000364	.000236	.000151	.000096	.000060	28

TABLE I

X	A=46	A=47	A=48	A=49	A=50	A=51	A=52	A=53	X
29	.001976	.001357	.000919	.000615	.000406	.000265	.000171	.000110	29
30	.003031	.002125	.001470	.001004	.000677	.000451	.000297	.000194	30
31	.004497	.003222	.002277	.001587	.001092	.000742	.000499	.000331	31
32	.006464	.004733	.003415	.002430	.001707	.001183	.000810	.000548	32
33	.009011	.006741	.004968	.003609	.002586	.001829	.001277	.000881	33
34	.012191	.009318	.007013	.005201	.003803	.002743	.001953	.001373	34
35	.016023	.012513	.009618	.007281	.005432	.003997	.002901	.002079	35
36	.020474	.016336	.012824	.009910	.007545	.005662	.004191	.003060	36
37	.025454	.020751	.016636	.013125	.010196	.007804	.005889	.004384	37
38	.030813	.025666	.021014	.016924	.013416	.010474	.008059	.006114	38
39	.036343	.030931	.025864	.021263	.017200	.013697	.010746	.008309	39
40	.041795	.036344	.031036	.026048	.021500	.017464	.013969	.011010	40
41	.046892	.041662	.036335	.031130	.026219	.021723	.017717	.014232	41
42	.051358	.046622	.041526	.036318	.031213	.026379	.021936	.017960	42
43	.054941	.050959	.046354	.041386	.036294	.031286	.026527	.022136	43
44	.057438	.054433	.050568	.046089	.041244	.036264	.031350	.026664	44
45	.058714	.056853	.053940	.050186	.045826	.041099	.036226	.031405	45
46	.058714	.058089	.056285	.053459	.049811	.045566	.040952	.036184	46
47	.057465	.058089	.057482	.055734	.052991	.049444	.045308	.040803	47
48	.055071	.056878	.057482	.056895	.055199	.052534	.049084	.045053	48
49	.051699	.054557	.056309	.056895	.056325	.054678	.052089	.048731	49
50	.047563	.051283	.054057	.055757	.056325	.055772	.054173	.051655	50
51	.042900	.047261	.050877	.053570	.055221	.055772	.055235	.053681	51
52	.037950	.042717	.046964	.050480	.053097	.054699	.055235	.054713	52
53	.032938	.037881	.042533	.046670	.050091	.052635	.054193	.054713	53
54	.028058	.032971	.037807	.042349	.046381	.049711	.052185	.053700	54
55	.023467	.028175	.032995	.037729	.042164	.046096	.049339	.051747	55
56	.019276	.023647	.028282	.033013	.037647	.041980	.045815	.048975	56
57	.015556	.019498	.023816	.028379	.033023	.037561	.041796	.045538	57
58	.012338	.015800	.019710	.023976	.028468	.033028	.037472	.041612	58
59	.009619	.012587	.016035	.019912	.024126	.028549	.033026	.037381	59
60	.007375	.009860	.012828	.016261	.020105	.024267	.028623	.033019	60
61	.005561	.007597	.010094	.013063	.016479	.020289	.024400	.028689	61
62	.004126	.005759	.007815	.010324	.013290	.016689	.020464	.024524	62
63	.003013	.004296	.005954	.008029	.010547	.013510	.016891	.020632	63
64	.002165	.003155	.004466	.006148	.008240	.010766	.013724	.017086	64
65	.001532	.002281	.003298	.004634	.006339	.008447	.010979	.013931	65
66	.001068	.001625	.002398	.003441	.004802	.006527	.008650	.011187	66
67	.000733	.001140	.001718	.002516	.003584	.004969	.006714	.008850	67
68	.000496	.000788	.001213	.001813	.002635	.003726	.005134	.006898	68
69	.000331	.000537	.000844	.001288	.001909	.002754	.003869	.005298	69
70	.000217	.000360	.000579	.000901	.001364	.002007	.002874	.004011	70
71	.000141	.000238	.000391	.000622	.000960	.001441	.002105	.002994	71
72	.000090	.000156	.000261	.000423	.000667	.001021	.001520	.002204	72
73	.000057	.000100	.000171	.000284	.000457	.000713	.001083	.001600	73
74	.000035	.000064	.000111	.000188	.000309	.000492	.000761	.001146	74
75	.000022	.000040	.000071	.000123	.000206	.000334	.000528	.000810	75
76	.000013	.000025	.000045	.000079	.000135	.000224	.000361	.000565	76
77	.000008	.000015	.000028	.000050	.000088	.000149	.000244	.000389	77
78	.000005	.000009	.000017	.000032	.000056	.000097	.000163	.000264	78
79	.000003	.000005	.000010	.000020	.000036	.000063	.000107	.000177	79
80	.000002	.000003	.000006	.000012	.000022	.000040	.000070	.000117	80
81	.000001	.000002	.000004	.000007	.000014	.000025	.000045	.000077	81
82		.000001	.000002	.000004	.000008	.000016	.000028	.000050	82
83		.000001	.000001	.000003	.000005	.000010	.000018	.000032	83
84			.000001	.000001	.000003	.000006	.000011	.000020	84
85				.000001	.000002	.000004	.000007	.000012	85
86					.000001	.000002	.000004	.000008	86
87					.000001	.000001	.000002	.000005	87
88						.000001	.000001	.000003	88
89							.000001	.000002	89
90								.000001	90
91								.000001	91

X	A=54	A=55	A=56	A=57	A=58	A=59	A=60	A=61	X
23	.000001	.000001							23

TABLE I

X	A=54	A=55	A=56	A=57	A=58	A=59	A=60	A=61	X
24	.000002	.000001	.000001						24
25	.000005	.000003	.000002	.000001	.000001				25
26	.000010	.000006	.000003	.000002	.000001	.000001			26
27	.000019	.000012	.000007	.000004	.000002	.000001	.000001		27
28	.000037	.000023	.000014	.000008	.000005	.000003	.000002	.000001	28
29	.000069	.000043	.000027	.000017	.000010	.000006	.000004	.000002	29
30	.000125	.000080	.000050	.000031	.000020	.000012	.000007	.000004	30
31	.000217	.000141	.000091	.000058	.000036	.000023	.000014	.000009	31
32	.000367	.000243	.000159	.000103	.000066	.000042	.000026	.000017	32
33	.000600	.000405	.000270	.000178	.000116	.000075	.000048	.000031	33
34	.000954	.000655	.000444	.000298	.000198	.000130	.000085	.000055	34
35	.001471	.001029	.000711	.000486	.000329	.000220	.000146	.000096	35
36	.002207	.001572	.001106	.000769	.000529	.000360	.000243	.000162	36
37	.003221	.002336	.001674	.001185	.000830	.000575	.000394	.000267	37
38	.004577	.003381	.002467	.001778	.001267	.000892	.000622	.000429	38
39	.006337	.004768	.003542	.002599	.001884	.001350	.000956	.000670	39
40	.008555	.006556	.004959	.003703	.002731	.001991	.001435	.001022	40
41	.011267	.008795	.006773	.005148	.003864	.002865	.002099	.001521	41
42	.014486	.011518	.009031	.006987	.005336	.004025	.002999	.002209	42
43	.018192	.014732	.011761	.009262	.007197	.005522	.004185	.003134	43
44	.022327	.018415	.014969	.011998	.009488	.007405	.005707	.004345	44
45	.026792	.022507	.018628	.015197	.012228	.009709	.007609	.005889	45
46	.031451	.026910	.022677	.018832	.015418	.012452	.009925	.007810	46
47	.036136	.031491	.027020	.022838	.019027	.015632	.012670	.010136	47
48	.040653	.036083	.031523	.027121	.022991	.019214	.015837	.012881	48
49	.044801	.040501	.036026	.031548	.027214	.023135	.019393	.016036	49
50	.048385	.044552	.040349	.035965	.031568	.027299	.023271	.019564	50
51	.051231	.048046	.044305	.040196	.035901	.031582	.027378	.023400	51
52	.053202	.050818	.047713	.044061	.040043	.035833	.031590	.027450	52
53	.054205	.052735	.050414	.047387	.043821	.039889	.035762	.031593	53
54	.054205	.053712	.052281	.050019	.047067	.043583	.039736	.035689	54
55	.053220	.053712	.053232	.051838	.049634	.046753	.043348	.039582	55
56	.051319	.052753	.053232	.052764	.051407	.049257	.046444	.043116	56
57	.048618	.050902	.052298	.052764	.052308	.050985	.048889	.046142	57
58	.045265	.048269	.050494	.051854	.052308	.051865	.050575	.048528	58
59	.041429	.044996	.047927	.050097	.051422	.051865	.051432	.050173	59
60	.037286	.041247	.044732	.047592	.049708	.051000	.051432	.051010	60
61	.033008	.037190	.041065	.044471	.047263	.049328	.050589	.051010	61
62	.028748	.032991	.037091	.040885	.044214	.046941	.048957	.050187	62
63	.024642	.028802	.032970	.036991	.040705	.043961	.046625	.048594	63
64	.020791	.024751	.028849	.032945	.036889	.040526	.043711	.046316	64
65	.017273	.020943	.024854	.028890	.032916	.036785	.040349	.043466	65
66	.014132	.017453	.021088	.024951	.028926	.032884	.036681	.040173	66
67	.011390	.014327	.017626	.021227	.025041	.028958	.032849	.036575	67
68	.009045	.011588	.014516	.017793	.021358	.025125	.028984	.032810	68
69	.007079	.009237	.011781	.014698	.017953	.021484	.025203	.029006	69
70	.005461	.007257	.009425	.011969	.014876	.018108	.021603	.025277	70
71	.004153	.005622	.007434	.009609	.012152	.015047	.018256	.021717	71
72	.003115	.004295	.005782	.007607	.009789	.012330	.015213	.018399	72
73	.002304	.003236	.004435	.005940	.007778	.009966	.012564	.015374	73
74	.001681	.002405	.003356	.004575	.006096	.007946	.010138	.012673	74
75	.001211	.001764	.002506	.003477	.004714	.006250	.008111	.010308	75
76	.000860	.001276	.001847	.002608	.003598	.004852	.006403	.008273	76
77	.000603	.000912	.001343	.001930	.002710	.003718	.004990	.006554	77
78	.000418	.000643	.000964	.001411	.002015	.002812	.003838	.005126	78
79	.000285	.000448	.000683	.001018	.001479	.002100	.002915	.003958	79
80	.000193	.000308	.000478	.000725	.001073	.001549	.002186	.003018	80
81	.000128	.000209	.000331	.000510	.000768	.001128	.001619	.002273	81
82	.000085	.000140	.000226	.000355	.000543	.000812	.001185	.001691	82
83	.000055	.000093	.000152	.000244	.000380	.000577	.000857	.001243	83
84	.000035	.000061	.000102	.000165	.000262	.000405	.000612	.000902	84
85	.000022	.000039	.000067	.000111	.000179	.000281	.000432	.000648	85
86	.000014	.000025	.000044	.000073	.000121	.000193	.000301	.000459	86
87	.000009	.000016	.000028	.000048	.000080	.000131	.000208	.000322	87
88	.000005	.000010	.000018	.000031	.000053	.000088	.000142	.000223	88
89	.000003	.000006	.000011	.000020	.000035	.000058	.000096	.000153	89
90	.000002	.000004	.000007	.000013	.000022	.000038	.000064	.000104	90

TABLE I

X	A=54	A=55	A=56	A=57	A=58	A=59	A=60	A=61	X
91	.000001	.000002	.000004	.000008	.000014	.000025	.000042	.000070	91
92	.000001	.000001	.000003	.000005	.000009	.000016	.000027	.000046	92
93		.000001	.000002	.000003	.000006	.000010	.000018	.000030	93
94			.000001	.000002	.000003	.000006	.000011	.000020	94
95			.000001	.000001	.000002	.000004	.000007	.000013	95
96				.000001	.000001	.000002	.000004	.000008	96
97					.000001	.000001	.000003	.000005	97
98						.000001	.000002	.000003	98
99						.000001	.000001	.000002	99
100							.000001	.000001	100
101								.000001	101

X	A=62	A=63	A=64	A=65	A=66	A=67	A=68	A=69	X
28	.000001								28
29	.000001	.000001							29
30	.000003	.000002	.000001	.000001					30
31	.000005	.000003	.000002	.000001	.000001				31
32	.000010	.000006	.000004	.000002	.000001	.000001			32
33	.000019	.000012	.000007	.000005	.000003	.000002	.000001	.000001	33
34	.000035	.000022	.000014	.000009	.000005	.000003	.000002	.000001	34
35	.000062	.000040	.000026	.000016	.000010	.000006	.000004	.000002	35
36	.000107	.000070	.000045	.000029	.000019	.000012	.000007	.000005	36
37	.000179	.000119	.000079	.000051	.000033	.000021	.000014	.000009	37
38	.000292	.000198	.000132	.000088	.000058	.000038	.000024	.000016	38
39	.000465	.000319	.000217	.000146	.000098	.000065	.000042	.000027	39
40	.000721	.000503	.000347	.000238	.000161	.000108	.000072	.000047	40
41	.001090	.000773	.000542	.000377	.000259	.000177	.000119	.000080	41
42	.001609	.001159	.000826	.000583	.000407	.000282	.000193	.000131	42
43	.002320	.001698	.001230	.000881	.000625	.000439	.000305	.000210	43
44	.003269	.002431	.001788	.001302	.000937	.000668	.000472	.000330	44
45	.004504	.003404	.002544	.001880	.001375	.000995	.000713	.000506	45
46	.006070	.004662	.003539	.002656	.001972	.001449	.001054	.000759	46
47	.008007	.006249	.004819	.003674	.002770	.002066	.001525	.001114	47
48	.010343	.008201	.006425	.004975	.003809	.002884	.002160	.001602	48
49	.013087	.010545	.008392	.006599	.005130	.003943	.002998	.002255	49
50	.016228	.013286	.010742	.008579	.006772	.005284	.004077	.003112	50
51	.019728	.016412	.013480	.010934	.008763	.006941	.005436	.004211	51
52	.023521	.019884	.016591	.013668	.011122	.008944	.007109	.005587	52
53	.027516	.023636	.020034	.016763	.013851	.011306	.009121	.007274	53
54	.031592	.027575	.023744	.020177	.016929	.014028	.011485	.009294	54
55	.035613	.031586	.027629	.023846	.020314	.017088	.014200	.011660	55
56	.039428	.035534	.031576	.027678	.023942	.020445	.017243	.014367	56
57	.042887	.039275	.035454	.031563	.027722	.024032	.020570	.017391	57
58	.045845	.042661	.039122	.035372	.031546	.027761	.024117	.020690	58
59	.048176	.045553	.042437	.038969	.035288	.031525	.027796	.024197	59
60	.049782	.047831	.045266	.042217	.038817	.035203	.031502	.027826	60
61	.050598	.049399	.047493	.044985	.041999	.038666	.035117	.031475	61
62	.050598	.050196	.049025	.047162	.044709	.041784	.038515	.035029	62
63	.049795	.050196	.049803	.048659	.046838	.044437	.041572	.038365	63
64	.048238	.049411	.049803	.049419	.048301	.046520	.044170	.041362	64
65	.046012	.047891	.049037	.049419	.049044	.047951	.046209	.043908	65
66	.043223	.045714	.047551	.048671	.049044	.048678	.047609	.045904	66
67	.039998	.042985	.045422	.047218	.048312	.048678	.048320	.047274	67
68	.036469	.039824	.042750	.045135	.046891	.047962	.048320	.047969	68
69	.032769	.036361	.039652	.042518	.044853	.046572	.047619	.047969	69
70	.029024	.032725	.036253	.039481	.042290	.044576	.046259	.047284	70
71	.025345	.029038	.032679	.036145	.039312	.042065	.044304	.045952	71
72	.021825	.025408	.029048	.032631	.036036	.039144	.041843	.044037	72
73	.018536	.021927	.025467	.029055	.032580	.035926	.038977	.041624	73
74	.015530	.018668	.022025	.025521	.029058	.032528	.035817	.038812	74
75	.012838	.015681	.018795	.022118	.025571	.029058	.032474	.035707	75
76	.010473	.012999	.015827	.018917	.022206	.025617	.029055	.032418	76
77	.008433	.010635	.013155	.015969	.019034	.022290	.025659	.029050	77
78	.006703	.008590	.010794	.013307	.016106	.019147	.022370	.025698	78
79	.005261	.006850	.008744	.010949	.013455	.016238	.019255	.022445	79
80	.004077	.005395	.006996	.008896	.011101	.013600	.016367	.019359	80

TABLE I

X	A=62	A=63	A=64	A=65	A=66	A=67	A=68	A=69	X
81	.003121	.004196	.005527	.007139	.009045	.011249	.013740	.016491	81
82	.002360	.003224	.004314	.005659	.007280	.009191	.011394	.013876	82
83	.001763	.002447	.003326	.004432	.005789	.007419	.009335	.011536	83
84	.001301	.001835	.002534	.003429	.004549	.005918	.007557	.009476	84
85	.000949	.001360	.001908	.002622	.003532	.004665	.006045	.007692	85
86	.000684	.000996	.001420	.001982	.002710	.003634	.004780	.006172	86
87	.000488	.000722	.001045	.001481	.002056	.002799	.003736	.004895	87
88	.000343	.000517	.000760	.001094	.001542	.002131	.002887	.003838	88
89	.000239	.000366	.000546	.000799	.001144	.001604	.002206	.002975	89
90	.000165	.000256	.000389	.000577	.000839	.001194	.001667	.002281	90
91	.000112	.000177	.000273	.000412	.000608	.000879	.001245	.001730	91
92	.000076	.000121	.000190	.000291	.000436	.000640	.000921	.001297	92
93	.000050	.000082	.000131	.000203	.000310	.000461	.000673	.000962	93
94	.000033	.000055	.000089	.000141	.000217	.000329	.000487	.000707	94
95	.000022	.000037	.000060	.000096	.000151	.000232	.000349	.000513	95
96	.000014	.000024	.000040	.000065	.000104	.000162	.000247	.000369	96
97	.000009	.000016	.000026	.000044	.000071	.000112	.000173	.000262	97
98	.000006	.000010	.000017	.000029	.000048	.000076	.000120	.000185	98
99	.000004	.000006	.000011	.000019	.000032	.000052	.000082	.000129	99
100	.000002	.000004	.000007	.000012	.000021	.000035	.000056	.000089	100
101	.000001	.000003	.000005	.000008	.000014	.000023	.000038	.000061	101
102	.000001	.000002	.000003	.000005	.000009	.000015	.000025	.000041	102
103		.000001	.000002	.000003	.000006	.000010	.000017	.000028	103
104		.000001	.000001	.000002	.000004	.000006	.000011	.000018	104
105			.000001	.000001	.000002	.000004	.000007	.000012	105
106				.000001	.000001	.000003	.000005	.000008	106
107					.000001	.000002	.000003	.000005	107
108					.000001	.000001	.000002	.000003	108
109						.000001	.000001	.000002	109
110							.000001	.000001	110
111								.000001	111

X	A=70	A=71	A=72	A=73	A=74	A=75	A=76	A=77	X
34	.000001								34
35	.000001	.000001	.000001						35
36	.000003	.000002	.000001	.000001					36
37	.000005	.000003	.000002	.000001	.000001				37
38	.000010	.000006	.000004	.000002	.000001	.000001	.000001		38
39	.000018	.000011	.000007	.000005	.000003	.000002	.000001	.000001	39
40	.000031	.000020	.000013	.000008	.000005	.000003	.000002	.000001	40
41	.000053	.000035	.000023	.000015	.000009	.000006	.000004	.000002	41
42	.000088	.000059	.000039	.000026	.000017	.000011	.000007	.000004	42
43	.000144	.000097	.000065	.000043	.000029	.000019	.000012	.000008	43
44	.000229	.000157	.000107	.000072	.000048	.000032	.000021	.000014	44
45	.000356	.000248	.000171	.000117	.000079	.000053	.000036	.000024	45
46	.000541	.000382	.000268	.000186	.000128	.000087	.000059	.000040	46
47	.000806	.000578	.000410	.000288	.000201	.000139	.000095	.000065	47
48	.001175	.000854	.000615	.000439	.000310	.000217	.000151	.000104	48
49	.001679	.001238	.000904	.000653	.000468	.000333	.000234	.000163	49
50	.002351	.001758	.001301	.000954	.000693	.000499	.000356	.000252	50
51	.003227	.002447	.001837	.001366	.001005	.000733	.000530	.000380	51
52	.004343	.003341	.002544	.001917	.001431	.001058	.000775	.000563	52
53	.005737	.004476	.003455	.002641	.001998	.001497	.001111	.000817	53
54	.007436	.005885	.004607	.003570	.002738	.002079	.001564	.001165	54
55	.009465	.007597	.006031	.004738	.003684	.002835	.002161	.001632	55
56	.011831	.009631	.007754	.006176	.004868	.003797	.002933	.002244	56
57	.014529	.011997	.009795	.007910	.006319	.004996	.003911	.003031	57
58	.017535	.014686	.012159	.009955	.008063	.006461	.005124	.004024	58
59	.020804	.017673	.014839	.012318	.010112	.008213	.006601	.005251	59
60	.024271	.020913	.017806	.014986	.012472	.010266	.008361	.006739	60
61	.027852	.024341	.021017	.017935	.015130	.012623	.010417	.008507	61
62	.031446	.027875	.024407	.021117	.018058	.015269	.012769	.010565	62
63	.034940	.031415	.027894	.024468	.021211	.018178	.015404	.012912	63
64	.038216	.034851	.031381	.027909	.024526	.021302	.018292	.015535	64
65	.041156	.038067	.034760	.031344	.027922	.024579	.021388	.018403	65
66	.043650	.040951	.037920	.034669	.031306	.027931	.024629	.021470	66

TABLE I

X	A=70	A=71	A=72	A=73	A=74	A=75	A=76	A=77	X
67	.045604	.043396	.040750	.037773	.034577	.031266	.027937	.024675	67
68	.046946	.045311	.043147	.040551	.037628	.034484	.031224	.027941	68
69	.047626	.046624	.045023	.042902	.040354	.037483	.034391	.031180	69
70	.047626	.047290	.046309	.044740	.042660	.040160	.037339	.034298	70
71	.046955	.047290	.046961	.046000	.044463	.042423	.039969	.037197	71
72	.045651	.046633	.046961	.046639	.045698	.044191	.042189	.039780	72
73	.043775	.045356	.046318	.046639	.046324	.045401	.043923	.041959	73
74	.041409	.043517	.045066	.046009	.046324	.046015	.045110	.043660	74
75	.038648	.041196	.043264	.044782	.045706	.046015	.045712	.044825	75
76	.035597	.038486	.040987	.043014	.044503	.045409	.045712	.045415	76
77	.032361	.035487	.038325	.040780	.042770	.044230	.045118	.045415	77
78	.029042	.032302	.035377	.038166	.040576	.042529	.043961	.044832	78
79	.025733	.029031	.032242	.035267	.038008	.040375	.042292	.043697	79
80	.022517	.025765	.029018	.032181	.035158	.037852	.040177	.042059	80
81	.019459	.022584	.025794	.029003	.032119	.035048	.037697	.039982	81
82	.016611	.019555	.022648	.025820	.028986	.032056	.034939	.037544	82
83	.014009	.016727	.019647	.022709	.025843	.028966	.031992	.034830	83
84	.011674	.014139	.016840	.019735	.022766	.025863	.028945	.031927	84
85	.009614	.011810	.014265	.016949	.019820	.022820	.025880	.028922	85
86	.007826	.009750	.011942	.014387	.017054	.019901	.022871	.025896	86
87	.006296	.007957	.009883	.012072	.014506	.017156	.019979	.022919	87
88	.005009	.006420	.008086	.010014	.012198	.014622	.017255	.020054	88
89	.003939	.005121	.006542	.008214	.010142	.012322	.014735	.017350	89
90	.003064	.004040	.005233	.006662	.008339	.010268	.012442	.014844	90
91	.002357	.003152	.004141	.005344	.006781	.008463	.010392	.012560	91
92	.001793	.002433	.003241	.004241	.005455	.006899	.008584	.010513	92
93	.001350	.001857	.002509	.003329	.004340	.005564	.007015	.008704	93
94	.001005	.001403	.001922	.002585	.003417	.004439	.005672	.007130	94
95	.000741	.001048	.001456	.001986	.002661	.003505	.004537	.005779	95
96	.000540	.000775	.001092	.001511	.002052	.002738	.003592	.004635	96
97	.000390	.000568	.000811	.001137	.001565	.002117	.002814	.003679	97
98	.000278	.000411	.000596	.000847	.001182	.001620	.002183	.002891	98
99	.000197	.000295	.000433	.000624	.000883	.001227	.001676	.002249	99
100	.000138	.000209	.000312	.000456	.000654	.000921	.001273	.001731	100
101	.000095	.000147	.000222	.000329	.000479	.000684	.000958	.001320	101
102	.000066	.000102	.000157	.000236	.000347	.000503	.000714	.000996	102
103	.000045	.000071	.000110	.000167	.000250	.000366	.000527	.000745	103
104	.000030	.000048	.000076	.000117	.000178	.000264	.000385	.000552	104
105	.000020	.000033	.000052	.000082	.000125	.000189	.000279	.000404	105
106	.000013	.000022	.000035	.000056	.000087	.000133	.000200	.000294	106
107	.000009	.000014	.000024	.000038	.000060	.000093	.000142	.000211	107
108	.000006	.000010	.000016	.000026	.000041	.000065	.000100	.000151	108
109	.000004	.000006	.000010	.000017	.000028	.000045	.000070	.000106	109
110	.000002	.000004	.000007	.000012	.000019	.000030	.000048	.000075	110
111	.000001	.000003	.000004	.000008	.000013	.000021	.000033	.000052	111
112	.000001	.000002	.000003	.000005	.000008	.000014	.000022	.000036	112
113	.000001	.000001	.000002	.000003	.000005	.000009	.000015	.000024	113
114		.000001	.000001	.000002	.000004	.000006	.000010	.000016	114
115			.000001	.000001	.000002	.000004	.000007	.000011	115
116				.000001	.000001	.000003	.000004	.000007	116
117				.000001	.000001	.000002	.000003	.000005	117
118					.000001	.000001	.000002	.000003	118
119						.000001	.000001	.000002	119
120							.000001	.000001	120
121								.000001	121
122								.000001	122

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X
40	.000001								40
41	.000002	.000001	.000001						41
42	.000003	.000002	.000001	.000001					42
43	.000005	.000003	.000002	.000001	.000001				43
44	.000009	.000006	.000004	.000002	.000001	.000001	.000001		44
45	.000016	.000010	.000007	.000004	.000003	.000002	.000001	.000001	45
46	.000026	.000017	.000011	.000007	.000005	.000003	.000002	.000001	46
47	.000044	.000029	.000019	.000013	.000008	.000005	.000004	.000002	47
48	.000071	.000048	.000032	.000022	.000014	.000009	.000006	.000004	48

TABLE I

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X
49	.000113	.000078	.000053	.000036	.000024	.000016	.000011	.000007	49
50	.000176	.000123	.000085	.000058	.000039	.000027	.000018	.000012	50
51	.000270	.000190	.000133	.000092	.000063	.000043	.000029	.000020	51
52	.000405	.000289	.000204	.000143	.000100	.000069	.000047	.000032	52
53	.000596	.000431	.000309	.000219	.000155	.000108	.000075	.000052	53
54	.000861	.000630	.000457	.000329	.000235	.000166	.000117	.000081	54
55	.001221	.000905	.000665	.000484	.000350	.000251	.000178	.000126	55
56	.001700	.001276	.000950	.000701	.000512	.000372	.000267	.000191	56
57	.002326	.001769	.000133	.000996	.000737	.000541	.000394	.000285	57
58	.003129	.002410	.000183	.001390	.001042	.000774	.000571	.000417	58
59	.004136	.003226	.000249	.001909	.001448	.001089	.000812	.000601	59
60	.005377	.004248	.003324	.002577	.001979	.001507	.001137	.000851	60
61	.006875	.005502	.004359	.003422	.002661	.002050	.001566	.001186	61
62	.008650	.007010	.005625	.004470	.003519	.002745	.002122	.001626	62
63	.010709	.008790	.007143	.005747	.004580	.003616	.002829	.002193	63
64	.013052	.010851	.008929	.007274	.005868	.004690	.003713	.002913	64
65	.015662	.013188	.010989	.009064	.007403	.005988	.004798	.003810	65
66	.018510	.015785	.013320	.011125	.009198	.007531	.006107	.004906	66
67	.021549	.018612	.015905	.013449	.011257	.009329	.007656	.006224	67
68	.024718	.021623	.018711	.016020	.013575	.011387	.009458	.007780	68
69	.027942	.024757	.021694	.018806	.016132	.013697	.011514	.009584	69
70	.031135	.027940	.024793	.021762	.018898	.016241	.013817	.011638	70
71	.034205	.031088	.027936	.024827	.021826	.018986	.016347	.013933	71
72	.037055	.034111	.031040	.027930	.024857	.021887	.019071	.016449	72
73	.039593	.036914	.034017	.030991	.027922	.024885	.021945	.019153	73
74	.041733	.039409	.036075	.033923	.030940	.027912	.024910	.022000	74
75	.043402	.041510	.039226	.036636	.033828	.030889	.027900	.024933	75
76	.044545	.043149	.041291	.039047	.036499	.033734	.030836	.027886	76
77	.045123	.044270	.042900	.041075	.038869	.036363	.033640	.030783	77
78	.045123	.044837	.044000	.042655	.040862	.038694	.036227	.033546	78
79	.044552	.044837	.044557	.043735	.042414	.040653	.038520	.036093	79
80	.043438	.044277	.044557	.044281	.043474	.042177	.040446	.038349	80
81	.041829	.043183	.044007	.044281	.044011	.043219	.041944	.040243	81
82	.039789	.041604	.042933	.043741	.044011	.043746	.042967	.041715	82
83	.037392	.039599	.041381	.042687	.043481	.043746	.043485	.042720	83
84	.034721	.037242	.039411	.041163	.042446	.043225	.043485	.043229	84
85	.031862	.034613	.037093	.039226	.040947	.042208	.042973	.043229	85
86	.028898	.031795	.034505	.036945	.039043	.040735	.041974	.042726	86
87	.025908	.028872	.031729	.034397	.036799	.038863	.040527	.041744	87
88	.022964	.025919	.028844	.031661	.034290	.036654	.038685	.040321	88
89	.020126	.023007	.025927	.028815	.031593	.034183	.036511	.038509	89
90	.017443	.020195	.023046	.025934	.028785	.031525	.034077	.036369	90
91	.014951	.017532	.020261	.023084	.025938	.028753	.031456	.033971	91
92	.012676	.015054	.017618	.020324	.023119	.025940	.028721	.031387	92
93	.010631	.012788	.015155	.017701	.020384	.023151	.025941	.028687	93
94	.008822	.010747	.012898	.015253	.017782	.020442	.023181	.025940	94
95	.007243	.008937	.010862	.013005	.015349	.017860	.020497	.023210	95
96	.005885	.007355	.009051	.010973	.013110	.015441	.017935	.020550	96
97	.004732	.005990	.007465	.009163	.011083	.013213	.015531	.018008	97
98	.003766	.004829	.006094	.007574	.009273	.011190	.013313	.015619	98
99	.002968	.003853	.004924	.006197	.007681	.009382	.011296	.013410	99
100	.002315	.003044	.003939	.005019	.006298	.007787	.009488	.011399	100
101	.001788	.002381	.003120	.004025	.005114	.006399	.007891	.009593	101
102	.001367	.001844	.002447	.003197	.004111	.005207	.006499	.007994	102
103	.001035	.001414	.001901	.002514	.003273	.004196	.005300	.006597	103
104	.000776	.001074	.001462	.001958	.002580	.003349	.004281	.005392	104
105	.000577	.000808	.001114	.001510	.002015	.002647	.003425	.004365	105
106	.000424	.000602	.000841	.001154	.001559	.002073	.002714	.003500	106
107	.000309	.000445	.000629	.000874	.001195	.001608	.002130	.002780	107
108	.000223	.000325	.000466	.000655	.000907	.001236	.001657	.002188	108
109	.000160	.000236	.000342	.000487	.000682	.000941	.001277	.001707	109
110	.000113	.000169	.000249	.000359	.000509	.000710	.000975	.001319	110
111	.000080	.000121	.000179	.000262	.000376	.000531	.000738	.001010	111
112	.000055	.000085	.000128	.000189	.000275	.000393	.000553	.000766	112
113	.000038	.000059	.000091	.000136	.000200	.000289	.000411	.000576	113
114	.000026	.000041	.000064	.000096	.000144	.000210	.000303	.000430	114
115	.000018	.000028	.000044	.000068	.000102	.000152	.000221	.000318	115

TABLE I

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X
116	.000012	.000019	.000030	.000047	.000072	.000109	.000160	.000233	116
117	.000008	.000013	.000021	.000033	.000051	.000077	.000115	.000169	117
118	.000005	.000009	.000014	.000023	.000035	.000054	.000082	.000122	118
119	.000003	.000006	.000010	.000015	.000024	.000038	.000058	.000087	119
120	.000002	.000004	.000006	.000010	.000017	.000026	.000040	.000062	120
121	.000001	.000002	.000004	.000007	.000011	.000018	.000028	.000043	121
122	.000001	.000002	.000003	.000005	.000008	.000012	.000019	.000030	122
123	.000001	.000001	.000002	.000003	.000005	.000008	.000013	.000021	123
124		.000001	.000001	.000002	.000003	.000006	.000009	.000014	124
125			.000001	.000001	.000002	.000004	.000006	.000010	125
126				.000001	.000001	.000002	.000004	.000007	126
127				.000001	.000001	.000002	.000003	.000004	127
128					.000001	.000001	.000002	.000003	128
129						.000001	.000001	.000002	129
130							.000001	.000001	130
131								.000001	131
132								.000001	132

X	A=86	A=87	A=88	A=89	A=90	A=91	A=92	A=93	X
46	.000001								46
47	.000001	.000001	.000001						47
48	.000003	.000002	.000001	.000001					48
49	.000005	.000003	.000002	.000001	.000001				49
50	.000008	.000005	.000003	.000002	.000001	.000001	.000001		50
51	.000013	.000009	.000006	.000004	.000002	.000002	.000001	.000001	51
52	.000022	.000015	.000010	.000006	.000004	.000003	.000002	.000001	52
53	.000035	.000024	.000016	.000011	.000007	.000005	.000003	.000002	53
54	.000056	.000039	.000026	.000018	.000012	.000008	.000005	.000004	54
55	.000088	.000061	.000042	.000029	.000020	.000013	.000009	.000006	55
56	.000135	.000095	.000066	.000046	.000032	.000022	.000015	.000010	56
57	.000204	.000145	.000102	.000072	.000050	.000034	.000024	.000016	57
58	.000302	.000217	.000155	.000110	.000077	.000054	.000037	.000026	58
59	.000441	.000321	.000231	.000166	.000118	.000083	.000058	.000041	59
60	.000632	.000465	.000340	.000246	.000177	.000126	.000090	.000063	60
61	.000890	.000663	.000490	.000359	.000261	.000188	.000135	.000096	61
62	.001235	.000930	.000695	.000515	.000379	.000277	.000200	.000144	62
63	.001686	.001285	.000971	.000728	.000541	.000400	.000293	.000213	63
64	.002265	.001747	.001335	.001012	.000761	.000568	.000421	.000309	64
65	.002997	.002338	.001808	.001386	.001054	.000795	.000595	.000442	65
66	.003906	.003082	.002410	.001869	.001438	.001097	.000830	.000623	66
67	.005013	.004001	.003166	.002483	.001931	.001489	.001140	.000865	67
68	.006340	.005120	.004097	.003250	.002556	.001993	.001542	.001183	68
69	.007902	.006455	.005225	.004192	.003334	.002629	.002056	.001595	69
70	.009709	.008023	.006569	.005330	.004286	.003417	.002002	.002118	70
71	.011760	.009831	.008141	.006681	.005433	.004380	.003501	.002775	71
72	.014046	.011879	.009950	.008258	.006791	.005536	.004473	.003584	72
73	.016548	.014157	.011995	.010068	.008373	.006901	.005638	.004566	73
74	.019231	.016644	.014264	.012109	.010183	.008486	.007009	.005739	74
75	.022052	.019307	.016737	.014369	.012220	.010297	.008598	.007116	75
76	.024954	.022101	.019380	.016827	.014471	.012329	.010408	.008707	76
77	.027870	.024972	.022148	.019449	.016914	.014570	.012435	.010517	77
78	.030729	.027853	.024987	.022192	.019516	.016999	.014667	.012539	78
79	.033451	.030673	.027834	.025001	.022234	.019581	.017081	.014761	79
80	.035960	.033357	.030618	.027814	.025013	.022273	.019643	.017160	80
81	.038180	.035828	.033264	.030561	.027792	.025023	.022310	.019702	81
82	.040042	.038013	.035698	.033170	.030504	.027770	.025031	.022345	82
83	.041490	.039845	.037848	.035568	.033077	.030446	.027746	.025038	83
84	.042478	.041268	.039650	.037685	.035439	.032983	.030388	.027720	84
85	.042977	.042239	.041050	.039458	.037524	.035312	.032890	.030329	85
86	.042977	.042730	.042004	.040835	.039269	.037365	.035185	.032798	86
87	.042483	.042730	.042487	.041774	.040623	.039083	.037207	.035060	87
88	.041518	.042245	.042487	.042248	.041546	.040415	.038899	.037052	88
89	.040118	.041295	.042010	.042248	.042013	.041323	.040210	.038717	89
90	.038335	.039919	.041076	.041779	.042013	.041782	.041103	.040008	90
91	.036229	.038164	.039722	.040861	.041552	.041782	.041555	.040887	91
92	.033866	.036090	.037995	.039528	.040648	.041328	.041555	.041331	92
93	.031317	.033762	.035952	.037828	.039337	.040439	.041108	.041331	93

TABLE I

X	A=86	A=87	A=88	A=89	A=90	A=91	A=92	A=93	X
94	.028652	.031247	.033657	.035816	.037663	.039149	.040233	.040892	94
95	.025937	.028616	.031177	.033554	.035681	.037500	.038963	.040031	95
96	.023236	.025933	.028579	.031107	.033451	.035547	.037339	.038780	96
97	.020601	.023260	.025928	.028542	.031037	.033348	.035415	.037181	97
98	.018078	.020649	.023282	.025920	.028503	.030966	.033247	.035284	98
99	.015704	.018146	.020695	.023302	.025912	.028464	.030896	.033145	99
100	.013506	.015787	.018212	.020739	.023321	.025902	.028424	.030825	100
101	.011500	.013599	.015868	.018275	.020781	.023338	.025891	.028384	101
102	.009696	.011599	.013690	.015946	.018336	.020821	.023353	.025879	102
103	.008096	.009797	.011696	.013778	.016022	.018395	.020859	.023367	103
104	.006694	.008196	.009897	.011791	.013865	.016096	.018452	.020895	104
105	.005483	.006791	.008294	.009994	.011884	.013950	.016168	.018507	105
106	.004449	.005574	.006886	.008392	.010090	.011976	.014032	.016237	106
107	.003575	.004532	.005663	.006980	.008487	.010185	.012065	.014113	107
108	.002847	.003651	.004614	.005752	.007073	.008582	.010278	.012153	108
109	.002246	.002914	.003725	.004697	.005840	.007165	.008675	.010369	109
110	.001756	.002305	.002980	.003800	.004778	.005927	.007255	.008766	110
111	.001361	.001806	.002363	.003047	.003874	.004859	.006013	.007345	111
112	.001045	.001403	.001856	.002421	.003113	.003948	.004940	.006099	112
113	.000795	.001080	.001446	.001907	.002479	.003179	.004022	.005019	113
114	.000600	.000824	.001116	.001489	.001957	.002538	.003245	.004095	114
115	.000449	.000624	.000854	.001152	.001532	.002008	.002596	.003311	115
116	.000333	.000468	.000648	.000884	.001189	.001575	.002059	.002655	116
117	.000244	.000348	.000487	.000672	.000914	.001225	.001619	.002110	117
118	.000178	.000256	.000363	.000507	.000697	.000945	.001262	.001663	118
119	.000129	.000187	.000269	.000379	.000527	.000723	.000976	.001300	119
120	.000092	.000136	.000197	.000281	.000396	.000548	.000748	.001007	120
121	.000066	.000098	.000143	.000207	.000294	.000412	.000569	.000774	121
122	.000046	.000070	.000103	.000151	.000217	.000307	.000429	.000590	122
123	.000032	.000049	.000074	.000109	.000159	.000227	.000321	.000446	123
124	.000022	.000035	.000052	.000078	.000115	.000167	.000238	.000335	124
125	.000015	.000024	.000037	.000056	.000083	.000122	.000175	.000249	125
126	.000011	.000017	.000026	.000039	.000059	.000088	.000128	.000184	126
127	.000007	.000011	.000018	.000028	.000042	.000063	.000093	.000135	127
128	.000005	.000008	.000012	.000019	.000030	.000045	.000067	.000098	128
129	.000003	.000005	.000008	.000013	.000021	.000032	.000048	.000070	129
130	.000002	.000003	.000006	.000009	.000014	.000022	.000034	.000050	130
131	.000001	.000002	.000004	.000006	.000010	.000015	.000024	.000036	131
132	.000001	.000002	.000003	.000004	.000007	.000011	.000016	.000025	132
133	.000001	.000001	.000002	.000003	.000005	.000007	.000011	.000018	133
134		.000001	.000001	.000002	.000003	.000005	.000008	.000012	134
135			.000001	.000001	.000002	.000003	.000005	.000008	135
136				.000001	.000001	.000002	.000004	.000006	136
137				.000001	.000001	.000001	.000002	.000004	137
138					.000001	.000001	.000002	.000003	138
139						.000001	.000001	.000002	139
140							.000001	.000001	140
141								.000001	141
142								.000001	142

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
52	.000001								52
53	.000001	.000001	.000001						53
54	.000002	.000001	.000001	.000001					54
55	.000004	.000003	.000002	.000001	.000001				55
56	.000007	.000004	.000003	.000002	.000001	.000001	.000001		56
57	.000011	.000007	.000005	.000003	.000002	.000001	.000001	.000001	57
58	.000018	.000012	.000008	.000005	.000004	.000002	.000002	.000001	58
59	.000028	.000019	.000013	.000009	.000006	.000004	.000003	.000002	59
60	.000044	.000031	.000021	.000014	.000010	.000007	.000004	.000003	60
61	.000068	.000048	.000033	.000023	.000016	.000011	.000007	.000005	61
62	.000103	.000073	.000051	.000036	.000025	.000017	.000012	.000008	62
63	.000154	.000110	.000078	.000055	.000039	.000027	.000019	.000013	63
64	.000225	.000163	.000117	.000084	.000059	.000042	.000029	.000020	64
65	.000326	.000239	.000173	.000125	.000090	.000064	.000045	.000032	65
66	.000464	.000343	.000252	.000184	.000133	.000096	.000068	.000048	66

TABLE I

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
67	.000652	.000487	.000361	.000266	.000195	.000141	.000102	.000073	67
68	.000901	.000680	.000510	.000380	.000281	.000206	.000150	.000109	68
69	.001227	.000937	.000710	.000534	.000399	.000295	.000217	.000159	69
70	.001648	.001271	.000973	.000740	.000558	.000418	.000311	.000229	70
71	.002181	.001701	.001316	.001011	.000770	.000582	.000437	.000326	71
72	.002848	.002245	.001755	.001351	.001048	.000801	.000608	.000458	72
73	.003667	.002921	.002308	.001809	.001407	.001086	.000832	.000633	73
74	.004658	.003750	.002994	.002371	.001863	.001453	.001125	.000864	74
75	.005838	.004750	.003832	.003067	.002435	.001918	.001499	.001163	75
76	.007221	.005937	.004841	.003914	.003140	.002499	.001973	.001546	76
77	.008816	.007325	.006035	.004931	.003996	.003212	.002562	.002028	77
78	.010624	.008922	.007428	.006132	.005021	.004077	.003285	.002626	78
79	.012641	.010729	.009027	.007529	.006228	.005110	.004158	.003357	79
80	.014853	.012740	.010832	.009129	.007630	.006323	.005198	.004239	80
81	.017237	.014943	.012838	.010933	.009231	.007728	.006417	.005285	81
82	.019760	.017311	.015029	.012933	.011032	.009331	.007826	.006510	82
83	.022378	.019814	.017384	.015114	.013026	.011129	.009429	.007922	83
84	.025042	.022409	.019867	.017453	.015197	.013117	.011225	.009525	84
85	.027694	.025045	.022438	.019917	.017521	.015277	.013205	.011318	85
86	.030270	.027666	.025047	.022465	.019966	.017586	.015355	.013292	86
87	.032706	.030210	.027638	.025047	.022490	.020012	.017649	.015431	87
88	.034935	.032614	.030151	.027609	.025046	.022513	.020056	.017711	88
89	.036898	.034812	.032522	.030090	.027578	.025043	.022535	.020099	89
90	.038538	.036746	.034690	.032431	.030030	.027547	.025039	.022555	90
91	.039809	.038361	.036596	.034569	.032340	.029969	.027515	.025034	91
92	.040674	.039612	.038187	.036448	.034449	.032249	.029908	.027483	92
93	.041111	.040464	.039419	.038015	.036301	.034330	.032159	.029847	93
94	.041111	.040895	.040258	.039229	.037846	.036156	.034212	.032069	94
95	.040679	.040895	.040682	.040054	.039041	.037678	.036012	.034095	95
96	.039831	.040469	.040682	.040472	.039854	.038856	.037513	.035871	96
97	.038599	.039634	.040262	.040472	.040265	.039657	.038673	.037350	97
98	.037024	.038421	.039440	.040059	.040265	.040061	.039462	.038493	98
99	.035154	.036869	.038245	.039249	.039858	.040061	.039861	.039271	99
100	.033045	.035025	.036715	.038072	.039061	.039661	.039861	.039663	100
101	.030754	.032945	.034898	.036564	.037901	.038875	.039466	.039663	101
102	.028342	.030684	.032845	.034772	.036415	.037732	.038692	.039275	102
103	.025866	.028300	.030613	.032746	.034647	.036267	.037566	.038512	103
104	.023379	.025851	.028258	.030542	.032648	.034523	.036121	.037401	104
105	.020929	.023389	.025836	.028215	.030471	.032550	.034401	.035976	105
106	.018560	.020962	.023399	.025820	.028172	.030401	.032453	.034279	106
107	.016305	.018611	.020993	.023406	.025802	.028128	.030330	.032357	107
108	.014192	.016371	.018661	.021022	.023413	.025784	.028084	.030260	108
109	.012239	.014268	.016435	.018708	.021050	.023418	.025765	.028039	109
110	.010458	.012323	.014343	.016497	.018754	.021077	.023423	.025745	110
111	.008857	.010546	.012405	.014416	.016557	.018798	.021101	.023426	111
112	.007433	.008946	.010633	.012486	.014488	.016616	.018841	.021125	112
113	.006183	.007521	.009033	.010718	.012565	.014557	.016673	.018881	113
114	.005099	.006267	.007607	.009120	.010801	.012642	.014625	.016728	114
115	.004168	.005177	.006350	.007692	.009204	.010883	.012718	.014692	115
116	.003377	.004240	.005255	.006432	.007776	.009288	.010964	.012792	116
117	.002713	.003443	.004312	.005333	.006513	.007859	.009371	.011043	117
118	.002161	.002772	.003508	.004384	.005409	.006594	.007941	.009452	118
119	.001707	.002213	.002830	.003573	.004455	.005486	.006673	.008022	119
120	.001337	.001752	.002264	.002888	.003638	.004526	.005561	.006752	120
121	.001039	.001375	.001796	.002315	.002947	.003703	.004596	.005636	121
122	.000801	.001071	.001413	.001841	.002367	.003005	.003767	.004666	122
123	.000612	.000827	.001103	.001452	.001886	.002418	.003063	.003831	123
124	.000464	.000634	.000854	.001136	.001490	.001931	.002470	.003121	124
125	.000349	.000482	.000656	.000881	.001168	.001529	.001976	.002521	125
126	.000260	.000363	.000500	.000678	.000909	.001202	.001568	.002021	126
127	.000193	.000272	.000378	.000518	.000701	.000937	.001235	.001607	127
128	.000141	.000202	.000283	.000393	.000537	.000724	.000965	.001268	128
129	.000103	.000148	.000211	.000295	.000408	.000556	.000748	.000993	129
130	.000075	.000108	.000156	.000220	.000308	.000423	.000575	.000771	130
131	.000053	.000079	.000114	.000163	.000230	.000320	.000439	.000595	131
132	.000038	.000057	.000083	.000120	.000171	.000240	.000333	.000455	132
133	.000027	.000040	.000060	.000087	.000126	.000179	.000250	.000346	133
134	.000019	.000029	.000043	.000063	.000092	.000132	.000187	.000260	134

TABLE I.

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
135	.000013	.000020	.000031	.000045	.000067	.000097	.000138	.000195	135
136	.000009	.000014	.000022	.000032	.000048	.000070	.000102	.000145	136
137	.000006	.000010	.000015	.000023	.000034	.000051	.000074	.000107	137
138	.000004	.000007	.000011	.000016	.000024	.000037	.000054	.000078	138
139	.000003	.000005	.000007	.000011	.000017	.000026	.000039	.000057	139
140	.000002	.000003	.000005	.000008	.000012	.000018	.000028	.000041	140
141	.000001	.000002	.000003	.000005	.000008	.000013	.000020	.000029	141
142	.000001	.000001	.000002	.000004	.000006	.000009	.000014	.000021	142
143	.000001	.000001	.000002	.000002	.000004	.000006	.000010	.000015	143
144		.000001	.000001	.000002	.000003	.000004	.000007	.000010	144
145			.000001	.000001	.000002	.000003	.000005	.000007	145
146				.000001	.000001	.000002	.000003	.000005	146
147					.000001	.000001	.000002	.000003	147
148					.000001	.000001	.000001	.000002	148
149						.000001	.000001	.000002	149
150							.000001	.000001	150
151								.000001	151

X	A=102	A=103	A=104	A=105	A=106	A=107	A=108	A=109	X
58	.000001								58
59	.000001	.000001							59
60	.000002	.000001	.000001	.000001					60
61	.000003	.000002	.000001	.000001	.000001				61
62	.000005	.000004	.000002	.000002	.000001	.000001			62
63	.000009	.000006	.000004	.000003	.000002	.000001	.000001	.000001	63
64	.000014	.000010	.000007	.000004	.000003	.000002	.000001	.000001	64
65	.000022	.000015	.000011	.000007	.000005	.000003	.000002	.000002	65
66	.000034	.000024	.000017	.000012	.000008	.000005	.000004	.000002	66
67	.000052	.000037	.000026	.000018	.000013	.000009	.000006	.000004	67
68	.000078	.000056	.000040	.000028	.000020	.000014	.000009	.000006	68
69	.000115	.000083	.000060	.000042	.000030	.000021	.000015	.000010	69
70	.000168	.000122	.000089	.000064	.000045	.000032	.000023	.000016	70
71	.000241	.000178	.000130	.000094	.000068	.000049	.000035	.000025	71
72	.000342	.000254	.000187	.000137	.000100	.000072	.000052	.000037	72
73	.000478	.000358	.000267	.000197	.000145	.000106	.000077	.000055	73
74	.000659	.000499	.000375	.000280	.000208	.000153	.000112	.000082	74
75	.000896	.000685	.000520	.000392	.000294	.000219	.000162	.000119	75
76	.001203	.000929	.000712	.000542	.000410	.000308	.000230	.000170	76
77	.001593	.001242	.000962	.000739	.000564	.000428	.000322	.000241	77
78	.002083	.001640	.001282	.000995	.000767	.000587	.000446	.000337	78
79	.002690	.002139	.001688	.001322	.001029	.000795	.000610	.000464	79
80	.003430	.002754	.002194	.001736	.001363	.001063	.000823	.000633	80
81	.004319	.003502	.002817	.002250	.001784	.001404	.001097	.000852	81
82	.005372	.004398	.003573	.002881	.002306	.001832	.001445	.001132	82
83	.006602	.005458	.004478	.003645	.002945	.002362	.001880	.001487	83
84	.008017	.006693	.005544	.004556	.003716	.003008	.002418	.001929	84
85	.009620	.008110	.006783	.005628	.004634	.003787	.003072	.002474	85
86	.011410	.009713	.008202	.006872	.005712	.004712	.003858	.003135	86
87	.013377	.011500	.009805	.008293	.006959	.005795	.004789	.003928	87
88	.015505	.013460	.011588	.009895	.008383	.007046	.005877	.004866	88
89	.017770	.015577	.013541	.011674	.009984	.008471	.007132	.005959	89
90	.020139	.017827	.015647	.013620	.011759	.010072	.008559	.007217	90
91	.022574	.020178	.017883	.015715	.013697	.011842	.010157	.008644	91
92	.025027	.022591	.020215	.017936	.015782	.013773	.011924	.010242	92
93	.027449	.025020	.022606	.020251	.017988	.015847	.013847	.012004	93
94	.029785	.027415	.025011	.022620	.020284	.018038	.015909	.013919	94
95	.031980	.029724	.027381	.025001	.022633	.020317	.018086	.015971	95
96	.033979	.031891	.029662	.027345	.024991	.022644	.020347	.018133	96
97	.035730	.033864	.031803	.029600	.027309	.024979	.022655	.020376	97
98	.037189	.035592	.033750	.031715	.029539	.027273	.024966	.022664	98
99	.038316	.037030	.035454	.033637	.031627	.029477	.027236	.024953	99
100	.039082	.038141	.036873	.035319	.033525	.031540	.029415	.027199	100
101	.039469	.038896	.037968	.036718	.035185	.033414	.031454	.029353	101
102	.039469	.039277	.038712	.037797	.036564	.035052	.033304	.031367	102
103	.039086	.039277	.039088	.038531	.037629	.036413	.034920	.033195	103
104	.038334	.038899	.039088	.038902	.038353	.037463	.036264	.034791	104
105	.037239	.038159	.038716	.038902	.038718	.038177	.037300	.036116	105

TABLE I

X	A=102	A=103	A=104	A=105	A=106	A=107	A=108	A=109	X
106	.035834	.037079	.037985	.038535	.038718	.038537	.038003	.037138	106
107	.034159	.035692	.036920	.037815	.038356	.038537	.038359	.037832	107
108	.032261	.034040	.035553	.036764	.037646	.038180	.038359	.038183	108
109	.030190	.032166	.033922	.035415	.036610	.037480	.038007	.038183	109
110	.027994	.030119	.032072	.033805	.035279	.036458	.037316	.037835	110
111	.025724	.027949	.030049	.031978	.033690	.035144	.036307	.037154	111
112	.023427	.025703	.027903	.029979	.031885	.033575	.035010	.036159	112
113	.021147	.023428	.025681	.027857	.029910	.031792	.033461	.034879	113
114	.018921	.021168	.023428	.025658	.027811	.029840	.031700	.033349	114
115	.016782	.018959	.021187	.023427	.025634	.027764	.029771	.031609	115
116	.014757	.016834	.018995	.021205	.023424	.025610	.027717	.029701	116
117	.012865	.014820	.016885	.019030	.021222	.023421	.025585	.027671	117
118	.011120	.012936	.014881	.016934	.019064	.021238	.023417	.025560	118
119	.009532	.011197	.013006	.014941	.016981	.019096	.021253	.023412	119
120	.008102	.009610	.011271	.013074	.015000	.017027	.019127	.021266	120
121	.006830	.008181	.009688	.011345	.013141	.015057	.017072	.019157	121
122	.005710	.006907	.008259	.009764	.011417	.013206	.015113	.017116	122
123	.004735	.005784	.006983	.008335	.009839	.011488	.013270	.015168	123
124	.003895	.004804	.005857	.007058	.008411	.009913	.011558	.013333	124
125	.003178	.003959	.004873	.005929	.007132	.008486	.009986	.011626	125
126	.002573	.003236	.004022	.004941	.006000	.007206	.008559	.010058	126
127	.002066	.002624	.003293	.004085	.005008	.006071	.007279	.008632	127
128	.001647	.002112	.002676	.003351	.004147	.005075	.006142	.007351	128
129	.001302	.001686	.002157	.002727	.003408	.004210	.005142	.006211	129
130	.001022	.001336	.001726	.002203	.002779	.003465	.004272	.005208	130
131	.000795	.001050	.001370	.001766	.002248	.002830	.003522	.004333	131
132	.000615	.000820	.001080	.001405	.001806	.002294	.002881	.003578	132
133	.000471	.000635	.000844	.001109	.001439	.001846	.002340	.002932	133
134	.000359	.000488	.000655	.000869	.001138	.001474	.001886	.002385	134
135	.000271	.000372	.000505	.000676	.000894	.001168	.001509	.001926	135
136	.000203	.000282	.000386	.000522	.000697	.000919	.001198	.001544	136
137	.000151	.000212	.000293	.000400	.000539	.000718	.000944	.001228	137
138	.000112	.000158	.000221	.000304	.000414	.000557	.000739	.000970	138
139	.000082	.000117	.000165	.000230	.000316	.000428	.000574	.000761	139
140	.000060	.000086	.000123	.000172	.000239	.000327	.000443	.000592	140
141	.000043	.000063	.000091	.000128	.000180	.000248	.000339	.000458	141
142	.000031	.000046	.000066	.000095	.000134	.000187	.000258	.000351	142
143	.000022	.000033	.000048	.000070	.000099	.000140	.000195	.000268	143
144	.000016	.000024	.000035	.000051	.000073	.000104	.000146	.000203	144
145	.000011	.000017	.000025	.000037	.000054	.000077	.000109	.000152	145
146	.000008	.000012	.000018	.000026	.000039	.000056	.000081	.000114	146
147	.000005	.000008	.000013	.000019	.000028	.000041	.000059	.000084	147
148	.000004	.000006	.000009	.000013	.000020	.000030	.000043	.000062	148
149	.000003	.000004	.000006	.000009	.000014	.000021	.000031	.000045	149
150	.000002	.000003	.000004	.000007	.000010	.000015	.000023	.000033	150
151	.000001	.000002	.000003	.000005	.000007	.000011	.000016	.000024	151
152	.000001	.000001	.000002	.000003	.000005	.000008	.000011	.000017	152
153	.000001	.000001	.000001	.000002	.000003	.000005	.000008	.000012	153
154		.000001	.000001	.000001	.000002	.000004	.000006	.000009	154
155			.000001	.000001	.000002	.000003	.000004	.000006	155
156				.000001	.000001	.000002	.000003	.000004	156
157					.000001	.000001	.000002	.000003	157
158						.000001	.000001	.000002	158
159						.000001	.000001	.000001	159
160							.000001	.000001	160
161								.000001	161

X	A=110	A=111	A=112	A=113	A=114	A=115	A=116	A=117	X
64	.000001								64
65	.000001	.000001							65
66	.000002	.000001	.000001						66
67	.000003	.000002	.000001	.000001	.000001				67
68	.000004	.000003	.000002	.000001	.000001	.000001			68
69	.000007	.000005	.000003	.000002	.000002	.000001	.000001		69
70	.000011	.000008	.000005	.000004	.000002	.000002	.000001	.000001	70
71	.000017	.000012	.000008	.000006	.000004	.000003	.000002	.000001	71

TABLE I

X	A=110	A=111	A=112	A=113	A=114	A=115	A=116	A=117	X
72	.000026	.000019	.000013	.000009	.000006	.000004	.000003	.000002	72
73	.000040	.000028	.000020	.000014	.000010	.000007	.000005	.000003	73
74	.000059	.000042	.000030	.000022	.000015	.000011	.000007	.000005	74
75	.000087	.000063	.000045	.000032	.000023	.000016	.000012	.000008	75
76	.000125	.000092	.000067	.000048	.000035	.000025	.000018	.000012	76
77	.000179	.000132	.000097	.000071	.000051	.000037	.000026	.000019	77
78	.000252	.000188	.000139	.000103	.000075	.000055	.000039	.000028	78
79	.000352	.000264	.000198	.000147	.000108	.000079	.000058	.000042	79
80	.000483	.000367	.000277	.000207	.000154	.000114	.000084	.000061	80
81	.000656	.000503	.000382	.000289	.000217	.000162	.000120	.000089	81
82	.000881	.000680	.000522	.000398	.000302	.000227	.000170	.000126	82
83	.001167	.000910	.000705	.000542	.000414	.000315	.000238	.000178	83
84	.001528	.001202	.000940	.000729	.000562	.000431	.000328	.000248	84
85	.001978	.001570	.001238	.000970	.000754	.000583	.000448	.000342	85
86	.002530	.002027	.001612	.001274	.001000	.000780	.000604	.000465	86
87	.003199	.002586	.002076	.001655	.001310	.001030	.000805	.000625	87
88	.003998	.003262	.002642	.002125	.001697	.001347	.001061	.000831	88
89	.004942	.004068	.003325	.002698	.002174	.001740	.001383	.001092	89
90	.006040	.005017	.004137	.003387	.002754	.002223	.001783	.001420	90
91	.007301	.006120	.005092	.004206	.003450	.002810	.002273	.001826	91
92	.008729	.007383	.006199	.005166	.004275	.003512	.002866	.002322	92
93	.010325	.008813	.007465	.006277	.005240	.004343	.003574	.002921	93
94	.012082	.010406	.008895	.007546	.006355	.005313	.004411	.003636	94
95	.013990	.012159	.010486	.008976	.007626	.006432	.005386	.004478	95
96	.016030	.014059	.012234	.010565	.009056	.007705	.006508	.005458	96
97	.018178	.016088	.014126	.012308	.010643	.009134	.007783	.006583	97
98	.020404	.018222	.016144	.014192	.012380	.010719	.009212	.007859	98
99	.022671	.020431	.018264	.016199	.014256	.012451	.010794	.009288	99
100	.024939	.022678	.020456	.018305	.016252	.014319	.012521	.010867	100
101	.027161	.024923	.022684	.020480	.018344	.016304	.014380	.012589	101
102	.029291	.027123	.024908	.022688	.020502	.018382	.016354	.014440	102
103	.031282	.029229	.027084	.024891	.022692	.020523	.018418	.016403	103
104	.033086	.031197	.029167	.027045	.024804	.022694	.020544	.018454	104
105	.034662	.032979	.031112	.029105	.027006	.024856	.022696	.020562	105
106	.035970	.034535	.032873	.031028	.029044	.026966	.024837	.022696	106
107	.036979	.035826	.034409	.032767	.030944	.028982	.026926	.024818	107
108	.037663	.036821	.035683	.034284	.032663	.030860	.028920	.026886	108
109	.038009	.037497	.036665	.035543	.034161	.032559	.030778	.028859	109
110	.038009	.037838	.037332	.036512	.035403	.034039	.032456	.030695	110
111	.037666	.037838	.037668	.037170	.036360	.035266	.033918	.032355	111
112	.036994	.037500	.037668	.037502	.037010	.036210	.035130	.033799	112
113	.036012	.036836	.037335	.037502	.037337	.036851	.036062	.034995	113
114	.034748	.035867	.036680	.037173	.037337	.037175	.036695	.035916	114
115	.033237	.034619	.035723	.036526	.037012	.037175	.037014	.036541	115
116	.031518	.033127	.034491	.035582	.036374	.036854	.037014	.036856	116
117	.029632	.031428	.033017	.034365	.035442	.036224	.036698	.036856	117
118	.027623	.029564	.031339	.032909	.034240	.035303	.036076	.036544	118
119	.025534	.027576	.029495	.031250	.032801	.034117	.035166	.035929	119
120	.023406	.025508	.027529	.029427	.031161	.032695	.033994	.035031	120
121	.021279	.023400	.025481	.027481	.029359	.031074	.032589	.033873	121
122	.019186	.021290	.023393	.025454	.027434	.029291	.030987	.032485	122
123	.017158	.019213	.021301	.023384	.025426	.027386	.029223	.030900	123
124	.015221	.017199	.019239	.021310	.023376	.025398	.027338	.029156	124
125	.013394	.015272	.017238	.019264	.021319	.023366	.025370	.027290	125
126	.011693	.013454	.015323	.017277	.019288	.021326	.023356	.025341	126
127	.010128	.011759	.013513	.015302	.017314	.019311	.021333	.023345	127
128	.008704	.010197	.011824	.013571	.015420	.017350	.019333	.021339	128
129	.007422	.008775	.010266	.011888	.013627	.015467	.017385	.019354	129
130	.006280	.007492	.008844	.010333	.011950	.013682	.015513	.017419	130
131	.005273	.006348	.007562	.008913	.010399	.012011	.013736	.015557	131
132	.004394	.005338	.006416	.007630	.008981	.010464	.012071	.013789	132
133	.003634	.004455	.005403	.006483	.007698	.009048	.010528	.012130	133
134	.002984	.003691	.004516	.005467	.006549	.007765	.009114	.010591	134
135	.002431	.003034	.003746	.004576	.005530	.006615	.007831	.009179	135
136	.001966	.002477	.003085	.003802	.004636	.005593	.006680	.007897	136
137	.001579	.002007	.002522	.003136	.003857	.004695	.005656	.006744	137
138	.001258	.001614	.002047	.002568	.003187	.003913	.004754	.005718	138
139	.000996	.001289	.001649	.002088	.002613	.003237	.003967	.004813	139

TABLE I

X	A=110	A=111	A=112	A=113	A=114	A=115	A=116	A=117	X
140	.000782	.001022	.001320	.001685	.002128	.002659	.003287	.004022	140
141	.000610	.000804	.001048	.001350	.001721	.002169	.002705	.003338	141
142	.000473	.000629	.000827	.001075	.001381	.001756	.002209	.002750	142
143	.000364	.000488	.000647	.000849	.001101	.001412	.001792	.002250	143
144	.000278	.000376	.000504	.000666	.000872	.001128	.001444	.001828	144
145	.000211	.000288	.000389	.000519	.000685	.000895	.001155	.001475	145
146	.000159	.000219	.000298	.000402	.000535	.000705	.000918	.001182	146
147	.000119	.000165	.000227	.000309	.000415	.000551	.000724	.000941	147
148	.000088	.000124	.000172	.000236	.000320	.000428	.000568	.000744	148
149	.000065	.000092	.000129	.000179	.000245	.000331	.000442	.000584	149
150	.000048	.000068	.000097	.000135	.000186	.000253	.000342	.000456	150
151	.000035	.000050	.000072	.000101	.000140	.000193	.000262	.000353	151
152	.000025	.000037	.000053	.000075	.000105	.000146	.000200	.000272	152
153	.000018	.000027	.000039	.000055	.000078	.000110	.000152	.000208	153
154	.000013	.000019	.000028	.000041	.000058	.000082	.000114	.000158	154
155	.000009	.000014	.000020	.000030	.000043	.000061	.000086	.000119	155
156	.000006	.000010	.000015	.000021	.000031	.000045	.000064	.000089	156
157	.000005	.000007	.000010	.000015	.000023	.000033	.000047	.000067	157
158	.000003	.000005	.000007	.000011	.000016	.000024	.000035	.000049	158
159	.000002	.000003	.000005	.000008	.000012	.000017	.000025	.000036	159
160	.000001	.000002	.000004	.000006	.000008	.000012	.000018	.000027	160
161	.000001	.000002	.000003	.000004	.000006	.000009	.000013	.000019	161
162	.000001	.000001	.000002	.000003	.000004	.000006	.000009	.000014	162
163		.000001	.000001	.000002	.000003	.000004	.000007	.000010	163
164		.000001	.000001	.000001	.000002	.000003	.000005	.000007	164
165			.000001	.000001	.000001	.000002	.000003	.000005	165
166				.000001	.000001	.000001	.000002	.000004	166
167					.000001	.000001	.000002	.000002	167
168						.000001	.000001	.000002	168
169							.000001	.000001	169
170							.000001	.000001	170
171								.000001	171

X	A=118	A=119	A=120	A=121	A=122	A=123	A=124	A=125	X
70	.000001								70
71	.000001	.000001							71
72	.000001	.000001	.000001						72
73	.000002	.000002	.000001	.000001					73
74	.000004	.000002	.000002	.000001	.000001	.000001			74
75	.000006	.000004	.000003	.000002	.000001	.000001	.000001		75
76	.000009	.000006	.000004	.000003	.000002	.000001	.000001	.000001	76
77	.000013	.000009	.000007	.000005	.000003	.000002	.000002	.000001	77
78	.000020	.000014	.000010	.000007	.000005	.000003	.000002	.000002	78
79	.000030	.000022	.000015	.000011	.000008	.000005	.000004	.000003	79
80	.000045	.000032	.000023	.000017	.000012	.000008	.000006	.000004	80
81	.000065	.000047	.000034	.000025	.000018	.000013	.000009	.000006	81
82	.000093	.000069	.000050	.000036	.000026	.000019	.000014	.000010	82
83	.000133	.000098	.000073	.000053	.000039	.000028	.000020	.000014	83
84	.000187	.000139	.000104	.000077	.000056	.000041	.000030	.000022	84
85	.000259	.000195	.000146	.000109	.000081	.000059	.000043	.000032	85
86	.000355	.000270	.000204	.000153	.000114	.000085	.000063	.000046	86
87	.000482	.000370	.000282	.000213	.000161	.000120	.000089	.000066	87
88	.000646	.000500	.000384	.000293	.000223	.000168	.000126	.000094	88
89	.000857	.000668	.000518	.000399	.000305	.000232	.000176	.000132	89
90	.001124	.000884	.000690	.000536	.000414	.000317	.000242	.000183	90
91	.001457	.001155	.000910	.000713	.000554	.000429	.000330	.000252	91
92	.001869	.001495	.001187	.000937	.000735	.000573	.000444	.000342	92
93	.002372	.001912	.001532	.001219	.000964	.000758	.000592	.000460	93
94	.002977	.002421	.001956	.001570	.001252	.000992	.000781	.000611	94
95	.003698	.003033	.002470	.001999	.001608	.001284	.001020	.000805	95
96	.004545	.003759	.003088	.002520	.002043	.001645	.001317	.001048	96
97	.005529	.004612	.003820	.003143	.002569	.002087	.001684	.001350	97
98	.006658	.005600	.004678	.003881	.003199	.002619	.002130	.001722	98
99	.007935	.006731	.005670	.004744	.003942	.003254	.002668	.002174	99
100	.009364	.008010	.006804	.005740	.004809	.004002	.003309	.002718	100
101	.010940	.009438	.008084	.006876	.005809	.004874	.004062	.003363	101
102	.012656	.011011	.009511	.008157	.006948	.005877	.004938	.004122	102

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TABLE I

X	A=118	A=119	A=120	A=121	A=122	A=123	A=124	A=125	X
103	.014499	.012721	.011081	.009583	.008229	.007018	.005945	.005002	103
104	.016451	.014556	.012785	.011149	.009654	.008301	.007088	.006012	104
105	.018487	.016497	.014612	.012848	.011217	.009724	.008371	.007157	105
106	.020580	.018520	.016542	.014666	.012910	.011283	.009792	.008440	106
107	.022696	.020597	.018552	.016585	.014720	.012970	.011348	.009860	107
108	.024798	.022695	.020613	.018582	.016628	.014772	.013029	.011412	108
109	.026845	.024777	.022693	.020628	.018611	.016669	.014823	.013087	109
110	.028797	.026804	.024756	.022690	.020641	.018639	.016709	.014872	110
111	.030614	.028736	.026763	.024734	.022687	.020654	.018666	.016748	111
112	.032254	.030532	.028675	.026722	.024712	.022683	.020666	.018692	112
113	.033681	.032153	.030451	.028614	.026681	.024690	.022678	.020677	113
114	.034862	.033564	.032054	.030371	.028553	.026639	.024667	.022672	114
115	.035772	.034731	.033448	.031955	.030291	.028492	.026597	.024643	115
116	.036389	.035629	.034601	.033333	.031858	.030212	.028432	.026555	116
117	.036700	.036238	.035488	.034472	.033219	.031761	.030133	.028371	117
118	.036700	.036545	.036090	.035349	.034345	.033107	.031665	.030054	118
119	.036391	.036545	.036393	.035943	.035211	.034220	.032995	.031570	119
120	.035785	.036241	.036393	.036242	.035798	.035075	.034095	.032885	120
121	.034898	.035642	.036092	.036242	.036094	.035655	.034940	.033972	121
122	.033753	.034765	.035501	.035945	.036094	.035947	.035513	.034807	122
123	.032381	.033635	.034635	.035361	.035800	.035947	.035802	.035373	123
124	.030814	.032279	.033517	.034505	.035223	.035657	.035802	.035659	124
125	.029089	.030729	.032177	.033401	.034378	.035087	.035516	.035659	125
126	.027242	.029022	.030644	.032076	.033286	.034251	.034952	.035376	126
127	.025311	.027194	.028955	.030560	.031976	.033172	.034126	.034819	127
128	.023334	.025282	.027146	.028889	.030477	.031877	.033060	.034003	128
129	.021344	.023322	.025252	.027098	.028823	.030394	.031778	.032948	129
130	.019374	.021349	.023309	.025222	.027049	.028757	.030312	.031681	130
131	.017451	.019393	.021352	.023296	.025191	.027001	.028692	.030230	131
132	.015600	.017483	.019411	.021355	.023283	.025160	.026953	.028627	132
133	.013841	.015643	.017514	.019428	.021357	.023268	.025129	.026905	133
134	.012188	.013892	.015684	.017543	.019444	.021358	.023254	.025098	134
135	.010654	.012245	.013941	.015724	.017572	.019460	.021359	.023239	135
136	.009243	.010715	.012301	.013990	.015763	.017600	.019474	.021359	136
137	.007962	.009307	.010775	.012356	.014037	.015801	.017627	.019488	137
138	.006808	.008025	.009369	.010834	.012410	.014084	.015838	.017652	138
139	.005779	.006871	.008089	.009431	.010892	.012463	.014129	.015874	139
140	.004871	.005840	.006933	.008151	.009492	.010949	.012514	.014174	140
141	.004076	.004929	.005900	.006995	.008213	.009551	.011006	.012565	141
142	.003387	.004131	.004986	.005960	.007056	.008273	.009610	.011061	142
143	.002795	.003437	.004184	.005043	.006020	.007116	.008334	.009669	143
144	.002291	.002841	.003487	.004238	.005100	.006079	.007176	.008393	144
145	.001864	.002331	.002886	.003536	.004291	.005156	.006137	.007235	145
146	.001507	.001900	.002372	.002931	.003586	.004344	.005212	.006195	146
147	.001209	.001538	.001936	.002412	.002976	.003635	.004397	.005268	147
148	.000964	.001237	.001570	.001972	.002453	.003021	.003684	.004449	148
149	.000764	.000988	.001264	.001602	.002009	.002494	.003066	.003732	149
150	.000601	.000784	.001011	.001292	.001634	.002045	.002534	.003110	150
151	.000469	.000618	.000804	.001035	.001320	.001666	.002081	.002575	151
152	.000364	.000483	.000635	.000824	.001059	.001348	.001698	.002117	152
153	.000281	.000376	.000498	.000652	.000845	.001084	.001376	.001730	153
154	.000215	.000291	.000388	.000512	.000669	.000865	.001108	.001404	154
155	.000164	.000223	.000300	.000400	.000527	.000687	.000886	.001132	155
156	.000124	.000170	.000231	.000310	.000412	.000541	.000704	.000907	156
157	.000093	.000129	.000177	.000239	.000320	.000424	.000556	.000722	157
158	.000070	.000097	.000134	.000183	.000247	.000330	.000437	.000572	158
159	.000052	.000073	.000101	.000139	.000190	.000255	.000341	.000449	159
160	.000038	.000054	.000076	.000105	.000145	.000196	.000264	.000351	160
161	.000028	.000040	.000057	.000079	.000110	.000150	.000203	.000273	161
162	.000020	.000029	.000042	.000059	.000083	.000114	.000156	.000210	162
163	.000015	.000021	.000031	.000044	.000062	.000086	.000118	.000161	163
164	.000011	.000016	.000023	.000032	.000046	.000064	.000089	.000123	164
165	.000008	.000011	.000016	.000024	.000034	.000048	.000067	.000093	165
166	.000005	.000008	.000012	.000017	.000025	.000036	.000050	.000070	166
167	.000004	.000006	.000009	.000013	.000018	.000026	.000037	.000052	167
168	.000003	.000004	.000006	.000009	.000013	.000019	.000028	.000039	168
169	.000002	.000003	.000004	.000006	.000010	.000014	.000020	.000029	169
170	.000001	.000002	.000003	.000005	.000007	.000010	.000015	.000021	170

TABLE I

X	A=118	A=119	A=12Ø	A=121	A=122	A=123	A=124	A=125	X
171	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØ11	.ØØØØ16	171
172	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ8	.ØØØØ11	172
173		.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	173
174			.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	174
175				.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	175
176				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	176
177					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	177
178						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	178
179							.ØØØØØ1	.ØØØØØ1	179
18Ø								.ØØØØØ1	18Ø

X	A=126	A=127	A=128	A=129	A=13Ø	A=131	A=132	A=133	X
77	.ØØØØØ1								77
78	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1						78
79	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1					79
8Ø	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1				8Ø
81	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			81
82	.ØØØØØ7	.ØØØØØ5	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	82
83	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	83
84	.ØØØØ15	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	84
85	.ØØØØ23	.ØØØØ17	.ØØØØ12	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	85
86	.ØØØØ34	.ØØØØ24	.ØØØØ18	.ØØØØ13	.ØØØØØ9	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	86
87	.ØØØØ49	.ØØØØ36	.ØØØØ26	.ØØØØ19	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	87
88	.ØØØØ7Ø	.ØØØØ51	.ØØØØ38	.ØØØØ28	.ØØØØ2Ø	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ7	88
89	.ØØØØ99	.ØØØØ73	.ØØØØ54	.ØØØØ4Ø	.ØØØØ29	.ØØØØ21	.ØØØØ15	.ØØØØ11	89
9Ø	.ØØØ138	.ØØØ1Ø4	.ØØØØ77	.ØØØØ57	.ØØØØ42	.ØØØØ31	.ØØØØ23	.ØØØØ16	9Ø
91	.ØØØ191	.ØØØ144	.ØØØ1Ø8	.ØØØØ81	.ØØØØ6Ø	.ØØØØ44	.ØØØØ33	.ØØØØ24	91
92	.ØØØ262	.ØØØ199	.ØØØ151	.ØØØ114	.ØØØØ85	.ØØØØ63	.ØØØØ47	.ØØØØ35	92
93	.ØØØ355	.ØØØ272	.ØØØ2Ø8	.ØØØ158	.ØØØ119	.ØØØØ89	.ØØØØ67	.ØØØØ49	93
94	.ØØØ476	.ØØØ368	.ØØØ283	.ØØØ216	.ØØØ164	.ØØØ124	.ØØØØ93	.ØØØØ7Ø	94
95	.ØØØ631	.ØØØ492	.ØØØ381	.ØØØ294	.ØØØ225	.ØØØ171	.ØØØ13Ø	.ØØØØ98	95
96	.ØØØ828	.ØØØ651	.ØØØ5Ø8	.ØØØ395	.ØØØ3Ø5	.ØØØ234	.ØØØ179	.ØØØ136	96
97	.ØØ1Ø76	.ØØØ852	.ØØØ671	.ØØØ525	.ØØØ4Ø8	.ØØØ316	.ØØØ243	.ØØØ186	97
98	.ØØ1383	.ØØ11Ø4	.ØØØ876	.ØØØ691	.ØØØ542	.ØØØ422	.ØØØ327	.ØØØ252	98
99	.ØØ176Ø	.ØØ1416	.ØØ1133	.ØØØ9ØØ	.ØØØ711	.ØØØ559	.ØØØ436	.ØØØ339	99
1ØØ	.ØØ2218	.ØØ1799	.ØØ145Ø	.ØØ1161	.ØØØ925	.ØØØ732	.ØØØ576	.ØØØ451	1ØØ
1Ø1	.ØØ2767	.ØØ2262	.ØØ1837	.ØØ1483	.ØØ119Ø	.ØØØ95Ø	.ØØØ753	.ØØØ594	1Ø1
1Ø2	.ØØ3418	.ØØ2816	.ØØ23Ø6	.ØØ1876	.ØØ1517	.ØØ1219	.ØØØ974	.ØØØ774	1Ø2
1Ø3	.ØØ4181	.ØØ3472	.ØØ2865	.ØØ235Ø	.ØØ1915	.ØØ1551	.ØØ1249	.ØØ1ØØØ	1Ø3
1Ø4	.ØØ5Ø66	.ØØ424Ø	.ØØ3527	.ØØ2914	.ØØ2393	.ØØ1954	.ØØ1585	.ØØ1278	1Ø4
1Ø5	.ØØ6Ø79	.ØØ5129	.ØØ4299	.ØØ3581	.ØØ2963	.ØØ2437	.ØØ1993	.ØØ1619	1Ø5
1Ø6	.ØØ7226	.ØØ6145	.ØØ5191	.ØØ4357	.ØØ3634	.ØØ3Ø12	.ØØ2481	.ØØ2Ø31	1Ø6
1Ø7	.ØØ85Ø9	.ØØ7293	.ØØ621Ø	.ØØ5253	.ØØ4416	.ØØ3688	.ØØ3Ø61	.ØØ2525	1Ø7
1Ø8	.ØØ9927	.ØØ8576	.ØØ736Ø	.ØØ6275	.ØØ5315	.ØØ4473	.ØØ3741	.ØØ311Ø	1Ø8
1Ø9	.Ø11475	.ØØ9993	.ØØ8643	.ØØ7426	.ØØ6339	.ØØ5376	.ØØ4531	.ØØ3794	1Ø9
11Ø	.Ø13144	.Ø11537	.Ø1ØØ57	.ØØ87Ø9	.ØØ7491	.ØØ64Ø2	.ØØ5437	.ØØ4588	11Ø
111	.Ø14921	.Ø132ØØ	.Ø11598	.Ø1Ø121	.ØØ8774	.ØØ7556	.ØØ6465	.ØØ5497	111
112	.Ø16786	.Ø14968	.Ø13255	.Ø11657	.Ø1Ø184	.ØØ8838	.ØØ762Ø	.ØØ6528	112
113	.Ø18717	.Ø16822	.Ø15Ø14	.Ø133Ø8	.Ø11716	.Ø1Ø246	.ØØ89Ø1	.ØØ7683	113
114	.Ø2Ø687	.Ø18741	.Ø16858	.Ø15Ø59	.Ø1336Ø	.Ø11774	.Ø1Ø3Ø7	.ØØ8964	114
115	.Ø22666	.Ø2Ø696	.Ø18763	.Ø16892	.Ø151Ø3	.Ø13412	.Ø1183Ø	.Ø1Ø367	115
116	.Ø2462Ø	.Ø22659	.Ø2Ø7Ø5	.Ø18785	.Ø16926	.Ø15146	.Ø13462	.Ø11886	116
117	.Ø26513	.Ø24595	.Ø22651	.Ø2Ø712	.Ø188Ø6	.Ø16958	.Ø15188	.Ø13511	117
118	.Ø28311	.Ø26471	.Ø24571	.Ø22643	.Ø2Ø719	.Ø18827	.Ø1699Ø	.Ø15229	118
119	.Ø29976	.Ø28251	.Ø26429	.Ø24546	.Ø22634	.Ø2Ø725	.Ø18846	.Ø17Ø2Ø	119
12Ø	.Ø31475	.Ø29899	.Ø28191	.Ø26387	.Ø2452Ø	.Ø22625	.Ø2Ø73Ø	.Ø18864	12Ø
121	.Ø32776	.Ø31381	.Ø29822	.Ø28131	.Ø26344	.Ø24495	.Ø22615	.Ø2Ø735	121
122	.Ø3385Ø	.Ø32667	.Ø31288	.Ø29745	.Ø28Ø72	.Ø263Ø2	.Ø24469	.Ø226Ø5	122
123	.Ø34676	.Ø3373Ø	.Ø3256Ø	.Ø31196	.Ø29669	.Ø28Ø12	.Ø26259	.Ø24442	123
124	.Ø35235	.Ø34546	.Ø33611	.Ø32454	.Ø311Ø5	.Ø29594	.Ø27953	.Ø26216	124
125	.Ø35517	.Ø35Ø99	.Ø34417	.Ø33493	.Ø32349	.Ø31Ø14	.Ø29518	.Ø27894	125
126	.Ø35517	.Ø35377	.Ø34964	.Ø3429Ø	.Ø33376	.Ø32245	.Ø3Ø924	.Ø29444	126
127	.Ø35237	.Ø35377	.Ø35239	.Ø3483Ø	.Ø34164	.Ø3326Ø	.Ø32142	.Ø3Ø835	127
128	.Ø34687	.Ø351Ø1	.Ø35239	.Ø351Ø2	.Ø34698	.Ø3404Ø	.Ø33146	.Ø32Ø39	128
129	.Ø3388Ø	.Ø34557	.Ø34966	.Ø351Ø2	.Ø34967	.Ø34568	.Ø33917	.Ø33Ø33	129
13Ø	.Ø32838	.Ø33759	.Ø34428	.Ø34832	.Ø34967	.Ø34834	.Ø34439	.Ø33795	13Ø

TABLE I

X	A=126	A=127	A=128	A=129	A=130	A=131	A=132	A=133	X
131	.031584	.032728	.033639	.034300	.034700	.034834	.034702	.034311	131
132	.030149	.031489	.032620	.033521	.034174	.034570	.034702	.034571	132
133	.028562	.030068	.031394	.032513	.033404	.034050	.034441	.034571	133
134	.026857	.028497	.029988	.031300	.032406	.033288	.033927	.034313	134
135	.025066	.026809	.028433	.029908	.031206	.032301	.033173	.033805	135
136	.023223	.025035	.026761	.028369	.029829	.031114	.032197	.033059	136
137	.021359	.023207	.025003	.026712	.028305	.029751	.031022	.032094	137
138	.019501	.021357	.023191	.024970	.026664	.028242	.029673	.030931	138
139	.017677	.019514	.021356	.023174	.024938	.026616	.028179	.029596	139
140	.015910	.017702	.019525	.021353	.023157	.024905	.026569	.028116	140
141	.014217	.015944	.017725	.019536	.021350	.023139	.024873	.026521	141
142	.012615	.014260	.015977	.017747	.019546	.021347	.023121	.024840	142
143	.011116	.012664	.014301	.016010	.017769	.019555	.021343	.023103	143
144	.009726	.011169	.012712	.014342	.016041	.017790	.019564	.021338	144
145	.008452	.009783	.011222	.012760	.014382	.016072	.017810	.019572	145
146	.007294	.008510	.009838	.011274	.012806	.014421	.016102	.017829	146
147	.006252	.007352	.008567	.009893	.011325	.012851	.014459	.016131	147
148	.005323	.006309	.007409	.008623	.009948	.011375	.012896	.014496	148
149	.004501	.005377	.006365	.007466	.008679	.010001	.011425	.012940	149
150	.003781	.004553	.005431	.006421	.007522	.008734	.010054	.011473	150
151	.003155	.003829	.004604	.005485	.006476	.007577	.008789	.010106	151
152	.002615	.003199	.003877	.004655	.005538	.006530	.007632	.008842	152
153	.002154	.002656	.003244	.003925	.004706	.005591	.006585	.007687	153
154	.001762	.002190	.002696	.003288	.003973	.004756	.005644	.006638	154
155	.001432	.001794	.002226	.002736	.003332	.004020	.004807	.005696	155
156	.001157	.001461	.001827	.002263	.002777	.003376	.004067	.004856	156
157	.000928	.001182	.001489	.001859	.002299	.002817	.003419	.004114	157
158	.000740	.000950	.001207	.001518	.001892	.002335	.002857	.003463	158
159	.000587	.000759	.000971	.001232	.001547	.001924	.002372	.002897	159
160	.000462	.000602	.000777	.000993	.001257	.001575	.001957	.002408	160
161	.000362	.000475	.000618	.000796	.001015	.001282	.001604	.001989	161
162	.000281	.000372	.000488	.000633	.000814	.001037	.001307	.001633	162
163	.000217	.000290	.000383	.000501	.000649	.000833	.001058	.001332	163
164	.000167	.000225	.000299	.000394	.000515	.000665	.000852	.001081	164
165	.000128	.000173	.000232	.000308	.000406	.000528	.000682	.000871	165
166	.000097	.000132	.000179	.000240	.000318	.000417	.000542	.000698	166
167	.000073	.000101	.000137	.000185	.000247	.000327	.000428	.000556	167
168	.000055	.000076	.000104	.000142	.000191	.000255	.000337	.000440	168
169	.000041	.000057	.000079	.000108	.000147	.000198	.000263	.000346	169
170	.000030	.000043	.000060	.000082	.000113	.000152	.000204	.000271	170
171	.000022	.000032	.000045	.000062	.000086	.000117	.000158	.000211	171
172	.000016	.000023	.000033	.000047	.000065	.000089	.000121	.000163	172
173	.000012	.000017	.000025	.000035	.000049	.000067	.000092	.000125	173
174	.000009	.000013	.000018	.000026	.000036	.000051	.000070	.000096	174
175	.000006	.000009	.000013	.000019	.000027	.000038	.000053	.000073	175
176	.000004	.000007	.000010	.000014	.000020	.000028	.000040	.000055	176
177	.000003	.000005	.000007	.000010	.000015	.000021	.000030	.000041	177
178	.000002	.000003	.000005	.000007	.000011	.000015	.000022	.000031	178
179	.000002	.000002	.000004	.000005	.000008	.000011	.000016	.000023	179
180	.000001	.000002	.000003	.000004	.000006	.000008	.000012	.000017	180
181	.000001	.000001	.000002	.000003	.000004	.000006	.000009	.000012	181
182	.000001	.000001	.000001	.000002	.000003	.000004	.000006	.000009	182
183		.000001	.000001	.000001	.000002	.000003	.000005	.000007	183
184			.000001	.000001	.000001	.000002	.000003	.000005	184
185				.000001	.000001	.000002	.000002	.000003	185
186					.000001	.000001	.000002	.000002	186
187						.000001	.000001	.000002	187
188						.000001	.000001	.000001	188
189							.000001	.000001	189
190								.000001	190

X	A=134	A=135	A=136	A=137	A=138	A=139	A=140	A=141	X
83	.000001								83
84	.000001	.000001							84
85	.000001	.000001	.000001						85
86	.000002	.000002	.000001	.000001	.000001				86

TABLE I

X	A=134	A=135	A=136	A=137	A=138	A=139	A=14Ø	A=141	X
87	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			87
88	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1		88
89	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	89
9Ø	.ØØØØ12	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	9Ø
91	.ØØØØ17	.ØØØØ13	.ØØØØØ9	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	91
92	.ØØØØ25	.ØØØØ18	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ2	92
93	.ØØØØ36	.ØØØØ27	.ØØØØ2Ø	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	93
94	.ØØØØ52	.ØØØØ38	.ØØØØ28	.ØØØØ21	.ØØØØ15	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	94
95	.ØØØØ73	.ØØØØ55	.ØØØØ41	.ØØØØ3Ø	.ØØØØ22	.ØØØØ16	.ØØØØ12	.ØØØØØ8	95
96	.ØØØ1Ø2	.ØØØØ77	.ØØØØ57	.ØØØØ43	.ØØØØ32	.ØØØØ23	.ØØØØ17	.ØØØØ12	96
97	.ØØØ141	.ØØØ1Ø7	.ØØØØ81	.ØØØØ6Ø	.ØØØØ45	.ØØØØ33	.ØØØØ25	.ØØØØ18	97
98	.ØØØ193	.ØØØ147	.ØØØ112	.ØØØØ84	.ØØØØ63	.ØØØØ47	.ØØØØ35	.ØØØØ26	98
99	.ØØØ262	.ØØØ2Ø1	.ØØØ154	.ØØØ117	.ØØØØ88	.ØØØØ66	.ØØØØ5Ø	.ØØØØ37	99
1ØØ	.ØØØ351	.ØØØ271	.ØØØ2Ø9	.ØØØ16Ø	.ØØØ122	.ØØØØ92	.ØØØØ69	.ØØØØ52	1ØØ
1Ø1	.ØØØ465	.ØØØ363	.ØØØ281	.ØØØ217	.ØØØ166	.ØØØ127	.ØØØØ96	.ØØØØ73	1Ø1
1Ø2	.ØØØ611	.ØØØ48Ø	.ØØØ375	.ØØØ291	.ØØØ225	.ØØØ173	.ØØØ132	.ØØØ1ØØ	1Ø2
1Ø3	.ØØØ795	.ØØØ629	.ØØØ495	.ØØØ387	.ØØØ3Ø1	.ØØØ233	.ØØØ18Ø	.ØØØ137	1Ø3
1Ø4	.ØØ1Ø25	.ØØØ817	.ØØØ647	.ØØØ51Ø	.ØØØ4ØØ	.ØØØ312	.ØØØ242	.ØØØ186	1Ø4
1Ø5	.ØØ13Ø8	.ØØ1Ø5Ø	.ØØØ839	.ØØØ666	.ØØØ526	.ØØØ413	.ØØØ322	.ØØØ25Ø	1Ø5
1Ø6	.ØØ1653	.ØØ1338	.ØØ1Ø76	.ØØØ86Ø	.ØØØ684	.ØØØ541	.ØØØ426	.ØØØ333	1Ø6
1Ø7	.ØØ2Ø7Ø	.ØØ1688	.ØØ1368	.ØØ11Ø2	.ØØØ883	.ØØØ7Ø3	.ØØØ557	.ØØØ439	1Ø7
1Ø8	.ØØ2569	.ØØ211Ø	.ØØ1722	.ØØ1398	.ØØ1128	.ØØØ9Ø5	.ØØØ722	.ØØØ573	1Ø8
1Ø9	.ØØ3158	.ØØ2613	.ØØ2149	.ØØ1757	.ØØ1428	.ØØ1154	.ØØØ927	.ØØØ741	1Ø9
11Ø	.ØØ3847	.ØØ32Ø7	.ØØ2657	.ØØ2188	.ØØ1791	.ØØ1458	.ØØ118Ø	.ØØØ95Ø	11Ø
111	.ØØ4644	.ØØ39ØØ	.ØØ3255	.ØØ27ØØ	.ØØ2227	.ØØ1826	.ØØ1488	.ØØ12Ø7	111
112	.ØØ5557	.ØØ47Ø1	.ØØ3952	.ØØ33Ø3	.ØØ2744	.ØØ2266	.ØØ1861	.ØØ1519	112
113	.ØØ6589	.ØØ5616	.ØØ4757	.ØØ4ØØ4	.ØØ3351	.ØØ2787	.ØØ23Ø5	.ØØ1895	113
114	.ØØ7745	.ØØ665Ø	.ØØ5675	.ØØ4812	.ØØ4Ø56	.ØØ3399	.ØØ2831	.ØØ2344	114
115	.ØØ9Ø25	.ØØ78Ø7	.ØØ6711	.ØØ5733	.ØØ4868	.ØØ41Ø8	.ØØ3446	.ØØ2874	115
116	.Ø1Ø426	.ØØ9Ø86	.ØØ7868	.ØØ6771	.ØØ5791	.ØØ4922	.ØØ4159	.ØØ3494	116
117	.Ø1194Ø	.Ø1Ø484	.ØØ9146	.ØØ7928	.ØØ683Ø	.ØØ5848	.ØØ4977	.ØØ421Ø	117
118	.Ø13559	.Ø11994	.Ø1Ø541	.ØØ92Ø5	.ØØ7988	.ØØ6889	.ØØ59Ø5	.ØØ5Ø31	118
119	.Ø15269	.Ø136Ø7	.Ø12Ø47	.Ø1Ø597	.ØØ9263	.ØØ8Ø47	.ØØ6947	.ØØ5961	119
12Ø	.Ø17Ø5Ø	.Ø153Ø7	.Ø13653	.Ø12Ø98	.Ø1Ø653	.ØØ9321	.ØØ81Ø5	.ØØ7ØØ4	12Ø
121	.Ø18882	.Ø17Ø79	.Ø15345	.Ø13698	.Ø12149	.Ø1Ø7Ø7	.ØØ9377	.ØØ8162	121
122	.Ø2Ø739	.Ø18898	.Ø171Ø6	.Ø15382	.Ø13743	.Ø12199	.Ø1Ø761	.ØØ9433	122
123	.Ø22594	.Ø2Ø742	.Ø18914	.Ø17133	.Ø15419	.Ø13786	.Ø12248	.Ø1Ø814	123
124	.Ø24416	.Ø22582	.Ø2Ø745	.Ø1893Ø	.Ø17159	.Ø15454	.Ø13829	.Ø12297	124
125	.Ø26174	.Ø24389	.Ø2257Ø	.Ø2Ø747	.Ø18944	.Ø17185	.Ø15488	.Ø1387Ø	125
126	.Ø27835	.Ø26131	.Ø24362	.Ø22558	.Ø2Ø748	.Ø18958	.Ø172Ø9	.Ø15522	126
127	.Ø2937Ø	.Ø27777	.Ø26Ø88	.Ø24334	.Ø22545	.Ø2Ø749	.Ø18971	.Ø17233	127
128	.Ø3Ø746	.Ø29296	.Ø27719	.Ø26Ø45	.Ø243Ø7	.Ø22532	.Ø2Ø749	.Ø18983	128
129	.Ø31938	.Ø3Ø659	.Ø29223	.Ø2766Ø	.Ø26ØØ2	.Ø24279	.Ø22518	.Ø2Ø749	129
13Ø	.Ø32921	.Ø31838	.Ø3Ø571	.Ø2915Ø	.Ø276Ø3	.Ø2596Ø	.Ø24251	.Ø225Ø4	13Ø
131	.Ø33675	.Ø3281Ø	.Ø31738	.Ø3Ø485	.Ø29Ø78	.Ø27545	.Ø25917	.Ø24222	131
132	.Ø34185	.Ø33556	.Ø327ØØ	.Ø3164Ø	.Ø3Ø399	.Ø29ØØ6	.Ø27487	.Ø25874	132
133	.Ø34442	.Ø34Ø6Ø	.Ø33438	.Ø32591	.Ø31542	.Ø3Ø314	.Ø28934	.Ø2743Ø	133
134	.Ø34442	.Ø34314	.Ø33937	.Ø33321	.Ø32484	.Ø31445	.Ø3Ø23Ø	.Ø28863	134
135	.Ø34187	.Ø34314	.Ø34188	.Ø33815	.Ø332Ø5	.Ø32377	.Ø31349	.Ø3Ø146	135
136	.Ø33684	.Ø34Ø62	.Ø34188	.Ø34Ø63	.Ø33694	.Ø33Ø91	.Ø32271	.Ø31254	136
137	.Ø32946	.Ø33565	.Ø33939	.Ø34Ø63	.Ø3394Ø	.Ø33574	.Ø32978	.Ø32167	137
138	.Ø31991	.Ø32835	.Ø33447	.Ø33816	.Ø3394Ø	.Ø33818	.Ø33456	.Ø32866	138
139	.Ø3Ø841	.Ø3189Ø	.Ø32725	.Ø3333Ø	.Ø33696	.Ø33818	.Ø33697	.Ø33339	139
14Ø	.Ø29519	.Ø3Ø751	.Ø3179Ø	.Ø32616	.Ø33214	.Ø33576	.Ø33697	.Ø33577	14Ø
141	.Ø28Ø53	.Ø29443	.Ø3Ø662	.Ø3169Ø	.Ø325Ø7	.Ø331ØØ	.Ø33458	.Ø33577	141
142	.Ø26473	.Ø27991	.Ø29367	.Ø3Ø574	.Ø31592	.Ø324ØØ	.Ø32986	.Ø33341	142
143	.Ø248Ø7	.Ø26425	.Ø27929	.Ø29292	.Ø3Ø487	.Ø31494	.Ø32294	.Ø32874	143
144	.Ø23Ø84	.Ø24774	.Ø26378	.Ø27868	.Ø29217	.Ø3Ø4Ø1	.Ø31397	.Ø32189	144
145	.Ø21333	.Ø23Ø65	.Ø2474Ø	.Ø2633Ø	.Ø278Ø6	.Ø29143	.Ø3Ø315	.Ø313Ø1	145
146	.Ø1958Ø	.Ø21327	.Ø23Ø46	.Ø247Ø7	.Ø26283	.Ø27745	.Ø29Ø69	.Ø3Ø23Ø	146
147	.Ø17848	.Ø19586	.Ø21321	.Ø23Ø26	.Ø24674	.Ø26235	.Ø27685	.Ø28996	147
148	.Ø1616Ø	.Ø17866	.Ø19593	.Ø21315	.Ø23ØØ6	.Ø2464Ø	.Ø26188	.Ø27624	148
149	.Ø14533	.Ø16187	.Ø17883	.Ø19598	.Ø213Ø8	.Ø22986	.Ø246Ø6	.Ø26141	149
15Ø	.Ø12983	.Ø14569	.Ø16214	.Ø179ØØ	.Ø196Ø3	.Ø213Ø1	.Ø22966	.Ø24573	15Ø
151	.Ø11521	.Ø13Ø25	.Ø146Ø3	.Ø1624Ø	.Ø17916	.Ø196Ø8	.Ø21293	.Ø22945	151
152	.Ø1Ø157	.Ø11568	.Ø13Ø66	.Ø14638	.Ø16266	.Ø17931	.Ø19612	.Ø21285	152
153	.ØØ8895	.Ø1Ø2Ø7	.Ø11614	.Ø131Ø7	.Ø14671	.Ø1629Ø	.Ø17946	.Ø19615	153
154	.ØØ774Ø	.ØØ8948	.Ø1Ø257	.Ø1166Ø	.Ø13147	.Ø147Ø3	.Ø16314	.Ø1796Ø	154

TABLE I

X	A=134	A=135	A=136	A=137	A=138	A=139	A=140	A=141	X
155	.ØØ6692	.ØØ7793	.ØØ9ØØØ	.Ø1Ø3Ø6	.Ø117Ø5	.Ø13186	.Ø14735	.Ø16337	155
156	.ØØ5748	.ØØ6744	.ØØ7846	.ØØ9Ø51	.Ø1Ø354	.Ø11749	.Ø13224	.Ø14766	156
157	.ØØ49Ø6	.ØØ5799	.ØØ6796	.ØØ7898	.ØØ91Ø1	.Ø1Ø4Ø2	.Ø11792	.Ø13262	157
158	.ØØ4161	.ØØ4955	.ØØ585Ø	.ØØ6848	.ØØ7949	.ØØ9151	.Ø1Ø449	.Ø11835	158
159	.ØØ35Ø6	.ØØ42Ø7	.ØØ5ØØ4	.ØØ59Ø1	.ØØ6899	.ØØ8ØØØ	.ØØ92ØØ	.Ø1Ø495	159
16Ø	.ØØ2937	.ØØ355Ø	.ØØ4253	.ØØ5Ø52	.ØØ5951	.ØØ695Ø	.ØØ8Ø5Ø	.ØØ9249	16Ø
161	.ØØ2444	.ØØ2976	.ØØ3593	.ØØ4299	.ØØ51ØØ	.ØØ6ØØØ	.ØØ7ØØØ	.ØØ81ØØ	161
162	.ØØ2Ø22	.ØØ248Ø	.ØØ3Ø16	.ØØ3636	.ØØ4345	.ØØ5148	.ØØ6Ø49	.ØØ7Ø5Ø	162
163	.ØØ1662	.ØØ2Ø54	.ØØ2517	.ØØ3Ø56	.ØØ3678	.ØØ439Ø	.ØØ5196	.ØØ6Ø98	163
164	.ØØ1358	.ØØ1384	.ØØ2Ø87	.ØØ2553	.ØØ3Ø95	.ØØ3721	.ØØ4435	.ØØ5243	164
165	.ØØ11Ø3	.ØØ1125	.ØØ172Ø	.ØØ212Ø	.ØØ2589	.ØØ3135	.ØØ3763	.ØØ448Ø	165
166	.ØØØ89Ø	.ØØØ91Ø	.ØØ14Ø9	.ØØ1749	.ØØ2152	.ØØ2625	.ØØ3174	.ØØ38Ø6	166
167	.ØØØ714	.ØØØ731	.ØØ1148	.ØØ1435	.ØØ1778	.ØØ2185	.ØØ2661	.ØØ3213	167
168	.ØØØ57Ø	.ØØØ584	.ØØØ929	.ØØ117Ø	.ØØ1461	.ØØ18Ø8	.ØØ2217	.ØØ2697	168
169	.ØØØ452	.ØØØ464	.ØØØ748	.ØØØ949	.ØØ1193	.ØØ1487	.ØØ1837	.ØØ225Ø	169
17Ø	.ØØØ356	.ØØØ366	.ØØØ598	.ØØØ764	.ØØØ968	.ØØ1216	.ØØ1513	.ØØ1866	17Ø
171	.ØØØ279	.ØØØ287	.ØØØ476	.ØØØ612	.ØØØ781	.ØØØ988	.ØØ1238	.ØØ1539	171
172	.ØØØ217	.ØØØ224	.ØØØ376	.ØØØ488	.ØØØ627	.ØØØ799	.ØØ1ØØ8	.ØØ1261	172
173	.ØØØ168	.ØØØ174	.ØØØ296	.ØØØ386	.ØØØ5ØØ	.ØØØ642	.ØØØ816	.ØØ1Ø28	173
174	.ØØØ13Ø	.ØØØ134	.ØØØ231	.ØØØ3Ø4	.ØØØ397	.ØØØ513	.ØØØ656	.ØØØ833	174
175	.ØØØØ99	.ØØØ1Ø3	.ØØØ18Ø	.ØØØ238	.ØØØ313	.ØØØ4Ø7	.ØØØ525	.ØØØ671	175
176	.ØØØØ76	.ØØØØ78	.ØØØ139	.ØØØ185	.ØØØ245	.ØØØ322	.ØØØ418	.ØØØ538	176
177	.ØØØØ57	.ØØØØ6Ø	.ØØØ1Ø7	.ØØØ143	.ØØØ191	.ØØØ252	.ØØØ33Ø	.ØØØ428	177
178	.ØØØØ43	.ØØØØ45	.ØØØØ81	.ØØØ11Ø	.ØØØ148	.ØØØ197	.ØØØ26Ø	.ØØØ339	178
179	.ØØØØ32	.ØØØØ34	.ØØØØ62	.ØØØØ85	.ØØØ114	.ØØØ153	.ØØØ2Ø3	.ØØØ267	179
18Ø	.ØØØØ24	.ØØØØ25	.ØØØØ47	.ØØØØ64	.ØØØØ88	.ØØØ118	.ØØØ158	.ØØØ2Ø9	18Ø
181	.ØØØØ18	.ØØØØ19	.ØØØØ35	.ØØØØ49	.ØØØØ67	.ØØØØ91	.ØØØ122	.ØØØ163	181
182	.ØØØØ13	.ØØØØ14	.ØØØØ26	.ØØØØ37	.ØØØØ51	.ØØØØ69	.ØØØØ94	.ØØØ126	182
183	.ØØØØ1Ø	.ØØØØ1Ø	.ØØØØ2Ø	.ØØØØ27	.ØØØØ38	.ØØØØ53	.ØØØØ72	.ØØØØ97	183
184	.ØØØØØ7	.ØØØØØ7	.ØØØØ14	.ØØØØ2Ø	.ØØØØ29	.ØØØØ4Ø	.ØØØØ55	.ØØØØ75	184
185	.ØØØØØ5	.ØØØØØ5	.ØØØØ11	.ØØØØ15	.ØØØØ21	.ØØØØ3Ø	.ØØØØ41	.ØØØØ57	185
186	.ØØØØØ4	.ØØØØØ4	.ØØØØØ8	.ØØØØ11	.ØØØØ16	.ØØØØ22	.ØØØØ31	.ØØØØ43	186
187	.ØØØØØ3	.ØØØØØ3	.ØØØØØ6	.ØØØØØ8	.ØØØØ12	.ØØØØ17	.ØØØØ23	.ØØØØ32	187
188	.ØØØØØ2	.ØØØØØ2	.ØØØØØ4	.ØØØØØ6	.ØØØØØ9	.ØØØØ12	.ØØØØ17	.ØØØØ24	188
189	.ØØØØØ1	.ØØØØØ1	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ9	.ØØØØ13	.ØØØØ18	189
19Ø	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ13	19Ø
191	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	191
192			.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	192
193			.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	193
194			.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	194
195				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	195
196					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	196
197						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	197
198							.ØØØØØ1	.ØØØØØ1	198
199								.ØØØØØ1	199
2ØØ								.ØØØØØ1	2ØØ

X	A=142	A=143	A=144	A=145	A=146	A=147	A=148	A=149	X
9Ø	.ØØØØØ1								9Ø
91	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1						91
92	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1					92
93	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1				93
94	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			94
95	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	95
96	.ØØØØØ9	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	96
97	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	97
98	.ØØØØ19	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	98
99	.ØØØØ27	.ØØØØ2Ø	.ØØØØ15	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	99
1ØØ	.ØØØØ39	.ØØØØ29	.ØØØØ21	.ØØØØ16	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	1ØØ
1Ø1	.ØØØØ55	.ØØØØ41	.ØØØØ3Ø	.ØØØØ22	.ØØØØ17	.ØØØØ12	.ØØØØØ9	.ØØØØØ6	1Ø1
1Ø2	.ØØØØ76	.ØØØØ57	.ØØØØ43	.ØØØØ32	.ØØØØ24	.ØØØØ17	.ØØØØ13	.ØØØØØ9	1Ø2
1Ø3	.ØØØ1Ø5	.ØØØØ79	.ØØØØ6Ø	.ØØØØ45	.ØØØØ34	.ØØØØ25	.ØØØØ18	.ØØØØ14	1Ø3
1Ø4	.ØØØ143	.ØØØ1Ø9	.ØØØØ83	.ØØØØ63	.ØØØØ47	.ØØØØ35	.ØØØØ26	.ØØØØ19	1Ø4
1Ø5	.ØØØ193	.ØØØ149	.ØØØ114	.ØØØØ86	.ØØØØ65	.ØØØØ49	.ØØØØ37	.ØØØØ28	1Ø5
1Ø6	.ØØØ259	.ØØØ2Ø1	.ØØØ154	.ØØØ118	.ØØØØ9Ø	.ØØØØ68	.ØØØØ52	.ØØØØ39	1Ø6
1Ø7	.ØØØ344	.ØØØ268	.ØØØ2Ø8	.ØØØ16Ø	.ØØØ123	.ØØØØ94	.ØØØØ71	.ØØØØ54	1Ø7
1Ø8	.ØØØ452	.ØØØ355	.ØØØ277	.ØØØ215	.ØØØ166	.ØØØ128	.ØØØØ98	.ØØØØ74	1Ø8

TABLE I

X	A=142	A=143	A=144	A=145	A=146	A=147	A=148	A=149	X
1Ø9	.ØØØ589	.ØØØ466	.ØØØ366	.ØØØ286	.ØØØ223	.ØØØ172	.ØØØ133	.ØØØ1Ø2	1Ø9
11Ø	.ØØØ76Ø	.ØØØ6Ø5	.ØØØ479	.ØØØ377	.ØØØ296	.ØØØ231	.ØØØ179	.ØØØ138	11Ø
111	.ØØØ973	.ØØØ78Ø	.ØØØ622	.ØØØ493	.ØØØ389	.ØØØ3Ø5	.ØØØ238	.ØØØ185	111
112	.ØØ1233	.ØØØ996	.ØØØ799	.ØØØ638	.ØØØ5Ø7	.ØØØ4Ø1	.ØØØ315	.ØØØ246	112
113	.ØØ155Ø	.ØØ126Ø	.ØØ1Ø19	.ØØØ819	.ØØØ655	.ØØØ521	.ØØØ413	.ØØØ325	113
114	.ØØ193Ø	.ØØ158Ø	.ØØ1287	.ØØ1Ø42	.ØØØ839	.ØØØ672	.ØØØ536	.ØØØ425	114
115	.ØØ2383	.ØØ1965	.ØØ1611	.ØØ1314	.ØØ1Ø65	.ØØØ859	.ØØØ689	.ØØØ55Ø	115
116	.ØØ2918	.ØØ2423	.ØØ2ØØØ	.ØØ1642	.ØØ1341	.ØØ1Ø89	.ØØØ879	.ØØØ7Ø7	116
117	.ØØ3541	.ØØ2961	.ØØ2462	.ØØ2Ø35	.ØØ1673	.ØØ1368	.ØØ1112	.ØØØ9ØØ	117
118	.ØØ4261	.ØØ3588	.ØØ3ØØ4	.ØØ25Ø1	.ØØ2Ø7Ø	.ØØ17Ø4	.ØØ1395	.ØØ1136	118
119	.ØØ5Ø85	.ØØ4312	.ØØ3635	.ØØ3Ø47	.ØØ254Ø	.ØØ21Ø5	.ØØ1735	.ØØ1423	119
12Ø	.ØØ6Ø17	.ØØ5138	.ØØ4362	.ØØ3682	.ØØ3Ø9Ø	.ØØ2579	.ØØ214Ø	.ØØ1766	12Ø
121	.ØØ7Ø61	.ØØ6Ø73	.ØØ5191	.ØØ4412	.ØØ3728	.ØØ3133	.ØØ2618	.ØØ2175	121
122	.ØØ8219	.ØØ7118	.ØØ6127	.ØØ5244	.ØØ4462	.ØØ3775	.ØØ3175	.ØØ2656	122
123	.ØØ9489	.ØØ8275	.ØØ7174	.ØØ6182	.ØØ5296	.ØØ4511	.ØØ3821	.ØØ3218	123
124	.Ø10866	.ØØ9543	.ØØ8331	.ØØ7229	.ØØ6236	.ØØ5348	.ØØ456Ø	.ØØ3867	124
125	.Ø12344	.Ø1Ø917	.ØØ9597	.ØØ8385	.ØØ7283	.ØØ6289	.ØØ54ØØ	.ØØ46Ø9	125
126	.Ø13911	.Ø1239Ø	.Ø1Ø968	.ØØ965Ø	.ØØ844Ø	.ØØ7338	.ØØ6342	.ØØ5451	126
127	.Ø15554	.Ø13951	.Ø12436	.Ø11Ø18	.ØØ97Ø2	.ØØ8493	.ØØ7391	.ØØ6395	127
128	.Ø17256	.Ø15586	.Ø1399Ø	.Ø12481	.Ø11Ø67	.ØØ9754	.ØØ8546	.ØØ7444	128
129	.Ø18995	.Ø17278	.Ø15617	.Ø14Ø29	.Ø12525	.Ø11115	.ØØ98Ø5	.ØØ8598	129
13Ø	.Ø2Ø748	.Ø19ØØ6	.Ø17299	.Ø15648	.Ø14Ø66	.Ø12568	.Ø11162	.ØØ9855	13Ø
131	.Ø2249Ø	.Ø2Ø747	.Ø19Ø16	.Ø1732Ø	.Ø15677	.Ø141Ø3	.Ø12611	.Ø112Ø9	131
132	.Ø24194	.Ø22475	.Ø2Ø745	.Ø19Ø26	.Ø1734Ø	.Ø157Ø6	.Ø14139	.Ø12652	132
133	.Ø25831	.Ø24165	.Ø2246Ø	.Ø2Ø742	.Ø19Ø35	.Ø17359	.Ø15734	.Ø14174	133
134	.Ø27373	.Ø25788	.Ø24136	.Ø22445	.Ø2Ø739	.Ø19Ø43	.Ø17378	.Ø15761	134
135	.Ø28793	.Ø27317	.Ø25746	.Ø241Ø7	.Ø22429	.Ø2Ø736	.Ø19Ø51	.Ø17396	135
136	.Ø3ØØ63	.Ø28723	.Ø2726Ø	.Ø257Ø3	.Ø24Ø78	.Ø22413	.Ø2Ø732	.Ø19Ø58	136
137	.Ø3116Ø	.Ø2998Ø	.Ø28653	.Ø272Ø4	.Ø2566Ø	.Ø24Ø49	.Ø22397	.Ø2Ø728	137
138	.Ø32Ø63	.Ø31Ø67	.Ø29899	.Ø28584	.Ø27148	.Ø25617	.Ø24Ø2Ø	.Ø2238Ø	138
139	.Ø32755	.Ø31961	.Ø3Ø974	.Ø29817	.Ø28515	.Ø27Ø92	.Ø25575	.Ø2399Ø	139
14Ø	.Ø33223	.Ø32646	.Ø31859	.Ø3Ø882	.Ø29737	.Ø28446	.Ø27Ø36	.Ø25532	14Ø
141	.Ø33459	.Ø331Ø9	.Ø32537	.Ø31758	.Ø3Ø791	.Ø29657	.Ø28378	.Ø26981	141
142	.Ø33459	.Ø33342	.Ø32995	.Ø32429	.Ø31659	.Ø3Ø7Ø1	.Ø29578	.Ø28311	142
143	.Ø33225	.Ø33342	.Ø33226	.Ø32883	.Ø32323	.Ø3156Ø	.Ø3Ø612	.Ø29499	143
144	.Ø32763	.Ø3311Ø	.Ø33226	.Ø33111	.Ø32772	.Ø32217	.Ø31462	.Ø3Ø523	144
145	.Ø32Ø86	.Ø32654	.Ø32997	.Ø33111	.Ø32998	.Ø32662	.Ø32113	.Ø31365	145
146	.Ø312Ø6	.Ø31983	.Ø32545	.Ø32885	.Ø32998	.Ø32886	.Ø32553	.Ø32Ø1Ø	146
147	.Ø3Ø145	.Ø31112	.Ø31881	.Ø32437	.Ø32773	.Ø32886	.Ø32774	.Ø32445	147
148	.Ø28923	.Ø3ØØ61	.Ø31Ø19	.Ø3178Ø	.Ø3233Ø	.Ø32663	.Ø32774	.Ø32664	148
149	.Ø27564	.Ø28851	.Ø29978	.Ø3Ø926	.Ø3168Ø	.Ø32225	.Ø32554	.Ø32664	149
15Ø	.Ø26Ø94	.Ø275Ø4	.Ø28779	.Ø29896	.Ø3Ø835	.Ø3158Ø	.Ø3212Ø	.Ø32447	15Ø
151	.Ø24539	.Ø26Ø47	.Ø27445	.Ø287Ø8	.Ø29814	.Ø3Ø744	.Ø31482	.Ø32Ø17	151
152	.Ø22924	.Ø245Ø5	.Ø26ØØØ	.Ø27386	.Ø28637	.Ø29733	.Ø3Ø654	.Ø31385	152
153	.Ø21276	.Ø229Ø3	.Ø24471	.Ø25954	.Ø27327	.Ø28567	.Ø29652	.Ø3Ø564	153
154	.Ø19618	.Ø21267	.Ø22882	.Ø24437	.Ø259Ø7	.Ø27268	.Ø28497	.Ø29572	154
155	.Ø17973	.Ø19621	.Ø21258	.Ø2286Ø	.Ø244Ø3	.Ø25861	.Ø2721Ø	.Ø28427	155
156	.Ø1636Ø	.Ø17986	.Ø19623	.Ø21248	.Ø22839	.Ø24369	.Ø25814	.Ø27152	156
157	.Ø14797	.Ø16382	.Ø17998	.Ø19624	.Ø21238	.Ø22817	.Ø24335	.Ø25768	157
158	.Ø13298	.Ø14827	.Ø164Ø3	.Ø18Ø1Ø	.Ø19625	.Ø21228	.Ø22794	.Ø243ØØ	158
159	.Ø11877	.Ø13335	.Ø14856	.Ø16424	.Ø18Ø21	.Ø19626	.Ø21217	.Ø22772	159
16Ø	.Ø1Ø541	.Ø11918	.Ø1337Ø	.Ø14884	.Ø16444	.Ø18Ø31	.Ø19626	.Ø212Ø6	16Ø
161	.ØØ9297	.Ø1Ø585	.Ø11958	.Ø134Ø5	.Ø14912	.Ø16463	.Ø18Ø41	.Ø19626	161
162	.ØØ8149	.ØØ9344	.Ø1Ø63Ø	.Ø11998	.Ø13439	.Ø14939	.Ø16482	.Ø18Ø51	162
163	.ØØ7Ø99	.ØØ8197	.ØØ9391	.Ø1Ø673	.Ø12Ø37	.Ø13473	.Ø14965	.Ø165Ø1	163
164	.ØØ6147	.ØØ7148	.ØØ8245	.ØØ9437	.Ø1Ø716	.Ø12Ø76	.Ø135Ø5	.Ø14991	164
165	.ØØ529Ø	.ØØ6195	.ØØ7196	.ØØ8293	.ØØ9482	.Ø1Ø759	.Ø12114	.Ø13538	165
166	.ØØ4525	.ØØ5336	.ØØ6242	.ØØ7244	.ØØ834Ø	.ØØ9527	.Ø1Ø8ØØ	.Ø12151	166
167	.ØØ3848	.ØØ457Ø	.ØØ5383	.ØØ629Ø	.ØØ7291	.ØØ8386	.ØØ9572	.Ø1Ø842	167
168	.ØØ3252	.ØØ389Ø	.ØØ4614	.ØØ5428	.ØØ6336	.ØØ7338	.ØØ8432	.ØØ9615	168
169	.ØØ2733	.ØØ3291	.ØØ3931	.ØØ4658	.ØØ5474	.ØØ6383	.ØØ7384	.ØØ8478	169
17Ø	.ØØ2283	.ØØ2768	.ØØ333Ø	.ØØ3973	.ØØ47Ø1	.ØØ5519	.ØØ6429	.ØØ743Ø	17Ø
171	.ØØ1895	.ØØ2315	.ØØ28Ø4	.ØØ3369	.ØØ4Ø14	.ØØ4745	.ØØ5564	.ØØ6474	171
172	.ØØ1565	.ØØ1925	.ØØ2348	.ØØ284Ø	.ØØ34Ø7	.ØØ4Ø55	.ØØ4788	.ØØ56Ø9	172
173	.ØØ1284	.ØØ1591	.ØØ1954	.ØØ238Ø	.ØØ2875	.ØØ3446	.ØØ4Ø96	.ØØ4831	173
174	.ØØ1Ø48	.ØØ13Ø8	.ØØ1617	.ØØ1983	.ØØ2413	.ØØ2911	.ØØ3484	.ØØ4136	174
175	.ØØØ851	.ØØ1Ø68	.ØØ1331	.ØØ1643	.ØØ2Ø13	.ØØ2445	.ØØ2946	.ØØ3522	175

TABLE I

X	A=142	A=143	A=144	A=145	A=146	A=147	A=148	A=149	X
176	.000686	.000868	.001089	.001354	.001670	.002042	.002478	.002982	176
177	.000551	.000701	.000886	.001109	.001377	.001696	.002072	.002510	177
178	.000439	.000563	.000717	.000904	.001130	.001401	.001722	.002101	178
179	.000348	.000450	.000576	.000732	.000921	.001150	.001424	.001749	179
180	.000275	.000358	.000461	.000590	.000747	.000939	.001171	.001448	180
181	.000216	.000283	.000367	.000472	.000603	.000763	.000957	.001192	181
182	.000168	.000222	.000290	.000376	.000484	.000616	.000779	.000976	182
183	.000131	.000173	.000228	.000298	.000386	.000495	.000630	.000794	183
184	.000101	.000135	.000179	.000235	.000306	.000395	.000506	.000643	184
185	.000077	.000104	.000139	.000184	.000242	.000314	.000405	.000518	185
186	.000059	.000080	.000108	.000144	.000190	.000248	.000322	.000415	186
187	.000045	.000061	.000083	.000111	.000148	.000195	.000255	.000331	187
188	.000034	.000047	.000064	.000086	.000115	.000153	.000201	.000262	188
189	.000025	.000035	.000048	.000066	.000089	.000119	.000157	.000207	189
190	.000019	.000027	.000037	.000050	.000068	.000092	.000123	.000162	190
191	.000014	.000020	.000028	.000038	.000052	.000071	.000095	.000126	191
192	.000010	.000015	.000021	.000029	.000040	.000054	.000073	.000098	192
193	.000008	.000011	.000015	.000022	.000030	.000041	.000056	.000076	193
194	.000006	.000008	.000011	.000016	.000023	.000031	.000043	.000058	194
195	.000004	.000006	.000008	.000012	.000017	.000024	.000032	.000044	195
196	.000003	.000004	.000006	.000009	.000013	.000018	.000025	.000034	196
197	.000002	.000003	.000005	.000007	.000009	.000013	.000018	.000026	197
198	.000002	.000002	.000003	.000005	.000007	.000010	.000014	.000019	198
199	.000001	.000002	.000002	.000003	.000005	.000007	.000010	.000014	199
200	.000001	.000001	.000002	.000003	.000004	.000005	.000008	.000011	200
201	.000001	.000001	.000001	.000002	.000003	.000004	.000006	.000008	201
202		.000001	.000001	.000001	.000002	.000003	.000004	.000006	202
203			.000001	.000001	.000001	.000002	.000003	.000004	203
204				.000001	.000001	.000001	.000002	.000003	204
205					.000001	.000001	.000002	.000002	205
206						.000001	.000001	.000002	206
207						.000001	.000001	.000001	207
208							.000001	.000001	208
209								.000001	209

X	A=150	A=151	A=152	A=153	A=154	A=155	A=156	A=157	X
96	.000001								96
97	.000001	.000001							97
98	.000001	.000001	.000001						98
99	.000002	.000001	.000001	.000001	.000001				99
100	.000003	.000002	.000002	.000001	.000001	.000001			100
101	.000005	.000003	.000002	.000002	.000001	.000001	.000001		101
102	.000007	.000005	.000004	.000003	.000002	.000001	.000001	.000001	102
103	.000010	.000007	.000005	.000004	.000003	.000002	.000001	.000001	103
104	.000014	.000011	.000008	.000006	.000004	.000003	.000002	.000001	104
105	.000020	.000015	.000011	.000008	.000006	.000004	.000003	.000002	105
106	.000029	.000022	.000016	.000012	.000009	.000006	.000005	.000003	106
107	.000041	.000030	.000023	.000017	.000012	.000009	.000007	.000005	107
108	.000056	.000043	.000032	.000024	.000018	.000013	.000010	.000007	108
109	.000078	.000059	.000045	.000033	.000025	.000019	.000014	.000010	109
110	.000106	.000081	.000062	.000047	.000035	.000026	.000020	.000015	110
111	.000143	.000110	.000084	.000064	.000049	.000037	.000028	.000021	111
112	.000192	.000148	.000114	.000088	.000067	.000051	.000038	.000029	112
113	.000254	.000198	.000154	.000119	.000091	.000070	.000053	.000040	113
114	.000335	.000263	.000205	.000159	.000123	.000095	.000073	.000055	114
115	.000437	.000345	.000271	.000212	.000165	.000128	.000098	.000076	115
116	.000565	.000449	.000355	.000280	.000219	.000171	.000132	.000102	116
117	.000724	.000579	.000461	.000366	.000288	.000226	.000177	.000137	117
118	.000920	.000742	.000594	.000474	.000376	.000297	.000233	.000182	118
119	.001160	.000941	.000759	.000610	.000487	.000387	.000306	.000241	119
120	.001450	.001184	.000962	.000777	.000625	.000500	.000398	.000315	120
121	.001798	.001478	.001208	.000983	.000795	.000640	.000513	.000409	121
122	.002210	.001829	.001505	.001232	.001004	.000813	.000656	.000526	122
123	.002695	.002245	.001860	.001533	.001257	.001025	.000832	.000671	123
124	.003260	.002734	.002280	.001892	.001561	.001281	.001046	.000850	124
125	.003913	.003303	.002773	.002315	.001923	.001589	.001306	.001068	125
126	.004658	.003958	.003345	.002811	.002350	.001954	.001617	.001330	126

TABLE I

X	A=150	A=151	A=152	A=153	A=154	A=155	A=156	A=157	X
127	.005501	.004706	.004003	.003387	.002850	.002385	.001986	.001645	127
128	.006447	.005552	.004754	.004048	.003429	.002888	.002420	.002017	128
129	.007496	.006498	.005602	.004802	.004093	.003471	.002927	.002455	129
130	.008650	.007548	.006550	.005651	.004849	.004138	.003512	.002965	130
131	.009904	.008701	.007599	.006600	.005700	.004896	.004182	.003553	131
132	.011255	.009953	.008751	.007650	.006650	.005749	.004943	.004227	132
133	.012693	.011300	.010001	.008801	.007700	.006700	.005798	.004989	133
134	.014209	.012734	.011344	.010048	.008850	.007750	.006749	.005846	134
135	.015788	.014243	.012773	.011388	.010095	.008898	.007799	.006798	135
136	.017413	.015814	.014276	.012812	.011431	.010141	.008946	.007848	136
137	.019065	.017430	.015839	.014308	.012850	.011474	.010187	.008994	137
138	.020723	.019071	.017446	.015863	.014340	.012887	.011516	.010232	138
139	.022363	.020718	.019077	.017461	.015887	.014371	.012924	.011557	139
140	.023960	.022346	.020712	.019082	.017476	.015910	.014401	.012960	140
141	.025490	.023931	.022328	.020707	.019087	.017490	.015933	.014431	141
142	.026926	.025447	.023901	.022311	.020700	.019091	.017504	.015955	142
143	.028244	.026871	.025405	.023871	.022293	.020694	.019095	.017517	143
144	.029421	.028177	.026816	.025363	.023841	.022274	.020686	.019099	144
145	.030435	.029343	.028111	.026762	.025320	.023810	.022256	.020679	145
146	.031269	.030348	.029266	.028045	.026708	.025278	.023780	.022237	146
147	.031907	.031174	.030262	.029190	.027980	.026654	.025236	.023750	147
148	.032338	.031806	.031079	.030176	.029114	.027915	.026600	.025194	148
149	.032555	.032233	.031705	.030986	.030091	.029039	.027850	.026547	149
150	.032555	.032448	.032128	.031606	.030893	.030007	.028964	.027786	150
151	.032340	.032448	.032341	.032024	.031507	.030801	.029923	.028890	151
152	.031914	.032234	.032341	.032235	.031922	.031409	.030710	.029840	152
153	.031289	.031813	.032129	.032235	.032130	.031820	.031313	.030620	153
154	.030476	.031193	.031712	.032026	.032130	.032027	.031719	.031217	154
155	.029493	.030388	.031098	.031612	.031923	.032027	.031924	.031619	155
156	.028358	.029414	.030301	.031005	.031514	.031821	.031924	.031822	156
157	.027094	.028290	.029336	.030215	.030912	.031416	.031721	.031822	157
158	.025722	.027037	.028222	.029258	.030129	.030819	.031319	.031621	158
159	.024266	.025676	.026979	.028154	.029182	.030044	.030728	.031223	159
160	.022750	.024232	.025630	.026923	.028087	.029105	.029960	.030638	160
161	.021195	.022727	.024198	.025585	.026866	.028021	.029029	.029876	161
162	.019625	.021184	.022704	.024163	.025539	.026810	.027954	.028954	162
163	.018060	.019624	.021172	.022681	.024129	.025494	.026754	.027888	163
164	.016518	.018069	.019623	.021160	.022658	.024095	.025449	.026698	164
165	.015017	.016535	.018077	.019621	.021147	.022635	.024061	.025404	165
166	.013569	.015041	.016552	.018084	.019619	.021135	.022611	.024026	166
167	.012188	.013600	.015065	.016568	.018091	.019616	.021122	.022588	167
168	.010882	.012224	.013631	.015089	.016584	.018098	.019613	.021109	168
169	.009659	.010922	.012259	.013660	.015112	.016599	.018104	.019610	169
170	.008522	.009701	.010961	.012294	.013690	.015134	.016613	.018110	170
171	.007476	.008567	.009743	.011000	.012329	.013718	.015156	.016628	171
172	.006520	.007521	.008611	.009785	.011038	.012362	.013746	.015177	172
173	.005653	.006564	.007565	.008654	.009826	.011076	.012395	.013774	173
174	.004873	.005697	.006609	.007609	.008697	.009867	.011113	.012428	174
175	.004177	.004915	.005740	.006653	.007653	.008739	.009907	.011150	175
176	.003560	.004217	.004957	.005783	.006696	.007696	.008781	.009946	176
177	.003017	.003598	.004257	.004999	.005826	.006740	.007739	.008822	177
178	.002542	.003052	.003635	.004297	.005041	.005869	.006783	.007781	178
179	.002130	.002575	.003087	.003673	.004337	.005082	.005911	.006825	179
180	.001775	.002160	.002607	.003122	.003710	.004376	.005123	.005953	180
181	.001471	.001802	.002189	.002639	.003157	.003747	.004415	.005164	181
182	.001213	.001495	.001828	.002219	.002671	.003192	.003785	.004454	182
183	.000994	.001233	.001519	.001855	.002248	.002703	.003226	.003821	183
184	.000810	.001012	.001254	.001542	.001881	.002277	.002735	.003261	184
185	.000657	.000826	.001031	.001276	.001566	.001908	.002306	.002767	185
186	.000530	.000671	.000842	.001049	.001297	.001590	.001934	.002336	186
187	.000425	.000542	.000685	.000858	.001068	.001318	.001614	.001961	187
188	.000339	.000435	.000554	.000699	.000875	.001087	.001339	.001638	188
189	.000269	.000348	.000445	.000566	.000713	.000891	.001105	.001360	189
190	.000212	.000276	.000356	.000455	.000578	.000727	.000907	.001124	190
191	.000167	.000218	.000283	.000365	.000466	.000590	.000741	.000924	191
192	.000130	.000172	.000224	.000291	.000374	.000476	.000602	.000756	192
193	.000101	.000134	.000177	.000230	.000298	.000382	.000487	.000615	193
194	.000078	.000105	.000138	.000182	.000237	.000306	.000391	.000497	194

TABLE I

X	A=15Ø	A=151	A=152	A=153	A=154	A=155	A=156	A=157	X
195	.ØØØØ6Ø	.ØØØØ81	.ØØØ1Ø8	.ØØØ143	.ØØØ187	.ØØØ243	.ØØØ313	.ØØØ4ØØ	195
196	.ØØØØ46	.ØØØØ62	.ØØØØ84	.ØØØ111	.ØØØ147	.ØØØ192	.ØØØ249	.ØØØ321	196
197	.ØØØØ35	.ØØØØ48	.ØØØØ65	.ØØØØ86	.ØØØ115	.ØØØ151	.ØØØ197	.ØØØ256	197
198	.ØØØØ27	.ØØØØ36	.ØØØØ5Ø	.ØØØØ67	.ØØØØ89	.ØØØ118	.ØØØ155	.ØØØ2Ø3	198
199	.ØØØØ2Ø	.ØØØØ28	.ØØØØ38	.ØØØØ51	.ØØØØ69	.ØØØØ92	.ØØØ122	.ØØØ16Ø	199
2ØØ	.ØØØØ15	.ØØØØ21	.ØØØØ29	.ØØØØ39	.ØØØØ53	.ØØØØ71	.ØØØØ95	.ØØØ126	2ØØ
2Ø1	.ØØØØ11	.ØØØØ16	.ØØØØ22	.ØØØØ3Ø	.ØØØØ41	.ØØØØ55	.ØØØØ74	.ØØØØ98	2Ø1
2Ø2	.ØØØØØ8	.ØØØØ12	.ØØØØ16	.ØØØØ23	.ØØØØ31	.ØØØØ42	.ØØØØ57	.ØØØØ76	2Ø2
2Ø3	.ØØØØØ6	.ØØØØØ9	.ØØØØ12	.ØØØØ17	.ØØØØ24	.ØØØØ32	.ØØØØ44	.ØØØØ59	2Ø3
2Ø4	.ØØØØØ5	.ØØØØØ6	.ØØØØØ9	.ØØØØ13	.ØØØØ18	.ØØØØ25	.ØØØØ33	.ØØØØ45	2Ø4
2Ø5	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ13	.ØØØØ19	.ØØØØ25	.ØØØØ35	2Ø5
2Ø6	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ14	.ØØØØ19	.ØØØØ26	2Ø6
2Ø7	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ15	.ØØØØ2Ø	2Ø7
2Ø8	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ8	.ØØØØ11	.ØØØØ15	2Ø8
2Ø9	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	.ØØØØ11	2Ø9
21Ø	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	21Ø
211		.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	211
212			.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	212
213			.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	213
214				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	214
215					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	215
216						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	216
217							.ØØØØØ1	.ØØØØØ1	217
218								.ØØØØØ1	218

X	A=158	A=159	A=16Ø	A=161	A=162	A=163	A=164	A=165	X
1Ø3	.ØØØØØ1								1Ø3
1Ø4	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1						1Ø4
1Ø5	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1					1Ø5
1Ø6	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1				1Ø6
1Ø7	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			1Ø7
1Ø8	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	1Ø8
1Ø9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	1Ø9
11Ø	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	11Ø
111	.ØØØØ15	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	111
112	.ØØØØ22	.ØØØØ16	.ØØØØ12	.ØØØØØ9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ3	.ØØØØØ3	112
113	.ØØØØ3Ø	.ØØØØ23	.ØØØØ17	.ØØØØ13	.ØØØØØ9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	113
114	.ØØØØ42	.ØØØØ32	.ØØØØ24	.ØØØØ18	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	114
115	.ØØØØ58	.ØØØØ44	.ØØØØ33	.ØØØØ25	.ØØØØ19	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ8	115
116	.ØØØØ79	.ØØØØ6Ø	.ØØØØ46	.ØØØØ35	.ØØØØ26	.ØØØØ2Ø	.ØØØØ15	.ØØØØ11	116
117	.ØØØ1Ø6	.ØØØØ82	.ØØØØ63	.ØØØØ48	.ØØØØ36	.ØØØØ27	.ØØØØ21	.ØØØØ15	117
118	.ØØØ142	.ØØØ11Ø	.ØØØØ85	.ØØØØ65	.ØØØØ5Ø	.ØØØØ38	.ØØØØ29	.ØØØØ22	118
119	.ØØØ189	.ØØØ147	.ØØØ114	.ØØØØ88	.ØØØØ68	.ØØØØ52	.ØØØØ39	.ØØØØ3Ø	119
12Ø	.ØØØ248	.ØØØ195	.ØØØ152	.ØØØ118	.ØØØØ91	.ØØØØ7Ø	.ØØØØ54	.ØØØØ41	12Ø
121	.ØØØ324	.ØØØ256	.ØØØ2Ø1	.ØØØ157	.ØØØ122	.ØØØØ95	.ØØØØ73	.ØØØØ56	121
122	.ØØØ42Ø	.ØØØ333	.ØØØ264	.ØØØ2Ø7	.ØØØ162	.ØØØ127	.ØØØØ98	.ØØØØ76	122
123	.ØØØ539	.ØØØ431	.ØØØ343	.ØØØ271	.ØØØ214	.ØØØ168	.ØØØ131	.ØØØ1Ø2	123
124	.ØØØ687	.ØØØ553	.ØØØ442	.ØØØ352	.ØØØ279	.ØØØ22Ø	.ØØØ173	.ØØØ135	124
125	.ØØØ869	.ØØØ7Ø3	.ØØØ566	.ØØØ454	.ØØØ362	.ØØØ288	.ØØØ227	.ØØØ179	125
126	.ØØ1Ø89	.ØØØ887	.ØØØ719	.ØØØ58Ø	.ØØØ466	.ØØØ372	.ØØØ296	.ØØØ234	126
127	.ØØ1355	.ØØ1111	.ØØØ9Ø6	.ØØØ735	.ØØØ594	.ØØØ477	.ØØØ382	.ØØØ3Ø4	127
128	.ØØ1673	.ØØ138Ø	.ØØ1133	.ØØØ925	.ØØØ752	.ØØØ6Ø8	.ØØØ489	.ØØØ392	128
129	.ØØ2Ø49	.ØØ17Ø1	.ØØ14Ø5	.ØØ1154	.ØØØ944	.ØØØ768	.ØØØ622	.ØØØ5Ø1	129
13Ø	.ØØ249Ø	.ØØ2Ø8Ø	.ØØ1729	.ØØ143Ø	.ØØ1176	.ØØØ963	.ØØØ785	.ØØØ636	13Ø
131	.ØØ3ØØ3	.ØØ2525	.ØØ2112	.ØØ1757	.ØØ1455	.ØØ1198	.ØØØ982	.ØØØ8Ø1	131
132	.ØØ3595	.ØØ3Ø41	.ØØ256Ø	.ØØ2143	.ØØ1785	.ØØ148Ø	.ØØ1221	.ØØ1ØØ2	132
133	.ØØ427Ø	.ØØ3636	.ØØ3Ø79	.ØØ2594	.ØØ2175	.ØØ1814	.ØØ15Ø5	.ØØ1243	133
134	.ØØ5Ø35	.ØØ4314	.ØØ3677	.ØØ3117	.ØØ2629	.ØØ22Ø6	.ØØ1842	.ØØ153Ø	134
135	.ØØ5893	.ØØ5Ø81	.ØØ4358	.ØØ3718	.ØØ3155	.ØØ2664	.ØØ2238	.ØØ187Ø	135
136	.ØØ6846	.ØØ594Ø	.ØØ5127	.ØØ44Ø1	.ØØ3758	.ØØ3193	.ØØ2698	.ØØ2269	136
137	.ØØ7896	.ØØ6894	.ØØ5987	.ØØ5172	.ØØ4444	.ØØ3798	.ØØ323Ø	.ØØ2733	137
138	.ØØ9Ø4Ø	.ØØ7944	.ØØ6942	.ØØ6Ø34	.ØØ5217	.ØØ4487	.ØØ3839	.ØØ3268	138
139	.Ø1Ø276	.ØØ9Ø86	.ØØ7991	.ØØ6989	.ØØ6Ø8Ø	.ØØ5261	.ØØ4529	.ØØ3879	139
14Ø	.Ø11597	.Ø1Ø32Ø	.ØØ9132	.ØØ8Ø37	.ØØ7Ø35	.ØØ6126	.ØØ53Ø6	.ØØ4571	14Ø
141	.Ø12995	.Ø11637	.Ø1Ø363	.ØØ9177	.ØØ8Ø83	.ØØ7Ø81	.ØØ6171	.ØØ535Ø	141
142	.Ø1446Ø	.Ø13Ø3Ø	.Ø11676	.Ø1Ø4Ø5	.ØØ9222	.ØØ8129	.ØØ7127	.ØØ6216	142
143	.Ø15977	.Ø14488	.Ø13Ø64	.Ø11715	.Ø1Ø447	.ØØ9266	.ØØ8174	.ØØ7172	143

TABLE I

X	A=158	A=159	A=160	A=161	A=162	A=163	A=164	A=165	X
144	.017530	.015997	.014516	.013098	.011753	.010488	.009309	.008218	144
145	.019101	.017542	.016018	.014543	.013131	.011790	.010529	.009352	145
146	.020671	.019104	.017553	.016037	.014570	.013163	.011827	.010569	146
147	.022218	.020663	.019106	.017565	.016056	.014596	.013195	.011863	147
148	.023719	.022199	.020655	.019107	.017575	.016075	.014621	.013226	148
149	.025152	.023689	.022180	.020646	.019109	.017585	.016093	.014646	149
150	.026494	.025110	.023658	.022160	.020637	.019109	.017595	.016110	150
151	.027722	.026441	.025069	.023628	.022141	.020628	.019110	.017604	151
152	.028816	.027658	.026388	.025027	.023597	.022121	.020618	.019110	152
153	.029758	.028743	.027595	.026336	.024985	.023567	.022101	.020609	153
154	.030531	.029676	.028670	.027533	.026283	.024944	.023536	.022081	154
155	.031122	.030442	.029595	.028598	.027470	.026231	.024903	.023505	155
156	.031521	.031027	.030354	.029515	.028527	.027408	.026180	.024861	156
157	.031721	.031423	.030934	.030267	.029435	.028456	.027347	.026128	157
158	.031721	.031622	.031326	.030842	.030181	.029356	.028385	.027286	158
159	.031522	.031622	.031523	.031230	.030750	.030095	.029278	.028315	159
160	.031128	.031424	.031523	.031425	.031134	.030659	.030010	.029200	160
161	.030548	.031034	.031327	.031425	.031328	.031040	.030569	.029926	161
162	.029794	.030459	.030940	.031231	.031328	.031232	.030946	.030480	162
163	.028880	.029711	.030371	.030848	.031136	.031232	.031136	.030854	163
164	.027823	.028806	.029630	.030283	.030756	.031041	.031136	.031042	164
165	.026643	.027758	.028732	.029549	.030197	.030665	.030948	.031042	165
166	.025359	.026588	.027694	.028659	.029469	.030111	.030575	.030855	166
167	.023992	.025314	.026533	.027629	.028587	.029390	.030026	.030485	167
168	.022564	.023958	.025269	.026478	.027566	.028515	.029311	.029941	168
169	.021095	.022540	.023924	.025225	.026424	.027502	.028443	.029232	169
170	.019606	.021082	.022516	.023889	.025180	.026370	.027440	.028373	170
171	.018116	.019602	.021068	.022492	.023855	.025136	.026316	.027377	171
172	.016641	.018121	.019598	.021054	.022468	.023821	.025092	.026263	172
173	.015198	.016654	.018125	.019594	.021040	.022444	.023787	.025048	173
174	.013801	.015219	.016667	.018130	.019589	.021025	.022420	.023753	174
175	.012460	.013827	.015238	.016679	.018133	.019583	.021011	.022395	175
176	.011186	.012492	.013853	.015258	.016691	.018137	.019578	.020996	176
177	.009985	.011221	.012523	.013879	.015277	.016702	.018140	.019572	177
178	.008863	.010023	.011256	.012553	.013903	.015295	.016713	.018143	178
179	.007823	.008904	.010061	.011291	.012583	.013928	.015313	.016724	179
180	.006867	.007865	.008944	.010099	.011325	.012612	.013952	.015330	180
181	.005995	.006909	.007906	.008983	.010136	.011358	.012641	.013975	181
182	.005204	.006036	.006950	.007947	.009022	.010172	.011391	.012670	182
183	.004493	.005244	.006077	.006991	.007987	.009061	.010208	.011424	183
184	.003858	.004532	.005284	.006117	.007032	.008027	.009099	.010244	184
185	.003295	.003895	.004570	.005324	.006158	.007072	.008066	.009136	185
186	.002799	.003329	.003931	.004608	.005363	.006198	.007112	.008105	186
187	.002365	.002831	.003364	.003967	.004646	.005402	.006237	.007151	187
188	.001988	.002394	.002863	.003398	.004004	.004684	.005441	.006277	188
189	.001662	.002014	.002423	.002894	.003432	.004039	.004721	.005479	189
190	.001382	.001686	.002041	.002453	.002926	.003465	.004075	.004758	190
191	.001143	.001403	.001710	.002067	.002482	.002957	.003499	.004111	191
192	.000941	.001162	.001425	.001734	.002094	.002511	.002989	.003533	192
193	.000770	.000957	.001181	.001446	.001758	.002120	.002540	.003020	193
194	.000627	.000785	.000974	.001200	.001468	.001782	.002147	.002569	194
195	.000508	.00[illegible]40	.000799	.000991	.001219	.001489	.001806	.002173	195
196	.000410	.000519	.000652	.000814	.001008	.001238	.001511	.001830	196
197	.000329	.000419	.000530	.000665	.000829	.001025	.001258	.001533	197
198	.000262	.000336	.000428	.000541	.000678	.000844	.001042	.001277	198
199	.000208	.000269	.000344	.000438	.000552	.000691	.000859	.001059	199
200	.000164	.000214	.000275	.000352	.000447	.000563	.000704	.000874	200
201	.000129	.000169	.000219	.000282	.000360	.000457	.000574	.000717	201
202	.000101	.000133	.000174	.000225	.000289	.000369	.000466	.000586	202
203	.000079	.000104	.000137	.000178	.000231	.000296	.000377	.000476	203
204	.000061	.000081	.000107	.000141	.000183	.000236	.000303	.000385	204
205	.000047	.000063	.000084	.000111	.000145	.000188	.000242	.000310	205
206	.000036	.000049	.000065	.000086	.000114	.000149	.000193	.000248	206
207	.000027	.000037	.000050	.000067	.000089	.000117	.000153	.000198	207
208	.000021	.000029	.000039	.000052	.000069	.000092	.000120	.000157	208
209	.000016	.000022	.000030	.000040	.000054	.000072	.000095	.000124	209
210	.000012	.000016	.000023	.000031	.000041	.000056	.000074	.000097	210

TABLE I

X	A=158	A=159	A=16Ø	A=161	A=162	A=163	A=164	A=165	X
211	.ØØØØØ9	.ØØØØ12	.ØØØØ17	.ØØØØ23	.ØØØØ32	.ØØØØ43	.ØØØØ57	.ØØØØ76	211
212	.ØØØØØ7	.ØØØØØ9	.ØØØØ13	.ØØØØ18	.ØØØØ24	.ØØØØ33	.ØØØØ44	.ØØØØ59	212
213	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ13	.ØØØØ19	.ØØØØ25	.ØØØØ34	.ØØØØ46	213
214	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ14	.ØØØØ19	.ØØØØ26	.ØØØØ35	214
215	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ8	.ØØØØ11	.ØØØØ15	.ØØØØ2Ø	.ØØØØ27	215
216	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	.ØØØØ11	.ØØØØ15	.ØØØØ21	216
217	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	.ØØØØ11	.ØØØØ16	217
218	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ9	.ØØØØ12	218
219	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ6	.ØØØØØ9	219
22Ø	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	22Ø
221		.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	221
222			.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	222
223				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	223
224					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	224
225						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	225
226						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	226
227							.ØØØØØ1	.ØØØØØ1	227
228								.ØØØØØ1	228

X	A=166	A=167	A=168	A=169	A=17Ø	A=171	A=172	A=173	X
1Ø9	.ØØØØØ1								1Ø9
11Ø	.ØØØØØ1	.ØØØØØ1							11Ø
111	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1						111
112	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1					112
113	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			113
114	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1		114
115	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	115
116	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	116
117	.ØØØØ12	.ØØØØØ9	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	117
118	.ØØØØ16	.ØØØØ12	.ØØØØØ9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	118
119	.ØØØØ23	.ØØØØ17	.ØØØØ13	.ØØØØØ9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	119
12Ø	.ØØØØ31	.ØØØØ24	.ØØØØ18	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	12Ø
121	.ØØØØ43	.ØØØØ33	.ØØØØ25	.ØØØØ19	.ØØØØ14	.ØØØØ1Ø	.ØØØØØ8	.ØØØØØ6	121
122	.ØØØØ58	.ØØØØ45	.ØØØØ34	.ØØØØ26	.ØØØØ2Ø	.ØØØØ15	.ØØØØ11	.ØØØØØ8	122
123	.ØØØØ79	.ØØØØ61	.ØØØØ46	.ØØØØ35	.ØØØØ27	.ØØØØ2Ø	.ØØØØ15	.ØØØØ12	123
124	.ØØØ1Ø5	.ØØØØ82	.ØØØØ63	.ØØØØ48	.ØØØØ37	.ØØØØ28	.ØØØØ21	.ØØØØ16	124
125	.ØØØ14Ø	.ØØØ1Ø9	.ØØØØ85	.ØØØØ65	.ØØØØ5Ø	.ØØØØ38	.ØØØØ29	.ØØØØ22	125
126	.ØØØ184	.ØØØ145	.ØØØ113	.ØØØØ88	.ØØØØ68	.ØØØØ52	.ØØØØ4Ø	.ØØØØ31	126
127	.ØØØ241	.ØØØ19Ø	.ØØØ149	.ØØØ117	.ØØØØ91	.ØØØØ7Ø	.ØØØØ54	.ØØØØ42	127
128	.ØØØ312	.ØØØ248	.ØØØ196	.ØØØ154	.ØØØ121	.ØØØØ94	.ØØØØ73	.ØØØØ56	128
129	.ØØØ4Ø2	.ØØØ321	.ØØØ255	.ØØØ2Ø2	.ØØØ159	.ØØØ125	.ØØØØ97	.ØØØØ76	129
13Ø	.ØØØ513	.ØØØ412	.ØØØ33Ø	.ØØØ262	.ØØØ2Ø8	.ØØØ164	.ØØØ129	.ØØØ1Ø1	13Ø
131	.ØØØ651	.ØØØ526	.ØØØ423	.ØØØ338	.ØØØ27Ø	.ØØØ214	.ØØØ169	.ØØØ133	131
132	.ØØØ818	.ØØØ665	.ØØØ538	.ØØØ433	.ØØØ347	.ØØØ277	.ØØØ22Ø	.ØØØ174	132
133	.ØØ1Ø21	.ØØØ835	.ØØØ68Ø	.ØØØ551	.ØØØ444	.ØØØ356	.ØØØ285	.ØØØ226	133
134	.ØØ1265	.ØØ1Ø41	.ØØØ852	.ØØØ694	.ØØØ563	.ØØØ455	.ØØØ365	.ØØØ292	134
135	.ØØ1556	.ØØ1287	.ØØ1Ø6Ø	.ØØØ869	.ØØØ7Ø9	.ØØØ576	.ØØØ465	.ØØØ374	135
136	.ØØ1899	.ØØ1581	.ØØ131Ø	.ØØ1Ø8Ø	.ØØØ886	.ØØØ724	.ØØØ589	.ØØØ476	136
137	.ØØ23Ø1	.ØØ1927	.ØØ16Ø6	.ØØ1332	.ØØ11ØØ	.ØØØ9Ø4	.ØØØ739	.ØØØ6Ø2	137
138	.ØØ2767	.ØØ2332	.ØØ1955	.ØØ1632	.ØØ1355	.ØØ112Ø	.ØØØ921	.ØØØ754	138
139	.ØØ33Ø5	.ØØ28Ø2	.ØØ2363	.ØØ1984	.ØØ1657	.ØØ1378	.ØØ114Ø	.ØØØ939	139
14Ø	.ØØ3919	.ØØ3342	.ØØ2836	.ØØ2395	.ØØ2Ø12	.ØØ1683	.ØØ14ØØ	.ØØ116Ø	14Ø
141	.ØØ4613	.ØØ3958	.ØØ3379	.ØØ287Ø	.ØØ2426	.ØØ2Ø41	.ØØ17Ø8	.ØØ1423	141
142	.ØØ5393	.ØØ4655	.ØØ3998	.ØØ3416	.ØØ29Ø4	.ØØ2457	.ØØ2Ø69	.ØØ1734	142
143	.ØØ6261	.ØØ5437	.ØØ4697	.ØØ4Ø37	.ØØ3453	.ØØ2939	.ØØ2489	.ØØ2Ø97	143
144	.ØØ7217	.ØØ63Ø5	.ØØ548Ø	.ØØ4738	.ØØ4Ø76	.ØØ349Ø	.ØØ2973	.ØØ252Ø	144
145	.ØØ8262	.ØØ7261	.ØØ6349	.ØØ5522	.ØØ4779	.ØØ4115	.ØØ3526	.ØØ3ØØ6	145
146	.ØØ9394	.ØØ83Ø6	.ØØ73Ø5	.ØØ6392	.ØØ5565	.ØØ482Ø	.ØØ4154	.ØØ3562	146
147	.Ø1Ø6Ø8	.ØØ9436	.ØØ8349	.ØØ7349	.ØØ6435	.ØØ56Ø7	.ØØ486Ø	.ØØ4193	147
148	.Ø11899	.Ø1Ø647	.ØØ9477	.ØØ8392	.ØØ7392	.ØØ6478	.ØØ5649	.ØØ49Ø1	148
149	.Ø13256	.Ø11934	.Ø1Ø686	.ØØ9518	.ØØ8434	.ØØ7435	.ØØ6521	.ØØ569Ø	149
15Ø	.Ø1467Ø	.Ø13286	.Ø11968	.Ø1Ø724	.ØØ9558	.ØØ8476	.ØØ7477	.ØØ6563	15Ø
151	.Ø16127	.Ø14694	.Ø13316	.Ø12ØØ2	.Ø1Ø761	.ØØ9598	.ØØ8517	.ØØ7519	151
152	.Ø17613	.Ø16144	.Ø14717	.Ø13344	.Ø12Ø35	.Ø1Ø798	.ØØ9637	.ØØ8558	152
153	.Ø191Ø9	.Ø17621	.Ø1616Ø	.Ø1474Ø	.Ø13373	.Ø12Ø68	.Ø1Ø834	.ØØ9676	153
154	.Ø2Ø598	.Ø191Ø9	.Ø17629	.Ø16176	.Ø14762	.Ø134ØØ	.Ø121Ø1	.Ø1Ø87Ø	154
155	.Ø22Ø6Ø	.Ø2Ø588	.Ø191Ø8	.Ø17637	.Ø16191	.Ø14784	.Ø13428	.Ø12132	155

TABLE I

X	A=166	A=167	A=168	A=169	A=17Ø	A=171	A=172	A=173	X
156	.Ø23474	.Ø22Ø4Ø	.Ø2Ø577	.Ø191Ø6	.Ø17644	.Ø162Ø5	.Ø148Ø5	.Ø13454	156
157	.Ø2482Ø	.Ø23444	.Ø22Ø19	.Ø2Ø567	.Ø191Ø5	.Ø1765Ø	.Ø16219	.Ø14826	157
158	.Ø26Ø77	.Ø24779	.Ø23413	.Ø21998	.Ø2Ø556	.Ø191Ø2	.Ø17656	.Ø16233	158
159	.Ø27225	.Ø26Ø26	.Ø24738	.Ø23382	.Ø21978	.Ø2Ø544	.Ø191ØØ	.Ø17662	159
16Ø	.Ø28246	.Ø27164	.Ø25975	.Ø24697	.Ø23351	.Ø21957	.Ø2Ø533	.Ø19Ø97	16Ø
161	.Ø29123	.Ø28177	.Ø271Ø4	.Ø25924	.Ø24657	.Ø2332Ø	.Ø21935	.Ø2Ø521	161
162	.Ø29842	.Ø29Ø46	.Ø281Ø8	.Ø27Ø45	.Ø25874	.Ø24616	.Ø2329Ø	.Ø21914	162
163	.Ø3Ø391	.Ø29759	.Ø2897Ø	.Ø28Ø4Ø	.Ø26985	.Ø25824	.Ø24575	.Ø23259	163
164	.Ø3Ø762	.Ø3Ø3Ø4	.Ø29677	.Ø28895	.Ø27973	.Ø26926	.Ø25774	.Ø24535	164
165	.Ø3Ø948	.Ø3Ø671	.Ø3Ø217	.Ø29596	.Ø2882Ø	.Ø279Ø5	.Ø26868	.Ø25725	165
166	.Ø3Ø948	.Ø3Ø856	.Ø3Ø581	.Ø3Ø13Ø	.Ø29515	.Ø28746	.Ø27839	.Ø268Ø9	166
167	.Ø3Ø763	.Ø3Ø856	.Ø3Ø764	.Ø3Ø491	.Ø3ØØ45	.Ø29435	.Ø28672	.Ø27773	167
168	.Ø3Ø397	.Ø3Ø672	.Ø3Ø764	.Ø3Ø673	.Ø3Ø4Ø3	.Ø2996Ø	.Ø29355	.Ø28599	168
169	.Ø29857	.Ø3Ø3Ø9	.Ø3Ø582	.Ø3Ø673	.Ø3Ø582	.Ø3Ø315	.Ø29876	.Ø29276	169
17Ø	.Ø29155	.Ø29774	.Ø3Ø222	.Ø3Ø492	.Ø3Ø582	.Ø3Ø493	.Ø3Ø228	.Ø29793	17Ø
171	.Ø283Ø2	.Ø29Ø78	.Ø29692	.Ø3Ø136	.Ø3Ø4Ø4	.Ø3Ø493	.Ø3Ø4Ø4	.Ø3Ø141	171
172	.Ø27315	.Ø28232	.Ø29ØØ1	.Ø2961Ø	.Ø3ØØ5Ø	.Ø3Ø316	.Ø3Ø4Ø4	.Ø3Ø316	172
173	.Ø2621Ø	.Ø27253	.Ø28163	.Ø28925	.Ø29529	.Ø29965	.Ø3Ø229	.Ø3Ø316	173
174	.Ø25ØØ5	.Ø26157	.Ø27192	.Ø28Ø94	.Ø2885Ø	.Ø29449	.Ø29881	.Ø3Ø142	174
175	.Ø23719	.Ø24961	.Ø261Ø4	.Ø27131	.Ø28Ø26	.Ø28775	.Ø29369	.Ø29798	175
176	.Ø22371	.Ø23685	.Ø24918	.Ø26Ø52	.Ø27Ø7Ø	.Ø27958	.Ø287Ø1	.Ø2929Ø	176
177	.Ø2Ø981	.Ø22347	.Ø23651	.Ø24874	.Ø26ØØØ	.Ø27Ø1Ø	.Ø27891	.Ø28628	177
178	.Ø19566	.Ø2Ø966	.Ø22322	.Ø23617	.Ø24831	.Ø25948	.Ø2695Ø	.Ø27824	178
179	.Ø18145	.Ø1956Ø	.Ø2Ø95Ø	.Ø22297	.Ø23583	.Ø24788	.Ø25897	.Ø26891	179
18Ø	.Ø16734	.Ø18147	.Ø19554	.Ø2Ø935	.Ø22273	.Ø23549	.Ø24746	.Ø25845	18Ø
181	.Ø15347	.Ø16744	.Ø18149	.Ø19547	.Ø2Ø919	.Ø22248	.Ø23515	.Ø247Ø3	181
182	.Ø13998	.Ø15364	.Ø16753	.Ø18151	.Ø1954Ø	.Ø2Ø9Ø3	.Ø22223	.Ø23481	182
183	.Ø12698	.Ø14Ø2Ø	.Ø1538Ø	.Ø16762	.Ø18152	.Ø19533	.Ø2Ø887	.Ø22198	183
184	.Ø11455	.Ø12725	.Ø14Ø42	.Ø15396	.Ø16771	.Ø18153	.Ø19525	.Ø2Ø871	184
185	.Ø1Ø279	.Ø11487	.Ø12752	.Ø14Ø64	.Ø15411	.Ø16779	.Ø18153	.Ø19517	185
186	.ØØ9174	.Ø1Ø314	.Ø11518	.Ø12779	.Ø14Ø85	.Ø15426	.Ø16787	.Ø18153	186
187	.ØØ8144	.ØØ9211	.Ø1Ø348	.Ø11549	.Ø128Ø5	.Ø141Ø6	.Ø1544Ø	.Ø16794	187
188	.ØØ7191	.ØØ8182	.ØØ9247	.Ø1Ø381	.Ø11579	.Ø1283Ø	.Ø14126	.Ø15454	188
189	.ØØ6316	.ØØ7229	.ØØ8219	.ØØ9283	.Ø1Ø415	.Ø116Ø8	.Ø12855	.Ø14146	189
19Ø	.ØØ5518	.ØØ6354	.ØØ7268	.ØØ8257	.ØØ9318	.Ø1Ø448	.Ø11638	.Ø1288Ø	19Ø
191	.ØØ4796	.ØØ5556	.ØØ6393	.ØØ73Ø6	.ØØ8294	.ØØ9354	.Ø1Ø48Ø	.Ø11666	191
192	.ØØ4146	.ØØ4832	.ØØ5594	.ØØ6431	.ØØ7344	.ØØ8331	.ØØ9388	.Ø1Ø512	192
193	.ØØ3566	.ØØ4181	.ØØ4869	.ØØ5631	.ØØ6468	.ØØ7381	.ØØ8367	.ØØ9423	193
194	.ØØ3Ø51	.ØØ3599	.ØØ4216	.ØØ49Ø5	.ØØ5668	.ØØ65Ø6	.ØØ7418	.ØØ84Ø3	194
195	.ØØ2598	.ØØ3Ø83	.ØØ3633	.ØØ4251	.ØØ4941	.ØØ57Ø5	.ØØ6543	.ØØ7455	195
196	.ØØ22ØØ	.ØØ2626	.ØØ3114	.ØØ3666	.ØØ4286	.ØØ4977	.ØØ5742	.ØØ658Ø	196
197	.ØØ1854	.ØØ2227	.ØØ2655	.ØØ3145	.ØØ3699	.ØØ4321	.ØØ5Ø13	.ØØ5778	197
198	.ØØ1554	.ØØ1878	.ØØ2253	.ØØ2684	.ØØ3176	.ØØ3731	.ØØ4355	.ØØ5Ø49	198
199	.ØØ1296	.ØØ1576	.ØØ19Ø2	.ØØ2279	.ØØ2713	.ØØ32Ø6	.ØØ3764	.ØØ4389	199
2ØØ	.ØØ1Ø76	.ØØ1316	.ØØ1598	.ØØ1926	.ØØ23Ø6	.ØØ2741	.ØØ3237	.ØØ3797	2ØØ
2Ø1	.ØØØ889	.ØØ1Ø93	.ØØ1335	.ØØ1619	.ØØ195Ø	.ØØ2332	.ØØ277Ø	.ØØ3268	2Ø1
2Ø2	.ØØØ73Ø	.ØØØ9Ø4	.ØØ1111	.ØØ1355	.ØØ1641	.ØØ1974	.ØØ2359	.ØØ2799	2Ø2
2Ø3	.ØØØ597	.ØØØ744	.ØØØ919	.ØØ1128	.ØØ1374	.ØØ1663	.ØØ1998	.ØØ2385	2Ø3
2Ø4	.ØØØ486	.ØØØ6Ø9	.ØØØ757	.ØØØ934	.ØØ1145	.ØØ1394	.ØØ1685	.ØØ2Ø23	2Ø4
2Ø5	.ØØØ394	.ØØØ496	.ØØØ62Ø	.ØØØ77Ø	.ØØØ95Ø	.ØØ1163	.ØØ1414	.ØØ17Ø7	2Ø5
2Ø6	.ØØØ317	.ØØØ4Ø2	.ØØØ5Ø6	.ØØØ632	.ØØØ784	.ØØØ965	.ØØ118Ø	.ØØ1433	2Ø6
2Ø7	.ØØØ254	.ØØØ324	.ØØØ411	.ØØØ516	.ØØØ644	.ØØØ797	.ØØØ981	.ØØ1198	2Ø7
2Ø8	.ØØØ2Ø3	.ØØØ26Ø	.ØØØ332	.ØØØ419	.ØØØ526	.ØØØ656	.ØØØ811	.ØØØ996	2Ø8
2Ø9	.ØØØ161	.ØØØ2Ø8	.ØØØ267	.ØØØ339	.ØØØ428	.ØØØ536	.ØØØ667	.ØØØ825	2Ø9
21Ø	.ØØØ127	.ØØØ165	.ØØØ213	.ØØØ273	.ØØØ346	.ØØØ437	.ØØØ547	.ØØØ679	21Ø
211	.ØØØ1ØØ	.ØØØ131	.ØØØ17Ø	.ØØØ218	.ØØØ279	.ØØØ354	.ØØØ446	.ØØØ557	211
212	.ØØØØ78	.ØØØ1Ø3	.ØØØ135	.ØØØ174	.ØØØ224	.ØØØ286	.ØØØ362	.ØØØ455	212
213	.ØØØØ61	.ØØØØ81	.ØØØ1Ø6	.ØØØ138	.ØØØ179	.ØØØ229	.ØØØ292	.ØØØ369	213
214	.ØØØØ47	.ØØØØ63	.ØØØØ83	.ØØØ1Ø9	.ØØØ142	.ØØØ183	.ØØØ235	.ØØØ298	214
215	.ØØØØ37	.ØØØØ49	.ØØØØ65	.ØØØØ86	.ØØØ112	.ØØØ146	.ØØØ188	.ØØØ24Ø	215
216	.ØØØØ28	.ØØØØ38	.ØØØØ51	.ØØØØ67	.ØØØØ88	.ØØØ115	.ØØØ149	.ØØØ192	216
217	.ØØØØ22	.ØØØØ29	.ØØØØ39	.ØØØØ52	.ØØØØ69	.ØØØØ91	.ØØØ118	.ØØØ153	217
218	.ØØØØ16	.ØØØØ22	.ØØØØ3Ø	.ØØØØ41	.ØØØØ54	.ØØØØ71	.ØØØØ93	.ØØØ122	218
219	.ØØØØ12	.ØØØØ17	.ØØØØ23	.ØØØØ31	.ØØØØ42	.ØØØØ56	.ØØØØ73	.ØØØØ96	219
22Ø	.ØØØØØ9	.ØØØØ13	.ØØØØ18	.ØØØØ24	.ØØØØ32	.ØØØØ43	.ØØØØ57	.ØØØØ76	22Ø
221	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ13	.ØØØØ18	.ØØØØ25	.ØØØØ33	.ØØØØ45	.ØØØØ59	221
222	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ14	.ØØØØ19	.ØØØØ26	.ØØØØ35	.ØØØØ46	222
223	.ØØØØØ4	.ØØØØØ5	.ØØØØØ8	.ØØØØ11	.ØØØØ15	.ØØØØ2Ø	.ØØØØ27	.ØØØØ36	223

TABLE I

X	A=166	A=167	A=168	A=169	A=170	A=171	A=172	A=173	X
224	.000003	.000004	.000006	.000008	.000011	.000015	.000020	.000028	224
225	.000002	.000003	.000004	.000006	.000008	.000011	.000016	.000021	225
226	.000002	.000002	.000003	.000004	.000006	.000009	.000012	.000016	226
227	.000001	.000002	.000002	.000003	.000005	.000007	.000009	.000012	227
228	.000001	.000001	.000002	.000002	.000003	.000005	.000007	.000009	228
229	.000001	.000001	.000001	.000002	.000003	.000004	.000005	.000007	229
230		.000001	.000001	.000001	.000002	.000003	.000004	.000005	230
231			.000001	.000001	.000001	.000002	.000003	.000004	231
232				.000001	.000001	.000001	.000002	.000003	232
233				.000001	.000001	.000001	.000002	.000002	233
234					.000001	.000001	.000001	.000002	234
235						.000001	.000001	.000001	235
236							.000001	.000001	236
237								.000001	237

X	A=174	A=175	A=176	A=177	A=178	A=179	A=180	A=181	X
116	.000001								116
117	.000001	.000001							117
118	.000001	.000001	.000001	.000001					118
119	.000002	.000001	.000001	.000001	.000001				119
120	.000003	.000002	.000002	.000001	.000001	.000001			120
121	.000004	.000003	.000002	.000002	.000001	.000001	.000001		121
122	.000006	.000005	.000003	.000002	.000002	.000001	.000001	.000001	122
123	.000009	.000006	.000005	.000004	.000003	.000002	.000001	.000001	123
124	.000012	.000009	.000007	.000005	.000004	.000003	.000002	.000001	124
125	.000017	.000013	.000010	.000007	.000005	.000004	.000003	.000002	125
126	.000023	.000018	.000013	.000010	.000007	.000006	.000004	.000003	126
127	.000032	.000024	.000018	.000014	.000010	.000008	.000006	.000004	127
128	.000043	.000033	.000025	.000019	.000015	.000011	.000008	.000006	128
129	.000058	.000045	.000035	.000026	.000020	.000015	.000011	.000009	129
130	.000078	.000061	.000047	.000036	.000028	.000021	.000016	.000012	130
131	.000104	.000081	.000063	.000049	.000037	.000029	.000022	.000017	131
132	.000137	.000107	.000084	.000065	.000050	.000039	.000030	.000023	132
133	.000179	.000141	.000111	.000087	.000067	.000052	.000040	.000031	133
134	.000233	.000185	.000146	.000114	.000090	.000070	.000054	.000042	134
135	.000300	.000239	.000190	.000150	.000118	.000093	.000072	.000056	135
136	.000384	.000308	.000246	.000195	.000155	.000122	.000096	.000075	136
137	.000487	.000393	.000316	.000252	.000201	.000159	.000126	.000099	137
138	.000615	.000499	.000403	.000324	.000259	.000207	.000164	.000130	138
139	.000769	.000628	.000510	.000412	.000332	.000266	.000212	.000169	139
140	.000956	.000785	.000641	.000521	.000422	.000340	.000273	.000218	140
141	.001180	.000974	.000800	.000654	.000533	.000432	.000348	.000280	141
142	.001446	.001200	.000992	.000816	.000668	.000544	.000442	.000357	142
143	.001759	.001469	.001220	.001009	.000831	.000681	.000556	.000452	143
144	.002126	.001785	.001492	.001241	.001027	.000847	.000695	.000568	144
145	.002551	.002154	.001811	.001515	.001261	.001045	.000863	.000709	145
146	.003040	.002582	.002183	.001836	.001538	.001282	.001064	.000878	146
147	.003599	.003074	.002613	.002211	.001862	.001561	.001302	.001082	147
148	.004231	.003635	.003108	.002644	.002239	.001888	.001584	.001323	148
149	.004941	.004269	.003671	.003141	.002675	.002268	.001913	.001607	149
150	.005731	.004981	.004307	.003707	.003175	.002706	.002296	.001939	150
151	.006604	.005772	.005020	.004345	.003742	.003208	.002737	.002324	151
152	.007560	.006646	.005813	.005059	.004382	.003778	.003241	.002768	152
153	.008598	.007601	.006687	.005853	.005098	.004420	.003813	.003274	153
154	.009714	.008638	.007642	.006727	.005893	.005137	.004457	.003848	154
155	.010905	.009752	.008677	.007682	.006767	.005933	.005176	.004494	155
156	.012164	.010940	.009790	.008716	.007722	.006807	.005972	.005214	156
157	.013481	.012194	.010974	.009826	.008755	.007761	.006847	.006011	157
158	.014846	.013506	.012225	.011008	.009863	.008793	.007800	.006886	158
159	.016246	.014865	.013532	.012254	.011041	.009899	.008830	.007839	159
160	.017668	.016259	.014885	.013556	.012284	.011074	.009934	.008868	160
161	.019094	.017673	.016272	.014904	.013581	.012312	.011107	.009969	161
162	.020509	.019091	.017678	.016284	.014922	.013604	.012341	.011139	162
163	.021893	.020497	.019088	.017682	.016295	.014940	.013628	.012369	163
164	.023228	.021871	.020484	.019084	.017686	.016306	.014957	.013651	164
165	.024495	.023197	.021850	.020472	.019080	.017690	.016317	.014974	165

TABLE I

X	A=174	A=175	A=176	A=177	A=178	A=179	A=180	A=181	X
166	.025675	.024455	.023166	.021828	.020459	.019005	.017693	.016528	166
167	.026751	.025626	.024415	.023135	.021806	.020446	.019071	.017696	167
168	.027707	.026694	.025577	.024375	.023104	.021785	.020433	.019066	168
169	.028527	.027642	.026637	.025529	.024335	.023074	.021763	.020419	169
170	.029198	.028455	.027577	.026580	.025480	.024295	.023043	.021741	170
171	.029710	.029120	.028383	.027512	.026523	.025432	.024256	.023012	171
172	.030056	.029628	.029043	.028312	.027448	.026467	.025384	.024216	172
173	.030229	.029971	.029547	.028967	.028242	.027385	.026411	.025336	173
174	.030229	.030143	.029886	.029466	.028891	.028172	.027322	.026355	174
175	.030057	.030143	.030057	.029803	.029386	.028816	.028102	.027259	175
176	.029715	.029972	.030057	.029972	.029720	.029307	.028741	.028033	176
177	.029211	.029633	.029887	.029972	.029888	.029638	.029228	.028667	177
178	.028555	.029133	.029552	.029804	.029888	.029804	.029557	.029150	178
179	.027757	.028482	.029056	.029471	.029721	.029804	.029722	.029476	179
180	.026832	.027691	.028411	.028980	.029391	.029639	.029722	.029639	180
181	.025794	.026773	.027626	.028339	.028904	.029311	.029557	.029639	181
182	.024661	.025744	.026715	.027561	.028268	.028828	.029233	.029477	182
183	.023448	.024618	.025693	.026657	.027496	.028198	.028753	.029154	183
184	.022173	.023414	.024576	.025643	.026599	.027432	.028128	.028679	184
185	.020855	.022148	.023380	.024534	.025593	.026542	.027368	.028059	185
186	.019509	.020839	.022123	.023347	.024492	.025543	.026485	.027305	186
187	.018153	.019501	.020822	.022098	.023313	.024451	.025494	.026429	187
188	.016801	.018153	.019493	.020805	.022073	.023280	.024409	.025445	188
189	.015468	.016808	.018152	.019484	.020789	.022048	.023247	.024368	189
190	.014165	.015481	.016815	.018151	.019476	.020772	.022023	.023213	190
191	.012905	.014184	.015494	.016821	.018150	.019467	.020755	.021998	191
192	.011695	.012928	.014203	.015507	.016827	.018149	.019458	.020738	192
193	.010543	.011723	.012952	.014221	.015519	.016832	.018147	.019448	193
194	.009457	.010575	.011750	.012905	.014239	.015531	.016837	.018145	194
195	.008438	.009490	.010605	.011777	.012998	.014256	.015542	.016842	195
196	.007491	.008473	.009523	.010636	.011804	.013020	.014273	.015553	196
197	.006616	.007527	.008508	.009556	.010666	.011830	.013042	.014290	197
198	.005814	.006653	.007563	.008542	.009588	.010695	.011856	.013063	198
199	.005084	.005850	.006689	.007598	.008576	.009620	.010724	.011882	199
200	.004423	.005119	.005886	.006724	.007633	.008610	.009652	.010753	200
201	.003829	.004457	.005154	.005921	.006760	.007668	.008643	.009683	201
202	.003298	.003861	.004490	.005188	.005956	.006795	.007702	.008676	202
203	.002827	.003329	.003893	.004524	.005223	.005991	.006829	.007736	203
204	.002411	.002855	.003359	.003925	.004557	.005257	.006026	.006864	204
205	.002047	.002438	.002884	.003389	.003957	.004590	.005291	.006060	205
206	.001729	.002071	.002464	.002912	.003419	.003989	.004623	.005325	206
207	.001453	.001751	.002095	.002490	.002940	.003449	.004020	.004656	207
208	.001216	.001473	.001772	.002119	.002516	.002968	.003479	.004052	208
209	.001012	.001233	.001493	.001794	.002143	.002542	.002996	.003509	209
210	.000839	.001028	.001251	.001512	.001816	.002167	.002568	.003024	210
211	.000691	.000852	.001043	.001269	.001532	.001838	.002191	.002594	211
212	.000568	.000704	.000866	.001059	.001287	.001552	.001860	.002215	212
213	.000464	.000578	.000716	.000880	.001075	.001304	.001572	.001882	213
214	.000377	.000473	.000589	.000728	.000894	.001091	.001322	.001592	214
215	.000305	.000385	.000482	.000599	.000740	.000908	.001107	.001340	215
216	.000246	.000312	.000393	.000491	.000610	.000753	.000923	.001123	216
217	.000197	.000251	.000318	.000401	.000500	.000621	.000765	.000937	217
218	.000157	.000202	.000257	.000325	.000409	.000510	.000632	.000778	218
219	.000125	.000161	.000207	.000263	.000332	.000417	.000519	.000643	219
220	.000099	.000128	.000165	.000211	.000269	.000339	.000425	.000529	220
221	.000078	.000102	.000132	.000169	.000216	.000275	.000346	.000433	221
222	.000061	.000080	.000104	.000135	.000174	.000221	.000281	.000353	222
223	.000048	.000063	.000082	.000107	.000139	.000178	.000226	.000287	223
224	.000037	.000049	.000065	.000085	.000110	.000142	.000182	.000232	224
225	.000029	.000038	.000051	.000067	.000087	.000113	.000146	.000186	225
226	.000022	.000030	.000039	.000052	.000069	.000089	.000116	.000149	226
227	.000017	.000023	.000031	.000041	.000054	.000071	.000092	.000119	227
228	.000013	.000017	.000024	.000032	.000042	.000055	.000073	.000094	228
229	.000010	.000013	.000018	.000024	.000033	.000043	.000057	.000075	229
230	.000007	.000010	.000014	.000019	.000025	.000034	.000045	.000059	230
231	.000006	.000008	.000011	.000014	.000019	.000026	.000035	.000046	231
232	.000004	.000006	.000008	.000011	.000015	.000020	.000027	.000036	232
233	.000003	.000004	.000006	.000008	.000011	.000015	.000021	.000028	233

TABLE I

X	A=174	A=175	A=176	A=177	A=178	A=179	A=180	A=181	X
234	.000002	.000003	.000005	.000006	.000009	.000012	.000016	.000022	234
235	.000002	.000002	.000003	.000005	.000007	.000009	.000012	.000017	235
236	.000001	.000002	.000003	.000004	.000005	.000007	.000009	.000013	236
237	.000001	.000001	.000002	.000003	.000004	.000005	.000007	.000010	237
238	.000001	.000001	.000001	.000002	.000003	.000004	.000005	.000007	238
239		.000001	.000001	.000001	.000002	.000003	.000004	.000006	239
240		.000001	.000001	.000001	.000002	.000002	.000003	.000004	240
241			.000001	.000001	.000001	.000002	.000002	.000003	241
242				.000001	.000001	.000001	.000002	.000002	242
243					.000001	.000001	.000001	.000002	243
244						.000001	.000001	.000001	244
245							.000001	.000001	245
246								.000001	246
247								.000001	247

X	A=182	A=183	A=184	A=185	A=186	A=187	A=188	A=189	X
123	.000001	.000001							123
124	.000001	.000001	.000001						124
125	.000002	.000001	.000001	.000001					125
126	.000002	.000002	.000001	.000001	.000001				126
127	.000003	.000002	.000002	.000001	.000001	.000001			127
128	.000005	.000003	.000003	.000002	.000001	.000001	.000001	.000001	128
129	.000006	.000005	.000004	.000003	.000002	.000001	.000001	.000001	129
130	.000009	.000007	.000005	.000004	.000003	.000002	.000002	.000001	130
131	.000013	.000009	.000007	.000005	.000004	.000003	.000002	.000002	131
132	.000017	.000013	.000010	.000007	.000006	.000004	.000003	.000002	132
133	.000024	.000018	.000014	.000010	.000008	.000006	.000004	.000003	133
134	.000032	.000025	.000019	.000014	.000011	.000008	.000006	.000005	134
135	.000043	.000033	.000026	.000020	.000015	.000011	.000009	.000006	135
136	.000058	.000045	.000035	.000027	.000020	.000016	.000012	.000009	136
137	.000077	.000060	.000047	.000036	.000028	.000021	.000016	.000012	137
138	.000102	.000080	.000062	.000048	.000037	.000029	.000022	.000017	138
139	.000133	.000105	.000083	.000064	.000050	.000039	.000030	.000023	139
140	.000173	.000137	.000108	.000085	.000067	.000052	.000040	.000031	140
141	.000224	.000178	.000142	.000112	.000088	.000069	.000054	.000042	141
142	.000287	.000230	.000183	.000146	.000115	.000091	.000071	.000056	142
143	.000365	.000294	.000236	.000188	.000150	.000119	.000094	.000074	143
144	.000462	.000374	.000301	.000242	.000194	.000154	.000122	.000096	144
145	.000580	.000472	.000383	.000309	.000248	.000199	.000158	.000126	145
146	.000722	.000591	.000482	.000391	.000316	.000255	.000204	.000163	146
147	.000894	.000736	.000603	.000493	.000400	.000324	.000261	.000209	147
148	.001100	.000910	.000750	.000616	.000503	.000409	.000331	.000267	148
149	.001343	.001118	.000927	.000764	.000628	.000513	.000418	.000339	149
150	.001630	.001364	.001137	.000943	.000779	.000640	.000524	.000427	150
151	.001965	.001653	.001385	.001155	.000959	.000793	.000653	.000535	151
152	.002353	.001990	.001677	.001406	.001173	.000975	.000807	.000665	152
153	.002798	.002381	.002016	.001700	.001427	.001192	.000992	.000821	153
154	.003307	.002829	.002409	.002042	.001723	.001447	.001211	.001008	154
155	.003883	.003340	.002860	.002437	.002068	.001746	.001468	.001229	155
156	.004531	.003918	.003373	.002890	.002465	.002093	.001770	.001489	156
157	.005252	.004567	.003953	.003406	.002921	.002493	.002119	.001793	157
158	.006050	.005290	.004604	.003988	.003438	.002951	.002521	.002145	158
159	.006925	.006088	.005327	.004640	.004022	.003471	.002981	.002549	159
160	.007877	.006964	.006126	.005365	.004676	.004056	.003503	.003011	160
161	.008905	.007915	.007002	.006164	.005402	.004711	.004090	.003535	161
162	.010004	.008941	.007953	.007040	.006202	.005438	.004747	.004124	162
163	.011170	.010038	.008977	.007990	.007077	.006239	.005475	.004782	163
164	.012396	.011201	.010072	.009013	.008026	.007114	.006276	.005511	164
165	.013673	.012423	.011232	.010105	.009048	.008063	.007151	.006313	165
166	.014991	.013695	.012450	.011262	.010138	.009083	.008099	.007188	166
167	.016338	.015007	.013717	.012476	.011291	.010171	.009117	.008134	167
168	.017699	.016347	.015023	.013738	.012501	.011321	.010203	.009151	168
169	.019061	.017702	.016357	.015039	.013759	.012527	.011350	.010234	169
170	.020406	.019055	.017704	.016366	.015054	.013779	.012551	.011378	170
171	.021719	.020392	.019050	.017706	.016374	.015068	.013799	.012576	171
172	.022981	.021697	.020379	.019044	.017707	.016383	.015083	.013819	172

TABLE I

X	A=182	A=183	A=184	A=185	A=186	A=187	A=188	A=189	X
173	.024177	.022951	.021674	.020365	.019038	.017708	.016391	.015097	173
174	.025289	.024138	.022920	.021652	.020351	.019031	.017709	.016398	174
175	.026300	.025241	.024099	.022889	.021630	.020336	.019025	.017710	175
176	.027197	.026245	.025194	.024060	.022859	.021607	.020322	.019018	176
177	.027965	.027135	.026191	.025147	.024021	.022828	.021585	.020307	177
178	.028593	.027897	.027073	.026136	.025101	.023982	.022798	.021562	178
179	.029073	.028520	.027830	.027012	.026082	.025054	.023944	.022767	179
180	.029396	.028996	.028448	.027763	.026952	.026029	.025008	.023905	180
181	.029558	.029316	.028920	.028376	.027696	.026891	.025975	.024962	181
182	.029558	.029477	.029237	.028844	.028305	.027630	.026831	.025922	182
183	.029397	.029477	.029397	.029159	.028769	.028234	.027565	.026772	183
184	.029077	.029317	.029397	.029318	.029082	.028694	.028164	.027499	184
185	.028605	.029000	.029238	.029318	.029239	.029005	.028621	.028094	185
186	.027990	.028532	.028924	.029160	.029239	.029161	.028928	.028547	186
187	.027242	.027922	.028460	.028848	.029082	.029161	.029083	.028853	187
188	.026372	.027179	.027854	.028388	.028773	.029005	.029083	.029006	188
189	.025396	.026317	.027117	.027787	.028316	.028699	.028929	.029006	189
190	.024326	.025347	.026261	.027056	.027720	.028245	.028625	.028853	190
191	.023180	.024285	.025299	.026206	.026995	.027654	.028175	.028551	191
192	.021973	.023147	.024244	.025250	.026151	.026934	.027588	.028105	192
193	.020721	.021948	.023114	.024204	.025202	.026096	.026873	.027523	193
194	.019439	.020703	.021922	.023081	.024163	.025155	.026042	.026813	194
195	.018143	.019429	.020686	.021897	.023048	.024123	.025107	.025988	195
196	.016847	.018140	.019419	.020668	.021872	.023015	.024083	.025060	196
197	.015564	.016851	.018138	.019409	.020651	.021847	.022982	.024042	197
198	.014306	.015575	.016855	.018135	.019399	.020633	.021822	.022950	198
199	.013084	.014322	.015585	.016859	.018132	.019389	.020615	.021796	199
200	.011907	.013105	.014338	.015595	.016863	.018129	.019378	.020598	200
201	.010781	.011931	.013125	.014353	.015604	.016866	.018125	.019368	201
202	.009714	.010809	.011956	.013145	.014368	.015613	.016869	.018121	202
203	.008709	.009744	.010837	.011980	.013165	.014383	.015622	.016872	203
204	.007770	.008741	.009774	.010864	.012003	.013184	.014397	.015631	204
205	.006898	.007803	.008773	.009804	.010891	.012027	.013203	.014411	205
206	.006094	.006932	.007836	.008805	.009834	.010917	.012050	.013222	206
207	.005358	.006128	.006965	.007869	.008836	.009863	.010944	.012072	207
208	.004689	.005392	.006162	.006999	.007901	.008867	.009891	.010969	208
209	.004083	.004721	.005425	.006195	.007032	.007933	.008897	.009920	209
210	.003538	.004114	.004753	.005458	.006228	.007065	.007965	.008928	210
211	.003052	.003568	.004145	.004785	.005490	.006261	.007097	.007997	211
212	.002620	.003080	.003597	.004176	.004817	.005523	.006294	.007129	212
213	.002239	.002646	.003108	.003627	.004206	.004849	.005555	.006326	213
214	.001904	.002263	.002672	.003135	.003656	.004237	.004880	.005587	214
215	.001612	.001926	.002287	.002698	.003163	.003685	.004267	.004911	215
216	.001358	.001632	.001948	.002311	.002724	.003190	.003714	.004297	216
217	.001139	.001376	.001652	.001970	.002334	.002749	.003218	.003743	217
218	.000951	.001155	.001394	.001672	.001992	.002358	.002775	.003245	218
219	.000790	.000965	.001171	.001412	.001692	.002014	.002382	.002800	219
220	.000654	.000803	.000980	.001187	.001430	.001712	.002036	.002406	220
221	.000538	.000665	.000816	.000994	.001204	.001448	.001732	.002057	221
222	.000441	.000548	.000676	.000828	.001009	.001220	.001466	.001752	222
223	.000360	.000450	.000558	.000687	.000841	.001023	.001236	.001485	223
224	.000293	.000367	.000458	.000568	.000698	.000854	.001038	.001253	224
225	.000237	.000299	.000375	.000467	.000577	.000710	.000867	.001052	225
226	.000191	.000242	.000305	.000382	.000475	.000587	.000721	.000880	226
227	.000153	.000195	.000247	.000311	.000389	.000484	.000597	.000733	227
228	.000122	.000157	.000200	.000253	.000318	.000397	.000492	.000607	228
229	.000097	.000125	.000160	.000204	.000258	.000324	.000404	.000501	229
230	.000077	.000100	.000128	.000164	.000209	.000263	.000330	.000412	230
231	.000060	.000079	.000102	.000131	.000168	.000213	.000269	.000337	231
232	.000047	.000062	.000081	.000105	.000135	.000172	.000218	.000275	232
233	.000037	.000049	.000064	.000083	.000108	.000138	.000176	.000223	233
234	.000029	.000038	.000050	.000066	.000085	.000110	.000141	.000180	234
235	.000022	.000030	.000039	.000052	.000068	.000088	.000113	.000145	235
236	.000017	.000023	.000031	.000041	.000053	.000070	.000090	.000116	236
237	.000013	.000018	.000024	.000032	.000042	.000055	.000071	.000092	237
238	.000010	.000014	.000018	.000025	.000033	.000043	.000056	.000073	238
239	.000008	.000010	.000014	.000019	.000025	.000034	.000044	.000058	239
240	.000006	.000008	.000011	.000015	.000020	.000026	.000035	.000046	240

TABLE I

X	A=182	A=183	A=184	A=185	A=186	A=187	A=188	A=189	X
241	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	.ØØØØ11	.ØØØØ15	.ØØØØ2Ø	.ØØØØ27	.ØØØØ36	241
242	.ØØØØØ3	.ØØØØØ5	.ØØØØØ6	.ØØØØØ9	.ØØØØ12	.ØØØØ16	.ØØØØ21	.ØØØØ28	242
243	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ12	.ØØØØ16	.ØØØØ22	243
244	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ13	.ØØØØ17	244
245	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ13	245
246	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	246
247	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	247
248	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	248
249		.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	249
25Ø			.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	25Ø
251				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	251
252					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	252
253						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	253
254						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	254
255							.ØØØØØ1	.ØØØØØ1	255
256								.ØØØØØ1	256

X	A=19Ø	A=191	A=192	A=193	A=194	A=195	A=196	A=197	X
129	.ØØØØØ1								129
13Ø	.ØØØØØ1	.ØØØØØ1							13Ø
131	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1						131
132	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1					132
133	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1			133
134	.ØØØØØ3	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1		134
135	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	135
136	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1	136
137	.ØØØØØ9	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	.ØØØØØ1	137
138	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	.ØØØØØ2	138
139	.ØØØØ18	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ8	.ØØØØØ6	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2	139
14Ø	.ØØØØ24	.ØØØØ18	.ØØØØ14	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ5	.ØØØØØ3	14Ø
141	.ØØØØ32	.ØØØØ25	.ØØØØ19	.ØØØØ15	.ØØØØ11	.ØØØØØ8	.ØØØØØ6	.ØØØØØ5	141
142	.ØØØØ43	.ØØØØ34	.ØØØØ26	.ØØØØ2Ø	.ØØØØ15	.ØØØØ12	.ØØØØØ9	.ØØØØØ7	142
143	.ØØØØ58	.ØØØØ45	.ØØØØ35	.ØØØØ27	.ØØØØ21	.ØØØØ16	.ØØØØ12	.ØØØØØ9	143
144	.ØØØØ76	.ØØØØ59	.ØØØØ46	.ØØØØ36	.ØØØØ28	.ØØØØ22	.ØØØØ17	.ØØØØ13	144
145	.ØØØØ99	.ØØØØ78	.ØØØØ61	.ØØØØ48	.ØØØØ37	.ØØØØ29	.ØØØØ22	.ØØØØ17	145
146	.ØØØ129	.ØØØ1Ø2	.ØØØØ81	.ØØØØ63	.ØØØØ5Ø	.ØØØØ39	.ØØØØ3Ø	.ØØØØ23	146
147	.ØØØ167	.ØØØ133	.ØØØ1Ø6	.ØØØØ83	.ØØØØ66	.ØØØØ51	.ØØØØ4Ø	.ØØØØ31	147
148	.ØØØ215	.ØØØ172	.ØØØ137	.ØØØ1Ø9	.ØØØØ86	.ØØØØ68	.ØØØØ53	.ØØØØ41	148
149	.ØØØ274	.ØØØ22Ø	.ØØØ176	.ØØØ141	.ØØØ112	.ØØØØ88	.ØØØØ7Ø	.ØØØØ55	149
15Ø	.ØØØ347	.ØØØ28Ø	.ØØØ226	.ØØØ181	.ØØØ145	.ØØØ115	.ØØØØ91	.ØØØØ72	15Ø
151	.ØØØ436	.ØØØ355	.ØØØ287	.ØØØ231	.ØØØ186	.ØØØ149	.ØØØ118	.ØØØØ94	151
152	.ØØØ546	.ØØØ446	.ØØØ363	.ØØØ294	.ØØØ237	.ØØØ191	.ØØØ153	.ØØØ122	152
153	.ØØØ678	.ØØØ556	.ØØØ455	.ØØØ371	.ØØØ3Ø1	.ØØØ243	.ØØØ195	.ØØØ157	153
154	.ØØØ836	.ØØØ69Ø	.ØØØ567	.ØØØ465	.ØØØ379	.ØØØ3Ø8	.ØØØ249	.ØØØ2ØØ	154
155	.ØØ1Ø25	.ØØØ85Ø	.ØØØ7Ø3	.ØØØ578	.ØØØ474	.ØØØ387	.ØØØ315	.ØØØ255	155
156	.ØØ1248	.ØØ1Ø41	.ØØØ865	.ØØØ716	.ØØØ59Ø	.ØØØ484	.ØØØ395	.ØØØ322	156
157	.ØØ151Ø	.ØØ1267	.ØØ1Ø58	.ØØØ88Ø	.ØØØ729	.ØØØ6Ø1	.ØØØ493	.ØØØ4Ø4	157
158	.ØØ1816	.ØØ1531	.ØØ1286	.ØØ1Ø75	.ØØØ895	.ØØØ741	.ØØØ612	.ØØØ5Ø3	158
159	.ØØ217Ø	.ØØ184Ø	.ØØ1552	.ØØ13Ø4	.ØØ1Ø91	.ØØØ9Ø9	.ØØØ754	.ØØØ623	159
16Ø	.ØØ2577	.ØØ2196	.ØØ1863	.ØØ1574	.ØØ1323	.ØØ11Ø8	.ØØØ924	.ØØØ768	16Ø
161	.ØØ3Ø42	.ØØ26Ø5	.ØØ2222	.ØØ1886	.ØØ1595	.ØØ1342	.ØØ1125	.ØØØ939	161
162	.ØØ3567	.ØØ3Ø72	.ØØ2633	.ØØ2247	.ØØ191Ø	.ØØ1616	.ØØ1361	.ØØ1142	162
163	.ØØ4158	.ØØ3599	.ØØ31Ø2	.ØØ2661	.ØØ2273	.ØØ1933	.ØØ1637	.ØØ138Ø	163
164	.ØØ4817	.ØØ4192	.ØØ3631	.ØØ3131	.ØØ2689	.ØØ2298	.ØØ1956	.ØØ1658	164
165	.ØØ5547	.ØØ4852	.ØØ4225	.ØØ3663	.ØØ3161	.ØØ2716	.ØØ2324	.ØØ198Ø	165
166	.ØØ6349	.ØØ5583	.ØØ4887	.ØØ4259	.ØØ3694	.ØØ3191	.ØØ2744	.ØØ2349	166
167	.ØØ7224	.ØØ6385	.ØØ5619	.ØØ4922	.ØØ4292	.ØØ3726	.ØØ322Ø	.ØØ2771	167
168	.ØØ817Ø	.ØØ726Ø	.ØØ6421	.ØØ5654	.ØØ4956	.ØØ4325	.ØØ3757	.ØØ325Ø	168
169	.ØØ9185	.ØØ82Ø5	.ØØ7295	.ØØ6457	.ØØ5689	.ØØ499Ø	.ØØ4357	.ØØ3788	169
17Ø	.Ø1Ø265	.ØØ9218	.ØØ8239	.ØØ733Ø	.ØØ6492	.ØØ5724	.ØØ5Ø24	.ØØ439Ø	17Ø
171	.Ø114Ø6	.Ø1Ø296	.ØØ9251	.ØØ8273	.ØØ7365	.ØØ6527	.ØØ5758	.ØØ5Ø57	171
172	.Ø126ØØ	.Ø11434	.Ø1Ø327	.ØØ9284	.ØØ83Ø7	.ØØ74ØØ	.ØØ6562	.ØØ5793	172
173	.Ø13838	.Ø12623	.Ø11461	.Ø1Ø357	.ØØ9316	.ØØ8341	.ØØ7434	.ØØ6596	173
174	.Ø1511Ø	.Ø13857	.Ø12647	.Ø11488	.Ø1Ø387	.ØØ9348	.ØØ8374	.ØØ7468	174
175	.Ø164Ø5	.Ø15124	.Ø13875	.Ø12669	.Ø11514	.Ø1Ø416	.ØØ9379	.ØØ84Ø7	175
176	.Ø1771Ø	.Ø16412	.Ø15136	.Ø13893	.Ø12692	.Ø1154Ø	.Ø1Ø445	.ØØ941Ø	176

TABLE I

X	A=19Ø	A=191	A=192	A=193	A=194	A=195	A=196	A=197	X
177	.Ø19Ø11	.Ø17711	.Ø16419	.Ø15149	.Ø13911	.Ø12714	.Ø11566	.Ø1Ø473	177
178	.Ø2Ø293	.Ø19ØØ4	.Ø17711	.Ø16426	.Ø15161	.Ø13928	.Ø12735	.Ø11591	178
179	.Ø2154Ø	.Ø2Ø278	.Ø18997	.Ø1771Ø	.Ø16432	.Ø15173	.Ø13945	.Ø12757	179
18Ø	.Ø22737	.Ø21517	.Ø2Ø263	.Ø18989	.Ø1771Ø	.Ø16437	.Ø15185	.Ø13962	18Ø
181	.Ø23867	.Ø227Ø6	.Ø21495	.Ø2Ø248	.Ø18982	.Ø177Ø9	.Ø16443	.Ø15196	181
182	.Ø24916	.Ø23829	.Ø22676	.Ø21472	.Ø2Ø233	.Ø18974	.Ø177Ø8	.Ø16448	182
183	.Ø25869	.Ø24871	.Ø23791	.Ø22645	.Ø21449	.Ø2Ø218	.Ø18966	.Ø177Ø7	183
184	.Ø26713	.Ø25817	.Ø24825	.Ø23753	.Ø22615	.Ø21427	.Ø2Ø2Ø3	.Ø18958	184
185	.Ø27435	.Ø26654	.Ø25765	.Ø2478Ø	.Ø23715	.Ø22585	.Ø214Ø4	.Ø2Ø187	185
186	.Ø28Ø25	.Ø27371	.Ø26596	.Ø25713	.Ø24735	.Ø23678	.Ø22555	.Ø21381	186
187	.Ø28474	.Ø27956	.Ø273Ø7	.Ø26538	.Ø25661	.Ø24691	.Ø2364Ø	.Ø22524	187
188	.Ø28777	.Ø284Ø2	.Ø27888	.Ø27244	.Ø2648Ø	.Ø2561Ø	.Ø24646	.Ø236Ø3	188
189	.Ø2893Ø	.Ø287Ø3	.Ø28331	.Ø2782Ø	.Ø27181	.Ø26423	.Ø25559	.Ø246Ø2	189
19Ø	.Ø2893Ø	.Ø28854	.Ø28629	.Ø28259	.Ø27753	.Ø27118	.Ø26366	.Ø255Ø8	19Ø
191	.Ø28778	.Ø28854	.Ø28779	.Ø28555	.Ø28189	.Ø27686	.Ø27Ø56	.Ø2631Ø	191
192	.Ø28478	.Ø287Ø4	.Ø28779	.Ø287Ø4	.Ø28482	.Ø28119	.Ø2762Ø	.Ø26995	192
193	.Ø28Ø36	.Ø284Ø6	.Ø2863Ø	.Ø287Ø4	.Ø2863Ø	.Ø2841Ø	.Ø28Ø49	.Ø27554	193
194	.Ø27458	.Ø27967	.Ø28334	.Ø28556	.Ø2863Ø	.Ø28557	.Ø28338	.Ø2798Ø	194
195	.Ø26754	.Ø27393	.Ø27899	.Ø28263	.Ø28483	.Ø28557	.Ø28484	.Ø28267	195
196	.Ø25935	.Ø26694	.Ø27329	.Ø27831	.Ø28193	.Ø28411	.Ø28484	.Ø28411	196
197	.Ø25Ø13	.Ø25881	.Ø26636	.Ø27266	.Ø27763	.Ø28122	.Ø28339	.Ø28411	197
198	.Ø24ØØ2	.Ø24966	.Ø25828	.Ø26577	.Ø272Ø2	.Ø27696	.Ø28Ø53	.Ø28268	198
199	.Ø22917	.Ø23963	.Ø2492Ø	.Ø25776	.Ø26519	.Ø2714Ø	.Ø2763Ø	.Ø27984	199
2ØØ	.Ø21771	.Ø22884	.Ø23923	.Ø24874	.Ø25723	.Ø26461	.Ø27Ø77	.Ø27564	2ØØ
2Ø1	.Ø2Ø58Ø	.Ø21746	.Ø22852	.Ø23884	.Ø24828	.Ø25671	.Ø264Ø4	.Ø27Ø16	2Ø1
2Ø2	.Ø19357	.Ø2Ø562	.Ø21721	.Ø22819	.Ø23844	.Ø24782	.Ø2562Ø	.Ø26347	2Ø2
2Ø3	.Ø18117	.Ø19346	.Ø2Ø544	.Ø21695	.Ø22787	.Ø238Ø5	.Ø24736	.Ø25568	2Ø3
2Ø4	.Ø16874	.Ø18113	.Ø19335	.Ø2Ø525	.Ø2167Ø	.Ø22755	.Ø23766	.Ø24691	2Ø4
2Ø5	.Ø15639	.Ø16876	.Ø181Ø9	.Ø19324	.Ø2Ø5Ø7	.Ø21645	.Ø22723	.Ø23727	2Ø5
2Ø6	.Ø14425	.Ø15647	.Ø16878	.Ø181Ø5	.Ø19313	.Ø2Ø489	.Ø2162Ø	.Ø22691	2Ø6
2Ø7	.Ø1324Ø	.Ø14438	.Ø15655	.Ø1688Ø	.Ø181ØØ	.Ø193Ø1	.Ø2Ø471	.Ø21594	2Ø7
2Ø8	.Ø12Ø94	.Ø13258	.Ø14451	.Ø15663	.Ø16882	.Ø18Ø95	.Ø1929Ø	.Ø2Ø452	2Ø8
2Ø9	.Ø1Ø995	.Ø12116	.Ø13276	.Ø14464	.Ø1567Ø	.Ø16883	.Ø18Ø9Ø	.Ø19278	2Ø9
21Ø	.ØØ9948	.Ø11Ø2Ø	.Ø12138	.Ø13293	.Ø14476	.Ø15677	.Ø16884	.Ø18Ø85	21Ø
211	.ØØ8958	.ØØ9975	.Ø11Ø45	.Ø12159	.Ø1331Ø	.Ø14488	.Ø15684	.Ø16885	211
212	.ØØ8Ø28	.ØØ8987	.Ø1ØØØ3	.Ø11Ø69	.Ø1218Ø	.Ø13326	.Ø145ØØ	.Ø1569Ø	212
213	.ØØ7161	.ØØ8Ø59	.ØØ9Ø17	.Ø1ØØ3Ø	.Ø11Ø93	.Ø122ØØ	.Ø13343	.Ø14512	213
214	.ØØ6358	.ØØ7193	.ØØ8Ø9Ø	.ØØ9Ø46	.Ø1ØØ56	.Ø11117	.Ø1222Ø	.Ø13359	214
215	.ØØ5619	.ØØ639Ø	.ØØ7224	.ØØ812Ø	.ØØ9Ø74	.Ø1ØØ83	.Ø1114Ø	.Ø1224Ø	215
216	.ØØ4942	.ØØ565Ø	.ØØ6422	.ØØ7255	.ØØ815Ø	.ØØ91Ø3	.Ø1Ø1Ø9	.Ø11164	216
217	.ØØ4327	.ØØ4973	.ØØ5682	.ØØ6453	.ØØ7286	.ØØ818Ø	.ØØ9131	.Ø1Ø135	217
218	.ØØ3772	.ØØ4357	.ØØ5ØØ4	.ØØ5713	.ØØ6484	.ØØ7317	.ØØ82Ø9	.ØØ9158	218
219	.ØØ3272	.ØØ38ØØ	.ØØ4387	.ØØ5Ø35	.ØØ5744	.ØØ6515	.ØØ7347	.ØØ8238	219
22Ø	.ØØ2826	.ØØ3299	.ØØ3829	.ØØ4417	.ØØ5Ø65	.ØØ5775	.ØØ6546	.ØØ7377	22Ø
221	.ØØ243Ø	.ØØ2851	.ØØ3326	.ØØ3857	.ØØ4446	.ØØ5Ø95	.ØØ58Ø5	.ØØ6576	221
222	.ØØ2Ø79	.ØØ2453	.ØØ2877	.ØØ3353	.ØØ3885	.ØØ4476	.ØØ5125	.ØØ5835	222
223	.ØØ1772	.ØØ21Ø1	.ØØ2477	.ØØ29Ø2	.ØØ338Ø	.ØØ3914	.ØØ45Ø5	.ØØ5155	223
224	.ØØ15Ø3	.ØØ1792	.ØØ2123	.ØØ25Ø1	.ØØ2927	.ØØ34Ø7	.ØØ3942	.ØØ4534	224
225	.ØØ1269	.ØØ1521	.ØØ1812	.ØØ2145	.ØØ2524	.ØØ2953	.ØØ3434	.ØØ397Ø	225
226	.ØØ1Ø67	.ØØ1285	.ØØ1539	.ØØ1832	.ØØ2167	.ØØ2548	.ØØ2978	.ØØ346Ø	226
227	.ØØØ893	.ØØ1Ø82	.ØØ13Ø2	.ØØ1557	.ØØ1852	.ØØ2189	.ØØ2571	.ØØ3ØØ3	227
228	.ØØØ744	.ØØØ9Ø6	.ØØ1Ø96	.ØØ1318	.ØØ1576	.ØØ1872	.ØØ221Ø	.ØØ2595	228
229	.ØØØ617	.ØØØ756	.ØØØ919	.ØØ1111	.ØØ1335	.ØØ1594	.ØØ1892	.ØØ2232	229
23Ø	.ØØØ51Ø	.ØØØ628	.ØØØ767	.ØØØ932	.ØØ1126	.ØØ1351	.ØØ1612	.ØØ1912	23Ø
231	.ØØØ419	.ØØØ519	.ØØØ638	.ØØØ779	.ØØØ946	.ØØ1141	.ØØ1368	.ØØ163Ø	231
232	.ØØØ344	.ØØØ427	.ØØØ528	.ØØØ648	.ØØØ791	.ØØØ959	.ØØ1156	.ØØ1384	232
233	.ØØØ28Ø	.ØØØ35Ø	.ØØØ435	.ØØØ537	.ØØØ658	.ØØØ8Ø2	.ØØØ972	.ØØ1171	233
234	.ØØØ227	.ØØØ286	.ØØØ357	.ØØØ443	.ØØØ546	.ØØØ669	.ØØØ814	.ØØØ985	234
235	.ØØØ184	.ØØØ232	.ØØØ292	.ØØØ364	.ØØØ451	.ØØØ555	.ØØØ679	.ØØØ826	235
236	.ØØØ148	.ØØØ188	.ØØØ237	.ØØØ297	.ØØØ37Ø	.ØØØ458	.ØØØ564	.ØØØ69Ø	236
237	.ØØØ119	.ØØØ152	.ØØØ192	.ØØØ242	.ØØØ3Ø3	.ØØØ377	.ØØØ466	.ØØØ573	237
238	.ØØØØ95	.ØØØ122	.ØØØ155	.ØØØ196	.ØØØ247	.ØØØ3Ø9	.ØØØ384	.ØØØ474	238
239	.ØØØØ75	.ØØØØ97	.ØØØ125	.ØØØ159	.ØØØ2Ø1	.ØØØ252	.ØØØ315	.ØØØ391	239
24Ø	.ØØØØ6Ø	.ØØØØ77	.ØØØ1ØØ	.ØØØ128	.ØØØ162	.ØØØ2Ø5	.ØØØ257	.ØØØ321	24Ø
241	.ØØØØ47	.ØØØØ61	.ØØØØ79	.ØØØ1Ø2	.ØØØ131	.ØØØ166	.ØØØ2Ø9	.ØØØ262	241
242	.ØØØØ37	.ØØØØ48	.ØØØØ63	.ØØØØ81	.ØØØ1Ø5	.ØØØ134	.ØØØ169	.ØØØ214	242
243	.ØØØØ29	.ØØØØ38	.ØØØØ5Ø	.ØØØØ65	.ØØØØ84	.ØØØ1Ø7	.ØØØ137	.ØØØ173	243
244	.ØØØØ22	.ØØØØ3Ø	.ØØØØ39	.ØØØØ51	.ØØØØ66	.ØØØØ86	.ØØØ11Ø	.ØØØ14Ø	244

TABLE I

X	A=19Ø	A=191	A=192	A=193	A=194	A=195	A=196	A=197	X
245	.ØØØØ17	.ØØØØ23	.ØØØØ31	.ØØØØ4Ø	.ØØØØ53	.ØØØØ68	.ØØØØ88	.ØØØ112	245
246	.ØØØØ13	.ØØØØ18	.ØØØØ24	.ØØØØ32	.ØØØØ41	.ØØØØ54	.ØØØØ7Ø	.ØØØØ9Ø	246
247	.ØØØØ1Ø	.ØØØØ14	.ØØØØ19	.ØØØØ25	.ØØØØ33	.ØØØØ43	.ØØØØ56	.ØØØØ72	247
248	.ØØØØØ8	.ØØØØ11	.ØØØØ14	.ØØØØ19	.ØØØØ25	.ØØØØ34	.ØØØØ44	.ØØØØ57	248
249	.ØØØØØ6	.ØØØØØ8	.ØØØØ11	.ØØØØ15	.ØØØØ2Ø	.ØØØØ26	.ØØØØ35	.ØØØØ45	249
25Ø	.ØØØØØ5	.ØØØØØ6	.ØØØØØ9	.ØØØØ12	.ØØØØ15	.ØØØØ2Ø	.ØØØØ27	.ØØØØ36	25Ø
251	.ØØØØØ3	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ12	.ØØØØ16	.ØØØØ21	.ØØØØ28	251
252	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ12	.ØØØØ16	.ØØØØ22	252
253	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØØ9	.ØØØØ13	.ØØØØ17	253
254	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7	.ØØØØ1Ø	.ØØØØ13	254
255	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	.ØØØØ1Ø	255
256	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	.ØØØØØ8	256
257	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ6	257
258		.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ5	258
259			.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	.ØØØØØ3	259
26Ø				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ3	26Ø
261				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	261
262					.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	262
263						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	263
264							.ØØØØØ1	.ØØØØØ1	264
265								.ØØØØØ1	265

X	A=198	A=199	A=2ØØ
136	.ØØØØØ1		
137	.ØØØØØ1	.ØØØØØ1	
138	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1
139	.ØØØØØ2	.ØØØØØ1	.ØØØØØ1
14Ø	.ØØØØØ3	.ØØØØØ2	.ØØØØØ1
141	.ØØØØØ4	.ØØØØØ3	.ØØØØØ2
142	.ØØØØØ5	.ØØØØØ4	.ØØØØØ3
143	.ØØØØØ7	.ØØØØØ5	.ØØØØØ4
144	.ØØØØ1Ø	.ØØØØØ7	.ØØØØØ6
145	.ØØØØ13	.ØØØØ1Ø	.ØØØØØ8
146	.ØØØØ18	.ØØØØ14	.ØØØØ11
147	.ØØØØ24	.ØØØØ19	.ØØØØ14
148	.ØØØØ32	.ØØØØ25	.ØØØØ19
149	.ØØØØ43	.ØØØØ33	.ØØØØ26
15Ø	.ØØØØ57	.ØØØØ44	.ØØØØ35
151	.ØØØØ74	.ØØØØ58	.ØØØØ46
152	.ØØØØ97	.ØØØØ76	.ØØØØ6Ø
153	.ØØØ125	.ØØØØ99	.ØØØØ79
154	.ØØØ161	.ØØØ128	.ØØØ1Ø2
155	.ØØØ2Ø5	.ØØØ165	.ØØØ132
156	.ØØØ261	.ØØØ21Ø	.ØØØ169
157	.ØØØ329	.ØØØ267	.ØØØ216
158	.ØØØ412	.ØØØ336	.ØØØ273
159	.ØØØ513	.ØØØ42Ø	.ØØØ343
16Ø	.ØØØ635	.ØØØ523	.ØØØ429
161	.ØØØ781	.ØØØ646	.ØØØ533
162	.ØØØ954	.ØØØ794	.ØØØ658
163	.ØØ1159	.ØØØ969	.ØØØ8Ø7
164	.ØØ1399	.ØØ1176	.ØØØ984
165	.ØØ1679	.ØØ1419	.ØØ1193
166	.ØØ2ØØ3	.ØØ17Ø1	.ØØ1438
167	.ØØ2375	.ØØ2Ø26	.ØØ1722
168	.ØØ2799	.ØØ24ØØ	.ØØ2Ø5Ø
169	.ØØ3279	.ØØ2826	.ØØ2426
17Ø	.ØØ3819	.ØØ33Ø9	.ØØ2854
171	.ØØ4422	.ØØ385Ø	.ØØ3338
172	.ØØ5Ø91	.ØØ4455	.ØØ3881
173	.ØØ5827	.ØØ5124	.ØØ4487
174	.ØØ663Ø	.ØØ586Ø	.ØØ5157
175	.ØØ75Ø2	.ØØ6664	.ØØ5894
176	.ØØ8439	.ØØ7535	.ØØ6698
177	.ØØ9441	.ØØ8472	.ØØ7568
178	.Ø1Ø5Ø1	.ØØ9471	.ØØ85Ø3
179	.Ø11616	.Ø1Ø529	.ØØ95Ø1

TABLE I

X	A=198	A=199	A=200
180	.012778	.011641	.010557
181	.013978	.012798	.011665
182	.015207	.013994	.012819
183	.016453	.015217	.014009
184	.017705	.016458	.015228
185	.018949	.017703	.016462
186	.020172	.018941	.017701
187	.021358	.020156	.018932
188	.022494	.021335	.020140
189	.023566	.022464	.021313
190	.024558	.023528	.022434
191	.025458	.024514	.023491
192	.026253	.025408	.024470
193	.026933	.026198	.025358
194	.027489	.026873	.026142
195	.027912	.027424	.026812
196	.028197	.027844	.027360
197	.028340	.028126	.027776
198	.028340	.028268	.028057
199	.028197	.028268	.028198
200	.027915	.028127	.028198
201	.027499	.027847	.028057
202	.026954	.027434	.027780
203	.026290	.026893	.027369
204	.025517	.026234	.026832
205	.024646	.025466	.026178
206	.023689	.024601	.025416
207	.022659	.023650	.024556
208	.021569	.022627	.023612
209	.020434	.021544	.022595
210	.020397	.020416	.021519
211	.018079	.019254	.019266
212	.016885	.018074	.019243
213	.015696	.016886	.018068
214	.014523	.015702	.016886
215	.013374	.014534	.015708
216	.012260	.013390	.014544
217	.011186	.012279	.013405
218	.010160	.011209	.012298
219	.009186	.010185	.011231
220	.008267	.009213	.010210
221	.007407	.008296	.009240
222	.006606	.007436	.008324
223	.005866	.006636	.007466
224	.005185	.005895	.006666
225	.004563	.005214	.005925
226	.003997	.004591	.005244
227	.003487	.004025	.004620
228	.003028	.003513	.004052
229	.002618	.003053	.003539
230	.002254	.002641	.003078
231	.001932	.002275	.002665
232	.001649	.001952	.002297
233	.001401	.001667	.001972
234	.001185	.001418	.001685
235	.000999	.001200	.001434
236	.000838	.001012	.001215
237	.000700	.000850	.001026
238	.000582	.000711	.000862
239	.000483	.000592	.000721
240	.000398	.000491	.000601
241	.000327	.000405	.000499
242	.000268	.000333	.000412
243	.000218	.000273	.000339
244	.000177	.000223	.000278
245	.000143	.000181	.000227
246	.000115	.000146	.000185
247	.000092	.000118	.000149

TABLE I

X	A=198	A=199	A=200
248	.ØØØØ74	.ØØØØ95	.ØØØ121
249	.ØØØØ59	.ØØØØ76	.ØØØØ97
25Ø	.ØØØØ46	.ØØØØ6Ø	.ØØØØØ7
251	.ØØØØ37	.ØØØØ48	.ØØØØ62
252	.ØØØØ29	.ØØØØ38	.ØØØØ49
253	.ØØØØ22	.ØØØØ3Ø	.ØØØØ39
254	.ØØØØ18	.ØØØØ23	.ØØØØ3Ø
255	.ØØØØ14	.ØØØØ18	.ØØØØ24
256	.ØØØØ11	.ØØØØ14	.ØØØØ19
257	.ØØØØØ8	.ØØØØ11	.ØØØØ15
258	.ØØØØØ6	.ØØØØØ8	.ØØØØØ1
259	.ØØØØØ5	.ØØØØØ6	.ØØØØØ9
26Ø	.ØØØØØ4	.ØØØØØ5	.ØØØØØ7
261	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5
262	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4
263	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3
264	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2
265	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2
266	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1
267		.ØØØØØ1	.ØØØØØ1
268		.ØØØØØ1	.ØØØØØ1

TABLE II

X	A=.001	A=.002	A=.003	A=.004	A=.005	A=.006	A=.007	A=.008	X
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0
1	.0009995	.0019980	.0029955	.0039920	.0049875	.0059820	.0069756	.0079681	1
2	.0000005	.0000020	.0000045	.0000080	.0000125	.0000179	.0000244	.0000318	2
3							.0000001	.0000001	3

X	A=.009	A=.010	A=.02	A=.03	A=.04	A=.05	A=.06	A=.07	X
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0
1	.0089596	.0099502	.0198013	.0295545	.0392106	.0487706	.0582355	.0676062	1
2	.0000403	.0000497	.0001973	.0004411	.0007790	.0012091	.0017296	.0023386	2
3	.0000001	.0000002	.0000013	.0000044	.0000104	.0000201	.0000344	.0000542	3
4					.0000001	.0000003	.0000005	.0000009	4

X	A=.08	A=.09	A=.10	A=.11	A=.12	A=.13	A=.14	A=.15	X
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0
1	.0768837	.0860688	.0951626	.1041659	.1130796	.1219046	.1306418	.1302920	1
2	.0030343	.0038150	.0046788	.0056241	.0066491	.0077522	.0089316	.0101858	2
3	.0000804	.0001136	.0001547	.0002043	.0002633	.0003323	.0004119	.0005029	3
4	.0000016	.0000025	.0000038	.0000056	.0000079	.0000107	.0000143	.0000187	4
5				.0000001	.0000002	.0000003	.0000004	.0000006	5

X	A=.16	A=.17	A=.18	A=.19	A=.20	A=.21	A=.22	A=.23	X
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0
1	.1478562	.1563352	.1647298	.1730409	.1812692	.1894158	.1974812	.2054664	1
2	.0115132	.0129122	.0143812	.0159187	.0175231	.0191931	.0209271	.0227237	2
3	.0006058	.0007212	.0008498	.0009920	.0011485	.0013197	.0015060	.0017083	3
4	.0000240	.0000304	.0000379	.0000467	.0000568	.0000685	.0000819	.0000971	4
5	.0000008	.0000010	.0000014	.0000018	.0000023	.0000029	.0000036	.0000044	5
6				.0000001	.0000001	.0000001	.0000001	.0000002	6

X	A=.24	A=.25	A=.26	A=.27	A=.28	A=.29	A=.30	A=.40	X
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0
1	.2133721	.2211992	.2289484	.2366205	.2442163	.2517364	.2591818	.3296800	1
2	.0245815	.0264990	.0284750	.0305080	.0325968	.0347400	.0369363	.0615519	2
3	.0019266	.0021615	.0024135	.0026829	.0029701	.0032755	.0035995	.0079263	3
4	.0001142	.0001334	.0001548	.0001786	.0002049	.0002339	.0002658	.0007763	4
5	.0000054	.0000066	.0000080	.0000096	.0000113	.0000134	.0000158	.0000612	5
6	.0000002	.0000003	.0000003	.0000004	.0000005	.0000006	.0000008	.0000040	6
7								.0000002	7

X	A=.5	A=.6	A=.7	A=.8	A=.9	A=1.0	A=1.1	A=1.2	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.393469	.451188	.503415	.550671	.593430	.632121	.667129	.698806	1
2	.090204	.121901	.155805	.191208	.227518	.264241	.300971	.337373	2
3	.014388	.023115	.034142	.047423	.062857	.080301	.099584	.120513	3
4	.001752	.003358	.005753	.009080	.013459	.018988	.025742	.033769	4
5	.000172	.000394	.000786	.001411	.002344	.003660	.005435	.007746	5
6	.000014	.000039	.000090	.000184	.000343	.000594	.000968	.001500	6
7	.000001	.000003	.000009	.000021	.000043	.000083	.000149	.000251	7
8			.000001	.000002	.000005	.000010	.000020	.000037	8
9						.000001	.000002	.000005	9
10								.000001	10

X	A=1.3	A=1.4	A=1.5	A=1.6	A=1.7	A=1.8	A=1.9	A=2.0	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.727468	.753403	.776870	.798103	.817316	.834701	.850431	.864665	1
2	.373177	.408167	.442175	.475069	.506754	.537163	.566251	.593994	2
3	.142888	.166502	.191153	.216642	.242777	.269379	.296280	.323324	3
4	.043095	.053725	.065642	.078813	.093189	.108708	.125298	.142877	4
5	.010663	.014253	.018576	.023682	.029615	.036407	.044081	.052653	5
6	.002231	.003201	.004456	.006040	.007999	.010378	.013219	.016564	6
7	.000404	.000622	.000926	.001336	.001875	.002569	.003446	.004534	7
8	.000064	.000107	.000170	.000260	.000388	.000562	.000793	.001097	8
9	.000009	.000016	.000028	.000045	.000072	.000110	.000163	.000237	9

TABLE II

X	A=1.3	A=1.4	A=1.5	A=1.6	A=1.7	A=1.8	A=1.9	A=2.0	X
10	.000001	.000002	.000004	.000007	.000012	.000019	.000030	.000046	10
11			.000001	.000001	.000002	.000003	.000005	.000008	11
12							.000001	.000001	12

X	A=2.1	A=2.2	A=2.3	A=2.4	A=2.5	A=2.6	A=2.7	A=2.8	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.877544	.889197	.899741	.909282	.917915	.925726	.932794	.939190	1
2	.620385	.645430	.669146	.691559	.712703	.732615	.751340	.768922	2
3	.350369	.377286	.403961	.430291	.456187	.481570	.506375	.530546	3
4	.161357	.180648	.200653	.221277	.242424	.263998	.285908	.308063	4
5	.062126	.072496	.083751	.095869	.108822	.122577	.137092	.152324	5
6	.020449	.024910	.029976	.035673	.042021	.049037	.056732	.065110	6
7	.005862	.007461	.009362	.011594	.014187	.017170	.020569	.024411	7
8	.001486	.001978	.002589	.003339	.004247	.005334	.006621	.008131	8
9	.000337	.000470	.000642	.000862	.001140	.001487	.001914	.002433	9
10	.000069	.000101	.000144	.000202	.000277	.000376	.000501	.000660	10
11	.000013	.000020	.000029	.000043	.000062	.000087	.000120	.000164	11
12	.000002	.000004	.000006	.000008	.000013	.000018	.000026	.000037	12
13		.000001	.000001	.000002	.000002	.000004	.000005	.000008	13
14						.000001	.000001	.000002	14

X	A=2.9	A=3.0	A=3.1	A=3.2	A=3.3	A=3.4	A=3.5	A=3.6	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.944977	.950213	.954951	.959238	.963117	.966627	.969803	.972676	1
2	.785409	.800852	.815298	.828799	.841402	.853158	.864112	.874311	2
3	.554037	.576810	.598837	.620096	.640574	.660260	.679153	.697253	3
4	.330377	.352768	.375160	.397480	.419662	.441643	.463367	.484784	4
5	.168223	.184737	.201811	.219387	.237410	.255818	.274555	.293562	5
6	.074174	.083918	.094334	.105408	.117123	.129458	.142386	.155881	6
7	.028717	.033509	.038804	.044619	.050966	.057853	.065288	.073273	7
8	.009885	.011905	.014213	.016830	.019777	.023074	.026739	.030789	8
9	.003058	.003803	.004683	.005714	.006912	.008293	.009874	.011671	9
10	.000858	.001102	.001401	.001762	.002195	.002709	.003315	.004024	10
11	.000220	.000292	.000383	.000497	.000638	.000810	.001019	.001271	11
12	.000052	.000071	.000097	.000129	.000171	.000223	.000289	.000370	12
13	.000011	.000016	.000023	.000031	.000042	.000057	.000076	.000100	13
14	.000002	.000003	.000005	.000007	.000010	.000014	.000019	.000025	14
15		.000001	.000001	.000001	.000002	.000003	.000004	.000006	15
16						.000001	.000001	.000001	16

X	A=3.7	A=3.8	A=3.9	A=4.0	A=4.1	A=4.2	A=4.3	A=4.4	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.975276	.977629	.979758	.981684	.983427	.985004	.986431	.987723	1
2	.883799	.892620	.900815	.908422	.915479	.922023	.928087	.933702	2
3	.714567	.731103	.746875	.761897	.776186	.789762	.802645	.814858	3
4	.505847	.526515	.546753	.566530	.585818	.604597	.622846	.640552	4
5	.312781	.332156	.351635	.371163	.390692	.410173	.429562	.448816	5
6	.169912	.184444	.199442	.214870	.230688	.246857	.263338	.280088	6
7	.081809	.090892	.100517	.110674	.121352	.132536	.144210	.156355	7
8	.035241	.040107	.045402	.051134	.057312	.063943	.071032	.078579	8
9	.013703	.015984	.018533	.021363	.024492	.027932	.031698	.035803	9
10	.004848	.005799	.006890	.008132	.009540	.011127	.012906	.014890	10
11	.001572	.001929	.002349	.002840	.003410	.004069	.004825	.005688	11
12	.000470	.000592	.000739	.000915	.001125	.001374	.001666	.002008	12
13	.000130	.000168	.000216	.000274	.000345	.000431	.000534	.000658	13
14	.000034	.000045	.000059	.000076	.000098	.000126	.000160	.000201	14
15	.000008	.000011	.000015	.000020	.000026	.000034	.000045	.000058	15
16	.000002	.000003	.000004	.000005	.000007	.000009	.000012	.000016	16
17		.000001	.000001	.000001	.000002	.000002	.000003	.000004	17
18							.000001	.000001	18

X	A=4.5	A=4.6	A=4.7	A=4.8	A=4.9	A=5.0	A=5.1	A=5.2	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.988891	.989948	.990905	.991770	.992553	.993262	.993903	.994483	1
2	.938901	.943710	.948157	.952267	.956065	.959572	.962810	.965797	2

TABLE II

X	A=4.5	A=4.6	A=4.7	A=4.8	A=4.9	A=5.0	A=5.1	A=5.2	X
3	.826422	.837361	.847700	.857461	.866669	.875348	.883522	.891213	3
4	.657704	.674294	.690316	.705770	.720655	.734974	.748732	.761935	4
5	.467896	.486766	.505391	.523741	.541788	.559507	.576875	.593872	5
6	.297070	.314240	.331562	.348994	.366499	.384039	.401580	.419087	6
7	.168949	.181971	.195395	.209195	.223345	.237817	.252580	.267607	7
8	.086586	.095051	.103969	.113334	.123138	.133372	.144023	.155078	8
9	.040257	.045072	.050256	.055817	.061761	.068094	.074818	.081935	9
10	.017093	.019527	.022206	.025141	.028345	.031828	.035601	.039674	10
11	.006669	.007777	.009022	.010417	.011971	.013695	.015601	.017699	11
12	.002404	.002863	.003389	.003992	.004677	.005453	.006328	.007310	12
13	.000805	.000979	.001183	.001422	.001699	.002019	.002387	.002809	13
14	.000252	.000312	.000385	.000473	.000576	.000698	.000841	.001008	14
15	.000074	.000093	.000118	.000147	.000183	.000226	.000278	.000339	15
16	.000020	.000026	.000034	.000043	.000055	.000069	.000086	.000108	16
17	.000005	.000007	.000009	.000012	.000015	.000020	.000025	.000032	17
18	.000001	.000002	.000002	.000003	.000004	.000005	.000007	.000009	18
19			.000001	.000001	.000001	.000001	.000002	.000002	19
20								.000001	20

X	A=5.3	A=5.4	A=5.5	A=5.6	A=5.7	A=5.8	A=5.9	A=6.0	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.995008	.995483	.995913	.996302	.996654	.996972	.997261	.997521	1
2	.968553	.971094	.973436	.975594	.977582	.979413	.981098	.982649	2
3	.898446	.905242	.911624	.917612	.923227	.928489	.933418	.938031	3
4	.774590	.786709	.798301	.809378	.819952	.830037	.839647	.848796	4
5	.610482	.626689	.642482	.657850	.672785	.687282	.701335	.714943	5
6	.436527	.453868	.471081	.488139	.505015	.521685	.538127	.554320	6
7	.282866	.298329	.313964	.329742	.345634	.361609	.377639	.393697	7
8	.166523	.178341	.190515	.203025	.215851	.228974	.242371	.256020	8
9	.089446	.097350	.105643	.114322	.123382	.132814	.142611	.152763	9
10	.044056	.048755	.053777	.059130	.064817	.070844	.077212	.083924	10
11	.020000	.022514	.025251	.028222	.031436	.034901	.038627	.042621	11
12	.008409	.009632	.010988	.012487	.014138	.015950	.017931	.020092	12
13	.003289	.003835	.004451	.005144	.005922	.006790	.007756	.008827	13
14	.001202	.001427	.001685	.001981	.002319	.002703	.003138	.003626	14
15	.000412	.000498	.000599	.000716	.000852	.001010	.001192	.001400	15
16	.000133	.000164	.000200	.000244	.000295	.000356	.000426	.000509	16
17	.000041	.000051	.000063	.000078	.000096	.000118	.000144	.000175	17
18	.000012	.000015	.000019	.000024	.000030	.000037	.000046	.000057	18
19	.000003	.000004	.000005	.000007	.000009	.000011	.000014	.000018	19
20	.000001	.000001	.000001	.000002	.000002	.000003	.000004	.000005	20
21					.000001	.000001	.000001	.000001	21

X	A=6.1	A=6.2	A=6.3	A=6.4	A=6.5	A=6.6	A=6.7	A=6.8	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.997757	.997971	.998164	.998338	.998497	.998640	.998769	.998886	1
2	.984076	.985388	.986595	.987704	.988724	.989661	.990522	.991313	2
3	.942347	.946382	.950154	.953676	.956964	.960032	.962894	.965562	3
4	.857499	.865771	.873626	.881081	.888150	.894849	.901192	.907194	4
5	.728106	.740823	.753096	.764930	.776328	.787296	.797841	.807969	5
6	.570246	.585887	.601228	.616256	.630959	.645327	.659351	.673023	6
7	.409755	.425787	.441767	.457671	.473476	.489161	.504703	.520084	7
8	.269899	.283984	.298252	.312679	.327242	.341918	.356683	.371514	8
9	.163258	.174086	.185233	.196685	.208427	.220443	.232716	.245230	9
10	.090980	.098379	.106121	.114201	.122616	.131361	.140430	.149816	10
11	.046890	.051441	.056280	.061411	.066839	.072567	.078598	.084934	11
12	.022440	.024985	.027734	.030697	.033880	.037291	.040937	.044825	12
13	.010012	.011316	.012748	.014316	.016027	.017889	.019910	.022097	13
14	.004180	.004797	.005485	.006251	.007100	.008038	.009072	.010208	14
15	.001639	.001910	.002217	.002565	.002956	.003395	.003886	.004434	15
16	.000605	.000716	.000844	.000992	.001160	.001352	.001569	.001816	16
17	.000211	.000254	.000304	.000362	.000430	.000509	.000599	.000703	17
18	.000070	.000085	.000104	.000126	.000151	.000182	.000217	.000258	18
19	.000022	.000027	.000034	.000041	.000051	.000062	.000075	.000090	19
20	.000007	.000008	.000010	.000013	.000016	.000020	.000024	.000030	20

TABLE II

X	A=6.1	A=6.2	A=6.3	A=6.4	A=6.5	A=6.6	A=6.7	A=6.8	X
21	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ6	.ØØØØØ8	.ØØØØ1Ø	21
22	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	22
23						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	23

X	A=6.9	A=7.Ø	A=7.1	A=7.2	A=7.3	A=7.4	A=7.5	A=7.6	X
Ø	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	Ø
1	.998992	.999Ø88	.999175	.999253	.999324	.999389	.999447	.9995ØØ	1
2	.992Ø38	.992ØØ5	.993317	.993878	.994393	.994865	.995299	.995696	2
3	.968Ø48	.97Ø364	.97252Ø	.974526	.976393	.978129	.979743	.981243	3
4	.91287Ø	.918235	.9233Ø1	.928Ø83	.932594	.936847	.94Ø855	.944629	4
5	.817689	.827ØØ8	.835937	.844484	.85266Ø	.86Ø475	.867938	.875Ø61	5
6	.686338	.699292	.711881	.7241Ø3	.735957	.747443	.758564	.769319	6
7	.535285	.55Ø289	.56508Ø	.579644	.593968	.6Ø8Ø38	.621845	.635379	7
8	.386389	.4Ø1286	.416183	.431Ø59	.445893	.46Ø667	.475361	.489958	8
9	.257967	.27Ø9Ø9	.284Ø36	.297332	.31Ø776	.324349	.338Ø33	.3518Ø8	9
1Ø	.15Ø51Ø	.1695Ø4	.179788	.19Ø33Ø	.2Ø118Ø	.212265	.223592	.235149	1Ø
11	.Ø91575	.Ø98521	.1Ø5771	.113323	.121175	.129323	.137762	.146487	11
12	.Ø48961	.Ø5335Ø	.Ø57997	.Ø629Ø6	.Ø68Ø81	.Ø73526	.Ø79241	.Ø8523Ø	12
13	.Ø24458	.Ø27ØØØ	.Ø2973Ø	.Ø32655	.Ø35782	.Ø39117	.Ø42666	.Ø46434	13
14	.Ø11452	.Ø12811	.Ø14292	.Ø159Ø1	.Ø17645	.Ø19531	.Ø21565	.Ø23753	14
15	.ØØ5Ø42	.ØØ5717	.ØØ6463	.ØØ7285	.ØØ8188	.ØØ9178	.Ø1Ø26Ø	.Ø11441	15
16	.ØØ2Ø94	.ØØ24Ø7	.ØØ2757	.ØØ3149	.ØØ3586	.ØØ4Ø71	.ØØ46Ø8	.ØØ52Ø2	16
17	.ØØØ822	.ØØØ958	.ØØ1113	.ØØ1288	.ØØ1486	.ØØ17Ø9	.ØØ1959	.ØØ2239	17
18	.ØØØ3Ø6	.ØØØ362	.ØØØ426	.ØØØ5ØØ	.ØØØ584	.ØØØ68Ø	.ØØØ79Ø	.ØØØ915	18
19	.ØØØ1Ø8	.ØØØ13Ø	.ØØØ155	.ØØØ184	.ØØØ218	.ØØØ258	.ØØØ3Ø3	.ØØØ355	19
2Ø	.ØØØØ37	.ØØØØ44	.ØØØØ54	.ØØØØ65	.ØØØØ78	.ØØØØ93	.ØØØ111	.ØØØ132	2Ø
21	.ØØØØ12	.ØØØØ14	.ØØØØ18	.ØØØØ22	.ØØØØ26	.ØØØØ32	.ØØØØ39	.ØØØØ46	21
22	.ØØØØØ4	.ØØØØØ5	.ØØØØØ6	.ØØØØØ7	.ØØØØØ9	.ØØØØ11	.ØØØØ13	.ØØØØ16	22
23	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	23
24				.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	24

X	A=7.7	A=7.8	A=7.9	A=8.Ø	A=8.1	A=8.2	A=8.3	A=8.4	X
Ø	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	Ø
1	.999547	.99959Ø	.999629	.999665	.999696	.999725	.999751	.999775	1
2	.996Ø6Ø	.996394	.9967ØØ	.996981	.997238	.997473	.997689	.997886	2
3	.982636	.98393Ø	.985131	.986246	.98728Ø	.988239	.989129	.989953	3
4	.948181	.951523	.954666	.95762Ø	.96Ø395	.963ØØØ	.965446	.96774Ø	4
5	.881855	.88833Ø	.894497	.9ØØ368	.9Ø5951	.91126Ø	.9163Ø3	.921Ø92	5
6	.779713	.789749	.799431	.8Ø8764	.817753	.8264Ø6	.834727	.842723	6
7	.648631	.661593	.67426Ø	.686626	.698686	.71Ø438	.721879	.733ØØ7	7
8	.50444Ø	.518791	.532996	.547Ø39	.56Ø9Ø8	.574591	.588Ø74	.6Ø1348	8
9	.365657	.379559	.393497	.4Ø7453	.4214Ø8	.435347	.449252	.4631Ø6	9
1Ø	.24692Ø	.258891	.271Ø48	.283376	.295858	.3Ø8481	.321226	.33408Ø	1Ø
11	.155492	.16477Ø	.174314	.184114	.194163	.2Ø445Ø	.214965	.225699	11
12	.Ø91493	.Ø98Ø3Ø	.1Ø4841	.111924	.119278	.1269ØØ	.134787	.142934	12
13	.Ø5Ø427	.Ø54649	.Ø591Ø4	.Ø63797	.Ø68731	.Ø739Ø7	.Ø7933Ø	.Ø84999	13
14	.Ø261Ø3	.Ø2862Ø	.Ø31311	.Ø34181	.Ø37236	.Ø4Ø481	.Ø43923	.Ø47564	14
15	.Ø12725	.Ø14118	.Ø15627	.Ø17257	.Ø19Ø14	.Ø2Ø9Ø3	.Ø22931	.Ø251Ø3	15
16	.ØØ5857	.ØØ6577	.ØØ7367	.ØØ8231	.ØØ9174	.Ø1Ø2Ø1	.Ø11316	.Ø12525	16
17	.ØØ2552	.ØØ29Ø1	.ØØ3289	.ØØ3718	.ØØ4192	.ØØ4715	.ØØ5291	.ØØ5922	17
18	.ØØ1Ø55	.ØØ1215	.ØØ1393	.ØØ1594	.ØØ1819	.ØØ2Ø7Ø	.ØØ2349	.ØØ2659	18
19	.ØØØ415	.ØØØ484	.ØØØ562	.ØØØ65Ø	.ØØØØ51	.ØØØ864	.ØØØ992	.ØØ1136	19
2Ø	.ØØØ156	.ØØØ184	.ØØØ216	.ØØØ253	.ØØØ296	.ØØØ344	.ØØØ4ØØ	.ØØØ463	2Ø
21	.ØØØØ56	.ØØØØ67	.ØØØØ79	.ØØØØ94	.ØØØ111	.ØØØ131	.ØØØ154	.ØØØ18Ø	21
22	.ØØØØ19	.ØØØØ23	.ØØØØ28	.ØØØØ33	.ØØØØ4Ø	.ØØØØ48	.ØØØØ57	.ØØØØ67	22
23	.ØØØØØ6	.ØØØØØ8	.ØØØØØ9	.ØØØØ11	.ØØØØ14	.ØØØØ17	.ØØØØ2Ø	.ØØØØ24	23
24	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	.ØØØØØ4	.ØØØØØ5	.ØØØØØ6	.ØØØØØ7	.ØØØØØ8	24
25	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	.ØØØØØ2	.ØØØØØ2	.ØØØØØ3	25
26						.ØØØØØ1	.ØØØØØ1	.ØØØØØ1	26

X	A=8.5	A=8.6	A=8.7	A=8.8	A=8.9	A=9.Ø	A=9.1	A=9.2	X
Ø	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	1.ØØØØØØ	Ø
1	.999797	.999816	.999833	.999849	.999864	.999877	.999888	.999899	1
2	.998Ø67	.998233	.998384	.998523	.99865Ø	.998766	.998872	.998969	2
3	.99Ø717	.991424	.992Ø8Ø	.992686	.993248	.993768	.994249	.994693	3
4	.969891	.9719Ø7	.973797	.975566	.977223	.978774	.98Ø224	.98158Ø	4

TABLE II

X	A=8.5	A=8.6	A=8.7	A=8.8	A=8.9	A=9.0	A=9.1	A=9.2	X
5	.925636	.929946	.934032	.937902	.941567	.945036	.948318	.951420	5
6	.850403	.857772	.864840	.871613	.878100	.884309	.890249	.895926	6
7	.743822	.754324	.764512	.774390	.783958	.793319	.802177	.810835	7
8	.614403	.627229	.639819	.652166	.664262	.676103	.687684	.699000	8
9	.476895	.490603	.504216	.517719	.531101	.544347	.557448	.570391	9
10	.347026	.360049	.373132	.386260	.399419	.412592	.425765	.438924	10
11	.236638	.247772	.259089	.270577	.282222	.294012	.305933	.317974	11
12	.151338	.159992	.168892	.178030	.187399	.196992	.206800	.216815	12
13	.090917	.097084	.103499	.110162	.117072	.124227	.131624	.139261	13
14	.051411	.055467	.059736	.064221	.068925	.073851	.079001	.084376	14
15	.027425	.029902	.032540	.035343	.038317	.041466	.044795	.048309	15
16	.013833	.015245	.016767	.018402	.020157	.022036	.024044	.026188	16
17	.006613	.007367	.008190	.009084	.010055	.011106	.012242	.013468	17
18	.003002	.003382	.003800	.004261	.004766	.005320	.005924	.006584	18
19	.001297	.001478	.001679	.001903	.002151	.002426	.002731	.003066	19
20	.000535	.000616	.000707	.000811	.000926	.001056	.001201	.001362	20
21	.000211	.000245	.000285	.000330	.000381	.000439	.000505	.000579	21
22	.000079	.000094	.000110	.000129	.000150	.000175	.000203	.000235	22
23	.000029	.000034	.000041	.000048	.000057	.000067	.000078	.000092	23
24	.000010	.000012	.000014	.000017	.000021	.000025	.000029	.000034	24
25	.000003	.000004	.000005	.000006	.000007	.000009	.000010	.000012	25
26	.000001	.000001	.000002	.000002	.000002	.000003	.000004	.000004	26
27			.000001	.000001	.000001	.000001	.000001	.000001	27

X	A=9.3	A=9.4	A=9.5	A=9.6	A=9.7	A=9.8	A=9.9	A=10.0	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.999909	.999917	.999925	.999932	.999939	.999945	.999950	.999955	1
2	.999058	.999140	.999214	.999282	.999344	.999401	.999453	.999501	2
3	.995105	.995485	.995836	.996161	.996461	.996738	.996994	.997231	3
4	.982848	.984033	.985140	.986174	.987139	.988040	.988880	.989664	4
5	.954353	.957122	.959737	.962205	.964533	.966729	.968798	.970747	5
6	.901350	.906529	.911472	.916185	.920678	.924959	.929035	.932914	6
7	.819197	.827267	.835051	.842553	.849779	.856735	.863426	.869859	7
8	.710050	.720829	.731337	.741572	.751533	.761221	.770636	.779779	8
9	.583166	.595765	.608177	.620394	.632410	.644217	.655809	.667180	9
10	.452054	.465142	.478174	.491138	.504021	.516812	.529498	.542070	10
11	.330119	.342356	.354672	.367052	.379484	.391955	.404451	.416960	11
12	.227029	.237430	.248010	.258759	.269665	.280719	.291909	.303224	12
13	.147133	.155238	.163570	.172124	.180895	.189876	.199062	.208444	13
14	.089978	.095807	.101864	.108148	.114659	.121395	.128355	.135536	14
15	.052010	.055903	.059992	.064279	.068767	.073458	.078355	.083458	15
16	.028470	.030897	.033473	.036202	.039090	.042139	.045355	.048740	16
17	.014788	.016206	.017727	.019357	.021098	.022956	.024936	.027042	17
18	.007302	.008083	.008928	.009844	.010832	.011898	.013045	.014278	18
19	.003435	.003840	.004284	.004770	.005300	.005877	.006505	.007187	19
20	.001542	.001742	.001962	.002207	.002476	.002772	.003098	.003454	20
21	.000662	.000755	.000859	.000976	.001106	.001250	.001411	.001588	21
22	.000272	.000314	.000361	.000414	.000473	.000540	.000616	.000700	22
23	.000107	.000125	.000145	.000168	.000194	.000224	.000258	.000296	23
24	.000041	.000048	.000056	.000066	.000077	.000089	.000104	.000120	24
25	.000015	.000018	.000021	.000025	.000029	.000034	.000040	.000047	25
26	.000005	.000006	.000007	.000009	.000011	.000013	.000015	.000018	26
27	.000002	.000002	.000003	.000003	.000004	.000004	.000005	.000006	27
28	.000001	.000001	.000001	.000001	.000001	.000002	.000002	.000002	28
29						.000001	.000001	.000001	29

X	A=10.1	A=10.2	A=10.3	A=10.4	A=10.5	A=10.6	A=10.7	A=10.8	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.999959	.999963	.999966	.999970	.999972	.999975	.999977	.999980	1
2	.999544	.999584	.999620	.999653	.999683	.999711	.999736	.999759	2
3	.997449	.997650	.997836	.998007	.998165	.998311	.998446	.998570	3
4	.990395	.991076	.991711	.992302	.992853	.993365	.993843	.994287	4
5	.972583	.974312	.975938	.977468	.978906	.980259	.981529	.982723	5
6	.936604	.940112	.943446	.946613	.949620	.952473	.955179	.957745	6
7	.876039	.881974	.887669	.893131	.898367	.903384	.908187	.912784	7
8	.788652	.797257	.805597	.813673	.821489	.829050	.836358	.843417	8
9	.678327	.689244	.699928	.710377	.720587	.730557	.740285	.749771	9

TABLE II

X	A=10.1	A=10.2	A=10.3	A=10.4	A=10.5	A=10.6	A=10.7	A=10.8	X
10	.554517	.566829	.578997	.591013	.602867	.614554	.626066	.637396	10
11	.429469	.441966	.454438	.466874	.479262	.491591	.503851	.516031	11
12	.314652	.326183	.337805	.349506	.361275	.373100	.384969	.396872	12
13	.218015	.227768	.237695	.247787	.258036	.268432	.278966	.289630	13
14	.142935	.150550	.158378	.166413	.174651	.183088	.191718	.200536	14
15	.088770	.094292	.100022	.105963	.112112	.118470	.125035	.131806	15
16	.052300	.056036	.059952	.064051	.068335	.072807	.077468	.082321	16
17	.029277	.031647	.034156	.036808	.039606	.042555	.045658	.048918	17
18	.015599	.017015	.018527	.020142	.021862	.023692	.025636	.027698	18
19	.007925	.008723	.009584	.010512	.011511	.012584	.013734	.014965	19
20	.003845	.004271	.004736	.005242	.005791	.006387	.007031	.007728	20
21	.001784	.002001	.002239	.002501	.002788	.003102	.003445	.003820	21
22	.000794	.000898	.001014	.001143	.001286	.001444	.001618	.001810	22
23	.000339	.000387	.000441	.000502	.000570	.000645	.000730	.000823	23
24	.000139	.000160	.000184	.000212	.000242	.000277	.000316	.000360	24
25	.000055	.000064	.000074	.000086	.000099	.000115	.000132	.000152	25
26	.000021	.000024	.000029	.000034	.000039	.000046	.000053	.000061	26
27	.000008	.000009	.000011	.000013	.000015	.000018	.000021	.000024	27
28	.000003	.000003	.000004	.000005	.000005	.000007	.000008	.000009	28
29	.000001	.000001	.000001	.000002	.000002	.000002	.000003	.000003	29
30				.000001	.000001	.000001	.000001	.000001	30

X	A=10.9	A=11.0	A=11.1	A=11.2	A=11.3	A=11.4	A=11.5	A=11.6	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.999982	.999983	.999985	.999986	.999988	.999989	.999990	.999991	1
2	.999780	.999800	.999817	.999833	.999848	.999861	.999873	.999885	2
3	.998684	.998789	.998886	.998976	.999058	.999134	.999203	.999268	3
4	.994700	.995084	.995441	.995774	.996082	.996369	.996636	.996883	4
5	.983843	.984895	.985882	.986808	.987677	.988491	.989253	.989968	5
6	.960177	.962480	.964661	.966726	.968680	.970527	.972274	.973925	6
7	.917182	.921386	.925403	.929239	.932902	.936397	.939730	.942908	7
8	.850233	.856808	.863150	.869261	.875147	.880814	.886265	.891508	8
9	.759014	.768015	.776773	.785290	.793568	.801607	.809410	.816979	9
10	.648539	.659489	.670243	.680794	.691141	.701279	.711205	.720919	10
11	.528121	.540111	.551993	.563758	.575398	.586904	.598270	.609489	11
12	.408797	.420733	.432669	.444595	.456499	.468371	.480202	.491982	12
13	.300412	.311303	.322294	.333375	.344535	.355764	.367053	.378391	13
14	.209535	.218709	.228051	.237555	.247213	.257017	.266960	.277033	14
15	.138780	.145956	.153330	.160899	.168660	.176608	.184740	.193051	15
16	.087366	.092604	.098036	.103663	.109483	.115498	.121705	.128104	16
17	.052339	.055924	.059676	.063597	.067690	.071957	.076399	.081018	17
18	.029881	.032191	.034629	.037201	.039910	.042758	.045750	.048889	18
19	.016282	.017687	.019184	.020777	.022470	.024266	.026169	.028183	19
20	.008480	.009289	.010160	.011095	.012098	.013171	.014318	.015542	20
21	.004228	.004671	.005152	.005673	.006237	.006846	.007503	.008210	21
22	.002021	.002252	.002505	.002782	.003084	.003413	.003771	.004160	22
23	.000927	.001042	.001170	.001310	.001464	.001634	.001821	.002025	23
24	.000409	.000464	.000525	.000593	.000669	.000752	.000845	.000948	24
25	.000174	.000199	.000227	.000258	.000294	.000334	.000378	.000427	25
26	.000071	.000082	.000094	.000109	.000125	.000143	.000163	.000186	26
27	.000028	.000033	.000038	.000044	.000051	.000059	.000068	.000078	27
28	.000011	.000013	.000015	.000017	.000020	.000023	.000027	.000032	28
29	.000004	.000005	.000006	.000007	.000008	.000009	.000011	.000012	29
30	.000001	.000002	.000002	.000002	.000003	.000003	.000004	.000005	30
31		.000001	.000001	.000001	.000001	.000001	.000001	.000002	31
32						.000001	.000001	.000001	32

X	A=11.7	A=11.8	A=11.9	A=12.0	A=12.1	A=12.2	A=12.3	A=12.4	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.999992	.999992	.999993	.999994	.999994	.999995	.999995	.999996	1
2	.999895	.999904	.999912	.999920	.999927	.999934	.999939	.999945	2
3	.999327	.999381	.999432	.999478	.999520	.999559	.999595	.999628	3
4	.997113	.997326	.997524	.997708	.997879	.998037	.998183	.998319	4
5	.990637	.991264	.991851	.992400	.992913	.993393	.993842	.994262	5
6	.975484	.976957	.978347	.979659	.980897	.982064	.983164	.984200	6
7	.945936	.948819	.951565	.954178	.956663	.959026	.961272	.963406	7

TABLE II

X	A=11.7	A=11.8	A=11.9	A=12.0	A=12.1	A=12.2	A=12.3	A=12.4	X
8	.896547	.901388	.906035	.910496	.914774	.918875	.922806	.926570	8
9	.824317	.831426	.838310	.844972	.851416	.857645	.863663	.869475	9
10	.730417	.739698	.748762	.757608	.766235	.774644	.780835	.790810	10
11	.620554	.631460	.642200	.652771	.663166	.673383	.683417	.693266	11
12	.503700	.515349	.526920	.538403	.549790	.561075	.572250	.583307	12
13	.389768	.401174	.412600	.424035	.435470	.446896	.458303	.469682	13
14	.287228	.296538	.307953	.318464	.329064	.339743	.350492	.361302	14
15	.201535	.210188	.219003	.227975	.237099	.246366	.255772	.265308	15
16	.134694	.141472	.148436	.155584	.162913	.170420	.178101	.185953	16
17	.085816	.090794	.095952	.101291	.106811	.112511	.118392	.124453	17
18	.052177	.055618	.059213	.062966	.066879	.070953	.075191	.079594	18
19	.030312	.032558	.034925	.037417	.040036	.042786	.045670	.048691	19
20	.016847	.018236	.019713	.021280	.022941	.024700	.026559	.028523	20
21	.008970	.009786	.010661	.011598	.012599	.013667	.014806	.016019	21
22	.004582	.005039	.005532	.006065	.006640	.007258	.007922	.008635	22
23	.002248	.002492	.002758	.003047	.003362	.003703	.004073	.004474	23
24	.001061	.001185	.001322	.001473	.001638	.001818	.002015	.002230	24
25	.000482	.000543	.000611	.000686	.000768	.000860	.000960	.001071	25
26	.000211	.000240	.000272	.000308	.000348	.000392	.000441	.000496	26
27	.000089	.000102	.000117	.000133	.000152	.000173	.000196	.000222	27
28	.000036	.000042	.000049	.000056	.000064	.000073	.000084	.000096	28
29	.000014	.000017	.000020	.000023	.000026	.000030	.000035	.000040	29
30	.000006	.000006	.000008	.000009	.000010	.000012	.000014	.000016	30
31	.000002	.000002	.000003	.000003	.000004	.000005	.000005	.000006	31
32	.000001	.000001	.000001	.000001	.000001	.000002	.000002	.000002	32
33					.000001	.000001	.000001	.000001	33

X	A=12.5	A=12.6	A=12.7	A=12.8	A=12.9	A=13.0	A=13.1	A=13.2	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0
1	.999996	.999997	.999997	.999997	.999997	.999998	.999998	.999998	1
2	.999950	.999954	.999958	.999962	.999965	.999968	.999971	.999974	2
3	.999659	.999686	.999712	.999736	.999757	.999777	.999796	.999812	3
4	.998445	.998562	.998670	.998771	.998864	.998950	.999029	.999103	4
5	.994655	.995021	.995363	.995683	.995981	.996260	.996520	.996762	5
6	.985177	.986097	.986963	.987778	.988545	.989266	.989944	.990582	6
7	.965433	.967356	.969182	.970914	.972556	.974113	.975588	.976986	7
8	.930175	.933624	.936923	.940077	.943092	.945972	.948722	.951347	8
9	.875084	.880494	.885711	.890738	.895580	.900242	.904728	.909042	9
10	.798569	.806114	.813446	.820567	.827481	.834188	.840692	.846996	10
11	.702925	.712394	.721669	.730749	.739632	.748318	.756806	.765095	11
12	.594239	.605042	.615708	.626232	.636609	.646835	.656905	.666814	12
13	.481025	.492322	.503566	.514748	.525860	.536895	.547846	.558705	13
14	.372165	.383071	.394012	.404979	.415963	.426955	.437948	.448932	14
15	.274968	.284745	.294631	.304619	.314700	.324868	.335115	.345432	15
16	.193971	.202151	.210488	.218978	.227615	.236393	.245308	.254353	16
17	.130692	.137108	.143700	.150465	.157402	.164507	.171778	.179212	17
18	.084163	.088900	.093805	.098879	.104122	.109535	.115117	.120867	18
19	.051852	.055154	.058602	.062196	.065939	.069833	.073880	.078081	19
20	.030594	.032776	.035071	.037483	.040014	.042669	.045448	.048356	20
21	.017308	.018677	.020129	.021667	.023293	.025012	.026826	.028737	21
22	.009400	.010218	.011093	.012026	.013021	.014081	.015208	.016406	22
23	.004906	.005373	.005876	.006417	.006999	.007622	.008291	.009007	23
24	.002464	.002719	.002996	.003290	.003620	.003972	.004351	.004760	24
25	.001192	.001326	.001471	.001631	.001805	.001994	.002201	.002425	25
26	.000557	.000623	.000697	.000778	.000868	.000966	.001074	.001192	26
27	.000251	.000283	.000319	.000359	.000403	.000452	.000506	.000566	27
28	.000109	.000124	.000141	.000160	.000181	.000204	.000230	.000260	28
29	.000046	.000053	.000060	.000069	.000079	.000089	.000102	.000115	29
30	.000019	.000022	.000025	.000029	.000033	.000038	.000043	.000050	30
31	.000007	.000009	.000010	.000012	.000013	.000016	.000018	.000021	31
32	.000003	.000003	.000004	.000005	.000005	.000006	.000007	.000008	32
33	.000001	.000001	.000001	.000002	.000002	.000002	.000003	.000003	33
34			.000001	.000001	.000001	.000001	.000001	.000001	34

X	A=13.3	A=13.4	A=13.5	A=13.6	A=13.7	A=13.8	A=13.9	A=14.0	X
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0

TABLE II

X	A=13.3	A=13.4	A=13.5	A=13.6	A=13.7	A=13.8	A=13.9	A=14.0	X
1	.999998	.999999	.999999	.999999	.999999	.999999	.999999	.999999	1
2	.999976	.999978	.999980	.999982	.999983	.999985	.999986	.999988	2
3	.999828	.999842	.999855	.999867	.999878	.999888	.999898	.999906	3
4	.999171	.999235	.999293	.999347	.999397	.999443	.999486	.999526	4
5	.996988	.997199	.997396	.997579	.997750	.997909	.998057	.998195	5
6	.991181	.991744	.992273	.992769	.993235	.993673	.994083	.994468	6
7	.978309	.979561	.980746	.981868	.982928	.983930	.984877	.985772	7
8	.953851	.956240	.958517	.960687	.962754	.964723	.966597	.968380	8
9	.913190	.917176	.921005	.924680	.928207	.931591	.934835	.937945	9
10	.853103	.859015	.864736	.870270	.875619	.880788	.885781	.890601	10
11	.773186	.781079	.788774	.796271	.803574	.810681	.817596	.824319	11
12	.676560	.686138	.695547	.704783	.713844	.722728	.731434	.739960	12
13	.569465	.580122	.590667	.601096	.611402	.621582	.631630	.641542	13
14	.459900	.470843	.481753	.492623	.503445	.514212	.524917	.535552	14
15	.355812	.366247	.376729	.387249	.397801	.408376	.418966	.429563	15
16	.263522	.272809	.282207	.291711	.301313	.311006	.320784	.330640	16
17	.186805	.194554	.202455	.210503	.218695	.227025	.235489	.244082	17
18	.126785	.132871	.139122	.145537	.152114	.158852	.165748	.172799	18
19	.082438	.086951	.091622	.096451	.101439	.106586	.111892	.117357	19
20	.051394	.054565	.057872	.061316	.064900	.068624	.072492	.076505	20
21	.030750	.032867	.035091	.037424	.039870	.042431	.045110	.047908	21
22	.017676	.019022	.020446	.021951	.023541	.025218	.026985	.028844	22
23	.009772	.010588	.011459	.012386	.013373	.014421	.015533	.016712	23
24	.005201	.005675	.006184	.006731	.007316	.007943	.008613	.009328	24
25	.002668	.002932	.003217	.003526	.003859	.004218	.004604	.005020	25
26	.001321	.001461	.001615	.001782	.001964	.002161	.002376	.002608	26
27	.000631	.000704	.000783	.000870	.000966	.001070	.001184	.001309	27
28	.000292	.000328	.000367	.000411	.000459	.000512	.000571	.000635	28
29	.000131	.000148	.000167	.000188	.000211	.000237	.000266	.000298	29
30	.000057	.000064	.000073	.000083	.000094	.000107	.000120	.000136	30
31	.000024	.000027	.000031	.000036	.000041	.000046	.000053	.000060	31
32	.000010	.000011	.000013	.000015	.000017	.000020	.000022	.000026	32
33	.000004	.000004	.000005	.000006	.000007	.000008	.000009	.000011	33
34	.000001	.000002	.000002	.000002	.000003	.000003	.000004	.000004	34
35	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000002	35
36							.000001	.000001	36

X	A=14.1	A=14.2	A=14.3	A=14.4	A=14.5	A=14.6	A=14.7	A=14.8	X
0	1.000000	1.000000	1.000000	1.000000	1.000000				0
1	.999999	.999999	.999999	.999999	.999999	1.000000	1.000000	1.000000	1
2	.999989	.999990	.999991	.999991	.999992	.999993	.999993	.999994	2
3	.999914	.999921	.999928	.999934	.999939	.999944	.999949	.999953	3
4	.999562	.999596	.999627	.999656	.999683	.999708	.999730	.999751	4
5	.998323	.998443	.998554	.998658	.998754	.998844	.998927	.999004	5
6	.994829	.995167	.995484	.995782	.996060	.996321	.996565	.996793	6
7	.986617	.987415	.988168	.988879	.989550	.990182	.990778	.991340	7
8	.970077	.971690	.973223	.974680	.976064	.977378	.978626	.979810	8
9	.940924	.943777	.946508	.949121	.951621	.954011	.956296	.958418	9
10	.895251	.899736	.904061	.908227	.912241	.916105	.919823	.923400	10
11	.830853	.837199	.843361	.849340	.855139	.860761	.866209	.871485	11
12	.748305	.756469	.764451	.772251	.779869	.787305	.794563	.801635	12
13	.651312	.660939	.670417	.679745	.688918	.697934	.706791	.715487	13
14	.546112	.556590	.566980	.577276	.587472	.597563	.607545	.617411	14
15	.440161	.450751	.461326	.471879	.482403	.492891	.503336	.513731	15
16	.340567	.350557	.360603	.370699	.380837	.391010	.401211	.411432	16
17	.252799	.261634	.270582	.279636	.288792	.298043	.307383	.316807	17
18	.180004	.187357	.194858	.202501	.210284	.218201	.226250	.234426	18
19	.122980	.128761	.134699	.140793	.147040	.153441	.159992	.166692	19
20	.080663	.084969	.089422	.094024	.098776	.103677	.108729	.113930	20
21	.050830	.053876	.057049	.060351	.063784	.067350	.071050	.074886	21
22	.030799	.032851	.035004	.037261	.039623	.042094	.044675	.047370	22
23	.017960	.019281	.020675	.022147	.023699	.025333	.027052	.028859	23
24	.010090	.010902	.011767	.012685	.013660	.014694	.015788	.016947	24
25	.005466	.005945	.006458	.007007	.007594	.008221	.008890	.009602	25
26	.002859	.003130	.003422	.003737	.004077	.004441	.004833	.005253	26
27	.001444	.001592	.001752	.001926	.002115	.002319	.002539	.002778	27
28	.000706	.000783	.000868	.000960	.001061	.001171	.001291	.001421	28

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TABLE II

X	A=14.1	A=14.2	A=14.3	A=14.4	A=14.5	A=14.6	A=14.7	A=14.8	X
29	.000334	.000373	.000416	.000463	.000515	.000573	.000635	.000704	29
30	.000153	.000172	.000193	.000217	.000243	.000271	.000303	.000338	30
31	.000068	.000077	.000087	.000098	.000111	.000125	.000140	.000157	31
32	.000029	.000033	.000038	.000043	.000049	.000056	.000063	.000071	32
33	.000012	.000014	.000016	.000018	.000021	.000024	.000027	.000031	33
34	.000005	.000006	.000007	.000008	.000009	.000010	.000012	.000013	34
35	.000002	.000002	.000003	.000003	.000004	.000004	.000005	.000006	35
36	.000001	.000001	.000001	.000001	.000001	.000002	.000002	.000002	36
37					.000001	.000001	.000001	.000001	37

X	A=14.9	A=15.0	A=16	A=17	A=18	A=19	A=20	A=21	X
1	1.000000	1.000000	1.000000	1.000000					1
2	.999995	.999995	.999998	.999999	1.000000	1.000000			2
3	.999957	.999961	.999984	.999993	.999997	.999999	1.000000	1.000000	3
4	.999771	.999789	.999907	.999959	.999982	.999992	.999997	.999999	4
5	.999076	.999143	.999600	.999815	.999916	.999962	.999983	.999993	5
6	.997007	.997208	.998616	.999325	.999676	.999846	.999928	.999967	6
7	.991869	.992368	.995994	.997938	.998957	.999480	.999745	.999876	7
8	.980933	.981998	.990000	.994567	.997107	.998487	.999221	.999605	8
9	.960563	.962554	.978013	.987404	.992944	.996127	.997913	.998894	9
10	.926840	.930146	.956702	.973875	.984619	.991144	.995005	.997234	10
11	.876593	.881536	.922604	.950876	.969634	.981678	.989188	.993749	11
12	.808531	.815248	.873007	.915331	.945113	.965327	.978613	.987095	12
13	.724020	.732389	.806878	.864976	.908331	.939439	.960988	.975451	13
14	.627158	.636782	.725489	.799127	.857402	.901601	.933872	.956641	14
15	.524070	.534346	.632473	.719167	.791923	.850250	.895136	.928426	15
16	.421668	.431910	.533255	.628546	.713347	.785206	.843487	.888925	16
17	.326306	.335877	.434038	.532262	.624950	.707966	.778926	.837081	17
18	.242725	.251141	.340656	.435977	.531352	.621639	.702972	.773037	18
19	.173538	.180528	.257651	.345042	.437755	.530516	.618578	.698320	19
20	.119281	.124781	.187751	.263678	.349084	.439393	.529743	.615737	20
21	.078859	.082971	.131832	.194519	.269280	.352826	.440907	.529026	21
22	.050179	.053106	.089227	.138534	.200876	.274503	.356302	.442314	22
23	.030755	.032744	.058241	.095272	.144910	.206861	.279389	.359544	23
24	.018172	.019465	.036686	.063296	.101110	.150983	.212507	.283971	24
25	.010359	.011165	.022315	.040646	.068260	.106746	.156773	.217845	25
26	.005703	.006185	.013119	.025245	.044608	.073126	.112185	.162299	26
27	.003035	.003312	.007459	.015174	.028234	.048557	.077887	.117435	27
28	.001562	.001716	.004105	.008834	.017318	.031268	.052481	.082541	28
29	.000779	.000861	.002189	.004984	.010300	.019536	.034334	.056370	29
30	.000376	.000418	.001131	.002727	.005944	.011850	.021818	.037419	30
31	.000176	.000197	.000567	.001448	.003331	.006982	.013475	.024153	31
32	.000080	.000090	.000276	.000747	.001813	.003998	.008092	.015166	32
33	.000035	.000040	.000131	.000375	.000960	.002227	.004727	.009269	33
34	.000015	.000017	.000060	.000185	.000494	.001207	.002688	.005516	34
35	.000006	.000007	.000027	.000087	.000248	.000637	.001489	.003198	35
36	.000003	.000003	.000012	.000040	.000121	.000327	.000804	.001807	36
37	.000001	.000001	.000005	.000018	.000058	.000164	.000423	.000996	37
38			.000002	.000008	.000027	.000080	.000217	.000536	38
39			.000001	.000003	.000012	.000038	.000109	.000281	39
40				.000001	.000005	.000018	.000053	.000144	40
41				.000001	.000002	.000008	.000025	.000072	41
42					.000001	.000004	.000012	.000035	42
43						.000002	.000005	.000017	43
44						.000001	.000002	.000008	44
45							.000001	.000004	45
46								.000002	46
47								.000001	47

X	A=22	A=23	A=24	A=25	A=26	A=27	A=28	A=29	X
3	1.000000								3
4	.999999	1.000000	1.000000						4
5	.999997	.999999	.999999	1.000000	1.000000				5
6	.999985	.999993	.999997	.999999	.999999	1.000000	1.000000		6
7	.999941	.999972	.999987	.999994	.999997	.999999	.999999	1.000000	7

~~SECRET~~

TABLE II

X	A=22	A=23	A=24	A=25	A=26	A=27	A=28	A=29	X
8	.999803	.999903	.999953	.999977	.999989	.999995	.999998	.999999	8
9	.999423	.999703	.999849	.999925	.999963	.999982	.999991	.999996	9
10	.998495	.999194	.999575	.999779	.999886	.999942	.999971	.999986	10
11	.996453	.998023	.085513	.999414	.999687	.999836	.999914	.999956	11
12	.992370	.995573	.997476	.998584	.999218	.999574	.999771	.999878	12
13	.984884	.990878	.994598	.996856	.998200	.998985	.999436	.999690	13
14	.972215	.982572	.989284	.993533	.996164	.997762	.998714	.999271	14
15	.952307	.968926	.980175	.987598	.992383	.995403	.997270	.998403	15
16	.923108	.948002	.965600	.977707	.985830	.991156	.994574	.996725	16
17	.882960	.917923	.943728	.962252	.975182	.983991	.989857	.993682	17
18	.831004	.877229	.912874	.939525	.958895	.972610	.982088	.988492	18
19	.767502	.825231	.871721	.907959	.935371	.955539	.970003	.980131	19
20	.693973	.762286	.819739	.866425	.903179	.931281	.952193	.967369	20
21	.613091	.689900	.757361	.814508	.861330	.898532	.927259	.948863	21
22	.528358	.610619	.686072	.752701	.809517	.856426	.894014	.923308	22
23	.443625	.527734	.608302	.682467	.748283	.804750	.851702	.889622	23
24	.362576	.444850	.527150	.606124	.679063	.744087	.800191	.847149	24
25	.288281	.365419	.445999	.526602	.604073	.675842	.740096	.795826	25
26	.222901	.292343	.368093	.447079	.526085	.602137	.672789	.736293	26
27	.167580	.227698	.296181	.370614	.448096	.525597	.600305	.669889	27
28	.122503	.172631	.232258	.299814	.372996	.449057	.525136	.598567	28
29	.087086	.127397	.177468	.236599	.303260	.375251	.449967	.524698	29
30	.060217	.091521	.132124	.182104	.240738	.306535	.377390	.450829	30
31	.040514	.064017	.095848	.136691	.186553	.244690	.309651	.379422	31
32	.026531	.043611	.067764	.100068	.141107	.190825	.248468	.312622	32
33	.016918	.028943	.046701	.071456	.104182	.145377	.194933	.252085	33
34	.010509	.018721	.031383	.049780	.075089	.108192	.149509	.198885	34
35	.006362	.011806	.020570	.033842	.052842	.078663	.112101	.153509	35
36	.003755	.007261	.013155	.022458	.036316	.055883	.082175	.115912	36
37	.002162	.004358	.008212	.014552	.024380	.038798	.058899	.085625	37
38	.001215	.002553	.005006	.009211	.015993	.026331	.041285	.061887	38
39	.000667	.001461	.002980	.005696	.010254	.017473	.028306	.043771	39
40	.000357	.000817	.001734	.003444	.006429	.011340	.018987	.030300	40
41	.000187	.000446	.000987	.002036	.003942	.007200	.012465	.020533	41
42	.000096	.000238	.000549	.001177	.002365	.004474	.008010	.013625	42
43	.000048	.000125	.000299	.000666	.001389	.002722	.005040	.008856	43
44	.000024	.000064	.000159	.000369	.000798	.001622	.003107	.005639	44
45	.000011	.000032	.000083	.000200	.000450	.000946	.001876	.003519	45
46	.000005	.000016	.000042	.000106	.000248	.000541	.001110	.002152	46
47	.000002	.000008	.000021	.000055	.000134	.000303	.000644	.001291	47
48	.000001	.000004	.000010	.000028	.000071	.000167	.000367	.000759	48
49		.000002	.000005	.000014	.000037	.000090	.000205	.000438	49
50		.000001	.000002	.000007	.000019	.000048	.000112	.000248	50
51			.000001	.000003	.000009	.000025	.000061	.000138	51
52				.000002	.000005	.000013	.000032	.000075	52
53				.000001	.000002	.000006	.000016	.000040	53
54					.000001	.000003	.000008	.000021	54
55						.000001	.000004	.000011	55
56						.000001	.000002	.000006	56
57							.000001	.000003	57
58								.000001	58
59								.000001	59

X	A=30	A=31	A=32	A=33	A=34	A=35	A=36	A=37	X
7	1.000000								7
8	.999999	1.000000							8
9	.999998	.999999	1.000000	1.000000					9
10	.999993	.999997	.999998	.999999	1.000000	1.000000			10
11	.999978	.999989	.999994	.999997	.999999	.999999	1.000000	1.000000	11
12	.999936	.999967	.999983	.999991	.999996	.999998	.999999	.999999	12
13	.999832	.999910	.999952	.999975	.999987	.999993	.999997	.999998	13
14	.999593	.999775	.999877	.999934	.999965	.999981	.999990	.999995	14
15	.999079	.999476	.999706	.999837	.999911	.999951	.999974	.999986	15
16	.998053	.998859	.999340	.999623	.999788	.999882	.999935	.999964	16
17	.996127	.997662	.998608	.999183	.999526	.999729	.999846	.999914	17
18	.992730	.995479	.997231	.998328	.999004	.999414	.999660	.999805	18
19	.987067	.991721	.994782	.996760	.998017	.998802	.999286	.999579	19

TABLE II

X	A=30	A=31	A=32	A=33	A=34	A=35	A=36	A=37	X
20	.978127	.985588	.990658	.994038	.996251	.997675	.998578	.999141	20
21	.964715	.976082	.984059	.989546	.993249	.995703	.997303	.998330	21
22	.945557	.962050	.974004	.982487	.988388	.992417	.995119	.996901	22
23	.919431	.942278	.959379	.971899	.980876	.987187	.991544	.994498	23
24	.885354	.915628	.939031	.956707	.969771	.979230	.985948	.990632	24
25	.842758	.881205	.911899	.935819	.954040	.967626	.977554	.984672	25
26	.791643	.838521	.877171	.908246	.932644	.951380	.965467	.975852	26
27	.732663	.787628	.834429	.873249	.904666	.929510	.948731	.963299	27
28	.667131	.729196	.783772	.830475	.869434	.901160	.926417	.946098	28
29	.596918	.664502	.725877	.780064	.826653	.865723	.897727	.923367	29
30	.524283	.595348	.661994	.722699	.776495	.822953	.862112	.894366	30
31	.451648	.523888	.593852	.659597	.719650	.773058	.819374	.858598	31
32	.381337	.452428	.523512	.592424	.657303	.716722	.769743	.815907	32
33	.315459	.383202	.453171	.523153	.591060	.655105	.713908	.766545	33
34	.255551	.318170	.384963	.453881	.522810	.589754	.652997	.711201	34
35	.202692	.258877	.320766	.386646	.454559	.522481	.588503	.650973	35
36	.157383	.206361	.262072	.323254	.388258	.455208	.522167	.587303	36
37	.119627	.161138	.209900	.265144	.325641	.389804	.455830	.521865	37
38	.089013	.123249	.164778	.213317	.268101	.327934	.391286	.456427	38
39	.064844	.092339	.126780	.168309	.216618	.270950	.330140	.392711	39
40	.046253	.067770	.095603	.130225	.171735	.219810	.273696	.332262	40
41	.032310	.048728	.070661	.098806	.133585	.175062	.222898	.276347	41
42	.022107	.034331	.051194	.073517	.101948	.136863	.178294	.225887	42
43	.014820	.023705	.036362	.053647	.076337	.105030	.140062	.181435	43
44	.009735	.016044	.025324	.038399	.056086	.079120	.108054	.143185	44
45	.006269	.010647	.017297	.026962	.040438	.058509	.081865	.111020	45
46	.003958	.006929	.011588	.018575	.028615	.042479	.060915	.084573	46
47	.002450	.004423	.007617	.012559	.019877	.030282	.044518	.063301	47
48	.001488	.002770	.004914	.008334	.013555	.021199	.031960	.046555	48
49	.000887	.001703	.003111	.005430	.009077	.014576	.022540	.033646	49
50	.000519	.001027	.001934	.003474	.005970	.009846	.015620	.023899	50
51	.000298	.000609	.001181	.002183	.003857	.006534	.010638	.016686	51
52	.000168	.000354	.000708	.001348	.002449	.004261	.007121	.011453	52
53	.000093	.000202	.000417	.000817	.001528	.002732	.004686	.007729	53
54	.000051	.000114	.000242	.000487	.000937	.001722	.003032	.005130	54
55	.000027	.000063	.000138	.000286	.000565	.001067	.001929	.003349	55
56	.000014	.000034	.000077	.000165	.000335	.000650	.001208	.002151	56
57	.000007	.000018	.000042	.000093	.000195	.000390	.000744	.001359	57
58	.000004	.000010	.000023	.000052	.000112	.000230	.000451	.000845	58
59	.000002	.000005	.000012	.000029	.000063	.000134	.000269	.000517	59
60		.000003	.000006	.000015	.000035	.000076	.000158	.000312	60
61		.000001	.000003	.000008	.000019	.000043	.000091	.000185	61
62		.000001	.000002	.000004	.000010	.000024	.000052	.000108	62
63			.000001	.000002	.000006	.000013	.000029	.000062	63
64				.000001	.000003	.000007	.000016	.000035	64
65				.000001	.000001	.000004	.000009	.000020	65
66					.000001	.000002	.000005	.000011	66
67						.000001	.000002	.000006	67
68						.000001	.000001	.000003	68
69							.000001	.000002	69
70								.000001	70

X	A=38	A=39	A=40	A=41	A=42	A=43	A=44	A=45	X
12	1.000000								12
13	.999999	1.000000	1.000000						13
14	.999997	.999999	.999999	1.000000	1.000000				14
15	.999993	.999996	.999998	.999999	.999999	1.000000			15
16	.999981	.999990	.999995	.999997	.999999	.999999	1.000000	1.000000	16
17	.999952	.999974	.999986	.999992	.999996	.999998	.999999	.999999	17
18	.999889	.999938	.999965	.999981	.999990	.999994	.999997	.999998	18
19	.999755	.999859	.999920	.999955	.999975	.999986	.999992	.999996	19
20	.999487	.999698	.999824	.999898	.999942	.999967	.999982	.999990	20
21	.998979	.999383	.999632	.999783	.999873	.999927	.999958	.999976	21
22	.998059	.998799	.999266	.999557	.999735	.999843	.999908	.999947	22
23	.996469	.997763	.998601	.999135	.999472	.999681	.999809	.999887	23
24	.993843	.996008	.997445	.998385	.998991	.999377	.999620	.999770	24
25	.989685	.993154	.995517	.997102	.998150	.998833	.999272	.999551	25

TABLE II

X	A=38	A=39	A=40	A=41	A=42	A=43	A=44	A=45	X
26	.983364	.988703	.992434	.994999	.996737	.997896	.998660	.999156	26
27	.974126	.982026	.987689	.991683	.994454	.996348	.997624	.998473	27
28	.961125	.972382	.980661	.986646	.990902	.993882	.995937	.997334	28
29	.943481	.958948	.970620	.979272	.985576	.990095	.993285	.995503	29
30	.920360	.940883	.956771	.968846	.977861	.984479	.989261	.992663	30
31	.891074	.917398	.938306	.954597	.967060	.976431	.983359	.988402	31
32	.855176	.887852	.914479	.935751	.952427	.965266	.974983	.982218	32
33	.812546	.851843	.884696	.911606	.933221	.950264	.963466	.973520	33
34	.763457	.809287	.848596	.881607	.908777	.930715	.948109	.961661	34
35	.708593	.760472	.806124	.845431	.878581	.905993	.928236	.945964	35
36	.649026	.706079	.757586	.803055	.842347	.875619	.903252	.925782	36
37	.586151	.647153	.703654	.754792	.800073	.839339	.872717	.900556	37
38	.521575	.585042	.645349	.701312	.752087	.797176	.836404	.869874	38
39	.457000	.521297	.583976	.643610	.699049	.749465	.794358	.833541	39
40	.394081	.457551	.521029	.582949	.641931	.696860	.746922	.791618	40
41	.334307	.395399	.458082	.520771	.581958	.640310	.694742	.744455	41
42	.278907	.336279	.396670	.458593	.520522	.581002	.638744	.692690	42
43	.228784	.281382	.338163	.397896	.459086	.520282	.580079	.637228	43
44	.184488	.231592	.283776	.340021	.399078	.459562	.520050	.579187	44
45	.146233	.187459	.234315	.286093	.341799	.400221	.460021	.519826	45
46	.113929	.149211	.190350	.236958	.288338	.343518	.401326	.460465	46
47	.087243	.116783	.152119	.193164	.239525	.290513	.345183	.402395	47
48	.065667	.089875	.119583	.154961	.195906	.242019	.292623	.346796	48
49	.048586	.068012	.092469	.122329	.157739	.198577	.244444	.294671	49
50	.035339	.050611	.070335	.095025	.125024	.160454	.201181	.246802	50
51	.025272	.037038	.052628	.072636	.097544	.127668	.163109	.203720	51
52	.017771	.026659	.038740	.054636	.074913	.100025	.130263	.165706	52
53	.012289	.018874	.028057	.040444	.056634	.077167	.102470	.132809	53
54	.008359	.013146	.019995	.029466	.042149	.058621	.079397	.104878	54
55	.005593	.009009	.014022	.021130	.030883	.043853	.060596	.081602	55
56	.003683	.006076	.009679	.014916	.022280	.032308	.045556	.062558	56
57	.002386	.004033	.006576	.010367	.015827	.023442	.033738	.047255	57
58	.001522	.002635	.004399	.007095	.011073	.016754	.024616	.035174	58
59	.000955	.001695	.002898	.004781	.007630	.011796	.017696	.025800	59
60	.000590	.001074	.001880	.003174	.005179	.008182	.012535	.018651	60
61	.000359	.000670	.001201	.002075	.003464	.005592	.008750	.013289	61
62	.000215	.000412	.000756	.001337	.002282	.003767	.006027	.009334	62
63	.000127	.000249	.000469	.000849	.001482	.002500	.004083	.006463	63
64	.000074	.000149	.000287	.000531	.000949	.001636	.002730	.004412	64
65	.000042	.000087	.000173	.000328	.000599	.001056	.001799	.002970	65
66	.000024	.000051	.000103	.000199	.000372	.000672	.001170	.001972	66
67	.000013	.000029	.000060	.000119	.000228	.000421	.000750	.001291	67
68	.000007	.000016	.000035	.000071	.000138	.000261	.000474	.000834	68
69	.000004	.000009	.000020	.000041	.000083	.000159	.000296	.000532	69
70	.000002	.000005	.000011	.000024	.000049	.000096	.000182	.000334	70
71	.000001	.000003	.000006	.000013	.000028	.000057	.000111	.000207	71
72	.000001	.000001	.000003	.000008	.000016	.000033	.000066	.000127	72
73		.000001	.000002	.000004	.000009	.000019	.000039	.000077	73
74			.000001	.000002	.000005	.000011	.000023	.000046	74
75			.000001	.000001	.000003	.000006	.000013	.000027	75
76				.000001	.000002	.000003	.000008	.000016	76
77					.000001	.000002	.000004	.000009	77
78						.000001	.000002	.000005	78
79						.000001	.000001	.000003	79
80							.000001	.000002	80
81								.000001	81

X	A=46	A=47	A=48	A=49	A=50	A=51	A=52	A=53	X
17	1.000000								17
18	.999999	1.000000	1.000000						18
19	.999998	.999999	.999999	1.000000	1.000000				19
20	.999994	.999997	.999998	.999999	.999999	1.000000			20
21	.999987	.999993	.999996	.999998	.999999	.999999	1.000000	1.000000	21
22	.999970	.999983	.999990	.999995	.999997	.999998	.999999	.999999	22
23	.999934	.999962	.999978	.999987	.999993	.999996	.999998	.999999	23
24	.999862	.999919	.999952	.999972	.999984	.999991	.999995	.999997	24
25	.999726	.999834	.999901	.999941	.999965	.999980	.999988	.999993	25

TABLE II

X	A=46	A=47	A=48	A=49	A=50	A=51	A=52	A=53	X
26	.999474	.999676	.999802	.999880	.999928	.999958	.999975	.999985	26
27	.999029	.999389	.999620	.999766	.999857	.999914	.999948	.999969	27
28	.998270	.998891	.999296	.999558	.999725	.999831	.999897	.999938	28
29	.997024	.998054	.998741	.999194	.999489	.999680	.999801	.999878	29
30	.995048	.996697	.997822	.998579	.999083	.999414	.999630	.999768	30
31	.992017	.994572	.996351	.997575	.998406	.998963	.999332	.999574	31
32	.987520	.991349	.994075	.995988	.997314	.998221	.998834	.999243	32
33	.981056	.986616	.990659	.993558	.995607	.997038	.998024	.998695	33
34	.972045	.979876	.985692	.989949	.993021	.995209	.996747	.997814	34
35	.959853	.970558	.978679	.984748	.989219	.992466	.994794	.996441	35
36	.943830	.958045	.969061	.977467	.983786	.988469	.991893	.994363	36
37	.923356	.941709	.956237	.967556	.976241	.982807	.987702	.991302	37
38	.897902	.920958	.939601	.954432	.966045	.975003	.981813	.986918	38
39	.867090	.895292	.918587	.937508	.952629	.964529	.973754	.980804	39
40	.830746	.864361	.892724	.916244	.935430	.950831	.963008	.972495	40
41	.788951	.828017	.861687	.890197	.913930	.933367	.949039	.961485	41
42	.742060	.786355	.825352	.859067	.887711	.911644	.931321	.947252	42
43	.690702	.739733	.783826	.822748	.856498	.885265	.909386	.929293	43
44	.635762	.688774	.737472	.781362	.820203	.853979	.882859	.907156	44
45	.578324	.634341	.686903	.735273	.778960	.817716	.851509	.880492	45
46	.519609	.577488	.632964	.685087	.733134	.776617	.815283	.849087	46
47	.460895	.519399	.576679	.631628	.683322	.731051	.774331	.812903	47
48	.403430	.461311	.519196	.575894	.630332	.681607	.729023	.772100	48
49	.348359	.404432	.461714	.518999	.575133	.629073	.679939	.727047	49
50	.296660	.349875	.405404	.462104	.518808	.574395	.627850	.678316	50
51	.249097	.298592	.351347	.406347	.462483	.518623	.573678	.626661	51
52	.206197	.251331	.300470	.352777	.407263	.462851	.518443	.572981	52
53	.168247	.208614	.253507	.302297	.354166	.408152	.463208	.518268	53
54	.135309	.170733	.210974	.255627	.304075	.355516	.409016	.463555	54
55	.107251	.137762	.173167	.213278	.257694	.305805	.356830	.409856	55
56	.083784	.109588	.140171	.175549	.215530	.259710	.307491	.358109	56
57	.064508	.085941	.111890	.142537	.177883	.217730	.261676	.309134	57
58	.048951	.066443	.088073	.114157	.144859	.180169	.219881	.263596	58
59	.036613	.050642	.068363	.090181	.116391	.147141	.182408	.221984	59
60	.026994	.038056	.052328	.070269	.092265	.118591	.149382	.184603	60
61	.019619	.028196	.039500	.054008	.072160	.094324	.120759	.151584	61
62	.014058	.020600	.029406	.040945	.055681	.074035	.096359	.122895	62
63	.009932	.014841	.021591	.030622	.042391	.057346	.075895	.098370	63
64	.006919	.010545	.015637	.022592	.031843	.043836	.059004	.077739	64
65	.004754	.007390	.011171	.016445	.023603	.033070	.045280	.060653	65
66	.003221	.005108	.007873	.011810	.017265	.024623	.034300	.046722	66
67	.002153	.003484	.005475	.008370	.012463	.018095	.025650	.035534	67
68	.001420	.002344	.003757	.005854	.008879	.013127	.018936	.026685	68
69	.000924	.001556	.002544	.004040	.006244	.009400	.013802	.019787	69
70	.000593	.001020	.001700	.002753	.004335	.006646	.009933	.014489	70
71	.000376	.000660	.001122	.001851	.002971	.004639	.007059	.010478	71
72	.000235	.000421	.000730	.001229	.002010	.003198	.004954	.007483	72
73	.000145	.000265	.000470	.000806	.001343	.002177	.003434	.005279	73
74	.000089	.000165	.000298	.000522	.000886	.001463	.002351	.003679	74
75	.000053	.000101	.000187	.000334	.000578	.000972	.001590	.002532	75
76	.000032	.000062	.000116	.000211	.000372	.000637	.001062	.001722	76
77	.000019	.000037	.000071	.000131	.000237	.000413	.000701	.001158	77
78	.000011	.000022	.000043	.000081	.000149	.000265	.000457	.000769	78
79	.000006	.000013	.000026	.000049	.000092	.000167	.000295	.000505	79
80	.000004	.000007	.000015	.000030	.000057	.000105	.000188	.000327	80
81	.000002	.000004	.000009	.000018	.000034	.000065	.000118	.000210	81
82	.000001	.000002	.000005	.000010	.000021	.000040	.000074	.000133	82
83	.000001	.000001	.000003	.000006	.000012	.000024	.000045	.000083	83
84		.000001	.000002	.000003	.000007	.000014	.000028	.000052	84
85			.000001	.000002	.000004	.000008	.000017	.000032	85
86				.000001	.000002	.000005	.000010	.000019	86
87				.000001	.000001	.000003	.000006	.000012	87
88					.000001	.000002	.000003	.000007	88
89						.000001	.000002	.000004	89
90						.000001	.000001	.000002	90
91							.000001	.000001	91
92								.000001	92

TABLE II

X	A=54	A=55	A=56	A=57	A=58	A=59	A=60	A=61	X
22	1.000000								22
23	.999999	1.000000	1.000000						23
24	.999998	.999999	.999999	1.000000					24
25	.999996	.999998	.999999	.999999	1.000000	1.000000			25
26	.999992	.999995	.999997	.999998	.999999	.999999	1.000000		26
27	.999982	.999989	.999994	.999996	.999998	.999999	.999999	1.000000	27
28	.999963	.999978	.999987	.999992	.999996	.999997	.999998	.999999	28
29	.999925	.999955	.999973	.999984	.999991	.999994	.999997	.999998	29
30	.999856	.999911	.999946	.999967	.999980	.999988	.999993	.999996	30
31	.999731	.999832	.999896	.999936	.999961	.999976	.999986	.999992	31
32	.999514	.999691	.999805	.999878	.999924	.999954	.999972	.999983	32
33	.999147	.999448	.999646	.999775	.999858	.999912	.999945	.999966	33
34	.998547	.999043	.999376	.999597	.999742	.999836	.999897	.999936	34
35	.997593	.998389	.998932	.999299	.999544	.999706	.999812	.999881	35
36	.996122	.997360	.998221	.998813	.999215	.999486	.999666	.999785	36
37	.993915	.995788	.997115	.998043	.998686	.999126	.999424	.999623	37
38	.990695	.993452	.995441	.996858	.997856	.998551	.999030	.999356	38
39	.986118	.990071	.992974	.995080	.996589	.997659	.998408	.998928	39
40	.979781	.985303	.989432	.992481	.994705	.996309	.997452	.998257	40
41	.971227	.978746	.984473	.988778	.991974	.994318	.996017	.997235	41
42	.959960	.969951	.977700	.983630	.988110	.991453	.993918	.995714	42
43	.945473	.958434	.968669	.976643	.982774	.987428	.990919	.993505	43
44	.927281	.943702	.956908	.967381	.975577	.981906	.986734	.990371	44
45	.904954	.925287	.941939	.955383	.966089	.974501	.981027	.986027	45
46	.878162	.902781	.923312	.940186	.953861	.964793	.973418	.980137	46
47	.846711	.875870	.900635	.921354	.938442	.952340	.963493	.972328	47
48	.810575	.844380	.873615	.898516	.919415	.936709	.950823	.962191	48
49	.769922	.808297	.842092	.871395	.896425	.917495	.934986	.949310	49
50	.725122	.767795	.806066	.839847	.869211	.894360	.915593	.933274	50
51	.676736	.723244	.765717	.803882	.837643	.867061	.892322	.913710	51
52	.625505	.675198	.721412	.763685	.801742	.835479	.864944	.890311	52
53	.572304	.624380	.673699	.719624	.761699	.799646	.833354	.862861	53
54	.518098	.571645	.623285	.672237	.717879	.759757	.797592	.831267	54
55	.463893	.517933	.571004	.622218	.670812	.716174	.757857	.795579	55
56	.410673	.464221	.517772	.570379	.621178	.669421	.714509	.755997	56
57	.359354	.411468	.464540	.517615	.569771	.620164	.668064	.712881	57
58	.310735	.360566	.412243	.464851	.517463	.569179	.619176	.666739	58
59	.265470	.312297	.361748	.412997	.465154	.517314	.568601	.618211	59
60	.224041	.267301	.313821	.362901	.413732	.465450	.517169	.568038	60
61	.186755	.226054	.269090	.315309	.364025	.414449	.465738	.517028	61
62	.153747	.188864	.228024	.270838	.316761	.365121	.415149	.466018	62
63	.124999	.155874	.190933	.229953	.272548	.318180	.366192	.415831	63
64	.100357	.127072	.157963	.192963	.231843	.274220	.319567	.367238	64
65	.079566	.102321	.129115	.160018	.194954	.233693	.275855	.320922	65
66	.062293	.081377	.104260	.131128	.162038	.196908	.235506	.277456	66
67	.048161	.063924	.083172	.106177	.133111	.164024	.198826	.237284	67
68	.036771	.049597	.065546	.084950	.108071	.135066	.165977	.200709	68
69	.027726	.038009	.051030	.067157	.086712	.109941	.136993	.167898	69
70	.020647	.028773	.039249	.052459	.068759	.088458	.111790	.138892	70
71	.015186	.021515	.029825	.040490	.053883	.070350	.090187	.113616	71
72	.011033	.015893	.022391	.030882	.041731	.055303	.071931	.091899	72
73	.007918	.011598	.016610	.023275	.031942	.042973	.056717	.073500	73
74	.005614	.008363	.012174	.017335	.024165	.033007	.044213	.058126	74
75	.003932	.005958	.008818	.012760	.018069	.025061	.034075	.045453	75
76	.002721	.004194	.006312	.009283	.013355	.018811	.025964	.035145	76
77	.001861	.002918	.004465	.006675	.009757	.013959	.019561	.026872	77
78	.001258	.002006	.003122	.004744	.007047	.010241	.014571	.020317	78
79	.000840	.001364	.002158	.003334	.005032	.007428	.010733	.015192	79
80	.000555	.000916	.001475	.002316	.003553	.005328	.007818	.011234	80
81	.000362	.000608	.000996	.001591	.002480	.003779	.005632	.008216	81
82	.000234	.000400	.000665	.001080	.001712	.002650	.004012	.005943	82
83	.000149	.000259	.000439	.000726	.001169	.001839	.002827	.004253	83
84	.000094	.000167	.000287	.000482	.000789	.001262	.001971	.003010	84
85	.000059	.000106	.000185	.000317	.000527	.000856	.001359	.002108	85
86	.000036	.000066	.000118	.000206	.000348	.000575	.000927	.001460	86
87	.000022	.000041	.000075	.000132	.000228	.000382	.000626	.001001	87
88	.000013	.000025	.000047	.000084	.000147	.000251	.000418	.000679	88

TABLE II

X	A=54	A=55	A=56	A=57	A=58	A=59	A=60	A=61	X
89	.000008	.000015	.000029	.000053	.000094	.000163	.000276	.000456	89
90	.000005	.000009	.000018	.000033	.000060	.000105	.000181	.000303	90
91	.000003	.000006	.000011	.000020	.000037	.000067	.000117	.000199	91
92	.000002	.000003	.000006	.000012	.000023	.000042	.000075	.000130	92
93	.000001	.000002	.000004	.000007	.000014	.000026	.000047	.000083	93
94	.000001	.000001	.000002	.000004	.000009	.000016	.000030	.000053	94
95		.000001	.000001	.000003	.000005	.000010	.000019	.000034	95
96			.000001	.000002	.000003	.000006	.000011	.000021	96
97				.000001	.000002	.000004	.000007	.000013	97
98				.000001	.000001	.000002	.000004	.000008	98
99					.000001	.000001	.000003	.000005	99
100						.000001	.000001	.000003	100
101							.000001	.000002	101
102							.000001	.000001	102
103								.000001	103

X	A=62	A=63	A=64	A=65	A=66	A=67	A=68	A=69	X
28	1.000000	1.000000							28
29	.999999	.999999	1.000000						29
30	.999998	.999999	.999999	1.000000	1.000000				30
31	.999995	.999997	.999998	.999999	.999999	1.000000			31
32	.999990	.999994	.999996	.999998	.999999	.999999	1.000000	1.000000	32
33	.999979	.999988	.999993	.999996	.999997	.999998	.999999	.999999	33
34	.999960	.999976	.999985	.999991	.999995	.999997	.999998	.999999	34
35	.999925	.999953	.999971	.999982	.999989	.999993	.999996	.999998	35
36	.999863	.999913	.999946	.999966	.999979	.999987	.999992	.999995	36
37	.999756	.999843	.999900	.999937	.999960	.999975	.999985	.999991	37
38	.999577	.999724	.999822	.999886	.999927	.999954	.999971	.999982	38
39	.999284	.999527	.999689	.999798	.999870	.999917	.999947	.999967	39
40	.998819	.999207	.999472	.999652	.999772	.999852	.999905	.999939	40
41	.998099	.998704	.999125	.999414	.999611	.999744	.999833	.999892	41
42	.997009	.997932	.998583	.999038	.999352	.999567	.999714	.999812	42
43	.995400	.996773	.997757	.998455	.998945	.999286	.999521	.999681	43
44	.993080	.995075	.996527	.997574	.998320	.998847	.999215	.999470	44
45	.989811	.992644	.994739	.996272	.997383	.998179	.998743	.999140	45
46	.985308	.899240	.992195	.994392	.996008	.997184	.998030	.998634	46
47	.979238	.984578	.988656	.991736	.994035	.995734	.996977	.997876	47
48	.971231	.978329	.983838	.988062	.991266	.993668	.995452	.996762	48
49	.960888	.970128	.977412	.983087	.987457	.990785	.993291	.995160	49
50	.947801	.959583	.969020	.976488	.982327	.986842	.990294	.992905	50
51	.931574	.946297	.958279	.967908	.975556	.981558	.986217	.989793	51
52	.911846	.929885	.944799	.956974	.966792	.974617	.980781	.985582	52
53	.888325	.910001	.928208	.943306	.955670	.965673	.973672	.979995	53
54	.860809	.886365	.908174	.926543	.941819	.954367	.964551	.972721	54
55	.829217	.858790	.884430	.906366	.924891	.940339	.953066	.963427	55
56	.793605	.827203	.856801	.882520	.904577	.923251	.938866	.951767	56
57	.754177	.791669	.825224	.854842	.880635	.902806	.921623	.937400	57
58	.711290	.752394	.789770	.823279	.852913	.878774	.901053	.920009	58
59	.665445	.709733	.750648	.787907	.821367	.851013	.876936	.899319	59
60	.617269	.664180	.708211	.748938	.786079	.819487	.849141	.875122	60
61	.567488	.616350	.662944	.706721	.747261	.784284	.817639	.847296	61
62	.516890	.566951	.615452	.661736	.705262	.745618	.782522	.815821	62
63	.466292	.516755	.566427	.614574	.660554	.703834	.744007	.780792	63
64	.416498	.466560	.516624	.565915	.613716	.659397	.702435	.742427	64
65	.368259	.417149	.466821	.516496	.565415	.612877	.658265	.701064	65
66	.322247	.369258	.417784	.467076	.516370	.564925	.612056	.657157	66
67	.279024	.323544	.370234	.418406	.467326	.516247	.564447	.611253	67
68	.239026	.280559	.324812	.371188	.419013	.467570	.516128	.563979	68
69	.202557	.240735	.282062	.326054	.372122	.419607	.467808	.516010	69
70	.169789	.204373	.242410	.283536	.327269	.373035	.420189	.468041	70
71	.140765	.171648	.206157	.244054	.284979	.328459	.373930	.420757	71
72	.115420	.142610	.173478	.207910	.245668	.286395	.329626	.374806	72
73	.093595	.117202	.144430	.175279	.209632	.247251	.287783	.330768	73
74	.075059	.095275	.118964	.146225	.177052	.211325	.248806	.289144	74
75	.059529	.076607	.096938	.120704	.147994	.178797	.212989	.250333	75
76	.046691	.060926	.078143	.098586	.122423	.149739	.180516	.214626	76

TABLE II

X	A=62	A=63	A=64	A=65	A=66	A=67	A=68	A=69	X
77	.036217	.047927	.062316	.079669	.100217	.124122	.151460	.182208	77
78	.027784	.037292	.049161	.063700	.081183	.101832	.125801	.153158	78
79	.021081	.028701	.038367	.050393	.065077	.082685	.103431	.127460	79
80	.015820	.021851	.029623	.039444	.051622	.066447	.084176	.105015	80
81	.011743	.016456	.022627	.030548	.040521	.052847	.067810	.085656	81
82	.008623	.012261	.017100	.023409	.031476	.041598	.054070	.069165	82
83	.006263	.009037	.012786	.017750	.024196	.032407	.042676	.055288	83
84	.004500	.006590	.009459	.013318	.018407	.024987	.033341	.043753	84
85	.003199	.004755	.006925	.009889	.013858	.019070	.025784	.034277	85
86	.002250	.003395	.005016	.007267	.010326	.014405	.019738	.026585	86
87	.001566	.002398	.003596	.005285	.007616	.010771	.014958	.020413	87
88	.001079	.001677	.002552	.003804	.005560	.007972	.011222	.015518	88
89	.000735	.001160	.001792	.002710	.004017	.005841	.008335	.011680	89
90	.000496	.000795	.001245	.001911	.002874	.004237	.006129	.008705	90
91	.000331	.000539	.000857	.001334	.002035	.003043	.004463	.006424	91
92	.000219	.000361	.000584	.000922	.001427	.002164	.003217	.004694	92
93	.000143	.000240	.000394	.000631	.000991	.001523	.002297	.003397	93
94	.000093	.000158	.000263	.000428	.000681	.001062	.001624	.002434	94
95	.000060	.000103	.000174	.000287	.000463	.000733	.001137	.001728	95
96	.000038	.000066	.000114	.000190	.000312	.000501	.000788	.001214	96
97	.000024	.000042	.000074	.000125	.000209	.000340	.000541	.000846	97
98	.000015	.000027	.000047	.000082	.000138	.000228	.000368	.000583	98
99	.000009	.000017	.000030	.000053	.000090	.000151	.000248	.000398	99
100	.000006	.000010	.000019	.000034	.000059	.000100	.000166	.000270	100
101	.000003	.000006	.000012	.000021	.000038	.000065	.000110	.000181	101
102	.000002	.000004	.000007	.000013	.000024	.000042	.000072	.000120	102
103	.000001	.000002	.000004	.000008	.000015	.000027	.000047	.000079	103
104	.000001	.000001	.000003	.000005	.000009	.000017	.000030	.000052	104
105		.000001	.000002	.000003	.000006	.000011	.000019	.000033	105
106			.000001	.000002	.000004	.000007	.000012	.000021	106
107			.000001	.000001	.000002	.000004	.000008	.000014	107
108				.000001	.000001	.000003	.000005	.000009	108
109					.000001	.000002	.000003	.000005	109
110						.000001	.000002	.000003	110
111						.000001	.000001	.000002	111
112							.000001	.000001	112
113								.000001	113

X	A=70	A=71	A=72	A=73	A=74	A=75	A=76	A=77	X
33	1.000000								33
34	.999999	1.000000	1.000000						34
35	.999999	.999999	.999999	1.000000					35
36	.999997	.999998	.999999	.999999	1.000000				36
37	.999994	.999997	.999998	.999999	.999999	1.000000	1.000000		37
38	.999989	.999993	.999996	.999998	.999999	.999999	.999999	1.000000	38
39	.999979	.999987	.999992	.999995	.999997	.999998	.999999	.999999	39
40	.999961	.999976	.999985	.999991	.999994	.999996	.999998	.999999	40
41	.999930	.999956	.999972	.999982	.999989	.999993	.999996	.999997	41
42	.999877	.999921	.999949	.999968	.999979	.999987	.999992	.999995	42
43	.999789	.999862	.999910	.999942	.999963	.999976	.999985	.999991	43
44	.999645	.999764	.999845	.999898	.999934	.999957	.999973	.999983	44
45	.999417	.999607	.999738	.999826	.999886	.999925	.999952	.999969	45
46	.999061	.999360	.999567	.999709	.999806	.999872	.999916	.999945	46
47	.998520	.998978	.999299	.999524	.999679	.999785	.999857	.999906	47
48	.997714	.998400	.998889	.999235	.999478	.999646	.999762	.999841	48
49	.996539	.997546	.998274	.998797	.999167	.999429	.999611	.999737	49
50	.994859	.996308	.997371	.998143	.998699	.999096	.999377	.999574	50
51	.992509	.994550	.996070	.997189	.998006	.998597	.999021	.999322	51
52	.989282	.992103	.994233	.995823	.997001	.997864	.998491	.998942	52
53	.984939	.988762	.991689	.993906	.995570	.006806	.997716	.998379	53
54	.979202	.984287	.988234	.991266	.993572	.995309	.996604	.997562	54
55	.971766	.978402	.983626	.987696	.990834	.993230	.995040	.996397	55
56	.962301	.970805	.977595	.982958	.987151	.990394	.992879	.994765	56
57	.950471	.961174	.969841	.976782	.982283	.986597	.989946	.992521	57
58	.935942	.949177	.960046	.968873	.975964	.981601	.986036	.989491	58
59	.918407	.934491	.947886	.958917	.967901	.975140	.980911	.985467	59
60	.897603	.916818	.933048	.946600	.957789	.966927	.974310	.980216	60

TABLE II

X	A=70	A=71	A=72	A=73	A=74	A=75	A=76	A=77	X
61	.873332	.895905	.915242	.931613	.945317	.956660	.965949	.973477	61
62	.845479	.871563	.894224	.913678	.930187	.944038	.955532	.964970	62
63	.814033	.843688	.869817	.892562	.912128	.928769	.942763	.954406	63
64	.779093	.812274	.841924	.868093	.890917	.910591	.927359	.941493	64
65	.740877	.777423	.810543	.840184	.866391	.889289	.909066	.925958	65
66	.699721	.739356	.775783	.808840	.838470	.864710	.887678	.907555	66
67	.656071	.698404	.737863	.774171	.807163	.836779	.863050	.886085	67
68	.610467	.655008	.697113	.736398	.772587	.805513	.835113	.861410	68
69	.563521	.609697	.653966	.695847	.734959	.771029	.803889	.833469	69
70	.515895	.563073	.608944	.652945	.694605	.733546	.769497	.802289	70
71	.468269	.515783	.562635	.608205	.651944	.693386	.732158	.767991	71
72	.421314	.468493	.515673	.562205	.607482	.650963	.692189	.730794	72
73	.375663	.421860	.468712	.515565	.561784	.606772	.650000	.691015	73
74	.331889	.376504	.422394	.468926	.515460	.561371	.606077	.649055	74
75	.290480	.332987	.377327	.422917	.469136	.515356	.560967	.605395	75
76	.251832	.291791	.334064	.378135	.423430	.469342	.515255	.560570	76
77	.216235	.253305	.293077	.335120	.378926	.423932	.469543	.515156	77
78	.183874	.217818	.254752	.294340	.336157	.379703	.424425	.469741	78
79	.154832	.185516	.219375	.256174	.295580	.337174	.380464	.424909	79
80	.129099	.156485	.187133	.220907	.257572	.296799	.338172	.381212	80
81	.106583	.130719	.158114	.188726	.222415	.258947	.297995	.339153	81
82	.087124	.108135	.132321	.159723	.190296	.223899	.260298	.299171	82
83	.070513	.088581	.109672	.133903	.161310	.191843	.225360	.261628	83
84	.056503	.071853	.090026	.111194	.135467	.162876	.193367	.226798	84
85	.044829	.057714	.073186	.091459	.112701	.137013	.164422	.194871	85
86	.035215	.045904	.058921	.074510	.092881	.114193	.138542	.165948	86
87	.027389	.036154	.046979	.060123	.075827	.094292	.115671	.140053	87
88	.021093	.028197	.037095	.048052	.061321	.077136	.095691	.117133	88
89	.016084	.021778	.029009	.038037	.049123	.062514	.078436	.097079	89
90	.012145	.016656	.022467	.029824	.038980	.050192	.063702	.079729	90
91	.009081	.012616	.017234	.023161	.030641	.039924	.051259	.064885	91
92	.006724	.009464	.013093	.017817	.023860	.031461	.040868	.052324	92
93	.004931	.007031	.009852	.013576	.018405	.024562	.032284	.041812	93
94	.003581	.005174	.007344	.010247	.014065	.018999	.025268	.033108	94
95	.002576	.003771	.005422	.007662	.010648	.014559	.019597	.025978	95
96	.001835	.002722	.003966	.005676	.007987	.011055	.015059	.020199	96
97	.001295	.001947	.002873	.004165	.005935	.008317	.011467	.015564	97
98	.000906	.001379	.002062	.003029	.004370	.006200	.008653	.011885	98
99	.000627	.000968	.001467	.002182	.003188	.004580	.006470	.008994	99
100	.000430	.000673	.001034	.001557	.002305	.003352	.004794	.006745	100
101	.000293	.000464	.000722	.001102	.001651	.002432	.003521	.005014	101
102	.000197	.000317	.000499	.000772	.001172	.001748	.002563	.003694	102
103	.000132	.000214	.000342	.000536	.000825	.001246	.001849	.002697	103
104	.000087	.000144	.000233	.000369	.000575	.000880	.001322	.001952	104
105	.000057	.000095	.000157	.000252	.000398	.000616	.000937	.001401	105
106	.000037	.000063	.000105	.000170	.000272	.000427	.000658	.000997	106
107	.000024	.000041	.000069	.000114	.000185	.000294	.000459	.000703	107
108	.000015	.000027	.000045	.000076	.000125	.000200	.000317	.000491	108
109	.000010	.000017	.000029	.000050	.000083	.000135	.000217	.000341	109
110	.000006	.000011	.000019	.000033	.000055	.000091	.000147	.000234	110
111	.000004	.000007	.000012	.000021	.000036	.000060	.000099	.000159	111
112	.000002	.000004	.000008	.000014	.000023	.000040	.000066	.000108	112
113	.000001	.000003	.000005	.000009	.000015	.000026	.000044	.000072	113
114	.000001	.000002	.000003	.000005	.000010	.000017	.000029	.000048	114
115	.000001	.000001	.000002	.000003	.000006	.000011	.000019	.000032	115
116		.000001	.000001	.000002	.000004	.000007	.000012	.000021	116
117			.000001	.000001	.000002	.000004	.000008	.000013	117
118				.000001	.000002	.000003	.000005	.000009	118
119					.000001	.000002	.000003	.000006	119
120					.000001	.000001	.000002	.000003	120
121						.000001	.000001	.000002	121
122							.000001	.000001	122
123								.000001	123
124								.000001	124

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X,
39	1.000000								39

TABLE II

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X
40	.999999	1.000000	1.000000						40
41	.999998	.999999	.999999	1.000000					41
42	.999997	.999998	.999999	.999999	1.000000	1.000000			42
43	.999994	.999996	.999998	.999999	.999999	.999999	1.000000		43
44	.999989	.999993	.999996	.999997	.999998	.999999	.999999	1.000000	44
45	.999980	.999987	.999992	.999995	.999997	.999998	.999999	.999999	45
46	.999965	.999977	.999985	.999991	.999994	.999996	.999998	.999999	46
47	.999938	.999960	.999974	.999983	.999989	.999993	.999996	.999997	47
48	.999894	.999931	.999955	.999970	.999981	.999988	.999992	.999995	48
49	.999823	.999882	.999922	.999949	.999967	.999978	.999986	.999991	49
50	.999710	.999805	.999869	.999913	.999943	.999962	.999975	.999984	50
51	.999534	.999682	.999785	.999855	.999903	.999936	.999958	.999972	51
52	.999264	.999492	.999652	.999763	.999840	.999893	.999928	.999953	52
53	.998859	.999203	.999447	.999620	.999740	.999824	.999881	.999920	53
54	.998263	.998772	.999139	.999400	.999585	.999715	.999806	.999869	54
55	.997403	.998142	.998682	.999071	.999351	.999549	.999689	.999787	55
56	.996182	.997238	.998017	.998587	.999001	.999299	.999511	.999662	56
57	.994482	.995961	.997067	.997887	.998488	.998927	.999244	.999471	57
58	.992156	.994192	.995734	.996891	.997751	.998386	.998850	.999186	58
59	.989027	.991783	.993896	.995501	.996709	.997612	.998279	.998769	59
60	.984891	.988556	.991402	.993592	.995261	.996522	.997467	.998169	60
61	.979514	.984308	.988078	.991015	.993282	.995015	.996330	.997318	61
62	.972639	.978807	.983719	.987594	.990621	.992965	.994764	.996132	62
63	.963989	.971797	.978094	.983124	.987102	.990220	.992642	.994506	63
64	.953280	.963006	.970951	.977376	.982522	.986604	.989813	.992313	64
65	.940228	.952156	.962023	.970102	.976654	.981915	.986100	.989400	65
66	.924566	.938968	.951033	.961038	.969250	.975926	.981302	.985590	66
67	.906056	.923183	.937713	.949913	.960052	.968396	.975195	.980684	67
68	.884508	.904571	.921809	.936464	.948795	.959066	.967539	.974460	68
69	.859790	.882947	.903098	.920444	.935220	.947679	.958081	.966679	69
70	.831849	.858190	.881403	.901637	.919088	.933982	.946567	.957095	70
71	.800714	.830250	.856610	.879876	.900190	.917741	.932750	.945457	71
72	.766509	.799162	.828674	.855049	.878364	.898755	.916403	.931523	72
73	.729454	.765051	.797634	.827119	.853506	.876868	.897332	.915074	73
74	.689861	.728137	.763617	.796128	.825584	.851982	.875387	.895922	74
75	.648128	.688729	.726842	.762205	.794644	.824071	.850477	.873922	75
76	.604726	.647218	.687616	.725569	.760816	.793182	.822577	.848989	76
77	.560181	.604069	.646325	.686522	.724317	.759448	.791741	.821103	77
78	.515058	.559800	.603425	.645447	.685448	.723085	.758101	.790320	78
79	.469935	.514963	.559425	.602793	.644586	.684392	.721874	.756775	79
80	.425383	.470125	.514869	.559058	.602172	.643739	.683353	.720681	80
81	.381945	.425849	.470312	.514777	.558697	.601562	.642907	.682332	81
82	.340116	.382665	.426305	.470495	.514686	.558343	.600963	.642089	82
83	.300327	.341062	.383372	.426754	.470675	.514597	.557995	.600374	83
84	.262935	.301463	.341991	.384067	.427194	.470852	.514510	.557654	84
85	.228214	.264221	.302580	.342904	.384749	.427627	.471025	.514425	85
86	.196352	.229609	.265487	.303678	.343801	.385419	.428052	.471196	86
87	.167454	.197813	.230982	.266733	.304758	.344684	.386078	.428469	87
88	.141546	.168942	.199254	.232336	.267959	.305821	.345551	.386725	88
89	.118582	.143023	.170410	.200675	.233669	.269167	.306867	.346404	89
90	.098456	.120016	.144483	.171860	.202076	.234983	.270355	.307896	90
91	.081013	.099821	.121436	.145926	.173291	.203459	.236278	.271526	91
92	.066063	.082290	.101175	.122842	.147353	.174705	.204822	.237555	92
93	.053387	.067235	.083558	.102519	.124235	.148765	.176102	.206168	93
94	.042756	.054447	.068402	.084817	.103851	.125614	.150161	.177481	94
95	.033934	.043700	.055504	.069564	.086069	.105172	.126979	.151541	95
96	.026691	.034762	.044643	.056559	.070720	.087312	.106482	.128332	96
97	.020806	.027407	.035591	.045585	.057610	.071871	.088547	.107781	97
98	.016074	.021418	.028126	.036422	.046527	.058658	.073015	.089773	98
99	.012308	.016589	.022033	.028848	.037253	.047468	.059703	.074154	99
100	.009340	.012736	.017108	.022652	.029572	.038086	.048407	.060744	100
101	.007025	.009692	.013169	.017632	.023274	.030299	.038919	.049345	101
102	.005238	.007311	.010048	.013607	.018160	.023900	.031028	.039752	102
103	.003871	.005467	.007601	.010410	.014049	.018693	.024529	.031758	103
104	.002836	.004052	.005700	.007896	.010777	.014497	.019229	.025161	104
105	.002059	.002978	.004238	.005938	.008196	.011148	.014948	.019769	105
106	.001483	.002170	.003124	.004428	.006181	.008501	.011524	.015404	106
107	.001058	.001567	.002283	.003274	.004622	.006428	.008810	.011904	107

TABLE II

X	A=78	A=79	A=80	A=81	A=82	A=83	A=84	A=85	X
108	.000749	.001122	.001655	.002400	.003427	.004820	.006679	.009124	108
109	.000525	.000797	.001189	.001745	.002520	.003585	.005022	.006935	109
110	.000366	.000561	.000847	.001258	.001838	.002644	.003746	.005229	110
111	.000252	.000392	.000599	.000899	.001329	.001934	.002770	.003910	111
112	.000173	.000271	.000420	.000638	.000953	.001403	.002032	.002900	112
113	.000117	.000186	.000292	.000448	.000678	.001009	.001479	.002134	113
114	.000079	.000127	.000201	.000313	.000479	.000720	.001068	.001557	114
115	.000053	.000086	.000137	.000216	.000335	.000510	.000764	.001128	115
116	.000035	.000057	.000093	.000148	.000233	.000358	.000543	.000810	116
117	.000023	.000038	.000063	.000101	.000160	.000250	.000383	.000577	117
118	.000015	.000025	.000042	.000068	.000109	.000173	.000267	.000408	118
119	.000010	.000016	.000028	.000046	.000074	.000118	.000186	.000286	119
120	.000006	.000011	.000018	.000030	.000050	.000080	.000128	.000199	120
121	.000004	.000007	.000012	.000020	.000033	.000054	.000087	.000138	121
122	.000002	.000004	.000008	.000013	.000022	.000036	.000059	.000094	122
123	.000002	.000003	.000005	.000009	.000014	.000024	.000040	.000064	123
124	.000001	.000002	.000003	.000005	.000009	.000016	.000027	.000043	124
125	.000001	.000001	.000002	.000004	.000006	.000010	.000018	.000029	125
126		.000001	.000001	.000002	.000004	.000007	.000012	.000019	126
127			.000001	.000001	.000002	.000004	.000008	.000013	127
128				.000001	.000002	.000003	.000005	.000008	128
129				.000001	.000001	.000002	.000003	.000005	129
130					.000001	.000001	.000002	.000003	130
131						.000001	.000001	.000002	131
132							.000001	.000001	132
133								.000001	133
134								.000001	134

X	A=86	A=87	A=88	A=89	A=90	A=91	A=92	A=93	X
44	1.000000								44
45	.999999	1.000000							45
46	.999999	.999999	1.000000						46
47	.999998	.999999	.999999	1.000000	1.000000				47
48	.999997	.999998	.999999	.999999	.999999	1.000000			48
49	.999994	.999996	.999998	.999999	.999999	.999999	1.000000		49
50	.999990	.999993	.999996	.999997	.999998	.999999	.999999	1.000000	50
51	.999982	.999988	.999992	.999995	.999997	.999998	.999999	.999999	51
52	.999969	.999980	.999987	.999991	.999994	.999996	.999998	.999999	52
53	.999947	.999965	.999977	.999985	.999990	.999994	.999996	.999997	53
54	.999912	.999941	.999961	.999974	.999983	.999989	.999993	.999995	54
55	.999855	.999902	.999934	.999956	.999971	.999981	.999988	.999992	55
56	.999767	.999841	.999892	.999927	.999951	.999968	.999979	.999986	56
57	.999632	.999746	.999826	.999882	.999920	.999946	.999964	.999976	57
58	.999428	.999601	.999724	.999810	.999870	.999912	.999940	.999960	58
59	.999126	.999384	.999569	.999700	.999793	.999858	.999903	.999934	59
60	.998686	.999063	.999337	.999534	.999675	.999774	.999845	.999894	60
61	.998054	.998598	.998998	.999288	.999498	.999648	.999755	.999831	61
62	.997164	.997935	.998508	.998929	.999237	.999460	.999620	.999734	62
63	.995929	.997005	.997812	.998414	.998858	.999183	.999420	.999590	63
64	.994243	.995720	.996841	.997686	.998316	.998783	.999127	.999378	64
65	.991977	.993973	.995506	.996673	.997555	.998215	.998706	.999069	65
66	.988980	.991636	.993698	.995287	.996501	.997420	.998111	.998626	66
67	.985074	.988554	.991288	.993418	.995063	.996323	.997281	.998003	67
68	.980061	.984553	.988123	.990935	.993132	.994834	.996141	.997138	68
69	.973720	.979433	.984026	.987685	.990576	.992841	.994600	.995955	69
70	.965818	.972978	.978801	.983493	.987243	.990212	.992544	.994361	70
71	.956109	.964955	.972232	.978164	.982956	.986794	.989842	.992242	71
72	.944349	.955125	.964091	.971483	.977523	.982414	.986341	.989467	72
73	.930303	.943246	.954140	.963225	.970732	.976879	.981868	.985883	73
74	.913755	.929089	.942145	.953157	.962359	.969978	.976230	.981317	74
75	.894524	.912445	.927881	.941049	.952175	.961491	.969221	.975578	75
76	.872472	.893138	.911144	.926679	.939955	.951195	.960624	.968463	76
77	.847518	.871037	.891765	.909852	.925484	.938866	.950216	.959755	77
78	.819648	.846065	.869617	.890403	.908570	.924295	.937781	.949238	78
79	.788920	.818212	.844629	.868211	.889053	.907297	.923113	.936699	79
80	.755468	.787539	.816795	.843209	.866819	.887716	.906033	.921938	80

TABLE II

X	A=86	A=87	A=88	A=89	A=90	A=91	A=92	A=93	X
81	.719508	.754181	.786177	.815395	.841806	.865442	.886390	.904777	81
82	.681328	.718353	.752914	.784884	.814014	.840419	.864079	.885075	82
83	.641285	.680340	.717216	.751664	.783510	.812650	.839048	.862730	83
84	.599796	.640495	.679368	.716097	.750433	.782203	.811302	.837692	84
85	.557318	.599227	.639718	.678412	.714994	.749220	.780914	.809972	85
86	.514341	.556988	.598668	.638953	.677470	.713908	.748024	.779643	86
87	.471363	.514258	.556664	.598119	.638201	.676544	.712839	.746845	87
88	.428880	.471528	.514177	.556345	.597578	.637461	.675632	.711785	88
89	.387362	.429283	.471690	.514097	.556032	.597046	.636733	.674733	89
90	.347244	.387988	.429680	.471849	.514018	.555723	.596523	.636016	90
91	.308908	.348069	.388604	.430070	.472005	.513941	.555420	.596008	91
92	.272679	.309905	.348882	.389209	.430453	.472159	.513865	.555122	92
93	.238813	.273815	.310887	.349681	.389805	.430831	.472310	.513790	93
94	.207496	.240054	.274934	.311853	.350468	.390391	.431202	.472459	94
95	.178844	.208806	.241277	.276037	.312805	.351243	.390968	.431567	95
96	.152906	.180190	.210099	.242483	.277124	.313742	.352006	.391536	96
97	.129671	.154257	.181520	.211376	.243673	.278195	.314666	.352757	97
98	.109070	.130997	.155593	.182834	.212636	.244847	.27[illegible]251	.315576	98
99	.090992	.110348	.132311	.156914	.184133	.213880	.246005	.280292	99
100	.075288	.092202	.111616	.133612	.158221	.185416	.215109	.247147	100
101	.061782	.076415	.093404	.112873	.134900	.159514	.186685	.216322	101
102	.050282	.062816	.077536	.094598	.114119	.136176	.160794	.187939	102
103	.040586	.051217	.063847	.078652	.095783	.115356	.137441	.162059	103
104	.032490	.041420	.052151	.064873	.079761	.096960	.116582	.138693	104
105	.025796	.033224	.042254	.053082	.065896	.080865	.098130	.117798	105
106	.020313	.026434	.033960	.043088	.054012	.066915	.081962	.099291	106
107	.015864	.020860	.027074	.034696	.043921	.054939	.067930	.083053	107
108	.012289	.016328	.021411	.027716	.035434	.044755	.055865	.068941	108
109	.009442	.012678	.016796	.021964	.028361	.036173	.045587	.056788	109
110	.007195	.009764	.013071	.017268	.022521	.029008	.036912	.046419	110
111	.005439	.007460	.010091	.013468	.017743	.023081	.029657	.037653	111
112	.004078	.005653	.007728	.010421	.013869	.018222	.023644	.030308	112
113	.003033	.004250	.005871	.008000	.010756	.014274	.018704	.024209	113
114	.002238	.003170	.004426	.006093	.008277	.011095	.014683	.019190	114
115	.001638	.002346	.003310	.004605	.006319	.008557	.011437	.015095	115
116	.001190	.001722	.002456	.003452	.004787	.006549	.008841	.011784	116
117	.000857	.001254	.001808	.002568	.003598	.004973	.006782	.009129	117
118	.000613	.000906	.001320	.001896	.002684	.003748	.005163	.007018	118
119	.000435	.000650	.000957	.001389	.001987	.002803	.003900	.005355	119
120	.000306	.000462	.000688	.001010	.001459	.002080	.002924	.004056	120
121	.000214	.000326	.000491	.000728	.001064	.001532	.002176	.003048	121
122	.000148	.000229	.000348	.000521	.000770	.001120	.001607	.002274	122
123	.000102	.000159	.000245	.000370	.000553	.000813	.001178	.001684	123
124	.000069	.000110	.000171	.000261	.000394	.000585	.000857	.001238	124
125	.000047	.000075	.000118	.000183	.000279	.000418	.000619	.000903	125
126	.000032	.000051	.000081	.000127	.000196	.000297	.000444	.000654	126
127	.000021	.000034	.000055	.000088	.000136	.000209	.000316	.000470	127
128	.000014	.000023	.000037	.000060	.000094	.000146	.000223	.000335	128
129	.000009	.000015	.000025	.000041	.000065	.000101	.000156	.000238	129
130	.000006	.000010	.000017	.000027	.000044	.000070	.000109	.000167	130
131	.000004	.000007	.000011	.000018	.000030	.000048	.000075	.000117	131
132	.000002	.000004	.000007	.000012	.000020	.000032	.000052	.000081	132
133	.000002	.000003	.000005	.000008	.000013	.000022	.000035	.000056	133
134	.000001	.000002	.000003	.000005	.000009	.000015	.000024	.000038	134
135	.000001	.000001	.000002	.000003	.000006	.000010	.000016	.000026	135
136		.000001	.000001	.000002	.000004	.000006	.000011	.000017	136
137			.000001	.000001	.000002	.000004	.000007	.000012	137
138			.000001	.000001	.000002	.000003	.000005	.000008	138
139				.000001	.000001	.000002	.000003	.000005	139
140					.000001	.000001	.000002	.000003	140
141						.000001	.000001	.000002	141
142							.000001	.000001	142
143							.000001	.000001	143
144								.000001	144

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
51	1.000000	1.000000							51

TABLE II

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
52	.999999	.999999	1.000000						52
53	.999998	.999999	.999999	1.000000					53
54	.999997	.999998	.999999	.999999	1.000000	1.000000			54
55	.999995	.999997	.999998	.999999	.999999	.999999	1.000000		55
56	.999991	.999994	.999996	.999998	.999998	.999999	.999999	1.000000	56
57	.999984	.999990	.999993	.999996	.999997	.999998	.999999	.999999	57
58	.999973	.999982	.999988	.999992	.999995	.999997	.999998	.999999	58
59	.999956	.999970	.999980	.999987	.999991	.999994	.999996	.999998	59
60	.999928	.999951	.999967	.999978	.999985	.999990	.999994	.999996	60
61	.999884	.999920	.999946	.999964	.999976	.999984	.999989	.999993	61
62	.999816	.999873	.999913	.999941	.999960	.999973	.999982	.999988	62
63	.999713	.999800	.999861	.999905	.999935	.999956	.999970	.999980	63
64	.999559	.999690	.999783	.999849	.999896	.999929	.999951	.999967	64
65	.999334	.999527	.999666	.999766	.999837	.999887	.999922	.999947	65
66	.999008	.999288	.999492	.999640	.999747	.999823	.999877	.999915	66
67	.998543	.998945	.999240	.999457	.999614	.999727	.999809	.999866	67
68	.997892	.998458	.998879	.999190	.999419	.999586	.999707	.999793	68
69	.996991	.997777	.998369	.998811	.999139	.999380	.999557	.999685	69
70	.995764	.996840	.997659	.998277	.998740	.999085	.999339	.999526	70
71	.994117	.995569	.996685	.997537	.998182	.998667	.999029	.999297	71
72	.991935	.993868	.995369	.996527	.997412	.998084	.998591	.998971	72
73	.989087	.991623	.993614	.995165	.996364	.997283	.997984	.998513	73
74	.985420	.988702	.991306	.993356	.994957	.996197	.997151	.997880	74
75	.980762	.984952	.988312	.990985	.993093	.994744	.996027	.997016	75
76	.974923	.980202	.984480	.987918	.990658	.992826	.994527	.995853	76
77	.967702	.974265	.979639	.984003	.987519	.990327	.992554	.994307	77
78	.958887	.966940	.973604	.979072	.983523	.987115	.989992	.992278	78
79	.948263	.958018	.966176	.972940	.978502	.983038	.986707	.989652	79
80	.935622	.947289	.957149	.965411	.972274	.977928	.982549	.986295	80
81	.920769	.934549	.946318	.956281	.964644	.971605	.977351	.982056	81
82	.903532	.919606	.933480	.945348	.955413	.963877	.970934	.976771	82
83	.883772	.902295	.918450	.932416	.944381	.954546	.963108	.970261	83
84	.861394	.882480	.901067	.917301	.931356	.943417	.953679	.962339	84
85	.836352	.860071	.881200	.899848	.916159	.930300	.942455	.952814	85
86	.808658	.835026	.858762	.879931	.898638	.915023	.929250	.941496	86
87	.778388	.807360	.833715	.857466	.878673	.897437	.913895	.928204	87
88	.745682	.777149	.806077	.832419	.856183	.877425	.896245	.912773	88
89	.710747	.744536	.775927	.804810	.831137	.854912	.876189	.895062	89
90	.673849	.709723	.743405	.774720	.803559	.829869	.853654	.874963	90
91	.635310	.672977	.708715	.742289	.773529	.802322	.828615	.852408	91
92	.595502	.634616	.672119	.707721	.741189	.772353	.801100	.827374	92
93	.554828	.595003	.633931	.671273	.706740	.740104	.771192	.799892	93
94	.513717	.554539	.594512	.633258	.670439	.705774	.739033	.770045	94
95	.472605	.513644	.554255	.594029	.632594	.669618	.704821	.737976	95
96	.431927	.472750	.513573	.553975	.593553	.631940	.668808	.703881	96
97	.392096	.432281	.472892	.513503	.553699	.593084	.631295	.668010	97
98	.353497	.392647	.432629	.473031	.513434	.553427	.592622	.630661	98
99	.316473	.354225	.393189	.432973	.473169	.513366	.553160	.592167	99
100	.281319	.317357	.354944	.393723	.433311	.473304	.513299	.552897	100
101	.248275	.282332	.318228	.355651	.394249	.433643	.473438	.513233	101
102	.217520	.249387	.283330	.319087	.356349	.394768	.433971	.473570	102
103	.189178	.218703	.250485	.284315	.319934	.357036	.395279	.434295	103
104	.163312	.190403	.219872	.251569	.285287	.320769	.357713	.395783	104
105	.139933	.164552	.191614	.221027	.252639	.286246	.321593	.358382	105
106	.119004	.141162	.165778	.192812	.222168	.253696	.287192	.322405	106
107	.100444	.120200	.142380	.166992	.193996	.223295	.254739	.288126	107
108	.084139	.101589	.121387	.143586	.168194	.195167	.224408	.255769	108
109	.069947	.085218	.102726	.122563	.144781	.169383	.196325	.225509	109
110	.057709	.070950	.086291	.103855	.123730	.145964	.170560	.197470	110
111	.047250	.058627	.071948	.087358	.104976	.124888	.147137	.171725	111
112	.038394	.048081	.059543	.072942	.088419	.106090	.126036	.148300	112
113	.030960	.039135	.048910	.060456	.073931	.089474	.107195	.127175	113
114	.024777	.031614	.039877	.049738	.061366	.074916	.090522	.108294	114
115	.019678	.025347	.032270	.040619	.050565	.062274	.075897	.091565	115
116	.015511	.020170	.025920	.032927	.041361	.051391	.063179	.076874	116
117	.012134	.015930	.020664	.026494	.033585	.042103	.052215	.064082	117
118	.009420	.012487	.016352	.021162	.027071	.034244	.042845	.053039	118
119	.007259	.009716	.012844	.016778	.021662	.027650	.034904	.043587	119

TABLE II

X	A=94	A=95	A=96	A=97	A=98	A=99	A=100	A=101	X
120	.005552	.007503	.010014	.013205	.017207	.022164	.028230	.035565	120
121	.004214	.005751	.007750	.010317	.013569	.017639	.022669	.028813	121
122	.003175	.004376	.005954	.008001	.010622	.013936	.018073	.023177	122
123	.002375	.003305	.004541	.006160	.008255	.010931	.014306	.018511	123
124	.001763	.002478	.003437	.004708	.006369	.008513	.011244	.014680	124
125	.001299	.001844	.002583	.003573	.004879	.006582	.008774	.011559	125
126	.000950	.001362	.001927	.002691	.003711	.005053	.006798	.009038	126
127	.000690	.000999	.001428	.002013	.002802	.003851	.005229	.007017	127
128	.000498	.000728	.001050	.001495	.002100	.002915	.003995	.005409	128
129	.000356	.000526	.000766	.001102	.001563	.002190	.003030	.004141	129
130	.000253	.000378	.000556	.000807	.001156	.001634		.003148	130
131	.000179	.000269	.000400	.000586	.000848	.001211	.001707	.002377	131
132	.000125	.000190	.000286	.000423	.000618	.000891	.001268	.001782	132
133	.000087	.000134	.000203	.000303	.000447	.000651	.000935	.001327	133
134	.000060	.000093	.000143	.000216	.000321	.000472	.000685	.000981	134
135	.000041	.000065	.000100	.000153	.000229	.000340	.000498	.000721	135
136	.000028	.000044	.000070	.000107	.000163	.000244	.000360	.000526	136
137	.000019	.000030	.000048	.000075	.000114	.000173	.000258	.000381	137
138	.000013	.000021	.000033	.000052	.000080	.000122	.000184	.000274	138
139	.000009	.000014	.000022	.000036	.000056	.000086	.000130	.000196	139
140	.000006	.000009	.000015	.000024	.000038	.000060	.000092	.000139	140
141	.000004	.000006	.000010	.000016	.000026	.000041	.000064	.000098	141
142	.000002	.000004	.000007	.000011	.000018	.000028	.000044	.000069	142
143	.000002	.000003	.000004	.000007	.000012	.000019	.000031	.000048	143
144	.000001	.000002	.000003	.000005	.000008	.000013	.000021	.000033	144
145	.000001	.000001	.000002	.000003	.000005	.000009	.000014	.000023	145
146		.000001	.000001	.000002	.000004	.000006	.000010	.000016	146
147			.000001	.000001	.000002	.000004	.000006	.000011	147
148			.000001	.000001	.000002	.000003	.000004	.000008	148
149				.000001	.000001	.000002	.000003	.000005	149
150					.000001	.000001	.000002	.000004	150
151						.000001	.000001	.000003	151
152							.000001	.000002	152
153							.000001	.000001	153
154								.000001	154

APPENDIX 4

TABLE OF THE BINOMIAL DISTRIBUTION FOR p = 1/10

This table gives the probability that *r or more* hits will be obtained in N trials if the probability of a hit on any one trial is 1/10. In using the table, we first find the column corresponding to N, and then we locate the row corresponding to r (the observed number of hits); at the intersection is the desired probability.

Interpreting the entry. Probabilities are given to three significant figures. The number from 0 to 10 which directly follows the three significant figures gives the number of zeros to be placed between the decimal point and the first significant figure.

Example of use of the table. The entry corresponding to N $=480$, $r=65$ is .781−2. This means that the probability of getting 65 or more hits in 480 trials with a probability of a hit on one trial equal to 1/10 is .00781. (If desired one can also interpret the entry as $.781\times10^{-2}$.)

P=1/10

R	N= *	N=0010	N=0020	N=0030	N=0040	N=0050	N=0060	N=0070	N=0080	N=0090	R
1		.651- 0	.878- 0	.958- 0	.985- 0	.995- 0	.998- 0	.999- 0			1
2		.264- 0	.608- 0	.816- 0	.920- 0	.966- 0	.986- 0	.994- 0	.998- 0	.999- 0	2
3		.702- 1	.323- 0	.589- 0	.777- 0	.888- 0	.947- 0	.976- 0	.989- 0	.995- 0	3
4		.128- 1	.133- 0	.353- 0	.577- 0	.750- 0	.863- 0	.929- 0	.965- 0	.983- 0	4
5		.163- 2	.432- 1	.175- 0	.371- 0	.569- 0	.729- 0	.841- 0	.912- 0	.954- 0	5
6		.147- 3	.113- 1	.732- 1	.206- 0	.384- 0	.563- 0	.713- 0	.823- 0	.897- 0	6
7		.912- 5	.239- 2	.258- 1	.995- 1	.230- 0	.394- 0	.558- 0	.700- 0	.808- 0	7
8		.374- 6	.416- 3	.778- 2	.419- 1	.122- 0	.248- 0	.401- 0	.554- 0	.689- 0	8
9		.901- 8	.599- 4	.202- 2	.155- 1	.579- 1	.142- 0	.264- 0	.407- 0	.551- 0	9
10		.100-10	.715- 5	.454- 3	.506- 2	.245- 1	.731- 1	.159- 0	.277- 0	.412- 0	10
11			.709- 6	.891- 4	.147- 2	.935- 2	.342- 1	.873- 1	.173- 0	.287- 0	11
12			.582- 7	.153- 4	.381- 3	.322- 2	.146- 1	.441- 1	.100- 0	.186- 0	12
13			.392- 8	.230- 5	.884- 4	.100- 2	.568- 2	.205- 1	.538- 1	.113- 0	13
14			.215- 9	.305- 6	.185- 4	.285- 3	.203- 2	.876- 2	.267- 1	.634- 1	14
15				.356- 7	.348- 5	.738- 4	.666- 3	.347- 2	.123- 1	.333- 1	15
16				.366- 8	.593- 6	.175- 4	.201- 3	.127- 2	.530- 2	.163- 1	16
17				.331- 9	.914- 7	.380- 5	.563- 4	.434- 3	.213- 2	.750- 2	17
18				.263-10	.128- 7	.760- 6	.146- 4	.138- 3	.796- 3	.323- 2	18
19					.162- 8	.140- 6	.350- 5	.407- 4	.279- 3	.131- 2	19
20					.187- 9	.237- 7	.782- 6	.113- 4	.920- 4	.498- 3	20
21					.196-10	.371- 8	.163- 6	.291- 5	.285- 4	.179- 3	21
22						.536- 9	.315- 7	.707- 6	.828- 5	.607- 4	22
23						.717-10	.571- 8	.161- 6	.227- 5	.194- 4	23
24							.965- 9	.345- 7	.588- 6	.589- 5	24
25							.153- 9	.696- 8	.144- 6	.169- 5	25
26							.226-10	.132- 8	.333- 7	.462- 6	26
27								.236- 9	.729- 8	.120- 6	27
28								.398-10	.151- 8	.294- 7	28
29									.298- 9	.689- 8	29
30									.556-10	.154- 8	30
31										.326- 9	31
32										.661-10	32
33										.128-10	33

R	N=0100	N=0110	N=0120	N=0130	N=0140	N=0150	N=0160	N=0170	N=0180	N=0190	R
3	.998- 0	.999- 0									3
4	.992- 0	.996- 0	.998- 0	.999- 0							4
5	.976- 0	.988- 0	.994- 0	.997- 0	.999- 0	.999- 0					5
6	.942- 0	.969- 0	.984- 0	.992- 0	.996- 0	.998- 0	.999- 0				6
7	.883- 0	.932- 0	.962- 0	.979- 0	.989- 0	.994- 0	.997- 0	.999- 0	.999- 0		7
8	.794- 0	.870- 0	.922- 0	.954- 0	.974- 0	.986- 0	.993- 0	.996- 0	.998- 0	.999- 0	8
9	.679- 0	.782- 0	.859- 0	.912- 0	.947- 0	.969- 0	.983- 0	.990- 0	.995- 0	.997- 0	9
10	.549- 0	.671- 0	.771- 0	.848- 0	.903- 0	.940- 0	.964- 0	.979- 0	.988- 0	.993- 0	10
11	.417- 0	.546- 0	.664- 0	.762- 0	.838- 0	.894- 0	.933- 0	.959- 0	.975- 0	.986- 0	11
12	.297- 0	.421- 0	.544- 0	.658- 0	.753- 0	.829- 0	.886- 0	.926- 0	.954- 0	.972- 0	12
13	.198- 0	.305- 0	.424- 0	.543- 0	.652- 0	.746- 0	.821- 0	.878- 0	.920- 0	.949- 0	13
14	.124- 0	.209- 0	.313- 0	.427- 0	.541- 0	.647- 0	.738- 0	.813- 0	.871- 0	.913- 0	14
15	.726- 1	.134- 0	.218- 0	.319- 0	.429- 0	.540- 0	.642- 0	.732- 0	.806- 0	.864- 0	15
16	.399- 1	.814- 1	.144- 0	.227- 0	.325- 0	.432- 0	.539- 0	.638- 0	.726- 0	.799- 0	16
17	.206- 1	.465- 1	.899- 1	.153- 0	.235- 0	.331- 0	.434- 0	.537- 0	.634- 0	.720- 0	17
18	.100- 1	.251- 1	.531- 1	.979- 1	.161- 0	.242- 0	.336- 0	.436- 0	.536- 0	.631- 0	18
19	.458- 2	.128- 1	.297- 1	.596- 1	.106- 0	.169- 0	.249- 0	.340- 0	.438- 0	.535- 0	19
20	.198- 2	.616- 2	.158- 1	.344- 1	.659- 1	.113- 0	.176- 0	.255- 0	.344- 0	.439- 0	20
21	.808- 3	.281- 2	.794- 2	.189- 1	.392- 1	.721- 1	.120- 0	.183- 0	.261- 0	.348- 0	21
22	.312- 3	.122- 2	.380- 2	.991- 2	.222- 1	.440- 1	.781- 1	.127- 0	.190- 0	.266- 0	22
23	.114- 3	.500- 3	.173- 2	.494- 2	.120- 1	.256- 1	.487- 1	.840- 1	.133- 0	.196- 0	23
24	.396- 4	.195- 3	.750- 3	.235- 2	.623- 2	.143- 1	.291- 1	.534- 1	.896- 1	.139- 0	24
25	.131- 4	.726- 4	.310- 3	.107- 2	.308- 2	.765- 2	.167- 1	.327- 1	.581- 1	.951- 1	25
26	.410- 5	.257- 4	.122- 3	.464- 3	.146- 2	.392- 2	.919- 2	.192- 1	.362- 1	.627- 1	26
27	.122- 5	.869- 5	.461- 4	.193- 3	.662- 3	.192- 2	.485- 2	.108- 1	.218- 1	.398- 1	27
28	.348- 6	.280- 5	.166- 4	.767- 4	.288- 3	.907- 3	.246- 2	.589- 2	.126- 1	.244- 1	28
29	.944- 7	.864- 6	.573- 5	.292- 4	.120- 3	.411- 3	.120- 2	.308- 2	.701- 2	.144- 1	29
30	.244- 7	.255- 6	.189- 5	.107- 4	.481- 4	.179- 3	.565- 3	.155- 2	.377- 2	.822- 2	30
31	.605- 8	.718- 7	.599- 6	.375- 5	.185- 4	.749- 4	.256- 3	.753- 3	.195- 2	.453- 2	31
32	.143- 8	.194- 7	.182- 6	.126- 5	.686- 5	.302- 4	.111- 3	.352- 3	.976- 3	.241- 2	32
33	.323- 9	.502- 8	.531- 7	.410- 6	.244- 5	.117- 4	.468- 4	.159- 3	.472- 3	.124- 2	33
34	.700-10	.125- 8	.149- 7	.128- 6	.839- 6	.439- 5	.190- 4	.695- 4	.220- 3	.615- 3	34
35	.145-10	.297- 9	.400- 8	.383- 7	.277- 6	.159- 5	.743- 5	.293- 4	.994- 4	.296- 3	35
36		.680-10	.104- 8	.111- 7	.884- 7	.553- 6	.281- 5	.119- 4	.434- 4	.138- 3	36
37		.149-10	.258- 9	.308- 8	.272- 7	.186- 6	.103- 5	.470- 5	.183- 4	.621- 4	37
38			.619-10	.828- 9	.808- 8	.605- 7	.363- 6	.179- 5	.751- 5	.272- 4	38
39			.143-10	.214- 9	.232- 8	.190- 7	.124- 6	.663- 6	.298- 5	.115- 4	39
40				.536-10	.641- 9	.578- 8	.410- 7	.237- 6	.114- 5	.473- 5	40
41				.129-10	.172- 9	.170- 8	.132- 7	.822- 7	.427- 6	.189- 5	41
42					.445-10	.485- 9	.409- 8	.276- 7	.154- 6	.730- 6	42
43					.111-10	.134- 9	.123- 8	.901- 8	.542- 7	.274- 6	43
44						.357-10	.359- 9	.285- 8	.185- 7	.100- 6	44
45							.102- 9	.876- 9	.612- 8	.356- 7	45
46							.279-10	.261- 9	.197- 8	.123- 7	46
47								.757-10	.616- 9	.413- 8	47
48								.213-10	.187- 9	.135- 8	48
49									.555-10	.429- 9	49

R	N=Ø100	N=Ø110	N=Ø120	N=Ø130	N=Ø140	N=Ø150	N=Ø160	N=Ø170	N=Ø180	N=Ø190	R	P=1/1Ø
5Ø									.16Ø-1Ø	.133- 9	5Ø	
51										.4Ø1-1Ø	51	
52										.118-1Ø	52	

R	N=Ø200	N=Ø210	N=Ø220	N=Ø230	N=Ø240	N=Ø250	N=Ø260	N=Ø270	N=Ø280	N=Ø290	R
9	.999- Ø	.999- Ø									9
1Ø	.996- Ø	.998- Ø	.999- Ø	.999- Ø							1Ø
11	.992- Ø	.996- Ø	.998- Ø	.999- Ø	.999- Ø						11
12	.983- Ø	.99Ø- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø					12
13	.968- Ø	.981- Ø	.989- Ø	.993- Ø	.996- Ø	.998- Ø	.999- Ø	.999- Ø			13
14	.943- Ø	.964- Ø	.978- Ø	.987- Ø	.992- Ø	.995- Ø	.997- Ø	.999- Ø	.999- Ø		14
15	.907- Ø	.938- Ø	.961- Ø	.975·· Ø	.985- Ø	.991- Ø	.994- Ø	.997- Ø	.998- Ø	.999- Ø	15
16	.857- Ø	.9Ø1- Ø	.934- Ø	.956- Ø	.972- Ø	.982- Ø	.989- Ø	.994- Ø	.996- Ø	.998- Ø	16
17	.793- Ø	.851- Ø	.896- Ø	.929- Ø	.953- Ø	.969- Ø	.98Ø- Ø	.988- Ø	.993- Ø	.996- Ø	17
18	.715- Ø	.787- Ø	.845- Ø	.89Ø- Ø	.924- Ø	.949- Ø	.966- Ø	.978- Ø	.986- Ø	.991- Ø	18
19	.628- Ø	.71Ø- Ø	.781- Ø	.839- Ø	.885- Ø	.919- Ø	.945- Ø	.963- Ø	.976- Ø	.985- Ø	19
2Ø	.534- Ø	.625- Ø	.7Ø6- Ø	.776- Ø	.833- Ø	.879- Ø	.915- Ø	.941- Ø	.96Ø- Ø	.974- Ø	2Ø
21	.441- Ø	.534- Ø	.622- Ø	.7Ø1- Ø	.771- Ø	.828- Ø	.874- Ø	.91Ø- Ø	.937- Ø	.957- Ø	21
22	.352- Ø	.442- Ø	.533- Ø	.619- Ø	.697- Ø	.766- Ø	.823- Ø	.87Ø- Ø	.9Ø6- Ø	.934- Ø	22
23	.271- Ø	.355- Ø	.444- Ø	.532- Ø	.617- Ø	.694- Ø	.761- Ø	.818- Ø	.865- Ø	.9Ø2- Ø	23
24	.2Ø2- Ø	.276- Ø	.359- Ø	.445- Ø	.531- Ø	.614- Ø	.69Ø- Ø	.757- Ø	.814- Ø	.86Ø- Ø	24
25	.145- Ø	.2Ø7- Ø	.281- Ø	.361- Ø	.446- Ø	.531- Ø	.612- Ø	.687- Ø	.753- Ø	.8Ø9- Ø	25
26	.1ØØ- Ø	.15Ø- Ø	.213- Ø	.284- Ø	.364- Ø	.447- Ø	.53Ø- Ø	.61Ø- Ø	.684- Ø	.749- Ø	26
27	.672- 1	.1Ø6- Ø	.156- Ø	.217- Ø	.288- Ø	.366- Ø	.448- Ø	.53Ø- Ø	.6Ø8- Ø	.681- Ø	27
28	.434- 1	.717- 1	.111- Ø	.161- Ø	.222- Ø	.292- Ø	.369- Ø	.449- Ø	.529- Ø	.6Ø6- Ø	28
29	.271- 1	.47Ø- 1	.766- 1	.115- Ø	.166- Ø	.226- Ø	.296- Ø	.371- Ø	.45Ø- Ø	.529- Ø	29
3Ø	.163- 1	.298- 1	.5Ø6- 1	.8Ø3- 1	.12Ø- Ø	.17Ø- Ø	.231- Ø	.299- Ø	.373- Ø	.451- Ø	3Ø
31	.951- 2	.183- 1	.326- 1	.541- 1	.845- 1	.125- Ø	.175- Ø	.235- Ø	.3Ø2- Ø	.376- Ø	31
32	.535- 2	.1Ø9- 1	.2Ø3- 1	.354- 1	.577- 1	.886- 1	.129- Ø	.179- Ø	.239- Ø	.3Ø5- Ø	32
33	.292- 2	.625- 2	.123- 1	.224- 1	.382- 1	.611- 1	.926- 1	.133- Ø	.183- Ø	.242- Ø	33
34	.154- 2	.348- 2	.72Ø- 2	.138- 1	.245- 1	.41Ø- 1	.646- 1	.965- 1	.137- Ø	.187- Ø	34
35	.784- 3	.188- 2	.4Ø9- 2	.822- 2	.153- 1	.267- 1	.438- 1	.68Ø- 1	.1ØØ- Ø	.141- Ø	35
36	.388- 3	.981- 3	.225- 2	.475- 2	.928- 2	.169- 1	.289- 1	.466- 1	.713- 1	.1Ø4- Ø	36
37	.186- 3	.497- 3	.12Ø- 2	.267- 2	.547- 2	.1Ø4- 1	.185- 1	.311- 1	.494- 1	.746- 1	37
38	.863- 4	.245- 3	.625- 3	.146- 2	.313- 2	.623- 2	.116- 1	.2Ø2- 1	.333- 1	.522- 1	38
39	.389- 4	.117- 3	.315- 3	.773- 3	.174- 2	.362- 2	.7Ø3- 2	.128- 1	.219- 1	.356- 1	39
4Ø	.17Ø- 4	.541- 4	.154- 3	.398- 3	.942- 3	.2Ø5- 2	.416- 2	.788- 2	.14Ø- 1	.236- 1	4Ø
41	.722- 5	.244- 4	.734- 4	.2ØØ- 3	.496- 3	.113- 2	.239- 2	.473- 2	.876- 2	.153- 1	41
42	.298- 5	.1Ø7- 4	.34Ø- 4	.974- 4	.254- 3	.6Ø7- 3	.134- 2	.277- 2	.533- 2	.969- 2	42
43	.119- 5	.454- 5	.153- 4	.462- 4	.127- 3	.317- 3	.734- 3	.158- 2	.317- 2	.597- 2	43
44	.465- 6	.188- 5	.669- 5	.213- 4	.615- 4	.162- 3	.391- 3	.877- 3	.183- 2	.36Ø- 2	44
45	.176- 6	.756- 6	.285- 5	.96Ø- 5	.291- 4	.8Ø3- 4	.2Ø3- 3	.476- 3	.1Ø4- 2	.211- 2	45
46	.65Ø- 7	.296- 6	.118- 5	.42Ø- 5	.134- 4	.388- 4	.1Ø3- 3	.252- 3	.572- 3	.121- 2	46
47	.233- 7	.113- 6	.479- 6	.18Ø- 5	.6Ø3- 5	.183- 4	.5Ø9- 4	.13Ø- 3	.3Ø8- 3	.679- 3	47
48	.814- 8	.42Ø- 7	.189- 6	.747- 6	.264- 5	.844- 5	.245- 4	.655- 4	.162- 3	.372- 3	48
49	.277- 8	.152- 7	.724- 7	.3Ø3- 6	.113- 5	.379- 5	.116- 4	.323- 4	.831- 4	.199- 3	49
5Ø	.918- 9	.537- 8	.271- 7	.12Ø- 6	.472- 6	.166- 5	.531- 5	.155- 4	.417- 4	.1Ø4- 3	5Ø
51	.296- 9	.185- 8	.991- 8	.464- 7	.192- 6	.712- 6	.239- 5	.729- 5	.2Ø5- 4	.531- 4	51
52	.933-1Ø	.62Ø- 9	.353- 8	.175- 7	.764- 7	.298- 6	.1Ø5- 5	.335- 5	.981- 5	.266- 4	52
53	.286-1Ø	.2Ø3- 9	.123- 8	.644- 8	.297- 7	.122- 6	.449- 6	.15Ø- 5	.46Ø- 5	.13Ø- 4	53
54		.649-1Ø	.417- 9	.232- 8	.113- 7	.486- 7	.188- 6	.66Ø- 6	.211- 5	.621- 5	54
55		.2Ø2-1Ø	.138- 9	.813- 9	.418- 8	.19Ø- 7	.772- 7	.283- 6	.947- 6	.291- 5	55
56			.447-1Ø	.279- 9	.151- 8	.725- 8	.3Ø9- 7	.119- 6	.416- 6	.133- 5	56
57			.141-1Ø	.936-1Ø	.537- 9	.271- 8	.121- 7	.489- 7	.179- 6	.597- 6	57
58				.3Ø7-1Ø	.186- 9	.988- 9	.466- 8	.197- 7	.753- 7	.262- 6	58
59					.63Ø-1Ø	.353- 9	.175- 8	.775- 8	.31Ø- 7	.113- 6	59
6Ø					.2Ø9-1Ø	.123- 9	.644- 9	.299- 8	.125- 7	.476- 7	6Ø
61						.423-1Ø	.232- 9	.113- 8	.495- 8	.197- 7	61
62						.142-1Ø	.817-1Ø	.418- 9	.192- 8	.797- 8	62
63							.282-1Ø	.152- 9	.73Ø- 9	.316- 8	63
64								.54Ø-1Ø	.272- 9	.123- 8	64
65								.188-1Ø	.992-1Ø	.47Ø- 9	65
66									.355-1Ø	.176- 9	66
67									.125-1Ø	.647-1Ø	67
68										.233-1Ø	68

R	N=Ø300	N=Ø310	N=Ø320	N=Ø330	N=Ø340	N=Ø350	N=Ø360	N=Ø370	N=Ø380	N=Ø390	R
15	.999- Ø										15
16	.999- Ø	.999- Ø									16
17	.997- Ø	.998- Ø	.999- Ø	.999- Ø							17
18	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø						18
19	.99Ø- Ø	.994- Ø	.996- Ø	.998- Ø	.999- Ø	.999- Ø					19
2Ø	.983- Ø	.989- Ø	.993- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø			2Ø
21	.971- Ø	.981- Ø	.988- Ø	.992- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø		21
22	.954- Ø	.969- Ø	.979- Ø	.987- Ø	.991- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	22
23	.93Ø- Ø	.951- Ø	.967- Ø	.978- Ø	.985- Ø	.99Ø- Ø	.994- Ø	.996- Ø	.998- Ø	.999- Ø	23
24	.898- Ø	.926- Ø	.948- Ø	.964- Ø	.976- Ø	.984- Ø	.989- Ø	.993- Ø	.996- Ø	.997- Ø	24
25	.856- Ø	.894- Ø	.923- Ø	.945- Ø	.962- Ø	.974- Ø	.982- Ø	.988- Ø	.992- Ø	.995- Ø	25
26	.8Ø5- Ø	.852- Ø	.89Ø- Ø	.92Ø- Ø	.942- Ø	.959- Ø	.972- Ø	.981- Ø	.987- Ø	.992- Ø	26

R	N=Ø300	N=Ø310	N=Ø320	N=Ø330	N=Ø340	N=Ø350	N=Ø360	N=Ø370	N=Ø380	N=Ø390	R	P=1/1Ø
27	.745- Ø	.801- Ø	.848- Ø	.886- Ø	.916- Ø	.939- Ø	.957- Ø	.97Ø- Ø	.98Ø- Ø	.986- Ø	27	
28	.678- Ø	.742- Ø	.797- Ø	.844- Ø	.882- Ø	.913- Ø	.937- Ø	.955- Ø	.968- Ø	.978- Ø	28	
29	.6Ø4- Ø	.675- Ø	.738- Ø	.793- Ø	.84Ø- Ø	.879- Ø	.91Ø- Ø	.934- Ø	.952- Ø	.966- Ø	29	
3Ø	.528- Ø	.6Ø3- Ø	.672- Ø	.735- Ø	.79Ø- Ø	.836- Ø	.875- Ø	.9Ø6- Ø	.931- Ø	.95Ø- Ø	3Ø	
31	.452- Ø	.528- Ø	.6Ø1- Ø	.67Ø- Ø	.732- Ø	.786- Ø	.833- Ø	.872- Ø	.9Ø3- Ø	.928- Ø	31	
32	.377- Ø	.452- Ø	.527- Ø	.6ØØ- Ø	.667- Ø	.729- Ø	.783- Ø	.829- Ø	.868- Ø	.9ØØ- Ø	32	
33	.3Ø8- Ø	.379- Ø	.453- Ø	.527- Ø	.598- Ø	.665- Ø	.726- Ø	.78Ø- Ø	.826- Ø	.865- Ø	33	
34	.246- Ø	.311- Ø	.381- Ø	.454- Ø	.526- Ø	.597- Ø	.663- Ø	.723- Ø	.776- Ø	.823- Ø	34	
35	.191- Ø	.249- Ø	.314- Ø	.383- Ø	.455- Ø	.526- Ø	.595- Ø	.661- Ø	.72Ø- Ø	.773- Ø	35	
36	.145- Ø	.195- Ø	.253- Ø	.316- Ø	.385- Ø	.455- Ø	.526- Ø	.594- Ø	.659- Ø	.718- Ø	36	
37	.1Ø8- Ø	.149- Ø	.199- Ø	.256- Ø	.319- Ø	.386- Ø	.456- Ø	.525- Ø	.593- Ø	.657- Ø	37	
38	.779- 1	.111- Ø	.153- Ø	.2Ø2- Ø	.259- Ø	.321- Ø	.388 Ø	.456- Ø	.525- Ø	.592- Ø	38	
39	.549- 1	.811- 1	.115- Ø	.156- Ø	.2Ø6- Ø	.262- Ø	.323- Ø	.389- Ø	.457- Ø	.525- Ø	39	
4Ø	.378- 1	.577- 1	.842- 1	.118- Ø	.16Ø- Ø	.2Ø9- Ø	.265- Ø	.326- Ø	.391- Ø	.457- Ø	4Ø	
41	.254- 1	.4Ø1- 1	.6Ø4- 1	.873- 1	.122- Ø	.163- Ø	.212- Ø	.267- Ø	.328- Ø	.392- Ø	41	
42	.166- 1	.272- 1	.423- 1	.631- 1	.9Ø4- 1	.125- Ø	.166- Ø	.215- Ø	.27Ø- Ø	.33Ø- Ø	42	
43	.1Ø6- 1	.18Ø- 1	.29Ø- 1	.446- 1	.658- 1	.934- 1	.128- Ø	.17Ø- Ø	.218- Ø	.272- Ø	43	
44	.665- 2	.116- 1	.194- 1	.3Ø8- 1	.468- 1	.684- 1	.963- 1	.131- Ø	.173- Ø	.221- Ø	44	
45	.4Ø5- 2	.735- 2	.127- 1	.2Ø8- 1	.326- 1	.491- 1	.71Ø- 1	.992- 1	.134- Ø	.176- Ø	45	
46	.242- 2	.454- 2	.8Ø9- 2	.137- 1	.222- 1	.344- 1	.513- 1	.736- 1	.1Ø2- Ø	.137- Ø	46	
47	.141- 2	.274- 2	.5Ø6- 2	.886- 2	.148- 1	.236- 1	.363- 1	.535- 1	.762- 1	.1Ø5- Ø	47	
48	.8ØØ- 3	.162- 2	.3Ø9- 2	.56Ø- 2	.965- 2	.159- 1	.251- 1	.381- 1	.557- 1	.787- 1	48	
49	.444- 3	.933- 3	.185- 2	.346- 2	.616- 2	.1Ø5- 1	.17Ø- 1	.266- 1	.399- 1	.579- 1	49	
5Ø	.242- 3	.526- 3	.1Ø8- 2	.2Ø9- 2	.385- 2	.675- 2	.113- 1	.182- 1	.281- 1	.418- 1	5Ø	
51	.128- 3	.29Ø- 3	.617- 3	.124- 2	.236- 2	.426- 2	.736- 2	.122- 1	.193- 1	.296- 1	51	
52	.668- 4	.157- 3	.345- 3	.717- 3	.141- 2	.264- 2	.47Ø- 2	.8ØØ- 2	.131- 1	.2Ø5- 1	52	
53	.34Ø- 4	.828- 4	.189- 3	.4Ø7- 3	.828- 3	.16Ø- 2	.294- 2	.515- 2	.866- 2	.14Ø- 1	53	
54	.169- 4	.428- 4	.1Ø2- 3	.226- 3	.476- 3	.949- 3	.18Ø- 2	.325- 2	.563- 2	.934- 2	54	
55	.824- 5	.217- 4	.534- 4	.123- 3	.268- 3	.552- 3	.1Ø8- 2	.2Ø1- 2	.359- 2	.612- 2	55	
56	.393- 5	.1Ø8- 4	.275- 4	.657- 4	.148- 3	.315- 3	.636- 3	.122- 2	.224- 2	.394- 2	56	
57	.184- 5	.523- 5	.139- 4	.344- 4	.8Ø1- 4	.176- 3	.367- 3	.728- 3	.138- 2	.249- 2	57	
58	.841- 6	.249- 5	.685- 5	.176- 4	.425- 4	.967- 4	.2Ø8- 3	.425- 3	.828- 3	.154- 2	58	
59	.377- 6	.116- 5	.332- 5	.885- 5	.221- 4	.52Ø- 4	.116- 3	.244- 3	.489- 3	.936- 3	59	
6Ø	.166- 6	.531- 6	.158- 5	.436- 5	.113- 4	.274- 4	.63Ø- 4	.137- 3	.284- 3	.559- 3	6Ø	
61	.713- 7	.238- 6	.734- 6	.211- 5	.565- 5	.142- 4	.337- 4	.757- 4	.161- 3	.328- 3	61	
62	.3Ø1- 7	.1Ø5- 6	.335- 6	.998- 6	.277- 5	.723- 5	.177- 4	.411- 4	.9Ø2- 4	.189- 3	62	
63	.125- 7	.451- 7	.15Ø- 6	.464- 6	.134- 5	.36Ø- 5	.914- 5	.219- 4	.495- 4	.1Ø7- 3	63	
64	.5Ø7- 8	.191- 7	.661- 7	.212- 6	.633- 6	.177- 5	.463- 5	.114- 4	.267- 4	.593- 4	64	
65	.2Ø2- 8	.792- 8	.285- 7	.95Ø- 7	.294- 6	.849- 6	.23Ø- 5	.587- 5	.142- 4	.324- 4	65	
66	.79Ø- 9	.323- 8	.121- 7	.418- 7	.134- 6	.4Ø1- 6	.112- 5	.296- 5	.737- 5	.174- 4	66	
67	.3Ø3- 9	.129- 8	.5Ø2- 8	.18Ø- 7	.6ØØ- 7	.186- 6	.539- 6	.147- 5	.377- 5	.916- 5	67	
68	.114- 9	.5Ø6- 9	.2Ø5- 8	.765- 8	.264- 7	.848- 7	.254- 6	.715- 6	.19Ø- 5	.475- 5	68	
69	.422-1Ø	.195- 9	.823- 9	.319- 8	.114- 7	.38Ø- 7	.118- 6	.343- 6	.937- 6	.242- 5	69	
7Ø	.153-1Ø	.738-1Ø	.324- 9	.131- 8	.485- 8	.167- 7	.537- 7	.161- 6	.455- 6	.121- 5	7Ø	
71		.274-1Ø	.125- 9	.525- 9	.2Ø2- 8	.723- 8	.24Ø- 7	.746- 7	.218- 6	.598- 6	71	
72		.1ØØ-1Ø	.477-1Ø	.2Ø8- 9	.831- 9	.3Ø7- 8	.1Ø6- 7	.34Ø- 7	.1Ø2- 6	.29Ø- 6	72	
73			.178-1Ø	.8Ø6-1Ø	.335- 9	.129- 8	.458- 8	.152- 7	.473- 7	.138- 6	73	
74				.3Ø8-1Ø	.133- 9	.529- 9	.195- 8	.67Ø- 8	.215- 7	.649- 7	74	
75				.116-1Ø	.518-1Ø	.214- 9	.816- 9	.29Ø- 8	.963- 8	.3ØØ- 7	75	
76					.199-1Ø	.85Ø-1Ø	.336- 9	.124- 8	.424- 8	.136- 7	76	
77						.333-1Ø	.136- 9	.519- 9	.184- 8	.61Ø- 8	77	
78						.128-1Ø	.544-1Ø	.214- 9	.784- 9	.269- 8	78	
79							.214-1Ø	.87Ø-1Ø	.329- 9	.117- 8	79	
8Ø								.348-1Ø	.136- 9	.498- 9	8Ø	
81								.137-1Ø	.555-1Ø	.2Ø9- 9	81	
82									.223-1Ø	.867-1Ø	82	
83										.354-1Ø	83	
84										.142-1Ø	84	

R	N=Ø4ØØ	N=Ø41Ø	N=Ø42Ø	N=Ø43Ø	N=Ø44Ø	N=Ø45Ø	N=Ø46Ø	N=Ø47Ø	N=Ø48Ø	N=Ø49Ø	R
23	.999- Ø	.999- Ø									23
24	.998- Ø	.999- Ø	.999- Ø								24
25	.997- Ø	.998- Ø	.999- Ø	.999- Ø							25
26	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					26
27	.991- Ø	.994- Ø	.996- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø				27
28	.985- Ø	.99Ø- Ø	.993- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø			28
29	.977- Ø	.984- Ø	.989- Ø	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø		29
3Ø	.964- Ø	.975- Ø	.983- Ø	.988- Ø	.992- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	3Ø
31	.948- Ø	.962- Ø	.973- Ø	.982- Ø	.987- Ø	.991- Ø	.994- Ø	.996- Ø	.998- Ø	.998- Ø	31
32	.925- Ø	.945- Ø	.96Ø- Ø	.972- Ø	.98Ø- Ø	.986- Ø	.991- Ø	.994- Ø	.996- Ø	.997- Ø	32
33	.897- Ø	.923- Ø	.943- Ø	.959- Ø	.97Ø- Ø	.979- Ø	.985- Ø	.99Ø- Ø	.993- Ø	.995- Ø	33
34	.862- Ø	.894- Ø	.92Ø- Ø	.941- Ø	.957- Ø	.969- Ø	.978- Ø	.984- Ø	.989- Ø	.993- Ø	34
35	.82Ø- Ø	.859- Ø	.891- Ø	.917- Ø	.938- Ø	.955- Ø	.967- Ø	.976- Ø	.983- Ø	.988- Ø	35
36	.77Ø- Ø	.816- Ø	.856- Ø	.888- Ø	.915- Ø	.936- Ø	.953- Ø	.965- Ø	.975- Ø	.982- Ø	36
37	.715- Ø	.768- Ø	.813- Ø	.853- Ø	.885- Ø	.912- Ø	.934- Ø	.951- Ø	.964- Ø	.974- Ø	37
38	.655- Ø	.713- Ø	.765- Ø	.81Ø- Ø	.85Ø- Ø	.883- Ø	.91Ø- Ø	.932- Ø	.949- Ø	.962- Ø	38
39	.591- Ø	.653- Ø	.71Ø- Ø	.762- Ø	.8Ø8- Ø	.847- Ø	.88Ø- Ø	.9Ø7- Ø	.929- Ø	.947- Ø	39
4Ø	.524- Ø	.589- Ø	.651- Ø	.7Ø8- Ø	.759- Ø	.8Ø5- Ø	.844- Ø	.877- Ø	.9Ø5- Ø	.927- Ø	4Ø
41	.458- Ø	.524- Ø	.588- Ø	.649- Ø	.7Ø6- Ø	.757- Ø	.8Ø2- Ø	.841- Ø	.875- Ø	.9Ø2- Ø	41
42	.393- Ø	.459- Ø	.524- Ø	.587- Ø	.648- Ø	.7Ø4- Ø	.754- Ø	.8ØØ- Ø	.839- Ø	.872- Ø	42
43	.332- Ø	.395- Ø	.459- Ø	.524- Ø	.586- Ø	.646- Ø	.7Ø2- Ø	.752- Ø	.797- Ø	.836- Ø	43
44	.275- Ø	.334- Ø	.396- Ø	.46Ø- Ø	.523- Ø	.585- Ø	.645- Ø	.7ØØ- Ø	.75Ø- Ø	.794- Ø	44

P=1/1Ø

R	N=Ø400	N=Ø410	N=Ø420	N=Ø430	N=Ø440	N=Ø450	N=Ø460	N=Ø470	N=Ø480	N=Ø490	R
45	.224- Ø	.277- Ø	.336- Ø	.397- Ø	.460- Ø	.523- Ø	.584- Ø	.643- Ø	.698- Ø	.748- Ø	45
46	.179- Ø	.226- Ø	.280- Ø	.337- Ø	.398- Ø	.460- Ø	.523- Ø	.584- Ø	.642- Ø	.696- Ø	46
47	.140- Ø	.181- Ø	.229- Ø	.282- Ø	.339- Ø	.399- Ø	.461- Ø	.522- Ø	.583- Ø	.640- Ø	47
48	.108- Ø	.143- Ø	.184- Ø	.232- Ø	.284- Ø	.341- Ø	.400- Ø	.461- Ø	.522- Ø	.582- Ø	48
49	.812- 1	.110- Ø	.146- Ø	.187- Ø	.234- Ø	.286- Ø	.342- Ø	.401- Ø	.462- Ø	.522- Ø	49
5Ø	.601- 1	.837- 1	.113- Ø	.148- Ø	.190- Ø	.236- Ø	.288- Ø	.344- Ø	.402- Ø	.462- Ø	5Ø
51	.436- 1	.623- 1	.861- 1	.116- Ø	.151- Ø	.192- Ø	.239- Ø	.290- Ø	.345- Ø	.403- Ø	51
52	.311- 1	.455- 1	.644- 1	.885- 1	.118- Ø	.154- Ø	.195- Ø	.241- Ø	.292- Ø	.347- Ø	52
53	.217- 1	.326- 1	.473- 1	.666- 1	.909- 1	.121- Ø	.156- Ø	.197- Ø	.243- Ø	.294- Ø	53
54	.149- 1	.230- 1	.341- 1	.492- 1	.687- 1	.932- 1	.123- Ø	.159- Ø	.200- Ø	.246- Ø	54
55	.100- 1	.159- 1	.242- 1	.357- 1	.510- 1	.708- 1	.955- 1	.126- Ø	.161- Ø	.202- Ø	55
56	.664- 2	.108- 1	.168- 1	.254- 1	.372- 1	.528- 1	.728- 1	.978- 1	.128- Ø	.164- Ø	56
57	.431- 2	.717- 2	.115- 1	.178- 1	.267- 1	.388- 1	.546- 1	.749- 1	.100- Ø	.130- Ø	57
58	.274- 2	.469- 2	.772- 2	.123- 1	.188- 1	.280- 1	.403- 1	.565- 1	.770- 1	.102- Ø	58
59	.171- 2	.301- 2	.509- 2	.829- 2	.130- 1	.198- 1	.292- 1	.418- 1	.583- 1	.790- 1	59
6Ø	.105- 2	.190- 2	.330- 2	.551- 2	.887- 2	.138- 1	.208- 1	.305- 1	.434- 1	.601- 1	6Ø
61	.635- 3	.118- 2	.210- 2	.360- 2	.594- 2	.947- 2	.146- 1	.219- 1	.318- 1	.449- 1	61
62	.376- 3	.718- 3	.131- 2	.231- 2	.391- 2	.638- 2	.101- 1	.154- 1	.229- 1	.331- 1	62
63	.219- 3	.429- 3	.807- 3	.146- 2	.253- 2	.423- 2	.685- 2	.107- 1	.163- 1	.240- 1	63
64	.125- 3	.252- 3	.487- 3	.903- 3	.161- 2	.276- 2	.457- 2	.732- 2	.114- 1	.171- 1	64
65	.704- 4	.146- 3	.289- 3	.550- 3	.101- 2	.177- 2	.300- 2	.492- 2	.781- 2	.120- 1	65
66	.389- 4	.829- 4	.169- 3	.330- 3	.619- 3	.112- 2	.194- 2	.325- 2	.528- 2	.831- 2	66
67	.211- 4	.463- 4	.971- 4	.195- 3	.374- 3	.692- 3	.123- 2	.212- 2	.352- 2	.566- 2	67
68	.113- 4	.255- 4	.548- 4	.113- 3	.223- 3	.423- 3	.771- 3	.136- 2	.231- 2	.379- 2	68
69	.592- 5	.138- 4	.305- 4	.645- 4	.131- 3	.254- 3	.475- 3	.856- 3	.149- 2	.250- 2	69
7Ø	.306- 5	.731- 5	.167- 4	.362- 4	.753- 4	.150- 3	.288- 3	.531- 3	.946- 3	.163- 2	7Ø
71	.155- 5	.382- 5	.896- 5	.200- 4	.427- 4	.874- 4	.172- 3	.325- 3	.592- 3	.104- 2	71
72	.776- 6	.197- 5	.474- 5	.109- 4	.239- 4	.501- 4	.101- 3	.195- 3	.365- 3	.657- 3	72
73	.381- 6	.996- 6	.247- 5	.582- 5	.131- 4	.283- 4	.584- 4	.116- 3	.221- 3	.408- 3	73
74	.185- 6	.496- 6	.127- 5	.307- 5	.710- 5	.157- 4	.333- 4	.677- 4	.132- 3	.250- 3	74
75	.879- 7	.243- 6	.638- 6	.159- 5	.378- 5	.859- 5	.187- 4	.389- 4	.780- 4	.151- 3	75
76	.412- 7	.117- 6	.317- 6	.813- 6	.199- 5	.463- 5	.103- 4	.220- 4	.453- 4	.895- 4	76
77	.190- 7	.558- 7	.155- 6	.409- 6	.103- 5	.246- 5	.562- 5	.123- 4	.259- 4	.524- 4	77
78	.863- 8	.261- 7	.748- 7	.203- 6	.523- 6	.128- 5	.301- 5	.677- 5	.146- 4	.302- 4	78
79	.386- 8	.121- 7	.355- 7	.990- 7	.262- 6	.662- 6	.159- 5	.367- 5	.810- 5	.172- 4	79
8Ø	.170- 8	.547- 8	.166- 7	.476- 7	.130- 6	.336- 6	.830- 6	.196- 5	.444- 5	.964- 5	8Ø
81	.739- 9	.245- 8	.764- 8	.226- 7	.632- 7	.168- 6	.426- 6	.103- 5	.240- 5	.533- 5	81
82	.316- 9	.108- 8	.347- 8	.105- 7	.303- 7	.829- 7	.216- 6	.536- 6	.127- 5	.291- 5	82
83	.133- 9	.468- 9	.155- 8	.485- 8	.144- 7	.403- 7	.108- 6	.274- 6	.669- 6	.156- 5	83
84	.552-1Ø	.200- 9	.684- 9	.220- 8	.669- 8	.193- 7	.530- 7	.139- 6	.346- 6	.828- 6	84
85	.226-1Ø	.845-1Ø	.297- 9	.984- 9	.308- 8	.913- 8	.257- 7	.690- 7	.177- 6	.433- 6	85
86		.351-1Ø	.127- 9	.434- 9	.140- 8	.425- 8	.123- 7	.339- 7	.890- 7	.223- 6	86
87		.144-1Ø	.537-1Ø	.189- 9	.624- 9	.195- 8	.580- 8	.164- 7	.442- 7	.114- 6	87
88			.224-1Ø	.808-1Ø	.275- 9	.886- 9	.270- 8	.783- 8	.216- 7	.571- 7	88
89				.342-1Ø	.120- 9	.396- 9	.124- 8	.369- 8	.105- 7	.283- 7	89
9Ø				.142-1Ø	.513-1Ø	.175- 9	.562- 9	.172- 8	.499- 8	.138- 7	9Ø
91					.217-1Ø	.760-1Ø	.251- 9	.788- 9	.235- 8	.667- 8	91
92						.326-1Ø	.111- 9	.357- 9	.109- 8	.318- 8	92
93						.138-1Ø	.482-1Ø	.159- 9	.500- 9	.149- 8	93
94							.207-1Ø	.703-1Ø	.227- 9	.694- 9	94
95								.306-1Ø	.101- 9	.318- 9	95
96								.132-1Ø	.447-1Ø	.144- 9	96
97									.195-1Ø	.643-1Ø	97
98										.284-1Ø	98
99										.124-1Ø	99

R	N=Ø500	N=Ø510	N=Ø520	N=Ø530	N=Ø540	N=Ø550	N=Ø560	N=Ø570	N=Ø580	N=Ø590	R
3Ø	.999- Ø										3Ø
31	.999- Ø	.999- Ø									31
32	.998- Ø	.999- Ø	.999- Ø								32
33	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø						33
34	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					34
35	.992- Ø	.995- Ø	.996- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø				35
36	.988- Ø	.991- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø			36
37	.981- Ø	.987- Ø	.991- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	37
38	.973- Ø	.980- Ø	.986- Ø	.990- Ø	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	38
39	.961- Ø	.971- Ø	.979- Ø	.985- Ø	.990- Ø	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.998- Ø	39
4Ø	.945- Ø	.959- Ø	.970- Ø	.978- Ø	.984- Ø	.989- Ø	.992- Ø	.995- Ø	.996- Ø	.998- Ø	4Ø
41	.925- Ø	.943- Ø	.957- Ø	.969- Ø	.977- Ø	.984- Ø	.988- Ø	.992- Ø	.994- Ø	.996- Ø	41
42	.900- Ø	.923- Ø	.941- Ø	.956- Ø	.967- Ø	.976- Ø	.983- Ø	.988- Ø	.991- Ø	.994- Ø	42
43	.869- Ø	.898- Ø	.921- Ø	.939- Ø	.954- Ø	.966- Ø	.975- Ø	.982- Ø	.987- Ø	.991- Ø	43
44	.834- Ø	.867- Ø	.895- Ø	.918- Ø	.937- Ø	.953- Ø	.965- Ø	.974- Ø	.981- Ø	.986- Ø	44
45	.792- Ø	.831- Ø	.865- Ø	.893- Ø	.916- Ø	.936- Ø	.951- Ø	.963- Ø	.973- Ø	.980- Ø	45
46	.745- Ø	.790- Ø	.829- Ø	.862- Ø	.891- Ø	.914- Ø	.934- Ø	.949- Ø	.962- Ø	.971- Ø	46
47	.694- Ø	.743- Ø	.787- Ø	.826- Ø	.860- Ø	.888- Ø	.912- Ø	.932- Ø	.948- Ø	.960- Ø	47
48	.639- Ø	.692- Ø	.741- Ø	.785- Ø	.824- Ø	.858- Ø	.886- Ø	.910- Ø	.930- Ø	.946- Ø	48
49	.581- Ø	.638- Ø	.690- Ø	.739- Ø	.783- Ø	.822- Ø	.855- Ø	.884- Ø	.908- Ø	.928- Ø	49
5Ø	.522- Ø	.580- Ø	.636- Ø	.689- Ø	.737- Ø	.781- Ø	.819- Ø	.853- Ø	.882- Ø	.906- Ø	5Ø
51	.462- Ø	.522- Ø	.579- Ø	.635- Ø	.687- Ø	.735- Ø	.779- Ø	.817- Ø	.851- Ø	.880- Ø	51
52	.404- Ø	.463- Ø	.521- Ø	.579- Ø	.634- Ø	.685- Ø	.733- Ø	.776- Ø	.815- Ø	.849- Ø	52
53	.348- Ø	.405- Ø	.463- Ø	.521- Ø	.578- Ø	.633- Ø	.684- Ø	.731- Ø	.774- Ø	.813- Ø	53
54	.296- Ø	.350- Ø	.406- Ø	.464- Ø	.521- Ø	.577- Ø	.631- Ø	.682- Ø	.730- Ø	.772- Ø	54

R	N=Ø5ØØ	N=Ø51Ø	N=Ø52Ø	N=Ø53Ø	N=Ø54Ø	N=Ø55Ø	N=Ø56Ø	N=Ø57Ø	N=Ø58Ø	N=Ø59Ø	R	P=1/1Ø
55	.248- Ø	.298- Ø	.351- Ø	.4Ø7- Ø	.464- Ø	.521- Ø	.577- Ø	.63Ø- Ø	.681- Ø	.728- Ø	55	
56	.2Ø4- Ø	.25Ø- Ø	.299- Ø	.352- Ø	.4Ø8- Ø	.464- Ø	.521- Ø	.576- Ø	.629- Ø	.679- Ø	56	
57	.166- Ø	.2Ø6- Ø	.252- Ø	.3Ø1- Ø	.354- Ø	.4Ø9- Ø	.465- Ø	.52Ø- Ø	.575- Ø	.628- Ø	57	
58	.133- Ø	.168- Ø	.2Ø9- Ø	.254- Ø	.3Ø3- Ø	.355- Ø	.4Ø9- Ø	.465- Ø	.52Ø- Ø	.575- Ø	58	
59	.1Ø4- Ø	.135- Ø	.17Ø- Ø	.211- Ø	.256- Ø	.3Ø4- Ø	.356- Ø	.41Ø- Ø	.465- Ø	.52Ø- Ø	59	
6Ø	.81Ø- 1	.1Ø7- Ø	.137- Ø	.173- Ø	.213- Ø	.258- Ø	.3Ø6- Ø	.357- Ø	.411- Ø	.465- Ø	6Ø	
61	.618- 1	.83Ø- 1	.1Ø9- Ø	.139- Ø	.175- Ø	.215- Ø	.259- Ø	.3Ø8- Ø	.359- Ø	.412- Ø	61	
62	.465- 1	.636- 1	.849- 1	.111- Ø	.142- Ø	.177- Ø	.217- Ø	.261- Ø	.3Ø9- Ø	.36Ø- Ø	62	
63	.344- 1	.48Ø- 1	.654- 1	.869- 1	.113- Ø	.144- Ø	.179- Ø	.219- Ø	.263- Ø	.311- Ø	63	
64	.251- 1	.357- 1	.495- 1	.671- 1	.888- 1	.115- Ø	.146- Ø	.181- Ø	.221- Ø	.265- Ø	64	
65	.18Ø- 1	.261- 1	.37Ø- 1	.511- 1	.688- 1	.9Ø7- 1	.117- Ø	.148- Ø	.183- Ø	.223- Ø	65	
66	.127- 1	.188- 1	.272- 1	.383- 1	.526- 1	.7Ø6- 1	.926- 1	.119- Ø	.15Ø- Ø	.185- Ø	66	
67	.883- 2	.134- 1	.197- 1	.283- 1	.396- 1	.541- 1	.723- 1	.945- 1	.121- Ø	.152- Ø	67	
68	.6Ø5- 2	.936- 2	.141- 1	.2Ø6- 1	.294- 1	.4Ø9- 1	.556- 1	.74Ø- 1	.963- 1	.123- Ø	68	
69	.4Ø8- 2	.645- 2	.99Ø- 2	.148- 1	.215- 1	.3Ø5- 1	.422- 1	.571- 1	.756- 1	.981- 1	69	
7Ø	.271- 2	.438- 2	.686- 2	.1Ø4- 1	.155- 1	.224- 1	.316- 1	.435- 1	.586- 1	.773- 1	7Ø	
71	.177- 2	.293- 2	.468- 2	.728- 2	.11Ø- 1	.162- 1	.233- 1	.327- 1	.448- 1	.6Ø1- 1	71	
72	.114- 2	.193- 2	.315- 2	.5ØØ- 2	.771- 2	.116- 1	.17Ø- 1	.242- 1	.338- 1	.461- 1	72	
73	.727- 3	.125- 2	.2Ø9- 2	.338- 2	.533- 2	.816- 2	.122- 1	.177- 1	.252- 1	.349- 1	73	
74	.455- 3	.8Ø1- 3	.137- 2	.226- 2	.363- 2	.566- 2	.861- 2	.128- 1	.185- 1	.261- 1	74	
75	.28Ø- 3	.5Ø5- 3	.879- 3	.148- 2	.243- 2	.388- 2	.6Ø1- 2	.9Ø8- 2	.134- 1	.192- 1	75	
76	.17Ø- 3	.314- 3	.558- 3	.963- 3	.161- 2	.262- 2	.414- 2	.637- 2	.955- 2	.14Ø- 1	76	
77	.1Ø2- 3	.192- 3	.35Ø- 3	.616- 3	.1Ø5- 2	.174- 2	.281- 2	.441- 2	.673- 2	.1ØØ- 1	77	
78	.6Ø3- 4	.116- 3	.216- 3	.388- 3	.677- 3	.114- 2	.188- 2	.3Ø1- 2	.468- 2	.711- 2	78	
79	.351- 4	.691- 4	.131- 3	.241- 3	.429- 3	.741- 3	.124- 2	.2Ø3- 2	.321- 2	.497- 2	79	
8Ø	.2Ø1- 4	.4Ø6- 4	.788- 4	.148- 3	.269- 3	.474- 3	.81Ø- 3	.135- 2	.218- 2	.343- 2	8Ø	
81	.114- 4	.235- 4	.466- 4	.895- 4	.166- 3	.299- 3	.521- 3	.883- 3	.145- 2	.233- 2	81	
82	.636- 5	.134- 4	.272- 4	.534- 4	.1Ø1- 3	.186- 3	.331- 3	.571- 3	.959- 3	.157- 2	82	
83	.35Ø- 5	.755- 5	.157- 4	.314- 4	.6Ø8- 4	.114- 3	.2Ø7- 3	.365- 3	.624- 3	.1Ø4- 2	83	
84	.19Ø- 5	.419- 5	.89Ø- 5	.182- 4	.361- 4	.69Ø- 4	.128- 3	.23Ø- 3	.4Ø1- 3	.681- 3	84	
85	.1Ø2- 5	.23Ø- 5	.499- 5	.1Ø4- 4	.211- 4	.412- 4	.78Ø- 4	.143- 3	.254- 3	.44Ø- 3	85	
86	.538- 6	.124- 5	.276- 5	.59Ø- 5	.122- 4	.243- 4	.469- 4	.878- 4	.159- 3	.281- 3	86	
87	.28Ø- 6	.662- 6	.151- 5	.329- 5	.695- 5	.142- 4	.279- 4	.532- 4	.985- 4	.177- 3	87	
88	.144- 6	.349- 6	.811- 6	.181- 5	.391- 5	.814- 5	.164- 4	.319- 4	.6Ø1- 4	.11Ø- 3	88	
89	.731- 7	.181- 6	.431- 6	.986- 6	.217- 5	.462- 5	.948- 5	.188- 4	.363- 4	.677- 4	89	
9Ø	.366- 7	.929- 7	.226- 6	.529- 6	.119- 5	.259- 5	.542- 5	.11Ø- 4	.216- 4	.411- 4	9Ø	
91	.181- 7	.47Ø- 7	.117- 6	.28Ø- 6	.645- 6	.143- 5	.3Ø6- 5	.634- 5	.127- 4	.246- 4	91	
92	.884- 8	.235- 7	.599- 7	.146- 6	.345- 6	.781- 6	.171- 5	.361- 5	.737- 5	.146- 4	92	
93	.426- 8	.116- 7	.3Ø2- 7	.757- 7	.182- 6	.421- 6	.941- 6	.2Ø3- 5	.423- 5	.854- 5	93	
94	.2Ø3- 8	.565- 8	.151- 7	.386- 7	.949- 7	.224- 6	.512- 6	.113- 5	.24Ø- 5	.494- 5	94	
95	.951- 9	.272- 8	.742- 8	.194- 7	.488- 7	.118- 6	.275- 6	.618- 6	.134- 5	.282- 5	95	
96	.441- 9	.129- 8	.361- 8	.967- 8	.248- 7	.614- 7	.146- 6	.335- 6	.742- 6	.159- 5	96	
97	.2Ø2- 9	.6Ø5- 9	.173- 8	.475- 8	.125- 7	.315- 7	.766- 7	.179- 6	.4Ø6- 6	.887- 6	97	
98	.914-1Ø	.28Ø- 9	.822- 9	.231- 8	.62Ø- 8	.16Ø- 7	.397- 7	.95Ø- 7	.219- 6	.489- 6	98	
99	.4Ø8-1Ø	.128- 9	.385- 9	.111- 8	.3Ø4- 8	.8Ø2- 8	.2Ø3- 7	.497- 7	.117- 6	.266- 6	99	
1ØØ	.18Ø-1Ø	.581-1Ø	.178- 9	.524- 9	.147- 8	.397- 8	.1Ø3- 7	.257- 7	.617- 7	.143- 6	1ØØ	
1Ø1		.259-1Ø	.816-1Ø	.245- 9	.7Ø5- 9	.195- 8	.515- 8	.131- 7	.322- 7	.762- 7	1Ø1	
1Ø2		.114-1Ø	.369-1Ø	.113- 9	.334- 9	.941- 9	.255- 8	.663- 8	.166- 7	.4Ø1- 7	1Ø2	
1Ø3			.165-1Ø	.519-1Ø	.156- 9	.45Ø- 9	.124- 8	.331- 8	.846- 8	.2Ø9- 7	1Ø3	
1Ø4				.234-1Ø	.722-1Ø	.213- 9	.6Ø1- 9	.163- 8	.426- 8	.1Ø7- 7	1Ø4	
1Ø5				.1Ø5-1Ø	.33Ø-1Ø	.994-1Ø	.287- 9	.796- 9	.212- 8	.545- 8	1Ø5	
1Ø6					.149-1Ø	.459-1Ø	.136- 9	.384- 9	.1Ø5- 8	.274- 8	1Ø6	
1Ø7						.21Ø-1Ø	.633-1Ø	.183- 9	.51Ø- 9	.136- 8	1Ø7	
1Ø8							.292-1Ø	.864-1Ø	.245- 9	.67Ø- 9	1Ø8	
1Ø9							.133-1Ø	.4Ø3-1Ø	.117- 9	.326- 9	1Ø9	
11Ø								.186-1Ø	.551-1Ø	.157- 9	11Ø	
111									.257-1Ø	.746-1Ø	111	
112									.118-1Ø	.351-1Ø	112	
113										.164-1Ø	113	

R	N=Ø6ØØ	N=Ø61Ø	N=Ø62Ø	N=Ø63Ø	N=Ø64Ø	N=Ø65Ø	N=Ø66Ø	N=Ø67Ø	N=Ø68Ø	N=Ø69Ø	R
38	.999- Ø										38
39	.999- Ø	.999- Ø									39
4Ø	.998- Ø	.999- Ø	.999- Ø								4Ø
41	.997- Ø	.998- Ø	.999- Ø	.999- Ø							41
42	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					42
43	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø				43
44	.99Ø- Ø	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø			44
45	.985- Ø	.99Ø- Ø	.993- Ø	.995- Ø	.996- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	45
46	.979- Ø	.985- Ø	.989- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	46
47	.97Ø- Ø	.978- Ø	.984- Ø	.988- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	47
48	.959- Ø	.969- Ø	.977- Ø	.983- Ø	.988- Ø	.991- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	48
49	.945- Ø	.958- Ø	.968- Ø	.976- Ø	.982- Ø	.987- Ø	.991- Ø	.993- Ø	.995- Ø	.997- Ø	49
5Ø	.926- Ø	.943- Ø	.956- Ø	.967- Ø	.975- Ø	.982- Ø	.987- Ø	.99Ø- Ø	.993- Ø	.995- Ø	5Ø
51	.9Ø4- Ø	.925- Ø	.941- Ø	.955- Ø	.966- Ø	.974- Ø	.981- Ø	.986- Ø	.99Ø- Ø	.993- Ø	51
52	.878- Ø	.9Ø2- Ø	.923- Ø	.94Ø- Ø	.954- Ø	.965- Ø	.973- Ø	.98Ø- Ø	.985- Ø	.989- Ø	52
53	.847- Ø	.876- Ø	.9ØØ- Ø	.921- Ø	.938- Ø	.952- Ø	.963- Ø	.972- Ø	.979- Ø	.985- Ø	53
54	.811- Ø	.845- Ø	.874- Ø	.899- Ø	.92Ø- Ø	.937- Ø	.951- Ø	.962- Ø	.971- Ø	.978- Ø	54
55	.77Ø- Ø	.8Ø9- Ø	.842- Ø	.872- Ø	.897- Ø	.918- Ø	.935- Ø	.95Ø- Ø	.961- Ø	.97Ø- Ø	55
56	.726- Ø	.769- Ø	.8Ø7- Ø	.84Ø- Ø	.87Ø- Ø	.895- Ø	.916- Ø	.934- Ø	.948- Ø	.96Ø- Ø	56
57	.678- Ø	.724- Ø	.767- Ø	.8Ø5- Ø	.839- Ø	.868- Ø	.893- Ø	.914- Ø	.932- Ø	.947- Ø	57
58	.627- Ø	.677- Ø	.723- Ø	.765- Ø	.8Ø3- Ø	.837- Ø	.866- Ø	.891- Ø	.913- Ø	.931- Ø	58

P=1/1Ø

R	N=Ø600	N=Ø610	N=Ø620	N=Ø630	N=Ø640	N=Ø650	N=Ø660	N=Ø670	N=Ø680	N=Ø690	R
59	.574- Ø	.626- Ø	.675- Ø	.721- Ø	.763- Ø	.8Ø1- Ø	.835- Ø	.864- Ø	.889- Ø	.911- Ø	59
6Ø	.52Ø- Ø	.573- Ø	.625- Ø	.674- Ø	.72Ø- Ø	.761- Ø	.799- Ø	.833- Ø	.862- Ø	.888- Ø	6Ø
61	.466- Ø	.52Ø- Ø	.573- Ø	.624- Ø	.673- Ø	.718- Ø	.76Ø- Ø	.797- Ø	.831- Ø	.86Ø- Ø	61
62	.412- Ø	.466- Ø	.52Ø- Ø	.572- Ø	.623- Ø	.671- Ø	.716- Ø	.758- Ø	.796- Ø	.829- Ø	62
63	.361- Ø	.413- Ø	.466- Ø	.519- Ø	.572- Ø	.622- Ø	.67Ø- Ø	.715- Ø	.756- Ø	.794- Ø	63
64	.312- Ø	.362- Ø	.414- Ø	.467- Ø	.519- Ø	.571- Ø	.621- Ø	.669- Ø	.714- Ø	.755- Ø	64
65	.266- Ø	.313- Ø	.363- Ø	.414- Ø	.467- Ø	.519- Ø	.571- Ø	.62Ø- Ø	.668- Ø	.712- Ø	65
66	.225- Ø	.268- Ø	.315- Ø	.364- Ø	.415- Ø	.467- Ø	.519- Ø	.57Ø- Ø	.619- Ø	.667- Ø	66
67	.187- Ø	.226- Ø	.27Ø- Ø	.316- Ø	.365- Ø	.416- Ø	.467- Ø	.519- Ø	.57Ø- Ø	.619- Ø	67
68	.154- Ø	.189- Ø	.228- Ø	.271- Ø	.317- Ø	.366- Ø	.416- Ø	.468- Ø	.519- Ø	.569- Ø	68
69	.125- Ø	.156- Ø	.191- Ø	.23Ø- Ø	.273- Ø	.319- Ø	.367- Ø	.417- Ø	.468- Ø	.519- Ø	69
7Ø	.999- 1	.127- Ø	.158- Ø	.193- Ø	.232- Ø	.274- Ø	.32Ø- Ø	.368- Ø	.418- Ø	.468- Ø	7Ø
71	.79Ø- 1	.1Ø2- Ø	.129- Ø	.159- Ø	.195- Ø	.233- Ø	.276- Ø	.321- Ø	.369- Ø	.418- Ø	71
72	.616- 1	.8Ø6- 1	.1Ø3- Ø	.13Ø- Ø	.161- Ø	.196- Ø	.235- Ø	.277- Ø	.322- Ø	.37Ø- Ø	72
73	.474- 1	.631- 1	.823- 1	.1Ø5- Ø	.132- Ø	.163- Ø	.198- Ø	.237- Ø	.279- Ø	.324- Ø	73
74	.36Ø- 1	.487- 1	.646- 1	.839- 1	.1Ø7- Ø	.134- Ø	.165- Ø	.2ØØ- Ø	.238- Ø	.28Ø- Ø	74
75	.27Ø- 1	.372- 1	.5ØØ- 1	.66Ø- 1	.855- 1	.1Ø9- Ø	.136- Ø	.167- Ø	.2Ø1- Ø	.24Ø- Ø	75
76	.2ØØ- 1	.28Ø- 1	.383- 1	.513- 1	.675- 1	.871- 1	.11Ø- Ø	.137- Ø	.168- Ø	.2Ø3- Ø	76
77	.146- 1	.2Ø8- 1	.289- 1	.394- 1	.526- 1	.689- 1	.887- 1	.112- Ø	.139- Ø	.17Ø- Ø	77
78	.1Ø5- 1	.152- 1	.216- 1	.299- 1	.4Ø5- 1	.539- 1	.7Ø4- 1	.9Ø2- 1	.114- Ø	.141- Ø	78
79	.749- 2	.11Ø- 1	.159- 1	.224- 1	.3Ø8- 1	.416- 1	.552- 1	.718- 1	.918- 1	.115- Ø	79
8Ø	.526- 2	.788- 2	.115- 1	.165- 1	.232- 1	.318- 1	.428- 1	.565- 1	.732- 1	.933- 1	8Ø
81	.365- 2	.556- 2	.828- 2	.121- 1	.172- 1	.24Ø- 1	.327- 1	.439- 1	.578- 1	.746- 1	81
82	.25Ø- 2	.387- 2	.587- 2	.869- 2	.126- 1	.178- 1	.248- 1	.337- 1	.45Ø- 1	.59Ø- 1	82
83	.169- 2	.266- 2	.411- 2	.619- 2	.911- 2	.131- 1	.185- 1	.256- 1	.347- 1	.461- 1	83
84	.112- 2	.181- 2	.284- 2	.435- 2	.651- 2	.953- 2	.137- 1	.192- 1	.264- 1	.356- 1	84
85	.74Ø- 3	.121- 2	.194- 2	.3Ø2- 2	.46Ø- 2	.684- 2	.997- 2	.142- 1	.199- 1	.272- 1	85
86	.481- 3	.8Ø3- 3	.131- 2	.2Ø7- 2	.321- 2	.485- 2	.718- 2	.1Ø4- 1	.148- 1	.2Ø5- 1	86
87	.3Ø9- 3	.525- 3	.87Ø- 3	.14Ø- 2	.221- 2	.34Ø- 2	.512- 2	.753- 2	.1Ø9- 1	.153- 1	87
88	.196- 3	.339- 3	.572- 3	.939- 3	.15Ø- 2	.235- 2	.36Ø- 2	.538- 2	.788- 2	.113- 1	88
89	.123- 3	.216- 3	.371- 3	.621- 3	.1Ø1- 2	.161- 2	.25Ø- 2	.381- 2	.566- 2	.824- 2	89
9Ø	.76Ø- 4	.136- 3	.238- 3	.4Ø6- 3	.673- 3	.1Ø9- 2	.172- 2	.266- 2	.4Ø2- 2	.594- 2	9Ø
91	.464- 4	.849- 4	.151- 3	.262- 3	.442- 3	.727- 3	.117- 2	.184- 2	.282- 2	.423- 2	91
92	.28Ø- 4	.522- 4	.947- 4	.167- 3	.287- 3	.48Ø- 3	.785- 3	.125- 2	.195- 2	.298- 2	92
93	.167- 4	.317- 4	.586- 4	.1Ø5- 3	.184- 3	.313- 3	.521- 3	.845- 3	.134- 2	.2Ø8- 2	93
94	.985- 5	.191- 4	.358- 4	.655- 4	.117- 3	.2Ø2- 3	.341- 3	.563- 3	.9Ø8- 3	.143- 2	94
95	.573- 5	.113- 4	.217- 4	.4Ø3- 4	.73Ø- 4	.129- 3	.221- 3	.371- 3	.6Ø8- 3	.974- 3	95
96	.33Ø- 5	.663- 5	.129- 4	.245- 4	.452- 4	.811- 4	.142- 3	.242- 3	.4Ø3- 3	.655- 3	96
97	.188- 5	.384- 5	.764- 5	.147- 4	.277- 4	.5Ø5- 4	.899- 4	.156- 3	.264- 3	.436- 3	97
98	.1Ø5- 5	.22Ø- 5	.446- 5	.876- 5	.167- 4	.311- 4	.563- 4	.994- 4	.171- 3	.287- 3	98
99	.585- 6	.125- 5	.257- 5	.515- 5	.1ØØ- 4	.19Ø- 4	.349- 4	.626- 4	.11Ø- 3	.187- 3	99
1ØØ	.321- 6	.697- 6	.147- 5	.299- 5	.593- 5	.114- 4	.214- 4	.39Ø- 4	.694- 4	.12Ø- 3	1ØØ
1Ø1	.174- 6	.386- 6	.827- 6	.172- 5	.347- 5	.679- 5	.13Ø- 4	.24Ø- 4	.435- 4	.767- 4	1Ø1
1Ø2	.936- 7	.211- 6	.461- 6	.976- 6	.2ØØ- 5	.4ØØ- 5	.776- 5	.147- 4	.269- 4	.483- 4	1Ø2
1Ø3	.496- 7	.114- 6	.254- 6	.548- 6	.115- 5	.233- 5	.46Ø- 5	.883- 5	.165- 4	.3Ø1- 4	1Ø3
1Ø4	.26Ø- 7	.611- 7	.139- 6	.3Ø4- 6	.649- 6	.134- 5	.27Ø- 5	.527- 5	.999- 5	.186- 4	1Ø4
1Ø5	.135- 7	.323- 7	.747- 7	.167- 6	.363- 6	.764- 6	.156- 5	.311- 5	.598- 5	.113- 4	1Ø5
1Ø6	.693- 8	.169- 7	.398- 7	.9Ø8- 7	.2Ø1- 6	.43Ø- 6	.896- 6	.181- 5	.354- 5	.684- 5	1Ø6
1Ø7	.351- 8	.874- 8	.21Ø- 7	.488- 7	.11Ø- 6	.24Ø- 6	.5Ø8- 6	.1Ø5- 5	.2Ø6- 5	.4Ø8- 5	1Ø7
1Ø8	.176- 8	.447- 8	.11Ø- 7	.259- 7	.595- 7	.132- 6	.285- 6	.598- 6	.119- 5	.241- 5	1Ø8
1Ø9	.875- 9	.226- 8	.566- 8	.136- 7	.319- 7	.722- 7	.158- 6	.338- 6	.668- 6	.141- 5	1Ø9
11Ø	.429- 9	.113- 8	.289- 8	.71Ø- 8	.169- 7	.39Ø- 7	.871- 7	.189- 6	.367- 6	.817- 6	11Ø
111	.2Ø8- 9	.561- 9	.147- 8	.365- 8	.886- 8	.2Ø8- 7	.474- 7	.1Ø5- 6	.193- 6	.468- 6	111
112	.1ØØ- 9	.275- 9	.737- 9	.186- 8	.46Ø- 8	.11Ø- 7	.255- 7	.573- 7	.934- 7	.265- 6	112
113	.476-1Ø	.133- 9	.36Ø- 9	.938- 9	.236- 8	.576- 8	.136- 7	.311- 7	.689- 7	.149- 6	113
114	.224-1Ø	.64Ø-1Ø	.176- 9	.468- 9	.12Ø- 8	.298- 8	.716- 8	.167- 7	.376- 7	.826- 7	114
115	.1Ø4-1Ø	.3Ø4-1Ø	.853-1Ø	.231- 9	.6Ø4- 9	.153- 8	.373- 8	.885- 8	.2Ø3- 7	.454- 7	115
116		.143-1Ø	.4Ø9-1Ø	.113- 9	.3Ø1- 9	.774- 9	.193- 8	.465- 8	.1Ø9- 7	.247- 7	116
117			.194-1Ø	.546-1Ø	.148- 9	.389- 9	.986- 9	.242- 8	.577- 8	.133- 7	117
118				.261-1Ø	.723-1Ø	.193- 9	.499- 9	.125- 8	.3Ø2- 8	.711- 8	118
119				.124-1Ø	.349-1Ø	.95Ø-1Ø	.25Ø- 9	.637- 9	.157- 8	.375- 8	119
12Ø					.167-1Ø	.463-1Ø	.124- 9	.322- 9	.8Ø7- 9	.196- 8	12Ø
121						.223-1Ø	.6Ø9-1Ø	.161- 9	.411- 9	.1Ø2- 8	121
122						.1Ø7-1Ø	.296-1Ø	.797-1Ø	.2Ø7- 9	.522- 9	122
123							.143-1Ø	.391-1Ø	.1Ø3- 9	.265- 9	123
124								.19Ø-1Ø	.512-1Ø	.133- 9	124
125									.251-1Ø	.665-1Ø	125
126									.122-1Ø	.328-1Ø	126
127										.161-1Ø	127

R	N=Ø7ØØ	N=Ø71Ø	N=Ø72Ø	N=Ø73Ø	N=Ø74Ø	N=Ø75Ø	N=Ø76Ø	N=Ø77Ø	N=Ø78Ø	N=Ø79Ø	R
46	.999- Ø										46
47	.999- Ø	.999- Ø									47
48	.999- Ø	.999- Ø	.999- Ø								48
49	.998- Ø	.998- Ø	.999- Ø	.999- Ø							49
5Ø	.996- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					5Ø
51	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø				51
52	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø			52
53	.989- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø		53
54	.984- Ø	.988- Ø	.991- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	54
55	.978- Ø	.983- Ø	.988- Ø	.991- Ø	.993- Ø	.995- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	55
56	.969- Ø	.977- Ø	.983- Ø	.987- Ø	.99Ø- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	56

P=1/1Ø

R	N=Ø7ØØ	N=Ø71Ø	N=Ø72Ø	N=Ø73Ø	N=Ø74Ø	N=Ø75Ø	N=Ø76Ø	N=Ø77Ø	N=Ø78Ø	N=Ø79Ø	R
57	.959- Ø	.968- Ø	.976- Ø	.982- Ø	.986- Ø	.990- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	57
58	.945- Ø	.958- Ø	.967- Ø	.975- Ø	.981- Ø	.986- Ø	.990- Ø	.992- Ø	.994- Ø	.996- Ø	58
59	.929- Ø	.944- Ø	.956- Ø	.966- Ø	.974- Ø	.980- Ø	.985- Ø	.989- Ø	.992- Ø	.994- Ø	59
6Ø	.909- Ø	.928- Ø	.943- Ø	.955- Ø	.965- Ø	.973- Ø	.980- Ø	.985- Ø	.989- Ø	.992- Ø	6Ø
61	.886- Ø	.9Ø8- Ø	.926- Ø	.941- Ø	.954- Ø	.964- Ø	.973- Ø	.979- Ø	.984- Ø	.988- Ø	61
62	.859- Ø	.884- Ø	.9Ø6- Ø	.925- Ø	.94Ø- Ø	.953- Ø	.963- Ø	.972- Ø	.978- Ø	.984- Ø	62
63	.827- Ø	.857- Ø	.882- Ø	.9Ø5- Ø	.923- Ø	.939- Ø	.952- Ø	.962- Ø	.971- Ø	.978- Ø	63
64	.792- Ø	.826- Ø	.855- Ø	.881- Ø	.9Ø3- Ø	.922- Ø	.938- Ø	.951- Ø	.961- Ø	.97Ø- Ø	64
65	.753- Ø	.79Ø- Ø	.824- Ø	.853- Ø	.879- Ø	.9Ø1- Ø	.92Ø- Ø	.936- Ø	.949- Ø	.96Ø- Ø	65
66	.711- Ø	.752- Ø	.789- Ø	.822- Ø	.852- Ø	.877- Ø	.9ØØ- Ø	.919- Ø	.935- Ø	.948- Ø	66
67	.665- Ø	.7Ø9- Ø	.75Ø- Ø	.787- Ø	.82Ø- Ø	.85Ø- Ø	.876- Ø	.898- Ø	.917- Ø	.934- Ø	67
68	.618- Ø	.664- Ø	.7Ø8- Ø	.748- Ø	.785- Ø	.819- Ø	.848- Ø	.874- Ø	.897- Ø	.916- Ø	68
69	.569- Ø	.617- Ø	.663- Ø	.7Ø7- Ø	.747- Ø	.784- Ø	.817- Ø	.847- Ø	.873- Ø	.895- Ø	69
7Ø	.518- Ø	.568- Ø	.616- Ø	.662- Ø	.7Ø5- Ø	.746- Ø	.782- Ø	.815- Ø	.845- Ø	.871- Ø	7Ø
71	.468- Ø	.518- Ø	.568- Ø	.615- Ø	.661- Ø	.7Ø4- Ø	.744- Ø	.781- Ø	.814- Ø	.843- Ø	71
72	.419- Ø	.468- Ø	.518- Ø	.567- Ø	.615- Ø	.66Ø- Ø	.7Ø3- Ø	.743- Ø	.779- Ø	.812- Ø	72
73	.371- Ø	.419- Ø	.469- Ø	.518- Ø	.567- Ø	.614- Ø	.659- Ø	.7Ø2- Ø	.741- Ø	.778- Ø	73
74	.325- Ø	.372- Ø	.42Ø- Ø	.469- Ø	.518- Ø	.566- Ø	.613- Ø	.658- Ø	.7ØØ- Ø	.74Ø- Ø	74
75	.282- Ø	.326- Ø	.372- Ø	.42Ø- Ø	.469- Ø	.518- Ø	.566- Ø	.612- Ø	.657- Ø	.699- Ø	75
76	.241- Ø	.283- Ø	.327- Ø	.373- Ø	.421- Ø	.469- Ø	.518- Ø	.565- Ø	.612- Ø	.656- Ø	76
77	.2Ø5- Ø	.243- Ø	.284- Ø	.328- Ø	.374- Ø	.421- Ø	.47Ø- Ø	.518- Ø	.565- Ø	.611- Ø	77
78	.172- Ø	.2Ø6- Ø	.244- Ø	.286- Ø	.329- Ø	.375- Ø	.422- Ø	.47Ø- Ø	.517- Ø	.565- Ø	78
79	.143- Ø	.173- Ø	.2Ø8- Ø	.246- Ø	.287- Ø	.33Ø- Ø	.376- Ø	.422- Ø	.47Ø- Ø	.517- Ø	79
8Ø	.117- Ø	.144- Ø	.175- Ø	.21Ø- Ø	.247- Ø	.288- Ø	.331- Ø	.376- Ø	.423- Ø	.47Ø- Ø	8Ø
81	.948- 1	.119- Ø	.146- Ø	.177- Ø	.211- Ø	.249- Ø	.289- Ø	.332- Ø	.377- Ø	.423- Ø	81
82	.761- 1	.963- 1	.12Ø- Ø	.147- Ø	.178- Ø	.213- Ø	.25Ø- Ø	.291- Ø	.333- Ø	.378- Ø	82
83	.6Ø3- 1	.774- 1	.978- 1	.122- Ø	.149- Ø	.18Ø- Ø	.214- Ø	.252- Ø	.292- Ø	.334- Ø	83
84	.473- 1	.616- 1	.788- 1	.993- 1	.123- Ø	.151- Ø	.181- Ø	.216- Ø	.253- Ø	.293- Ø	84
85	.366- 1	.484- 1	.628- 1	.8Ø2- 1	.1Ø1- Ø	.125- Ø	.152- Ø	.183- Ø	.217- Ø	.254- Ø	85
86	.28Ø- 1	.376- 1	.495- 1	.641- 1	.816- 1	.1Ø2- Ø	.126- Ø	.154- Ø	.184- Ø	.218- Ø	86
87	.212- 1	.289- 1	.386- 1	.5Ø6- 1	.653- 1	.829- 1	.1Ø4- Ø	.128- Ø	.155- Ø	.186- Ø	87
88	.159- 1	.219- 1	.297- 1	.395- 1	.517- 1	.665- 1	.843- 1	.1Ø5- Ø	.129- Ø	.157- Ø	88
89	.118- 1	.165- 1	.226- 1	.3Ø5- 1	.4Ø5- 1	.528- 1	.678- 1	.856- 1	.1Ø7- Ø	.131- Ø	89
9Ø	.861- 2	.122- 1	.171- 1	.233- 1	.314- 1	.415- 1	.539- 1	.69Ø- 1	.869- 1	.1Ø8- Ø	9Ø
91	.623- 2	.898- 2	.127- 1	.176- 1	.241- 1	.322- 1	.424- 1	.55Ø- 1	.7Ø2- 1	.883- 1	91
92	.446- 2	.652- 2	.936- 2	.132- 1	.182- 1	.248- 1	.331- 1	.434- 1	.561- 1	.714- 1	92
93	.315- 2	.468- 2	.682- 2	.975- 2	.137- 1	.188- 1	.255- 1	.339- 1	.444- 1	.572- 1	93
94	.22Ø- 2	.333- 2	.492- 2	.713- 2	.1Ø1- 1	.142- 1	.194- 1	.262- 1	.348- 1	.454- 1	94
95	.152- 2	.234- 2	.351- 2	.516- 2	.744- 2	.1Ø5- 1	.147- 1	.2ØØ- 1	.269- 1	.356- 1	95
96	.1Ø4- 2	.162- 2	.247- 2	.369- 2	.54Ø- 2	.776- 2	.1Ø9- 1	.152- 1	.2Ø6- 1	.277- 1	96
97	.7Ø5- 3	.111- 2	.172- 2	.261- 2	.388- 2	.565- 2	.8Ø8- 2	.114- 1	.157- 1	.213- 1	97
98	.472- 3	.757- 3	.119- 2	.183- 2	.276- 2	.4Ø7- 2	.591- 2	.841- 2	.118- 1	.162- 1	98
99	.312- 3	.5Ø9- 3	.811- 3	.127- 2	.194- 2	.291- 2	.427- 2	.617- 2	.875- 2	.122- 1	99
1ØØ	.2Ø4- 3	.338- 3	.548- 3	.868- 3	.135- 2	.2Ø5- 2	.3Ø6- 2	.448- 2	.644- 2	.9Ø9- 2	1ØØ
1Ø1	.132- 3	.222- 3	.366- 3	.589- 3	.927- 3	.143- 2	.217- 2	.321- 2	.469- 2	.671- 2	1Ø1
1Ø2	.846- 4	.145- 3	.242- 3	.395- 3	.631- 3	.989- 3	.152- 2	.228- 2	.338- 2	.49Ø- 2	1Ø2
1Ø3	.536- 4	.931- 4	.158- 3	.262- 3	.425- 3	.676- 3	.1Ø5- 2	.161- 2	.241- 2	.354- 2	1Ø3
1Ø4	.336- 4	.593- 4	.1Ø2- 3	.172- 3	.284- 3	.457- 3	.723- 3	.112- 2	.17Ø- 2	.254- 2	1Ø4
1Ø5	.2Ø8- 4	.373- 4	.654- 4	.112- 3	.187- 3	.3Ø6- 3	.491- 3	.769- 3	.119- 2	.18Ø- 2	1Ø5
1Ø6	.128- 4	.233- 4	.414- 4	.719- 4	.122- 3	.2Ø3- 3	.33Ø- 3	.524- 3	.823- 3	.126- 2	1Ø6
1Ø7	.775- 5	.144- 4	.259- 4	.458- 4	.79Ø- 4	.133- 3	.22Ø- 3	.353- 3	.564- 3	.876- 3	1Ø7
1Ø8	.466- 5	.876- 5	.161- 4	.288- 4	.5Ø5- 4	.865- 4	.145- 3	.235- 3	.382- 3	.6Ø2- 3	1Ø8
1Ø9	.277- 5	.53Ø- 5	.988- 5	.18Ø- 4	.32Ø- 4	.556- 4	.946- 4	.155- 3	.257- 3	.41Ø- 3	1Ø9
11Ø	.163- 5	.317- 5	.6ØØ- 5	.111- 4	.2ØØ- 4	.354- 4	.611- 4	.1ØØ- 3	.171- 3	.277- 3	11Ø
111	.949- 6	.187- 5	.361- 5	.678- 5	.124- 4	.223- 4	.391- 4	.641- 4	.112- 3	.185- 3	111
112	.547- 6	.11Ø- 5	.215- 5	.41Ø- 5	.764- 5	.139- 4	.247- 4	.4Ø2- 4	.732- 4	.122- 3	112
113	.312- 6	.637- 6	.127- 5	.246- 5	.465- 5	.859- 5	.155- 4	.274- 4	.473- 4	.799- 4	113
114	.176- 6	.366- 6	.739- 6	.146- 5	.28Ø- 5	.525- 5	.962- 5	.172- 4	.3Ø2- 4	.518- 4	114
115	.985- 7	.2Ø8- 6	.427- 6	.855- 6	.167- 5	.318- 5	.591- 5	.1Ø8- 4	.191- 4	.333- 4	115
116	.545- 7	.117- 6	.244- 6	.497- 6	.985- 6	.191- 5	.36Ø- 5	.664- 5	.12Ø- 4	.212- 4	116
117	.299- 7	.652- 7	.138- 6	.286- 6	.576- 6	.113- 5	.217- 5	.4Ø6- 5	.744- 5	.133- 4	117
118	.162- 7	.36Ø- 7	.776- 7	.163- 6	.333- 6	.665- 6	.13Ø- 5	.246- 5	.458- 5	.832- 5	118
119	.871- 8	.197- 7	.431- 7	.92Ø- 7	.191- 6	.387- 6	.766- 6	.148- 5	.279- 5	.514- 5	119
12Ø	.464- 8	.1Ø6- 7	.237- 7	.514- 7	.1Ø9- 6	.223- 6	.449- 6	.879- 6	.168- 5	.315- 5	12Ø
121	.244- 8	.57Ø- 8	.129- 7	.285- 7	.611- 7	.128- 6	.26Ø- 6	.518- 6	.1Ø1- 5	.191- 5	121
122	.128- 8	.3Ø2- 8	.697- 8	.156- 7	.34Ø- 7	.722- 7	.15Ø- 6	.3Ø2- 6	.596- 6	.115- 5	122
123	.659- 9	.159- 8	.372- 8	.847- 8	.188- 7	.4Ø5- 7	.852- 7	.175- 6	.349- 6	.683- 6	123
124	.337- 9	.827- 9	.197- 8	.456- 8	.1Ø3- 7	.225- 7	.48Ø- 7	.1ØØ- 6	.2Ø3- 6	.4Ø3- 6	124
125	.171- 9	.427- 9	.1Ø3- 8	.243- 8	.556- 8	.124- 7	.268- 7	.567- 7	.117- 6	.236- 6	125
126	.859-1Ø	.218- 9	.537- 9	.128- 8	.298- 8	.675- 8	.149- 7	.319- 7	.668- 7	.136- 6	126
127	.428-1Ø	.11Ø- 9	.276- 9	.671- 9	.159- 8	.364- 8	.815- 8	.178- 7	.378- 7	.783- 7	127
128	.211-1Ø	.553-1Ø	.141- 9	.348- 9	.835- 9	.195- 8	.443- 8	.981- 8	.212- 7	.445- 7	128
129	.1Ø3-1Ø	.275-1Ø	.711-1Ø	.179- 9	.436- 9	.1Ø3- 8	.239- 8	.537- 8	.118- 7	.251- 7	129
13Ø		.135-1Ø	.356-1Ø	.9Ø9-1Ø	.225- 9	.544- 9	.127- 8	.291- 8	.647- 8	.14Ø- 7	13Ø
131			.177-1Ø	.459-1Ø	.116- 9	.283- 9	.674- 9	.156- 8	.353- 8	.777- 8	131
132				.229-1Ø	.587-1Ø	.146- 9	.354- 9	.832- 9	.191- 8	.426- 8	132
133				.113-1Ø	.296-1Ø	.747-1Ø	.184- 9	.439- 9	.1Ø2- 8	.232- 8	133
134					.147-1Ø	.379-1Ø	.946-1Ø	.23Ø- 9	.543- 9	.125- 8	134
135						.19Ø-1Ø	.483-1Ø	.119- 9	.286- 9	.669- 9	135
136							.245-1Ø	.613-1Ø	.149- 9	.354- 9	136
137							.123-1Ø	.312-1Ø	.773-1Ø	.186- 9	137
138								.158-1Ø	.397-1Ø	.97Ø-1Ø	138

R	N=Ø700	N=Ø710	N=Ø720	N=Ø730	N=Ø740	N=Ø750	N=Ø760	N=Ø770	N=Ø780	N=Ø790	R	P=1/1Ø
139									.2Ø2-1Ø	.5Ø1-1Ø	139	
14Ø									.1Ø2-1Ø	.257-1Ø	14Ø	
141										.13Ø-1Ø	141	

R	N=Ø800	N=Ø810	N=Ø820	N=Ø830	N=Ø840	N=Ø850	N=Ø860	N=Ø870	N=Ø880	N=Ø890	R
55	.999- Ø	.999- Ø									55
56	.999- Ø	.999- Ø	.999- Ø								56
57	.998- Ø	.999- Ø	.999- Ø	.999- Ø							57
58	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø						58
59	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø			59
6Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø		6Ø
61	.991- Ø	.994- Ø	.995- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø	61
62	.988- Ø	.991- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.998- Ø	62
63	.983- Ø	.987- Ø	.99Ø- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.998- Ø	63
64	.977- Ø	.982- Ø	.987- Ø	.99Ø- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	64
65	.969- Ø	.976- Ø	.982- Ø	.986- Ø	.99Ø- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.997- Ø	65
66	.959- Ø	.968- Ø	.975- Ø	.981- Ø	.986- Ø	.989- Ø	.992- Ø	.994- Ø	.995- Ø	.996- Ø	66
67	.947- Ø	.958- Ø	.967- Ø	.975- Ø	.981- Ø	.985- Ø	.989- Ø	.991- Ø	.994- Ø	.994- Ø	67
68	.932- Ø	.946- Ø	.957- Ø	.966- Ø	.974- Ø	.98Ø- Ø	.985- Ø	.988- Ø	.991- Ø	.993- Ø	68
69	.915- Ø	.931- Ø	.945- Ø	.956- Ø	.966- Ø	.973- Ø	.979- Ø	.984- Ø	.988- Ø	.99Ø- Ø	69
7Ø	.894- Ø	.913- Ø	.93Ø- Ø	.944- Ø	.955- Ø	.965- Ø	.972- Ø	.978- Ø	.983- Ø	.987- Ø	7Ø
71	.87Ø- Ø	.892- Ø	.912- Ø	.929- Ø	.943- Ø	.954- Ø	.964- Ø	.971- Ø	.978- Ø	.982- Ø	71
72	.842- Ø	.868- Ø	.891- Ø	.91Ø- Ø	.927- Ø	.941- Ø	.953- Ø	.963- Ø	.971- Ø	.976- Ø	72
73	.811- Ø	.84Ø- Ø	.866- Ø	.889- Ø	.909- Ø	.926- Ø	.94Ø- Ø	.952- Ø	.962- Ø	.969- Ø	73
74	.776- Ø	.8Ø9- Ø	.839- Ø	.865- Ø	.888- Ø	.9Ø8- Ø	.925- Ø	.939- Ø	.951- Ø	.96Ø- Ø	74
75	.739- Ø	.775- Ø	.8Ø8- Ø	.837- Ø	.863- Ø	.886- Ø	.9Ø6- Ø	.923- Ø	.938- Ø	.949- Ø	75
76	.698- Ø	.737- Ø	.773- Ø	.8Ø6- Ø	.836- Ø	.862- Ø	.885- Ø	.9Ø5- Ø	.922- Ø	.936- Ø	76
77	.655- Ø	.697- Ø	.736- Ø	.772- Ø	.8Ø5- Ø	.834- Ø	.86Ø- Ø	.883- Ø	.9Ø4- Ø	.92Ø- Ø	77
78	.61Ø- Ø	.654- Ø	.696- Ø	.735- Ø	.771- Ø	.8Ø3- Ø	.833- Ø	.859- Ø	.882- Ø	.9Ø1- Ø	78
79	.564- Ø	.61Ø- Ø	.653- Ø	.695- Ø	.733- Ø	.769- Ø	.8Ø2- Ø	.831- Ø	.857- Ø	.88Ø- Ø	79
8Ø	.517- Ø	.564- Ø	.6Ø9- Ø	.652- Ø	.694- Ø	.732- Ø	.768- Ø	.8ØØ- Ø	.83Ø- Ø	.856- Ø	8Ø
81	.47Ø- Ø	.517- Ø	.563- Ø	.6Ø8- Ø	.652- Ø	.693- Ø	.731- Ø	.766- Ø	.799- Ø	.828- Ø	81
82	.424- Ø	.47Ø- Ø	.517- Ø	.563- Ø	.6Ø8- Ø	.651- Ø	.691- Ø	.73Ø- Ø	.765- Ø	.797- Ø	82
83	.379- Ø	.424- Ø	.471- Ø	.517- Ø	.563- Ø	.6Ø7- Ø	.65Ø- Ø	.69Ø- Ø	.728- Ø	.763- Ø	83
84	.335- Ø	.379- Ø	.425- Ø	.471- Ø	.517- Ø	.562- Ø	.6Ø6- Ø	.649- Ø	.689- Ø	.727- Ø	84
85	.294- Ø	.336- Ø	.38Ø- Ø	.425- Ø	.471- Ø	.517- Ø	.562- Ø	.6Ø6- Ø	.648- Ø	.689- Ø	85
86	.256- Ø	.295- Ø	.337- Ø	.381- Ø	.426- Ø	.471- Ø	.516- Ø	.561- Ø	.6Ø5- Ø	.647- Ø	86
87	.22Ø- Ø	.257- Ø	.296- Ø	.338- Ø	.382- Ø	.426- Ø	.471- Ø	.516- Ø	.561- Ø	.6Ø5- Ø	87
88	.187- Ø	.221- Ø	.258- Ø	.297- Ø	.339- Ø	.382- Ø	.426- Ø	.471- Ø	.516- Ø	.561- Ø	88
89	.158- Ø	.189- Ø	.223- Ø	.259- Ø	.299- Ø	.34Ø- Ø	.383- Ø	.427- Ø	.471- Ø	.516- Ø	89
9Ø	.132- Ø	.16Ø- Ø	.19Ø- Ø	.224- Ø	.261- Ø	.3ØØ- Ø	.341- Ø	.383- Ø	.427- Ø	.472- Ø	9Ø
91	.1Ø9- Ø	.134- Ø	.161- Ø	.192- Ø	.225- Ø	.262- Ø	.3ØØ- Ø	.341- Ø	.384- Ø	.428- Ø	91
92	.896- 1	.111- Ø	.135- Ø	.162- Ø	.193- Ø	.227- Ø	.263- Ø	.3Ø1- Ø	.342- Ø	.385- Ø	92
93	.726- 1	.9Ø9- 1	.112- Ø	.136- Ø	.164- Ø	.194- Ø	.228- Ø	.264- Ø	.3Ø2- Ø	.343- Ø	93
94	.583- 1	.738- 1	.922- 1	.113- Ø	.138- Ø	.165- Ø	.196- Ø	.229- Ø	.265- Ø	.3Ø4- Ø	94
95	.463- 1	.594- 1	.75Ø- 1	.935- 1	.115- Ø	.139- Ø	.167- Ø	.197- Ø	.23Ø- Ø	.266- Ø	95
96	.365- 1	.473- 1	.6Ø5- 1	.762- 1	.947- 1	.116- Ø	.14Ø- Ø	.168- Ø	.198- Ø	.232- Ø	96
97	.284- 1	.373- 1	.483- 1	.616- 1	.774- 1	.96Ø- 1	.117- Ø	.142- Ø	.169- Ø	.2ØØ- Ø	97
98	.219- 1	.291- 1	.382- 1	.493- 1	.626- 1	.786- 1	.971- 1	.119- Ø	.143- Ø	.171- Ø	98
99	.167- 1	.225- 1	.299- 1	.39Ø- 1	.5Ø2- 1	.637- 1	.796- 1	.983- 1	.12Ø- Ø	.145- Ø	99
1ØØ	.126- 1	.172- 1	.231- 1	.3Ø6- 1	.399- 1	.512- 1	.647- 1	.8Ø7- 1	.995- 1	.121- Ø	1ØØ
1Ø1	.943- 2	.13Ø- 1	.177- 1	.238- 1	.313- 1	.4Ø7- 1	.52Ø- 1	.657- 1	.818- 1	.1Ø1- Ø	1Ø1
1Ø2	.698- 2	.978- 2	.135- 1	.183- 1	.244- 1	.321- 1	.414- 1	.529- 1	.667- 1	.832- 1	1Ø2
1Ø3	.512- 2	.726- 2	.1Ø1- 1	.139- 1	.188- 1	.25Ø- 1	.327- 1	.422- 1	.538- 1	.68Ø- 1	1Ø3
1Ø4	.371- 2	.534- 2	.755- 2	.1Ø5- 1	.144- 1	.193- 1	.255- 1	.334- 1	.43Ø- 1	.55Ø- 1	1Ø4
1Ø5	.267- 2	.389- 2	.557- 2	.784- 2	.1Ø9- 1	.148- 1	.198- 1	.261- 1	.341- 1	.441- 1	1Ø5
1Ø6	.19Ø- 2	.28Ø- 2	.4Ø7- 2	.58Ø- 2	.814- 2	.112- 1	.151- 1	.2Ø3- 1	.267- 1	.351- 1	1Ø6
1Ø7	.134- 2	.2ØØ- 2	.294- 2	.425- 2	.6Ø4- 2	.842- 2	.115- 1	.155- 1	.2Ø7- 1	.276- 1	1Ø7
1Ø8	.931- 3	.141- 2	.211- 2	.3Ø8- 2	.443- 2	.626- 2	.861- 2	.118- 1	.159- 1	.215- 1	1Ø8
1Ø9	.643- 3	.989- 3	.149- 2	.221- 2	.323- 2	.461- 2	.639- 2	.887- 2	.121- 1	.167- 1	1Ø9
11Ø	.44Ø- 3	.685- 3	.1Ø5- 2	.158- 2	.233- 2	.336- 2	.469- 2	.659- 2	.912- 2	.128- 1	11Ø
111	.298- 3	.47Ø- 3	.729- 3	.111- 2	.166- 2	.242- 2	.34Ø- 2	.484- 2	.678- 2	.968- 2	111
112	.199- 3	.32Ø- 3	.5Ø2- 3	.775- 3	.117- 2	.173- 2	.243- 2	.351- 2	.497- 2	.728- 2	112
113	.132- 3	.215- 3	.343- 3	.536- 3	.822- 3	.122- 2	.184- 2	.268- 2	.384- 2	.543- 2	113
114	.871- 4	.143- 3	.232- 3	.367- 3	.571- 3	.856- 3	.131- 2	.193- 2	.28Ø- 2	.4Ø1- 2	114
115	.567- 4	.947- 4	.155- 3	.249- 3	.392- 3	.591- 3	.923- 3	.138- 2	.2Ø3- 2	.293- 2	115
116	.366- 4	.619- 4	.1Ø3- 3	.167- 3	.267- 3	.4Ø3- 3	.645- 3	.976- 3	.145- 2	.213- 2	116
117	.234- 4	.4Ø1- 4	.675- 4	.111- 3	.18Ø- 3	.27Ø- 3	.446- 3	.684- 3	.1Ø3- 2	.153- 2	117
118	.148- 4	.257- 4	.439- 4	.734- 4	.12Ø- 3	.194- 3	.3Ø6- 3	.475- 3	.725- 3	.1Ø9- 2	118
119	.927- 5	.164- 4	.283- 4	.48Ø- 4	.797- 4	.13Ø- 3	.2Ø8- 3	.327- 3	.5Ø5- 3	.768- 3	119
12Ø	.576- 5	.1Ø3- 4	.181- 4	.311- 4	.523- 4	.865- 4	.14Ø- 3	.223- 3	.349- 3	.537- 3	12Ø
121	.354- 5	.643- 5	.114- 4	.199- 4	.34Ø- 4	.57Ø- 4	.936- 4	.151- 3	.239- 3	.372- 3	121
122	.216- 5	.398- 5	.717- 5	.127- 4	.219- 4	.372- 4	.619- 4	.1Ø1- 3	.162- 3	.256- 3	122
123	.13Ø- 5	.244- 5	.446- 5	.798- 5	.14Ø- 4	.241- 4	.4Ø6- 4	.671- 4	.1Ø9- 3	.174- 3	123
124	.781- 6	.148- 5	.274- 5	.498- 5	.885- 5	.154- 4	.264- 4	.442- 4	.727- 4	.118- 3	124
125	.463- 6	.89Ø- 6	.167- 5	.3Ø8- 5	.555- 5	.98Ø- 5	.17Ø- 4	.288- 4	.481- 4	.786- 4	125
126	.272- 6	.531- 6	.1Ø1- 5	.189- 5	.345- 5	.618- 5	.1Ø8- 4	.186- 4	.315- 4	.522- 4	126
127	.159- 6	.314- 6	.6Ø7- 6	.115- 5	.213- 5	.386- 5	.685- 5	.119- 4	.2Ø4- 4	.343- 4	127
128	.915- 7	.184- 6	.36Ø- 6	.691- 6	.13Ø- 5	.239- 5	.43Ø- 5	.759- 5	.132- 4	.224- 4	128
129	.523- 7	.1Ø7- 6	.212- 6	.412- 6	.785- 6	.146- 5	.267- 5	.478- 5	.839- 5	.144- 4	129
13Ø	.297- 7	.613- 7	.124- 6	.244- 6	.471- 6	.89Ø- 6	.165- 5	.299- 5	.531- 5	.925- 5	13Ø
131	.167- 7	.349- 7	.715- 7	.143- 6	.28Ø- 6	.536- 6	.1Ø1- 5	.185- 5	.333- 5	.588- 5	131

R	N=Ø800	N=Ø81Ø	N=Ø82Ø	N=Ø83Ø	N=Ø84Ø	N=Ø85Ø	N=Ø86Ø	N=Ø87Ø	N=Ø88Ø	N=Ø89Ø	R	P=1/1Ø
132	.929- 8	.197- 7	.41Ø- 7	.832- 7	.165- 6	.321- 6	.6Ø9- 6	.113- 5	.2Ø7- 5	.37Ø- 5	132	
133	.513- 8	.111- 7	.233- 7	.48Ø- 7	.965- 7	.19Ø- 6	.366- 6	.69Ø- 6	.128- 5	.231- 5	133	
134	.281- 8	.614- 8	.131- 7	.274- 7	.559- 7	.112- 6	.218- 6	.417- 6	.78Ø- 6	.143- 5	134	
135	.152- 8	.338- 8	.734- 8	.155- 7	.321- 7	.65Ø- 7	.129- 6	.249- 6	.473- 6	.879- 6	135	
136	.819- 9	.185- 8	.4Ø6- 8	.872- 8	.183- 7	.375- 7	.753- 7	.148- 6	.284- 6	.535- 6	136	
137	.437- 9	.1ØØ- 8	.223- 8	.486- 8	.1Ø3- 7	.215- 7	.437- 7	.87Ø- 7	.169- 6	.323- 6	137	
138	.231- 9	.536- 9	.121- 8	.268- 8	.579- 8	.122- 7	.252- 7	.5Ø7- 7	.1ØØ- 6	.194- 6	138	
139	.121- 9	.285- 9	.655- 9	.147- 8	.321- 8	.687- 8	.144- 7	.294- 7	.587- 7	.115- 6	139	
14Ø	.63Ø-1Ø	.151- 9	.351- 9	.798- 9	.177- 8	.384- 8	.813- 8	.168- 7	.341- 7	.678- 7	14Ø	
141	.325-1Ø	.788-1Ø	.186- 9	.43Ø- 9	.967- 9	.213- 8	.457- 8	.959- 8	.197- 7	.396- 7	141	
142	.166-1Ø	.4Ø9-1Ø	.981-1Ø	.229- 9	.524- 9	.117- 8	.254- 8	.541- 8	.113- 7	.23Ø- 7	142	
143		.21Ø-1Ø	.512-1Ø	.122- 9	.281- 9	.636- 9	.14Ø- 8	.3Ø3- 8	.639- 8	.132- 7	143	
144		.1Ø7-1Ø	.265-1Ø	.639-1Ø	.15Ø- 9	.344- 9	.769- 9	.168- 8	.36Ø- 8	.753- 8	144	
145			.136-1Ø	.333-1Ø	.793-1Ø	.184- 9	.418- 9	.927- 9	.2Ø1- 8	.426- 8	145	
146				.172-1Ø	.416-1Ø	.98Ø-1Ø	.225- 9	.5Ø7- 9	.111- 8	.239- 8	146	
147					.216-1Ø	.517-1Ø	.121- 9	.275- 9	.612- 9	.133- 8	147	
148					.111-1Ø	.27Ø-1Ø	.64Ø-1Ø	.148- 9	.333- 9	.734- 9	148	
149						.14Ø-1Ø	.337-1Ø	.788-1Ø	.18Ø- 9	.4Ø2- 9	149	
15Ø							.176-1Ø	.417-1Ø	.968-1Ø	.218- 9	15Ø	
151								.219-1Ø	.515-1Ø	.117- 9	151	
152								.114-1Ø	.272-1Ø	.621-1Ø	152	
153									.143-1Ø	.324-1Ø	153	
154										.165-1Ø	154	

R	N=Ø9ØØ	N=Ø91Ø	N=Ø92Ø	N=Ø93Ø	N=Ø94Ø	N=Ø95Ø	N=Ø96Ø	N=Ø97Ø	N=Ø98Ø	N=Ø99Ø	R
63	.999- Ø										63
64	.999- Ø	.999- Ø	.999- Ø								64
65	.998- Ø	.999- Ø	.999- Ø	.999- Ø							65
66	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø						66
67	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					67
68	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø				68
69	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø		69
7Ø	.991- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	7Ø
71	.987- Ø	.99Ø- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	71
72	.983- Ø	.987- Ø	.99Ø- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	72
73	.977- Ø	.982- Ø	.986- Ø	.989- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	73
74	.969- Ø	.976- Ø	.981- Ø	.985- Ø	.989- Ø	.992- Ø	.994- Ø	.995- Ø	.997- Ø	.997- Ø	74
75	.96Ø- Ø	.969- Ø	.975- Ø	.981- Ø	.985- Ø	.989- Ø	.991- Ø	.994- Ø	.995- Ø	.996- Ø	75
76	.949- Ø	.959- Ø	.968- Ø	.975- Ø	.98Ø- Ø	.985- Ø	.988- Ø	.991- Ø	.993- Ø	.995- Ø	76
77	.936- Ø	.948- Ø	.958- Ø	.967- Ø	.974- Ø	.98Ø- Ø	.984- Ø	.988- Ø	.991- Ø	.993- Ø	77
78	.92Ø- Ø	.935- Ø	.947- Ø	.958- Ø	.966- Ø	.973- Ø	.979- Ø	.984- Ø	.988- Ø	.99Ø- Ø	78
79	.9Ø1- Ø	.919- Ø	.933- Ø	.946- Ø	.957- Ø	.966- Ø	.973- Ø	.979- Ø	.983- Ø	.987- Ø	79
8Ø	.88Ø- Ø	.9ØØ- Ø	.917- Ø	.932- Ø	.945- Ø	.956- Ø	.965- Ø	.972- Ø	.978- Ø	.983- Ø	8Ø
81	.855- Ø	.878- Ø	.898- Ø	.916- Ø	.931- Ø	.944- Ø	.955- Ø	.964- Ø	.971- Ø	.978- Ø	81
82	.827- Ø	.854- Ø	.877- Ø	.897- Ø	.915- Ø	.93Ø- Ø	.943- Ø	.954- Ø	.963- Ø	.971- Ø	82
83	.796- Ø	.826- Ø	.852- Ø	.876- Ø	.896- Ø	.914- Ø	.929- Ø	.942- Ø	.953- Ø	.962- Ø	83
84	.763- Ø	.795- Ø	.824- Ø	.851- Ø	.874- Ø	.895- Ø	.913- Ø	.928- Ø	.941- Ø	.952- Ø	84
85	.726- Ø	.762- Ø	.794- Ø	.823- Ø	.849- Ø	.873- Ø	.894- Ø	.912- Ø	.927- Ø	.94Ø- Ø	85
86	.688- Ø	.725- Ø	.76Ø- Ø	.793- Ø	.822- Ø	.848- Ø	.872- Ø	.892- Ø	.91Ø- Ø	.926- Ø	86
87	.647- Ø	.687- Ø	.724- Ø	.759- Ø	.791- Ø	.82Ø- Ø	.847- Ø	.87Ø- Ø	.891- Ø	.9Ø9- Ø	87
88	.6Ø4- Ø	.646- Ø	.685- Ø	.723- Ø	.758- Ø	.79Ø- Ø	.819- Ø	.846- Ø	.869- Ø	.89Ø- Ø	88
89	.56Ø- Ø	.6Ø4- Ø	.645- Ø	.685- Ø	.722- Ø	.757- Ø	.789- Ø	.818- Ø	.844- Ø	.868- Ø	89
9Ø	.516- Ø	.56Ø- Ø	.6Ø3- Ø	.644- Ø	.684- Ø	.721- Ø	.755- Ø	.787- Ø	.817- Ø	.843- Ø	9Ø
91	.472- Ø	.516- Ø	.56Ø- Ø	.6Ø2- Ø	.644- Ø	.683- Ø	.72Ø- Ø	.754- Ø	.786- Ø	.815- Ø	91
92	.428- Ø	.472- Ø	.516- Ø	.56Ø- Ø	.6Ø2- Ø	.643- Ø	.682- Ø	.719- Ø	.753- Ø	.785- Ø	92
93	.385- Ø	.429- Ø	.472- Ø	.516- Ø	.559- Ø	.6Ø1- Ø	.642- Ø	.681- Ø	.718- Ø	.752- Ø	93
94	.344- Ø	.386- Ø	.429- Ø	.472- Ø	.516- Ø	.559- Ø	.6Ø1- Ø	.641- Ø	.68Ø- Ø	.717- Ø	94
95	.3Ø5- Ø	.345- Ø	.386- Ø	.429- Ø	.473- Ø	.516- Ø	.559- Ø	.6ØØ- Ø	.641- Ø	.679- Ø	95
96	.268- Ø	.3Ø6- Ø	.346- Ø	.387- Ø	.43Ø- Ø	.473- Ø	.516- Ø	.558- Ø	.6ØØ- Ø	.64Ø- Ø	96
97	.233- Ø	.269- Ø	.3Ø6- Ø	.346- Ø	.388- Ø	.43Ø- Ø	.473- Ø	.516- Ø	.558- Ø	.599- Ø	97
98	.2Ø1- Ø	.234- Ø	.27Ø- Ø	.3Ø8- Ø	.347- Ø	.388- Ø	.43Ø- Ø	.473- Ø	.516- Ø	.558- Ø	98
99	.172- Ø	.2Ø2- Ø	.235- Ø	.271- Ø	.3Ø8- Ø	.348- Ø	.389- Ø	.431- Ø	.473- Ø	.516- Ø	99
1ØØ	.146- Ø	.173- Ø	.2Ø3- Ø	.236- Ø	.272- Ø	.3Ø9- Ø	.349- Ø	.389- Ø	.431- Ø	.473- Ø	1ØØ
1Ø1	.123- Ø	.147- Ø	.174- Ø	.2Ø5- Ø	.238- Ø	.273- Ø	.31Ø- Ø	.349- Ø	.39Ø- Ø	.432- Ø	1Ø1
1Ø2	.1Ø2- Ø	.124- Ø	.148- Ø	.176- Ø	.2Ø6- Ø	.239- Ø	.274- Ø	.311- Ø	.35Ø- Ø	.391- Ø	1Ø2
1Ø3	.844- 1	.1Ø3- Ø	.125- Ø	.15Ø- Ø	.177- Ø	.2Ø7- Ø	.24Ø- Ø	.275- Ø	.312- Ø	.351- Ø	1Ø3
1Ø4	.69Ø- 1	.855- 1	.1Ø4- Ø	.126- Ø	.151- Ø	.178- Ø	.2Ø8- Ø	.241- Ø	.276- Ø	.313- Ø	1Ø4
1Ø5	.56Ø- 1	.7Ø1- 1	.865- 1	.1Ø6- Ø	.128- Ø	.152- Ø	.18Ø- Ø	.21Ø- Ø	.242- Ø	.277- Ø	1Ø5
1Ø6	.45Ø- 1	.57Ø- 1	.71Ø- 1	.878- 1	.1Ø7- Ø	.129- Ø	.153- Ø	.181- Ø	.211- Ø	.243- Ø	1Ø6
1Ø7	.358- 1	.459- 1	.578- 1	.722- 1	.889- 1	.1Ø8- Ø	.13Ø- Ø	.155- Ø	.182- Ø	.212- Ø	1Ø7
1Ø8	.283- 1	.366- 1	.466- 1	.588- 1	.732- 1	.9ØØ- 1	.1Ø9- Ø	.131- Ø	.156- Ø	.183- Ø	1Ø8
1Ø9	.221- 1	.289- 1	.372- 1	.475- 1	.598- 1	.742- 1	.911- 1	.11Ø- Ø	.132- Ø	.157- Ø	1Ø9
11Ø	.171- 1	.227- 1	.295- 1	.381- 1	.484- 1	.6Ø7- 1	.752- 1	.922- 1	.112- Ø	.134- Ø	11Ø
111	.131- 1	.176- 1	.231- 1	.3Ø2- 1	.388- 1	.492- 1	.616- 1	.763- 1	.933- 1	.113- Ø	111
112	.1ØØ- 1	.135- 1	.18Ø- 1	.238- 1	.3Ø9- 1	.396- 1	.5Ø1- 1	.626- 1	.773- 1	.943- 1	112
113	.754- 2	.1Ø3- 1	.139- 1	.186- 1	.244- 1	.315- 1	.4Ø3- 1	.5Ø9- 1	.635- 1	.783- 1	113
114	.563- 2	.781- 2	.1Ø6- 1	.143- 1	.19Ø- 1	.249- 1	.322- 1	.411- 1	.518- 1	.645- 1	114
115	.417- 2	.585- 2	.799- 2	.11Ø- 1	.147- 1	.195- 1	.255- 1	.329- 1	.418- 1	.526- 1	115
116	.3Ø6- 2	.434- 2	.598- 2	.835- 2	.113- 1	.152- 1	.2ØØ- 1	.261- 1	.335- 1	.426- 1	116
117	.223- 2	.32Ø- 2	.443- 2	.629- 2	.863- 2	.117- 1	.156- 1	.2Ø5- 1	.266- 1	.342- 1	117
118	.16Ø- 2	.233- 2	.324- 2	.469- 2	.651- 2	.891- 2	.12Ø- 1	.16Ø- 1	.21Ø- 1	.272- 1	118
119	.115- 2	.168- 2	.235- 2	.347- 2	.487- 2	.674- 2	.919- 2	.124- 1	.164- 1	.215- 1	119

R	N=Ø900	N=Ø910	N=Ø920	N=Ø930	N=Ø940	N=Ø950	N=Ø960	N=Ø970	N=Ø980	N=Ø990	R	P=1/1Ø
12Ø	.812- 3	.121- 2	.168- 2	.255- 2	.361- 2	.5Ø6- 2	.697- 2	.948- 2	.127- 1	.168- 1	12Ø	
121	.569- 3	.857- 3	.127- 2	.185- 2	.266- 2	.376- 2	.524- 2	.721- 2	.977- 2	.131- 1	121	
122	.396- 3	.6Ø3- 3	.9Ø4- 3	.133- 2	.194- 2	.277- 2	.391- 2	.543- 2	.744- 2	.1Ø1- 1	122	
123	.273- 3	.421- 3	.639- 3	.953- 3	.14Ø- 2	.2Ø3- 2	.289- 2	.4Ø6- 2	.562- 2	.769- 2	123	
124	.187- 3	.291- 3	.447- 3	.675- 3	.1ØØ- 2	.147- 2	.212- 2	.3Ø1- 2	.421- 2	.582- 2	124	
125	.126- 3	.2ØØ- 3	.31Ø- 3	.474- 3	.713- 3	.1Ø6- 2	.154- 2	.221- 2	.313- 2	.437- 2	125	
126	.849- 4	.136- 3	.214- 3	.33Ø- 3	.5Ø2- 3	.752- 3	.111- 2	.161- 2	.231- 2	.326- 2	126	
127	.565- 4	.915- 4	.146- 3	.228- 3	.351- 3	.532- 3	.793- 3	.117- 2	.169- 2	.241- 2	127	
128	.373- 4	.612- 4	.985- 4	.156- 3	.243- 3	.373- 3	.562- 3	.835- 3	.122- 2	.176- 2	128	
129	.244- 4	.4Ø5- 4	.661- 4	.1Ø6- 3	.167- 3	.259- 3	.395- 3	.594- 3	.878- 3	.128- 2	129	
13Ø	.158- 4	.266- 4	.439- 4	.713- 4	.114- 3	.178- 3	.275- 3	.418- 3	.626- 3	.923- 3	13Ø	
131	.1Ø2- 4	.173- 4	.29Ø- 4	.476- 4	.768- 4	.122- 3	.19Ø- 3	.293- 3	.443- 3	.66Ø- 3	131	
132	.65Ø- 5	.112- 4	.189- 4	.315- 4	.514- 4	.826- 4	.13Ø- 3	.2Ø3- 3	.31Ø- 3	.468- 3	132	
133	.411- 5	.717- 5	.123- 4	.2Ø7- 4	.342- 4	.555- 4	.887- 4	.14Ø- 3	.216- 3	.329- 3	133	
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138	.367- 6	.682- 6	.124- 5	.222- 5	.39Ø- 5	.673- 5	.114- 4	.19Ø- 4	.311- 4	.5Ø2- 4	138	
139	.221- 6	.416- 6	.767- 6	.139- 5	.247- 5	.431- 5	.739- 5	.125- 4	.2Ø6- 4	.337- 4	139	
14Ø	.132- 6	.251- 6	.469- 6	.86Ø- 6	.155- 5	.273- 5	.474- 5	.8Ø9- 5	.136- 4	.224- 4	14Ø	
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142	.458- 7	.896- 7	.172- 6	.322- 6	.595- 6	.1Ø8- 5	.191- 5	.334- 5	.573- 5	.967- 5	142	
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155	.22Ø-1Ø	.5Ø9-1Ø	.115- 9	.255- 9	.552- 9	.117- 8	.244- 8	.497- 8	.993- 8	.195- 7	155	
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157		.143-1Ø	.333-1Ø	.756-1Ø	.168- 9	.366- 9	.78Ø- 9	.163- 8	.334- 8	.67Ø- 8	157	
158			.177-1Ø	.4Ø8-1Ø	.917-1Ø	.2Ø2- 9	.436- 9	.922- 9	.191- 8	.389- 8	158	
159				.218-1Ø	.497-1Ø	.111- 9	.242- 9	.518- 9	.1Ø9- 8	.224- 8	159	
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162						.175-1Ø	.397-1Ø	.882-1Ø	.192- 9	.4Ø9- 9	162	
163							.214-1Ø	.481-1Ø	.1Ø6- 9	.229- 9	163	
164							.115-1Ø	.261-1Ø	.582-1Ø	.127- 9	164	
165								.141-1Ø	.317-1Ø	.7Ø1-1Ø	165	
166									.171-1Ø	.384-1Ø	166	
167										.2Ø9-1Ø	167	
168										.113-1Ø	168	

R	N=1ØØØ	N=1Ø1Ø	N=1Ø2Ø	N=1Ø3Ø	N=1Ø4Ø	N=1Ø5Ø	N=1Ø6Ø	N=1Ø7Ø	N=1Ø8Ø	N=1Ø9Ø	R
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72	.999- Ø	.999- Ø									72
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74	.998- Ø	.999- Ø	.999- Ø	.999- Ø							74
75	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø					75
76	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø				76
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79	.99Ø- Ø	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	79
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86	.939- Ø	.951- Ø	.96Ø- Ø	.968- Ø	.975- Ø	.98Ø- Ø	.984- Ø	.988- Ø	.991- Ø	.993- Ø	86
87	.925- Ø	.938- Ø	.95Ø- Ø	.959- Ø	.967- Ø	.974- Ø	.979- Ø	.984- Ø	.987- Ø	.99Ø- Ø	87
88	.9Ø8- Ø	.924- Ø	.937- Ø	.949- Ø	.959- Ø	.967- Ø	.973- Ø	.979- Ø	.983- Ø	.987- Ø	88
89	.889- Ø	.9Ø7- Ø	.923- Ø	.936- Ø	.948- Ø	.958- Ø	.966- Ø	.973- Ø	.978- Ø	.983- Ø	89
9Ø	.867- Ø	.887- Ø	.9Ø6- Ø	.922- Ø	.935- Ø	.947- Ø	.957- Ø	.965- Ø	.972- Ø	.978- Ø	9Ø
91	.842- Ø	.865- Ø	.886- Ø	.9Ø5- Ø	.921- Ø	.934- Ø	.946- Ø	.956- Ø	.965- Ø	.972- Ø	91
92	.814- Ø	.841- Ø	.864- Ø	.885- Ø	.9Ø4- Ø	.92Ø- Ø	.934- Ø	.945- Ø	.955- Ø	.964- Ø	92
93	.784- Ø	.813- Ø	.839- Ø	.863- Ø	.884- Ø	.9Ø2- Ø	.919- Ø	.933- Ø	.945- Ø	.955- Ø	93
94	.751- Ø	.783- Ø	.812- Ø	.838- Ø	.862- Ø	.883- Ø	.9Ø1- Ø	.918- Ø	.932- Ø	.944- Ø	94
95	.716- Ø	.75Ø- Ø	.782- Ø	.811- Ø	.837- Ø	.861- Ø	.882- Ø	.9ØØ- Ø	.917- Ø	.931- Ø	95
96	.678- Ø	.715- Ø	.749- Ø	.78Ø- Ø	.8Ø9- Ø	.836- Ø	.859- Ø	.88Ø- Ø	.899- Ø	.916- Ø	96
97	.639- Ø	.678- Ø	.714- Ø	.748- Ø	.779- Ø	.8Ø8- Ø	.835- Ø	.858- Ø	.879- Ø	.898- Ø	97
98	.599- Ø	.639- Ø	.677- Ø	.713- Ø	.747- Ø	.778- Ø	.8Ø7- Ø	.833- Ø	.857- Ø	.878- Ø	98
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103	.391- 0	.432- 0	.474- 0	.515- 0	.556- 0	.596- 0	.635- 0	.673- 0	.708- 0	.742- 0	103
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116	.535- 1	.663- 1	.813- 1	.986- 1	.118- 0	.140- 0	.165- 0	.192- 0	.222- 0	.253- 0	116
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127	.338- 2	.470- 2	.643- 2	.869- 2	.116- 1	.153- 1	.199- 1	.256- 1	.325- 1	.409- 1	127
128	.251- 2	.352- 2	.486- 2	.664- 2	.894- 2	.119- 1	.156- 1	.203- 1	.261- 1	.331- 1	128
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131	.970- 3	.140- 2	.200- 2	.282- 2	.392- 2	.538- 2	.728- 2	.973- 2	.129- 1	.168- 1	131
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173				.133-10	.292-10	.630-10	.134- 9	.278- 9	.568- 9	.114- 8	173
174					.160-10	.350-10	.750-10	.158- 9	.326- 9	.661- 9	174
175						.193-10	.418-10	.890-10	.186- 9	.381- 9	175
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178								.153-10	.331-10	.702-10	178
179									.184-10	.394-10	179
180									.101-10	.220-10	180
181										.122-10	181

R	N=1100	N=1110	N=1120	N=1130	N=1140	N=1150	N=1160	N=1170	N=1180	N=1190	R	P=1/10
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8Ø	.999- Ø	.999- Ø									8Ø	
81	.999- Ø	.999- Ø	.999- Ø								81	
82	.999- Ø	.999- Ø	.999- Ø	.999- Ø							82	
83	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø						83	
84	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø					84	
85	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø				85	
86	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø			86	
87	.992- Ø	.994- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	.999- Ø	87	
88	.99Ø- Ø	.992- Ø	.994- Ø	.995- Ø	.997- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	.999- Ø	88	
89	.987- Ø	.99Ø- Ø	.992- Ø	.994- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.999- Ø	.999- Ø	89	
9Ø	.982- Ø	.986- Ø	.989- Ø	.992- Ø	.994- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	.998- Ø	9Ø	
91	.977- Ø	.982- Ø	.986- Ø	.989- Ø	.991- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	.998- Ø	91	
92	.971- Ø	.977- Ø	.982- Ø	.986- Ø	.989- Ø	.991- Ø	.993- Ø	.995- Ø	.996- Ø	.997- Ø	92	
93	.963- Ø	.97Ø- Ø	.976- Ø	.981- Ø	.985- Ø	.988- Ø	.991- Ø	.993- Ø	.995- Ø	.996- Ø	93	
94	.954- Ø	.963- Ø	.97Ø- Ø	.976- Ø	.981- Ø	.985- Ø	.988- Ø	.991- Ø	.993- Ø	.994- Ø	94	
95	.943- Ø	.953- Ø	.962- Ø	.969- Ø	.975- Ø	.98Ø- Ø	.984- Ø	.988- Ø	.99Ø- Ø	.993- Ø	95	
96	.93Ø- Ø	.942- Ø	.952- Ø	.961- Ø	.969- Ø	.975- Ø	.98Ø- Ø	.984- Ø	.987- Ø	.99Ø- Ø	96	
97	.914- Ø	.929- Ø	.941- Ø	.952- Ø	.96Ø- Ø	.968- Ø	.974- Ø	.979- Ø	.984- Ø	.987- Ø	97	
98	.897- Ø	.913- Ø	.928- Ø	.94Ø- Ø	.951- Ø	.96Ø- Ø	.967- Ø	.974- Ø	.979- Ø	.983- Ø	98	
99	.877- Ø	.896- Ø	.913- Ø	.927- Ø	.939- Ø	.95Ø- Ø	.959- Ø	.967- Ø	.973- Ø	.978- Ø	99	
1ØØ	.855- Ø	.876- Ø	.895- Ø	.912- Ø	.926- Ø	.939- Ø	.949- Ø	.958- Ø	.966- Ø	.973- Ø	1ØØ	
1Ø1	.83Ø- Ø	.854- Ø	.875- Ø	.894- Ø	.911- Ø	.925- Ø	.938- Ø	.948- Ø	.958- Ø	.965- Ø	1Ø1	
1Ø2	.8Ø3- Ø	.829- Ø	.853- Ø	.874- Ø	.893- Ø	.91Ø- Ø	.924- Ø	.937- Ø	.948- Ø	.957- Ø	1Ø2	
1Ø3	.773- Ø	.8Ø1- Ø	.828- Ø	.851- Ø	.873- Ø	.892- Ø	.909- Ø	.923- Ø	.936- Ø	.947- Ø	1Ø3	
1Ø4	.741- Ø	.772- Ø	.8ØØ- Ø	.827- Ø	.85Ø- Ø	.872- Ø	.891- Ø	.9Ø8- Ø	.922- Ø	.935- Ø	1Ø4	
1Ø5	.7Ø7- Ø	.74Ø- Ø	.771- Ø	.799- Ø	.825- Ø	.849- Ø	.871- Ø	.89Ø- Ø	.9Ø7- Ø	.921- Ø	1Ø5	
1Ø6	.671- Ø	.7Ø6- Ø	.739- Ø	.77Ø- Ø	.798- Ø	.824- Ø	.848- Ø	.87Ø- Ø	.889- Ø	.9Ø6- Ø	1Ø6	
1Ø7	.633- Ø	.67Ø- Ø	.7Ø5- Ø	.738- Ø	.769- Ø	.797- Ø	.823- Ø	.847- Ø	.869- Ø	.888- Ø	1Ø7	
1Ø8	.594- Ø	.632- Ø	.669- Ø	.7Ø4- Ø	.737- Ø	.768- Ø	.796- Ø	.822- Ø	.846- Ø	.867- Ø	1Ø8	
1Ø9	.555- Ø	.594- Ø	.632- Ø	.668- Ø	.7Ø3- Ø	.736- Ø	.767- Ø	.795- Ø	.821- Ø	.845- Ø	1Ø9	
11Ø	.515- Ø	.554- Ø	.593- Ø	.631- Ø	.668- Ø	.7Ø2- Ø	.735- Ø	.766- Ø	.794- Ø	.82Ø- Ø	11Ø	
111	.475- Ø	.515- Ø	.554- Ø	.593- Ø	.631- Ø	.667- Ø	.7Ø2- Ø	.734- Ø	.765- Ø	.793- Ø	111	
112	.435- Ø	.475- Ø	.515- Ø	.554- Ø	.593- Ø	.63Ø- Ø	.666- Ø	.7Ø1- Ø	.733- Ø	.764- Ø	112	
113	.396- Ø	.435- Ø	.475- Ø	.515- Ø	.554- Ø	.592- Ø	.63Ø- Ø	.666- Ø	.7ØØ- Ø	.732- Ø	113	
114	.358- Ø	.396- Ø	.436- Ø	.475- Ø	.514- Ø	.554- Ø	.592- Ø	.629- Ø	.665- Ø	.699- Ø	114	
115	.322- Ø	.359- Ø	.397- Ø	.436- Ø	.475- Ø	.514- Ø	.553- Ø	.591- Ø	.629- Ø	.664- Ø	115	
116	.287- Ø	.322- Ø	.359- Ø	.397- Ø	.436- Ø	.475- Ø	.514- Ø	.553- Ø	.591- Ø	.628- Ø	116	
117	.254- Ø	.288- Ø	.323- Ø	.36Ø- Ø	.398- Ø	.436- Ø	.475- Ø	.514- Ø	.553- Ø	.591- Ø	117	
118	.224- Ø	.255- Ø	.289- Ø	.324- Ø	.361- Ø	.398- Ø	.437- Ø	.475- Ø	.514- Ø	.553- Ø	118	
119	.195- Ø	.225- Ø	.256- Ø	.29Ø- Ø	.325- Ø	.361- Ø	.399- Ø	.437- Ø	.476- Ø	.514- Ø	119	
12Ø	.169- Ø	.196- Ø	.226- Ø	.257- Ø	.29Ø- Ø	.325- Ø	.362- Ø	.399- Ø	.437- Ø	.476- Ø	12Ø	
121	.146- Ø	.171- Ø	.198- Ø	.227- Ø	.258- Ø	.291- Ø	.326- Ø	.362- Ø	.4ØØ- Ø	.437- Ø	121	
122	.125- Ø	.147- Ø	.172- Ø	.199- Ø	.228- Ø	.259- Ø	.292- Ø	.327- Ø	.363- Ø	.4ØØ- Ø	122	
123	.1Ø6- Ø	.126- Ø	.148- Ø	.173- Ø	.2ØØ- Ø	.229- Ø	.26Ø- Ø	.293- Ø	.328- Ø	.363- Ø	123	
124	.891- 1	.1Ø7- Ø	.127- Ø	.149- Ø	.174- Ø	.2Ø1- Ø	.23Ø- Ø	.261- Ø	.294- Ø	.328- Ø	124	
125	.744- 1	.9ØØ- 1	.1Ø8- Ø	.128- Ø	.15Ø- Ø	.175- Ø	.2Ø2- Ø	.231- Ø	.262- Ø	.294- Ø	125	
126	.618- 1	.753- 1	.91Ø- 1	.1Ø9- Ø	.129- Ø	.151- Ø	.176- Ø	.2Ø3- Ø	.232- Ø	.262- Ø	126	
127	.5Ø8- 1	.626- 1	.762- 1	.919- 1	.11Ø- Ø	.13Ø- Ø	.152- Ø	.177- Ø	.2Ø4- Ø	.232- Ø	127	
128	.415- 1	.516- 1	.634- 1	.771- 1	.929- 1	.111- Ø	.131- Ø	.153- Ø	.178- Ø	.2Ø5- Ø	128	
129	.337- 1	.422- 1	.523- 1	.642- 1	.78Ø- 1	.938- 1	.112- Ø	.132- Ø	.154- Ø	.179- Ø	129	
13Ø	.271- 1	.343- 1	.429- 1	.531- 1	.65Ø- 1	.789- 1	.947- 1	.113- Ø	.133- Ø	.155- Ø	13Ø	
131	.217- 1	.276- 1	.349- 1	.436- 1	.538- 1	.658- 1	.797- 1	.956- 1	.114- Ø	.134- Ø	131	
132	.172- 1	.221- 1	.282- 1	.355- 1	.442- 1	.546- 1	.666- 1	.8Ø6- 1	.967- 1	.115- Ø	132	
133	.135- 1	.176- 1	.226- 1	.287- 1	.361- 1	.449- 1	.553- 1	.674- 1	.816- 1	.975- 1	133	
134	.1Ø6- 1	.138- 1	.18Ø- 1	.23Ø- 1	.292- 1	.367- 1	.456- 1	.56Ø- 1	.684- 1	.823- 1	134	
135	.818- 2	.1Ø8- 1	.142- 1	.184- 1	.235- 1	.297- 1	.373- 1	.462- 1	.569- 1	.69Ø- 1	135	
136	.629- 2	.841- 2	.111- 1	.145- 1	.187- 1	.24Ø- 1	.3Ø3- 1	.379- 1	.47Ø- 1	.575- 1	136	
137	.48Ø- 2	.648- 2	.864- 2	.114- 1	.149- 1	.191- 1	.244- 1	.3Ø8- 1	.386- 1	.476- 1	137	
138	.364- 2	.496- 2	.667- 2	.888- 2	.117- 1	.152- 1	.195- 1	.249- 1	.315- 1	.391- 1	138	
139	.273- 2	.376- 2	.511- 2	.687- 2	.912- 2	.12Ø- 1	.155- 1	.199- 1	.253- 1	.319- 1	139	
14Ø	.2Ø4- 2	.284- 2	.389- 2	.527- 2	.7Ø7- 2	.936- 2	.123- 1	.159- 1	.2Ø4- 1	.258- 1	14Ø	
141	.151- 2	.212- 2	.294- 2	.4Ø2- 2	.544- 2	.727- 2	.96Ø- 2	.126- 1	.162- 1	.2Ø8- 1	141	
142	.111- 2	.157- 2	.22Ø- 2	.3Ø4- 2	.415- 2	.56Ø- 2	.747- 2	.985- 2	.128- 1	.166- 1	142	
143	.811- 3	.116- 2	.164- 2	.229- 2	.315- 2	.429- 2	.576- 2	.767- 2	.1Ø1- 1	.131- 1	143	
144	.588- 3	.849- 3	.121- 2	.17Ø- 2	.237- 2	.326- 2	.442- 2	.594- 2	.788- 2	.1Ø4- 1	144	
145	.423- 3	.616- 3	.887- 3	.126- 2	.177- 2	.246- 2	.336- 2	.456- 2	.611- 2	.8Ø9- 2	145	
146	.3Ø2- 3	.444- 3	.646- 3	.927- 3	.131- 2	.184- 2	.254- 2	.348- 2	.47Ø- 2	.628- 2	146	
147	.214- 3	.318- 3	.467- 3	.676- 3	.968- 3	.137- 2	.19Ø- 2	.263- 2	.359- 2	.485- 2	147	
148	.15Ø- 3	.226- 3	.335- 3	.49Ø- 3	.7Ø8- 3	.1Ø1- 2	.142- 2	.198- 2	.273- 2	.371- 2	148	
149	.1Ø5- 3	.159- 3	.238- 3	.352- 3	.514- 3	.74Ø- 3	.1Ø5- 2	.148- 2	.2Ø5- 2	.282- 2	149	
15Ø	.727- 4	.111- 3	.169- 3	.251- 3	.37Ø- 3	.538- 3	.773- 3	.11Ø- 2	.154- 2	.213- 2	15Ø	
151	.5ØØ- 4	.775- 4	.118- 3	.178- 3	.265- 3	.389- 3	.564- 3	.8Ø7- 3	.114- 2	.16Ø- 2	151	
152	.342- 4	.534- 4	.824- 4	.125- 3	.188- 3	.279- 3	.4Ø8- 3	.59Ø- 3	.842- 3	.119- 2	152	
153	.232- 4	.366- 4	.57Ø- 4	.876- 4	.133- 3	.199- 3	.294- 3	.428- 3	.617- 3	.878- 3	153	
154	.156- 4	.249- 4	.392- 4	.6Ø8- 4	.931- 4	.141- 3	.21Ø- 3	.3Ø9- 3	.449- 3	.645- 3	154	
155	.1Ø4- 4	.168- 4	.267- 4	.419- 4	.648- 4	.988- 4	.149- 3	.221- 3	.324- 3	.47Ø- 3	155	
156	.693- 5	.113- 4	.181- 4	.287- 4	.447- 4	.689- 4	.1Ø5- 3	.157- 3	.233- 3	.341- 3	156	
157	.457- 5	.751- 5	.122- 4	.195- 4	.3Ø7- 4	.477- 4	.732- 4	.111- 3	.166- 3	.245- 3	157	
158	.299- 5	.497- 5	.813- 5	.131- 4	.2Ø9- 4	.328- 4	.5Ø9- 4	.778- 4	.117- 3	.175- 3	158	
159	.194- 5	.326- 5	.539- 5	.879- 5	.141- 4	.224- 4	.351- 4	.541- 4	.825- 4	.124- 3	159	
16Ø	.125- 5	.213- 5	.355- 5	.585- 5	.95Ø- 5	.152- 4	.24Ø- 4	.374- 4	.576- 4	.874- 4	16Ø	
161	.8Ø3- 6	.138- 5	.232- 5	.386- 5	.634- 5	.1Ø2- 4	.163- 4	.257- 4	.399- 4	.612- 4	161	
162	.511- 6	.884- 6	.151- 5	.254- 5	.42Ø- 5	.686- 5	.11Ø- 4	.175- 4	.275- 4	.425- 4	162	
163	.323- 6	.565- 6	.973- 6	.165- 5	.276- 5	.456- 5	.741- 5	.119- 4	.188- 4	.294- 4	163	

R	N=11ØØ	N=111Ø	N=112Ø	N=113Ø	N=114Ø	N=115Ø	N=116Ø	N=117Ø	N=118Ø	N=119Ø	R	P=1/1Ø
164	.2Ø3- 6	.358- 6	.623- 6	.1Ø7- 5	.181- 5	.3Ø1- 5	.494- 5	.8ØØ- 5	.128- 4	.2Ø1- 4	164	
165	.126- 6	.225- 6	.397- 6	.687- 6	.117- 5	.197- 5	.327- 5	.534- 5	.862- 5	.137- 4	165	
166	.782- 7	.141- 6	.251- 6	.438- 6	.756- 6	.128- 5	.215- 5	.355- 5	.578- 5	.928- 5	166	
167	.481- 7	.876- 7	.157- 6	.278- 6	.484- 6	.83Ø- 6	.14Ø- 5	.234- 5	.385- 5	.624- 5	167	
168	.293- 7	.541- 7	.98Ø- 7	.175- 6	.3Ø8- 6	.533- 6	.911- 6	.153- 5	.255- 5	.417- 5	168	
169	.178- 7	.331- 7	.6Ø7- 7	.1Ø9- 6	.194- 6	.34Ø- 6	.587- 6	.998- 6	.167- 5	.276- 5	169	
17Ø	.1Ø7- 7	.2Ø2- 7	.373- 7	.68Ø- 7	.122- 6	.216- 6	.376- 6	.645- 6	.1Ø9- 5	.182- 5	17Ø	
171	.641- 8	.122- 7	.228- 7	.42Ø- 7	.761- 7	.136- 6	.239- 6	.414- 6	.7Ø8- 6	.119- 5	171	
172	.381- 8	.732- 8	.138- 7	.257- 7	.471- 7	.85Ø- 7	.151- 6	.264- 6	.456- 6	.776- 6	172	
173	.225- 8	.437- 8	.834- 8	.157- 7	.29Ø- 7	.528- 7	.948- 7	.168- 6	.292- 6	.5Ø1- 6	173	
174	.132- 8	.259- 8	.499- 8	.949- 8	.177- 7	.326- 7	.591- 7	.1Ø5- 6	.186- 6	.322- 6	174	
175	.769- 9	.152- 8	.297- 8	.57Ø- 8	.1Ø8- 7	.2ØØ- 7	.366- 7	.66Ø- 7	.117- 6	.2Ø5- 6	175	
176	.445- 9	.891- 9	.176- 8	.34Ø- 8	.649- 8	.122- 7	.225- 7	.41Ø- 7	.736- 7	.13Ø- 6	176	
177	.256- 9	.518- 9	.1Ø3- 8	.2Ø2- 8	.389- 8	.738- 8	.138- 7	.253- 7	.459- 7	.819- 7	177	
178	.146- 9	.299- 9	.6Ø2- 9	.119- 8	.232- 8	.444- 8	.837- 8	.155- 7	.284- 7	.513- 7	178	
179	.829-1Ø	.171- 9	.349- 9	.697- 9	.137- 8	.265- 8	.5Ø5- 8	.948- 8	.175- 7	.319- 7	179	
18Ø	.467-1Ø	.977-1Ø	.2Ø1- 9	.4Ø6- 9	.8Ø6- 9	.158- 8	.3Ø3- 8	.574- 8	.1Ø7- 7	.197- 7	18Ø	
181	.262-1Ø	.553-1Ø	.115- 9	.235- 9	.471- 9	.93Ø- 9	.181- 8	.346- 8	.651- 8	.121- 7	181	
182	.146-1Ø	.311-1Ø	.653-1Ø	.135- 9	.273- 9	.545- 9	.1Ø7- 8	.2Ø7- 8	.393- 8	.737- 8	182	
183		.174-1Ø	.369-1Ø	.769-1Ø	.158- 9	.318- 9	.63Ø- 9	.123- 8	.236- 8	.447- 8	183	
184			.2Ø7-1Ø	.436-1Ø	.9Ø3-1Ø	.184- 9	.368- 9	.726- 9	.141- 8	.269- 8	184	
185			.116-1Ø	.246-1Ø	.514-1Ø	.1Ø6- 9	.214- 9	.426- 9	.836- 9	.161- 8	185	
186				.138-1Ø	.291-1Ø	.6Ø5-1Ø	.124- 9	.249- 9	.492- 9	.96Ø- 9	186	
187					.164-1Ø	.344-1Ø	.71Ø-1Ø	.144- 9	.288- 9	.567- 9	187	
188						.194-1Ø	.4Ø5-1Ø	.83Ø-1Ø	.168- 9	.333- 9	188	
189						.1Ø9-1Ø	.229-1Ø	.475-1Ø	.97Ø-1Ø	.195- 9	189	
19Ø							.129-1Ø	.271-1Ø	.557-1Ø	.113- 9	19Ø	
191								.153-1Ø	.318-1Ø	.652-1Ø	191	
192									.181-1Ø	.374-1Ø	192	
193									.1Ø2-1Ø	.213-1Ø	193	
194										.121-1Ø	194	

R	N=12ØØ	R
88	.999- Ø	88
89	.999- Ø	89
9Ø	.999- Ø	9Ø
91	.998- Ø	91
92	.998- Ø	92
93	.997- Ø	93
94	.996- Ø	94
95	.994- Ø	95
96	.992- Ø	96
97	.99Ø- Ø	97
98	.987- Ø	98
99	.983- Ø	99
1ØØ	.978- Ø	1ØØ
1Ø1	.972- Ø	1Ø1
1Ø2	.965- Ø	1Ø2
1Ø3	.956- Ø	1Ø3
1Ø4	.946- Ø	1Ø4
1Ø5	.934- Ø	1Ø5
1Ø6	.921- Ø	1Ø6
1Ø7	.9Ø5- Ø	1Ø7
1Ø8	.887- Ø	1Ø8
1Ø9	.866- Ø	1Ø9
11Ø	.844- Ø	11Ø
111	.819- Ø	111
112	.792- Ø	112
113	.763- Ø	113
114	.732- Ø	114
115	.698- Ø	115
116	.664- Ø	116
117	.628- Ø	117
118	.59Ø- Ø	118
119	.552- Ø	119
12Ø	.514- Ø	12Ø
121	.476- Ø	121
122	.438- Ø	122
123	.4ØØ- Ø	123
124	.364- Ø	124
125	.329- Ø	125
126	.295- Ø	126
127	.263- Ø	127
128	.233- Ø	128
129	.2Ø5- Ø	129
13Ø	.18Ø- Ø	13Ø
131	.156- Ø	131
132	.135- Ø	132
133	.116- Ø	133
134	.984- 1	134
135	.832- 1	135
136	.699- 1	136

R	N=1200	R	P=1/10
137	.583- 1	137	
138	.482- 1	138	
139	.397- 1	139	
140	.324- 1	140	
141	.263- 1	141	
142	.212- 1	142	
143	.169- 1	143	
144	.134- 1	144	
145	.106- 1	145	
146	.831- 2	146	
147	.646- 2	147	
148	.499- 2	148	
149	.383- 2	149	
150	.292- 2	150	
151	.221- 2	151	
152	.166- 2	152	
153	.124- 2	153	
154	.916- 3	154	
155	.674- 3	155	
156	.492- 3	156	
157	.357- 3	157	
158	.258- 3	158	
159	.184- 3	159	
160	.131- 3	160	
161	.926- 4	161	
162	.650- 4	162	
163	.453- 4	163	
164	.313- 4	164	
165	.215- 4	165	
166	.147- 4	166	
167	.999- 5	167	
168	.673- 5	168	
169	.451- 5	169	
170	.300- 5	170	
171	.198- 5	171	
172	.130- 5	172	
173	.849- 6	173	
174	.550- 6	174	
175	.354- 6	175	
176	.227- 6	176	
177	.144- 6	177	
178	.910- 7	178	
179	.571- 7	179	
180	.356- 7	180	
181	.221- 7	181	
182	.136- 7	182	
183	.833- 8	183	
184	.507- 8	184	
185	.306- 8	185	
186	.184- 8	186	
187	.110- 8	187	
188	.652- 9	188	
189	.384- 9	189	
190	.225- 9	190	
191	.131- 9	191	
192	.760-10	192	
193	.438-10	193	
194	.250-10	194	
195	.142-10	195	

APPENDIX 5

PLAINTEXT AND RANDOM MATERIAL FOR SAMPLING PURPOSES

A. 1000 Letters of English Plain Text
B. 1000 Letters with Theoretical English Plaintext Frequencies
C. 1000 Letters of Random Text
D. 1000 Digits of Random Text
E. 1000 Characters of Random Text (32-Element Alphabet)
F. 25 Random 26-Letter Alphabets

* * * * *

In the study of a cryptographic system, the cryptanalyst often finds it expedient to encipher a known plain text and then examine the phenomena which arise. Or again, the analyst may use a certain kind of text as key and observe its effect in the resultant cipher text. This Appendix presents samples of the principal types of text useful in experimentation.

PLAINTEXT AND RANDOM MATERIAL FOR SAMPLING PURPOSES

A. 1000 Letters of English Plain Text

T H E F A	C T T H A	T T H E S	C I E N T	I F I C I	N V E S T
I G A T O	R W O R K	S F I F T	Y P E R C	E N T O F	H I S T I
M E B Y N	O N R A T	I O N A L	M E A N S	I S C M A	I T S E E
M S C M A	Q U I T E	I N S U F	F I C I E	N T L Y R	E C O G N
I Z E D P	D T H E R	E I S W I	T H O U T	T H E L E	A S T D O
U B T A N	I N S T I	N C T F O	R R E S E	A R C H C	M A A N D
O F T E N	T H E M O	S T S U C	C E S S F	U L I N V	E S T I G
A T O R S	O F N A T	U R E A R	E Q U I T	E U N A B	L E T O G
I V E A N	A C C O U	N T O F T	H E I R R	E A S O N	S F O R D
O I N G S	U C H A N	D S U C H	A N E X P	E R I M E	N T C M A
O R F O R	P L A C I	N G S I D	E B Y S I	D E T W O	A P P A R
E N T L Y	U N R E L	A T E D F	A C T S P	D A G A I	N C M A O
N E O F T	H E M O S	T S A L I	E N T T R	A I T S I	N T H E C
H A R A C	T E R O F	T H E S U	C C E S S	F U L S C	I E N T I
F I C W O	R K E R I	S T H E C	A P A C I	T Y F O R	K N O W I
N G T H A	T A P O I	N T I S P	R O V E D	W H E N I	T W O U L
D N O T A	P P E A R	T O B E P	R O V E D	T O A N O	U T S I D
E I N T E	L L I G E	N C E F U	N C T I O	N I N G I	N A P U R
E L Y R A	T I O N A	L M A N N	E R S E M	I C O L O	N T H U S
T H E I N	V E S T I	G A T O R	F E E L S	T H A T S	O M E P R
O P O S I	T I O N I	S T R U E	C M A A N	D P R O C	E E D S A
T O N C E	T O T H E	N E X T S	E T O F E	X P E R I	M E N T S
W I T H O	U T W A I	T I N G A	N D W A S	T I N G T	I M E I N
T H E E L	A B O R A	T I O N O	F T H E F	O R M A L	P R O O F
O F T H E	P O I N T	W H I C H	H E A V I	E R M I N	D S W O U
L D N E E	D P D Q U	E S T I O	N L E S S	S U C H A	S C I E N
T I F I C	I N T U I	T I O N M	A Y A N D	D O E S S	O M E T I
M E S L E	A D I N V	E S T I G	A T O R S	A S T R A	Y C M A B
U T I T I	S Q U I T	E C E R T	A I N T H	A T I F T	H E Y D I
D N O T W	I D E L Y	M A K E U	S E O F I	T C M A T	H E Y W O
U L D N O	T G E T A	Q U A R T	E R A S F	A R A S T	H E Y D O
P D E X P	E R I M E	N T S C O	N F I R M	E A C H O	T H E R C
M A A N D	A F A L S	E S T E P	I S U S U	A L L Y S	O O N D I
S C O V E	R E D P D				

Distribution:

A	81	G	15	L	27	Q	5	V	9
B	7	H	37	M	28	R	51	W	14
C	46	I	90	N	76	S	70	X	4
D	34	J	0	O	75	T	109	Y	14
E	112	K	4	P	25	U	33	Z	1
F	33								

B. 1000 Letters with Theoretical English Plaintext Frequencies

O I N S W	R T R O I	Y D N P D	A T T L S	C V U E A	T Z F D R
P A E P S	L L M S I	N A I S E	E S C E T	L M Y E D	N D P W D
P R I C M	T G A S C	S E L M R	E T E R N	T I O H T	S L O P S
I L T O V	I U L M D	A Y C E I	T A W E F	S R E Y C	I O G O E
G L N I H	N O F S E	A E T L N	R W E O O	E R S R R	S P E R E
M A I E W	A T E X I	F C R E N	S A U I E	U R S T I	M N S D T
E N T T N	F S X N U	N R U L M	O R O G I	E I A R E	R E E N N
R D E A E	T S P E R	L T A N E	I R R N O	E C Q A F	N T H U N
T O N B N	E I O F D	O S M E E	D T E S P	T T R E A	G H U E R
H S A O I	H R I H E	V F I W L	J S W L I	T N E A G	E R R O E
N N C I I	N E N L R	O T O E I	F C R N S	L P U I P	U S O I O
E C T S W	A N O T I	E C I T I	I R T E I	F S O E T	V T R N T
D E I E D	H S I H R	D I N C B	H T P N I	N R L M C	T F A W I
R E W D A	Y A A O S	N N I O G	M R K O F	F T S D O	H T A I A
A B E T T	O N T N F	O I T H F	L M N Y O	E W O T O	A L I N B
C T E P A	E S T V N	I M S O E	A S A D R	S U Y N R	D I C I V
S S M E O	I E A H F	U C Y N W	Y E N A T	R D T A D	M A D E E
M E H E E	T E P A C	Y W N O N	U P H T T	M O D M U	O F D O R
T R O O A	N N E E S	T G T O G	E I E E H	C T U O M	C I F H A
T E A D R	E E A T P	N O E N R	D R D S E	Y L E S Q	S F E X K
O R A A O	B S T N A	T I V R E	R F N O E	T O E A A	S I T C G
A I L R T	V A V C S	U R R R W	E T N A C	L K I G S	U E H R R
I L E O I	O E E O N	T I S A R	X N U D B	S A T E S	T P N I E
S H R E V	E I H Y N	A S I S B	L N R S A	G I L G E	S O U E H
D A U E L	H O T D S	T O H E G	R U E A U	I H G O I	U E D S E
F D R E L	J L F A E	R T T A I	R H N L E	T R N E T	T E F A C
I L C P I	A N O U V	I U N R G	R R R E T	E E O P E	E P R D E
O E B P A	E T I R S	D T S N H	W A T S W	P A A R R	N A O E E
D F O N D	T F D M E	S R N O O	T C A N E	Y L A R D	M I C O T
T V T O M	C Y T T X	N Y D L R	T E S A O	L E N V T	E E O T R
N Q N T N	Y H E O M	C E N P N	C H N N E	D A N V D	H O P E A
R R Y A O	B I P T D	V A R E E	M D H F E	O A Y E E	I T L P H
I R T S O	E T L H I	F N U O A	C Y H N E	M N I I T	P T O H A
O A A L F	O E I E S				

Distribution:

A	74	G	16	L	36	Q	3	V	15
B	10	H	34	M	25	R	76	W	16
C	31	I	74	N	79	S	61	X	5
D	42	J	2	O	75	T	92	Y	19
E	130	K	3	P	27	U	26	Z	1
F	28								

C. 1000 Letters of Random Text

A G L Q O	Y P F J Y	Z R U F D	K F W F A	B V L T W	P P J Q V
R D F Y N	L Q G S D	F S P D Z	H O K Z B	B H A O S	L W K P G
G X J F B	A Z O L Z	G A A Z I	W Z I G Y	Q S F N O	U Z N S S
B Y L N N	V K L M S	G U R E J	U H X X Q	T N Q Z J	I A T I K
D H A W K	J Q W K W	H Z X P S	V F Q S U	Q V X S Z	G T J N B
B R F J N	C R O S W	Y N T D N	Y T U H C	G E O F L	X D V K Y
B Q S V X	V W C R J	G W N E O	R O L M L	S T Y T O	H S C J B
J O X W F	D S S P Q	O C C E X	K J A B Y	L M N Q H	K A L N W
S K N S M	H M O J H	U M D W Z	U L Y K X	A G S A P	W J V J H
J W J J N	M H F Z A	U G V M Z	W I Q U F	Q Z V E V	X C W Q N
Z V U F M	X F O U J	X B P R E	R S I W N	P F L O M	F H V S L
Z N Y X I	F W P K O	D L J I T	Y V D L V	P U M A R	H F Q V T
T R A H J	L J M U K	G W J A C	V Z C R U	E S G N B	D A W Z O
E U A B B	Z F R B G	N H Z N A	D K R B P	C E Y N A	P X M U I
L H X C X	N R X R M	C O Q M Y	H T T I O	N E F X H	J Y W F I
A N O T C	R K Y O U	M W Q V J	P R S H W	E K S M B	D K I A L
D F C Y N	H C W M U	X L S F W	G A D B R	Z V Y E T	U K Z M U
H H E B G	E K W F E	Y P T T B	I K C R M	I Y F A R	D V P U O
P F Y G V	Q C Z G O	L L J F B	C F S O C	Y B E D S	X P J G N
X V H T U	U Y P D P	U H P Z K	V U S F J	E W A T O	H W M X E
Z D X Z V	J X W U O	T U B J D	G R L P A	K G B H U	E B Y R A
L D M U J	N Y W V J	F S D C P	C G S T D	I O V Z Q	T U C I Y
F F H W I	G D P A U	C K H B X	P Q G S P	O O D E J	G C B G A
K N B A F	E F E Y U	P F B H S	E O O E P	Z Z W H P	L M I O J
S Z Y P D	B Z O E C	M U B P X	Q T V Q N	E G R D B	J V F T I
E J M O N	H R H Z K	A E B Q F	H P W W C	H F L B U	Q I Y R Z
R H Y H S	M S I K W	K N Z O U	Q G Y P F	A A Y G A	N Z T G V
O M G C T	Z G Y K Y	N Y R W W	W P D L V	T M P K U	R I A V K
D Q D L E	Z B K E S	Q V W E Q	P M Y T C	X O C O K	S N M S C
R L D G U	G P N Q Q	R F Y C N	X Z X G K	W K F A I	V A C U T
K A D L V	J O N X R	B Y V H I	W F H F N	I J T O H	J K V G F
Z A X A V	O K D X M	T O L E N	W Y X D L	C A A D G	U Y Z O K
J W O N R	Q G W E E	R T W S F	Y J E P X	S N K X V	X S X A D
U Q P K X	V F G Q R				

Distribution:

A	43	G	40	L	33	Q	34	V	40
B	36	H	40	M	31	R	35	W	46
C	33	I	25	N	42	S	41	X	39
D	37	J	41	O	44	T	32	Y	42
E	35	K	41	P	40	U	41	Z	41
F	48								

D. 1000 Digits of Random Text

25401 46585 33365 29172 86725 30494
46675 03278 30680 00220 14984 66312
24287 01313 22197 38742 40168 43157
89376 02869 24794 78834 58446 51860
91923 25409 60368 99826 60597 10129
87642 93048 56608 39318 51264 24459
01674 52508 91414 85424 91555 72769
71164 20387 80270 62935 88043 35124
06511 34622 27851 23622 54125 25502
35993 73323 40120 80798 79750 08040
68728 77725 65824 31569 99089 84721
36375 62962 90430 40784 00925 95548
81820 91854 81424 09005 83579 09323
74854 37310 12822 03033 12949 89210
86816 47500 60746 70420 17187 06521
14367 22705 03154 98012 85583 47994
24149 52373 42101 03474 43960 92651
59350 35703 30915 00899 78279 88907
23310 81481 41213 41647 09537 84073
53605 83062 58392 49638 42762 84583
42536 71278 38614 53523 12046 63789
45129 89651 96264 00290 01423 79689
44300 80955 97918 08884 69845 37468
20141 86918 41393 48707 68790 71813
92620 16124 72388 82549 67540 30686
92396 84929 09351 44116 23105 69313
89698 98678 01154 26117 68947 46024
39841 14417 65001 61886 41819 36190
79557 51589 82394 88391 56288 72902
16806 05095 44993 38658 57924 73010
50235 57797 47452 91732 24764 01307
79818 51752 23604 26969 63282 97907
56620 90768 97989 23121 77332 72588
91511 93234

Distribution:

0	106	5	90
1	99	6	88
2	110	7	89
3	97	8	110
4	105	9	106

E. 1000 Characters of Random Text (32-Element Alphabet)

I H C Q N	7 Z G S K	Y V J 3 P	V 5 I Z V	J 9 S P N	E R A I C
8 7 W X K	V U K X U	N L P S V	C O H 8 Z	5 G Z C R	I N Q 8 U
T 3 3 V Y	P A Z I W	A T F R M	S L E K 7	U B R T E	J P 4 Z 9
W 8 3 U D	F M 3 F E	H L S B L	I D F 8 S	J 5 U Y U	B K P B E
4 A D 5 R	C X P E I	H E 9 K C	L V I 7 8	E M F 3 7	H 4 J S U
P 4 R J B	G B Z 9 V	F A 7 D H	D T 9 W Q	P R L M 3	U F I O U
G Z 7 X Q	X D W 8 E	C N E K F	T 9 W Q C	8 E E N C	7 M L H 7
Z Q V R A	C X B 4 C	8 N P W 3	P Y L R F	S F 4 O L	G X N A Y
F Z W V 8	L 5 5 Y Q	T J L R R	K U 5 L T	D Q D I S	P E U Z C
A G U T S	A 3 8 3 R	L W T I S	H F 8 Q 5	L E D B K	P X F V F
9 P 5 B 9	J Q 4 9 H	S T Y P B	I G O 5 D	V L T W 9	U D 4 C Y
Q K 4 R O	Q 5 A W K	P I L K I	9 J B 9 U	P B 9 N 4	A 5 G F A
I H V V G	N T E L 8	E I J P P	5 S 5 Z 5	V W Q E I	4 3 M B Y
Z X N C G	U L G T Y	9 8 7 U T	V T 4 7 9	J K T C S	W B F X X
A 9 X A E	P B T 8 V	E V D R 3	9 J R B C	Y D L M N	H L B G N
L Z 4 D T	T T O W 4	F N G C O	E W E Z 9	H 9 Y 5 Y	N A Y Z C
M B A 9 K	M D 9 3 8	8 V C W H	T I T O T	H O E 5 T	E P Z P L
R 4 J C 3	B C C X Y	L C Q P G	O O O 5 I	P U I Z Z	S Y G X 3
G I N N Q	Y F R R E	3 R I E 4	H 7 G Q G	4 X D X Z	U K R U 5
N D H 5 D	S B Y I N	5 8 Q X K	W J X 5 I	7 L I G 4	9 L N 7 I
U 4 F M T	7 D 4 X B	H M F F L	O R X U R	5 L E 5 K	F F M K V
4 T 9 C G	X J R M N	Z E R Q Y	Y I I Z 4	N C B 7 M	S H Y D J
I T F S S	L 8 I O X	A Z X T G	B K U Y F	F Z 3 J D	J Y Y B Q
Y U H 4 3	4 B 3 M 9	S Z V U N	D A W D S	F K V H H	A K 4 X X
K Z E 9 C	M P S H B	T K G F X	9 V T 4 D	J P W F C	Z K 8 H V
G P O Q D	5 D S 3 Y	X U E 9 T	Q P 3 Q H	Y N I Q N	T G B M O
8 5 E S I	B K 5 4 H	Q S K E 3	H A M P 4	X Y A J D	U O S Q P
M L O O W	J 8 N P A	Z L K T 3	8 I H K A	3 Q U X O	N 9 O Z K
W G 5 A Y	L 8 Q X D	Z 4 Z 8 N	Z B A N U	G U M 5 R	R O 8 E E
H P 5 W 5	O F 9 V B	4 U N Z P	L K R Y K	L M W 5 H	A M E 3 L
C K 3 X N	U 5 P I N	Y A E F P	L S B H 5	8 7 U 3 8	J W V Y H
U N E D L	E 7 9 G Y	N F K 3 K	Y L H I A	R A P V 9	3 C X 4 R
E 4 D 4 B	X P B B M	8 3 C 9 Y	3 H 7 F 3	B B S U F	W D D F V
M W H Y L	Y T F A L				

Distribution:

A	30	G	27	M	24	R	29	W	26	4	33
B	36	H	34	N	34	S	28	X	33	5	34
C	30	I	35	O	22	T	34	Y	37	7	19
D	32	J	22	P	38	U	35	Z	33	8	29
E	37	K	34	Q	27	V	28	3	32	9	32
F	37	L	39								

F. 25 Random 26-Letter Alphabets

1.	A M Q N O	D Z F R X	I L K P E	U Y C B G	W V J S H	T
2.	B S U G C	Q P N Y E	H M V L J	K A F R Z	I X O T D	W
3.	C O M K Q	A W V F Y	L I U B T	N R J H E	S X D Z G	P
4.	D M H T I	U L N Z F	Q K C R O	S J E X A	Y W P G V	B
5.	E F L I S	K O P R V	T N J U Z	Q C H W Y	D M G X B	A
6.	F Q N S K	L R P H D	M X Z Y E	T W J U C	A G I V B	O
7.	G M Q L C	T U V E N	Z B O X H	F J S K D	R I A W P	Y
8.	H M I W F	A K X T V	S B U Z D	P E J C Y	G L O Q N	R
9.	I K G J V	O W Y N X	T M F L U	P B S A R	Q E Z H D	C
10.	J X K P U	C R Y D E	I A W S H	O T N G F	Z M V L B	Q
11.	K T H L O	C P M G W	N X E V S	Z I F U B	J Q A Y D	R
12.	L R D E I	O V W Q K	X C B Y H	M F A P Z	N J U S T	G
13.	M I W R K	J L H E V	G Q A U Y	F Z S X P	N C B D T	O
14.	N F A K X	W G V P C	R I M L S	D J E Q O	Y H T Z U	B
15.	O L Y B Q	V P H U T	E W J R D	C X I M K	Z S A G N	F
16.	P H M L N	A T Q K O	Z Y G J F	B E U X S	I W C D V	R
17.	Q W U L E	X I V M O	Z F B T Y	P K C G A	N H J S D	R
18.	R E I F L	V M N T J	P W A K G	Y S Q O B	D H U X Z	C
19.	S K A V O	T R J H I	Y Z G M Q	W X U D B	N P F C E	L
20.	T I Z Q F	J X V R S	E H M K W	O L N P U	C B D Y G	A
21.	U D W Z G	B I E Y M	V C H P J	S N L O X	F A K Q R	T
22.	V L I C U	D Y K R X	Z A J Q W	T F M G P	E N B H O	S
23.	W Z D N E	J A T L M	K V P R G	Q U C S I	O H Y X F	B
24.	X I J F G	Q E K C Y	T U P W H	O V R N D	Z S L B A	M
25.	Y B G F C	L H Q X O	R S I P A	M W J E D	U Z T K N	V

Appendix 6

BASIC LETTER FREQUENCY DATA, 24 FOREIGN LANGUAGES

These data are based on selected samples of newspaper text, communiqué style, of between 50,000 and 67,000 letters in each language, with but one exception (Slovak, with 46,026 letters.)

BASIC LETTER FREQUENCY DATA, 24 FOREIGN LANGUAGES

1. ALBANIAN

1–a. *Absolute frequencies of single letters of Albanian plain text, arranged alphabetically, based on 57,851 letters of text.*

A	4,441	E	5 142	I	4 890	N	3 545	Rr	178	V	776
B	639	Ë	6,604	J	1 233	Nj	420	S	2,054	X	9
C	213	F	545	K	2,207	O	2,431	Sh	1,281	Xh	21
Ç	198	G	590	L	875	P	1,913	T	4,876	Y	296
D	1,161	Gj	325	Ll	381	Q	855	Th	207	Z	398
Dh	538	H	451	M	2,185	R	3,951	U	2,005	Zh	17

1–b. *Monographic kappa plain, Albanian language* =.0604 (I.C. =2.17)

1–c. *Frequency distribution of single letters based on 57,851 letters of Albanian plain text reduced to 1,000 letters, and arranged according to their frequencies.*

Ë	114	N	61	P	33	V	14	Nj	7	Th	4
E	89	O	42	Sh	22	B	11	Z	7	Ç	3
I	85	K	38	J	21	G	10	Ll	7	Rr	3
T	84	M	38	D	20	F	9	Gj	6	Xh	Ø
A	77	S	36	L	15	Dh	9	Y	5	Zh	Ø
R	68	U	35	Q	15	H	8	C	4	X	Ø

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 57,851 letters of Albanian plain text. Percentage of 7 most frequent letters in Albanian plain text.*

Vowels Ë, E, I, A, O, U, and Y =44.6%
High-Frequency Consonants T, R, N =21.4%
Medium-Frequency Consonants K, M, S, P, Sh, J, and D =20.8%
Low-Frequency Consonants L, Q, V, B, G, F, Dh, H, Nj, Z, Ll, Gj, C, Th, Ç, Rr, Xh, Zh, and X =13.2%
7 most frequent letters (in descending order of frequency) Ë, E, I, T, A, R, and N =58%

1–e. *Absolute frequencies of single letters as initial letters of 13,249 words in Albanian plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

T	1,252	Sh	571	Nj	338	J	187	Z	137	Rr	15
P	1,223	S	561	F	315	G	182	Ç	126	Y	10
K	1,050	Q	415	E	259	H	175	Ë	112	Xh	4
N	993	V	380	L	237	R	170	C	90	Ll	2
M	978	B	370	Gj	223	I	156	O	81		
D	605	A	351	Dh	221	U	138	Th	56		

2–a. *Frequency distribution of digraphs based on 57,851 letters of Albanian plain text, reduced to 5,000 digraphs.*

	A	B	C	Ç	D	Dh	E	Ë	F	G	Gj	H	I	J	K	L	Ll	M	N	Nj	O	P	Q	R	Rr	S	Sh	T	Th	U	V	X	Xh	Y	Z	Zh
A	3	6	2	2	5	4	11	2	4	4	1	2	2	17	19	17	4	23	49	3		15	11	59	6	19	19	51	3	3	11			1	6	1
B	17						6	11					4			4					4			3						7						
C	2						1	3					10								1															
Ç	4				2		1	3					2		1	1		1			1	1														
D	6						12	11					22	2							23			8						11				6		
Dh	3						35	3					1	3																1						
E	8	6	2	4	13	20	8	1	9	6	4	3	4	15	32	10	5	27	42	5	2	26	9	47	2	26	17	72	2	3	14				4	
Ë	9	12	3	4	21	6	13	2	10	6	9	8	6	7	28	9	2	36	84	10	2	35	11	79	2	46	37	66	2	4	19				7	
F	11						4	1					5	2		2					4			3			1	6		6	1					
G	22							3					1			2					6			12						5						
Gj	2						12	1					11								1									1				1		
H	5							16	4					3					1			3									4				1	
I	16	4	3	2	7	6	10	2	4	4	2	4	13	8	38	12	5	34	54	5	8	31	4	12		40	18	49	8	5	6				10	
J	18	1			2	1	27	9	1		1		2		2			9	9	1	3	2	1	1		1	2	10		3						
K	29						14	47					13	1		1	1		1		31	1		14		3		6	1	27				1		
L	9						12	7					29						1		4		1					1		7						
Ll	8						1	4					12		1				1		3							1								
M	19	13			1	1	43	34					38	1	2			1	2	1	10	4	1	1		1		4		12	1					
N	16	1	5	1	32	2	35	84	4	20	3	1	30	1	5			4	5	1	6	5	2		2	4	2	21		11	3	1		1	1	
Nj	2						5	27													2							1								
O	1	2	1		2	1	1	1	3	2	1	11	1	23	7	7	5	12	27	4	1	11	3	29	1	11	1	32	1	1	4		1		2	
P	28						8	48					6	3		2	1		1		25			20		1		8		14						
Q	1						9	33					26								1							1		1				1		
R	44	4	1	2	3	1	47	37	3	6	4	3	75	2	12	1		11	6	3	15	9	4	1		8	5	17	1	11	2			5		
Rr	3						2	4					3								1									2						
S	12	1			2	2	26	26	1		1		33	2	7	1	1	3	5	1	12	5	1	1		7	1	20		5	1			1		
Sh	4				1		2	7					11	1	15			3	1		4	5	19					33		6	1					
T	40	1			3	2	43	140	2	1		1	45	10	5	1		9	9	2	25	9	4	20		4	2	17		19	2			4	1	
Th	4						2	5					2					1			2									1						
U	30	2			3	2	1		5	1		6	1	3	13	3	7	10	30			5	1	26	2	6	4	5			2				2	
V	8				1		28	10					9	3		1					5			2						1						
X																																				
Xh	1																																			
Y							5								3		1	1	1			1	1	5		1	1	2								
Z	4	2					5	3			1		4	1				2			9									2				1	1	
Zh					1																															

2–b. *Digraphic kappa plain, Albanian language*=.0061 (I.C.=7.91)

2. ARABIC

1–a. *Absolute frequencies of single letters of Arabic plain text, arranged alphabetically, based on 53,472 letters of text.*

ا	10,391	د	1,721	ض	355	ك	967
ب	1,725	ذ	470	ط	535	ل	5,958
ت	3,411	ر	2,526	ظ	93	م	3,204
ث	253	ز	341	ع	1,833	ن	2,771
ج	657	س	1,330	غ	216	ه	1,935
ح	1,009	ش	463	ف	1,524	و	3,023
خ	432	ص	533	ق	1,260	ي	4,536

1–b. *Monographic kappa plain, Arabic language* =.0811 (I.C. =2.27)

1–c. *Frequency distribution of single letters based on 53,472 letters of Arabic plain text reduced to 1,000 letters, and arranged according to their frequencies.*

ا	194	و	57	د	32	ح	19	ذ	9	ث	5
ل	111	ن	52	ب	32	ك	18	ش	9	غ	4
ي	85	ر	47	ف	28	ج	12	خ	8	ظ	2
ت	64	ه	36	س	25	ط	10	ض	7		
م	60	ع	34	ق	24	ص	10	ز	6		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants, in 53,472 letters of Arabic plain text. Percentage of 8 most frequent letters in Arabic plain text.*

Vowels ا, ي, and و =33.6%
High-Frequency Consonants ل, ت, م, ن, and ر =33.4%
Medium-Frequency Consonants ه, ع, د, ب, ف, س, ق, ح, and ك =24.9%
Low-Frequency Consonants ج, ط, ص, ذ, ش, خ, ض, ز, ث, غ, and ظ =8.1%
8 most frequent letters (in descending order of frequency) ا, ل, ي, ت, م, و, ن, and ر =64.6%

1–e. *Absolute frequencies of single letters as initial letters of 11,496 words of Arabic plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

ا	4,122	ل	549	ه	214	ج	140	ذ	61	ض	27
م	905	ع	548	س	204	ر	133	ط	61	ز	25
و	880	ت	442	ك	203	د	109	ص	56	ظ	7
ف	776	ي	334	ح	202	ش	76	ث	42		
ب	591	ق	223	ن	152	خ	70	غ	29		

2–a. *Frequency distribution of digraphs based on 53,472 letters of Arabic plain text, reduced to 5,000 digraphs.*

	ا	ب	ت	ث	ج	ح	خ	د	ذ	ر	ز	س	ش	ص	ض	ط	ظ	ع	غ	ف	ق	گ	ل	م	ن	ه	و	ي
ا	78	27	49	6	10	17	9	25	9	39	4	27	7	10	7	8		26	1	24	18	12	348	50	86	12	35	32
ب	41	3	8		1	4		4	1	14		3	1	1	1	1		12	3	2	5	1	11	4	6	8	7	19
ت	75	14	8	2	5	11	6	5	1	13	1	8	4	6	1	3	1	17	1	17	9	6	13	23	6	14	31	17
ث	5		2							2								1			1		5	2	1	1	1	3
ج	12	2	5		1			4		6	1							2					6	5	3	4	4	6
ح	15	2	6	1	1			11		5	5	3		3	1	1				3	6	6	4	4	1	2	4	8
خ	10	2	3					1	1	4				2		2				1			4	4			2	2
د	37	3	17	1	1	2	2	3		7	1	2	1	1		1		5		3	3	2	4	11	5	9	16	25
ذ	16	1	1							2												2	6	1		8	1	5
ر	60	18	17		6	4	1	5		2	1	5	1	1	4	1		4	1	11	7	7	3	6	4	13	17	37
ز	11	4	1					1		2								1					1	2		1	2	6
س	23	7	24		2	1	1	1		2		1				7		6		4		4	7	6	4	3	7	17
ش	9		3			1	1	1		11								3		1		4	1	1		1	1	5
ص	9	3	2			4		4		7								1		2			6	1	1	2	5	3
ض	9	1	1			1		1		4								2		1			2	3		1	3	5
ط	14	1	4							4								2		1	1		4	1	3	2	3	9
ظ	2									3										1						1		1
ع	28	5	11			1		11	1	14	1	1	4		3	1	1	1		1	3	1	32	11	12	6	8	11
غ	2	1	1					2	1	4													1	1			2	4
ف	20	1	9		1	1	1	2		7		2	1	1	1		1	4		1	7	3	11	3	2	4	8	53
ق	25	7	9					21		6		1		2	3	2		4		3	1	1	6	3	1	2	13	10
گ	19	4	4	2				1	1	5	1	1						1		2			7	14	6	1	12	7
ل	115	19	33	3	11	23	6	16	6	8	2	23	10	8	2	5		25	7	16	15	17	12	65	11	15	23	62
م	58	7	21	2	4	7	3	8	2	16		16	4	4	1	2		16	1	8	5	3	12	9	38	9	24	20
ن	71	9	20		4	4	2	6	2	4	1	8	4	4	1	2	3	7	1	10	6	5	5	11	4	18	17	30
ه	69	4	4	1	2	2		3	14	5		1			1			6		7	2	2	7	18	3	2	18	8
و	63	7	7	1	5	5	3	10	1	15	8	9	2	4	7	4	1	11	2	10	18	4	30	20	19	6	1	13
ي	79	10	49	2	8	7	3	16	2	27	4	13	2	2	2	9		15	1	12	10	8	13	22	42	37	21	8

2–b. *Digraphic kappa plain, Arabic language*=.0101 (I.C.=7.92).

3. BULGARIAN

1–a. *Absolute frequencies of single letters of Bulgarian plain text, arranged alphabetically, based on 55,689 letters of text.*

А	6,543	Ж	426	М	1,166	Т	4,331	Ш	290
Б	849	З	1,203	Н	4,226	У	750	Щ	344
В	2,579	И	5,245	О	4,950	Ф	161	Ъ	988
Г	816	Й	350	П	1,561	Х	258	Ь	13
Д	1,949	К	1,723	Р	2,799	Ц	422	Ю	111
Е	4,987	Л	1,829	С	2,993	Ч	722	Я	1,105

1–b. *Monographic kappa plain, Bulgarian language*=.0647 (I.C.=1.94).

1–c. *Frequency distribution of single letters based on 55,689 letters of plain text, reduced to 1,000 letters and arranged according to their frequencies.*

А	117	С	54	П	28	Г	15	Щ	6
И	94	Р	50	З	22	У	13	Ш	5
Е	89	В	46	М	21	Ч	13	Х	5
О	89	Д	35	Я	20	Ж	8	Ф	3
Т	78	Л	33	Ъ	18	Ц	8	Ю	2
Н	76	К	31	Б	15	Й	6	Ь	0

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 55,689 letters of Bulgarian plain text. Percentage of 6 most frequent letters in Bulgarian plain text.*

Vowels А, И, Е, О, Я, Ъ, У, Й, Ю and Ь=45.0%
High-Frequency Consonants Т and Н=15.4%
Medium-Frequency Consonants С, Р, В, Д, Л, К, П, З, М, Б, Г, and Ч=36.2%
Low-Frequency Consonants Ж, Ц, Щ, Ш, Х and Ф=3.4%

6 most frequent letters (in descending order of frequency) А, И, Е, О, Т and Н=54.4%

1–e. *Absolute frequencies of single letters as initial letters of 10,421 words of Bulgarian plain text, arranged according to their frequencies.* (*One-letter words have been omitted*).

Н	1,284	О	611	Т	433	Р	304	А	117	Ф	93	Ш	25
С	1,191	К	518	Б	364	Г	232	У	114	Ц	82	Ю	18
П	1,022	В	488	М	340	Ч	197	Л	108	Ж	51	Я	13
Д	713	З	477	И	319	Е	126	Щ	100	Х	39		9,379

2–a. *Frequency distribution of digraphs based on 55,689 letters of Bulgarian plain text.*

	А	Б	В	Г	Д	Е	Ж	З	И	Й	К	Л	М	Н	О	П	Р	С	Т	У	Ф	Х	Ц	Ч	Ш	Щ	Ъ	Ь	Ю	Я
А	4	18	47	11	27	10	6	31	16	6	26	28	23	79	15	32	40	51	74	3	2	5	7	10	8	6			1	5
Б	5		3		1	9		1	11			6		2	12		8	1		1		1				4	10			2
В	53	2	3	2	2	32		2	29	1	3	3	2	10	30	4	8	15	3	2			1	2			20			5
Г	18		1			6			5		1	6		3	21	1	8			1							1			
Д	38		5		1	30			24		2	2	1	18	19	2	7	6	1	9				1			7			1
Е	3	8	18	13	45	3	7	15	13	3	12	32	21	83	13	22	25	45	36	3	1	2	4	10	5	7				1
Ж	7	2			8	12			6		1			2																
З	41	2	7	2	2	5		1	9		4	3	4	9	3	5	1	2	2	2		1					1			1
И	8	5	21	7	14	25	4	28	14	3	20	15	17	44	15	18	13	38	69	2	1	3	11	21	3	4				48
Й		1		1	1	1		1			3	1	1	5	1	2	1	5	4					1		1				
К	38	1	6		1	1		1	32			3		2	41		9	2	5	5			2				6			
Л	19	1	1	6	1	24	2	1	39		5			13	20	1		7	2	6				1	1		2		4	9
М	17	1	3	1	2	22		1	18		1	1	1	5	16	2	1	2	1	4				1			3			1
Н	139	1	2	3	5	40		3	85	1	5	1	1	5	46	2	1	13	11	2	1		4	2	1					5
О	1	20	49	13	36	7	9	9	18	12	13	25	13	36	8	20	21	42	65	2	4	1	3	5	3	4				7
П	12		1			10			7			5		1	42		52		1	4							4			1
Р	65	1	2	4	5	50	3	1	33		1	1	2	6	38	2		6	6	8	1	3		1	2		5			5
С	15	1	10	1	1	32		1	23		35	12	3	7	7	8	5	3	75	2				1			22			4
Т	70	2	25	4	5	68		2	36	1	6	2	3	22	82	6	22	10	4	6	1	1		2			8		1	3
У	1	3	7	2	5		4	1	1		2	3	4	4		3	6	9	2			3	1	4	1	1				
Ф	3					3			2						1		3													
Х	7		2		1	.			3					2	5					1										
Ц	6					5			23																					1
Ч	9		1			31			8		5	1		5	1		1		1	2										
Ш	4					9			5		1			4	1															
Щ	5					12			6					1	4					2										
Ъ		2	6	2	8		2	6		1	3	10	4	3	2	5	16	5	6				1	1		3			3	
Ь															1															
Ю		1		1				3						1		1							2	1						
Я	1	4	12	1	5	1		2	5	1	6	2	5	8	3	4	3	7	23		2	3	1	1	1	1				

2–b. *Digraphic kappa plain, Bulgarian language*=.0069 (I.C.=6.21).

4. CZECH

The frequencies of Ď, Ó, Ř, Ť, Ů, Ý, and Ž are included respectively with those of D, O, R, T, Ú, Y, and Z. The letter W is derived from the foreign words included in Czech plain text.

1–a. *Absolute frequencies of single letters of Czech plain text, arranged alphabetically, based on 62,107 letters of text.*

A 4,016	D 2,466	G 219	J 1,086	Ň 26	Š 434	W 39
Á 1,525	E 4,740	H 797	K 2,143	O 5,249	T 3,288	Y 2,091
B 1,251	É 728	Ch 528	L 2,847	P 2,050	U 1,988	Z 2,078
C 964	Ě 972	I 2,917	M 2,072	R 3,101	Ú 466	
Č 576	F 121	Í 1,697	N 4,285	S 2,771	V 2,576	

1–b. *Monographic kappa plain, Czech language*=.0465 (I.C.=1.53).

1–c. *Frequency distribution of single letters based on 62,107 letters of Czech plain text, reduced to 1,000 letters and arranged according to their frequencies.*

O 85	R 50	D 40	P 33	J 17	Č 9	F 2
E 76	I 47	K 34	U 32	C 16	Ch 8	W Ø
N 69	L 46	Y 34	Í 27	Ě 16	Ú 7	Ň Ø
A 65	S 45	Z 33	Á 25	H 13	Š 7	
T 53	V 42	M 33	B 20	É 12	G 4	

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants of 62,107 letters of Czech plain text. Percentage of 4 most frequent letters.*

Vowels O, E, A, I, Y, U, Í, Á, Ě, É, and Ú=42.5%
High-Frequency Consonants N, T, and R=17.2%
Medium-Frequency Consonants L, S, V, D, K, Z, M, P, B, and J=34.4%
Low-Frequency Consonants C, H, Č, Ch, Š, G, F, W, and Ň=5.9%
4 most frequent letters (in descending order of frequency) O, E, N, and A=29.5%

1–e. *Absolute frequencies of single letters as initial letters of 11,696 words in Czech plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

P 1,354	B 614	O 446	Č 143	I 44	W 16
S 1,056	D 603	A 401	L 130	Ch 40	Z 3
N 948	M 572	R 349	C 106	G 33	Y 3
Z 908	J 490	U 181	Ú 87	Š 32	Ě 1
V 780	K 460	H 158	F 71	E 27	Í 1
T 630					

2–a. *Frequency distribution of digraphs based on 62,107 letters of Czech plain text, reduced to 5,000 digraphs.*

	A	Á	B	C	Č	D	E	É	Ě	F	G	H	Ch	I	Í	J	K	L	M	N	Ň	O	P	R	S	Š	T	U	Ú	V	W	Y	Z
A	5		15	8	5	24				1	2	5	1			14	25	31	19	36		5	14	14	19	2	26	4	1	28	1	1	17
Á	1		3	3	1	13						2	1			2	5	9	5	20			3	9	10	3	10			16			7
B	4				1		6		3					4	1			3		3		10		8	1			11				44	
C	2		1				23						15	6		17			2		5	1				1		2	1				
Č	5	2					12							6	4		1	1		10							3	1	1				
D	16	9	1	1	1	1	23	1	10			1		9	4	1	3	9	3	28		33	4	8	4		1	6	5	4		12	1
E	6		15	9	8	35				1	1	5	7	1		14	16	25	27	67		6	15	32	29	2	17	2	1	16	1		23
É	2		2			2				1		8				1	2		8	3		2	6	2	6		4		1	3			6
Ě	2		1	4		3						1	2			5	4	9	6	9		2	3	5	7		8			3			3
F	1						1							2								3		2									
G	2	1					2							1				4				2		4				1					
H	5	2					3	1										12		1		29		6				2				2	
Ch	2	1	1	1		2						1		1		1	1	3	2	4		4	3	1	3		3	1	2	3			2
I	7	1	5	20	9	9	6			1	1	2	3	3	1	5	14	24	8	29		5	11	4	19	2	24	2	1	14			10
Í	5		3	7	2	6						4	6			3	8	3	15	8		4	12	7	12	1	7	1	1	9			13
J	9	1				1	38							11	11		1		3	3		1	1		3	2		1		1			
K	15	7	2	1		9	4	15						1		1	1	8	2	3		33	2	9	2		17	12	3	4		20	2
L	30	13	2	1		3	32	5				1		34	5	2	7	2	2	11		29	3	1	6	2	3	9	2	4		12	7
M	15	7	3	2	1	2	17	1	12					14	8	1	3	5	3	7		21	6	3	3		3	17	2	2		4	6
N	44	24	1	5	1	4	37	13	25	1	8	1		26	56	1	3		1	4		30	2	1	8	1	8	9	2	1		25	2
Ň																																	
O	5		23	8	6	44				1	1	13	2			16	13	22	23	24	1	5	21	23	38	2	23	36	1	47			28
P	10	2		1			4		3					4	1			5				57	1	65	2	1	1	3	2				
R	36	16	1	1	1	3	39	5			1	3	1	24	12		2	1	4	5		51	1		5	1	3	15	5	4		9	3
S	5	2				1	30					2	2	12	3	1	24	14	6	6		13	13	1	2		68	5	1	9		2	1
Š	3						11							3	8		1	2		1							5						
T	32	9	1	1	1	1	30	8	9					38	9	1	3	2	1	12		44	3	22	4	2	1	12	1	8		9	1
U	6		5	1	5	18					2	6	1			9	6	4	6	11		3	16	5	15	3	11	1		13			14
Ú	2		1		1	1										1	1	2	6	2		1	3	3	3		1			3			4
V	20	12	1	1	2	2	23	9	15			1	1	11	6	1	4	5	2	11		17	4	10	3	7	3	4	3			27	5
W	1													1						1													
Y	5		11	1	2	6	1					3	16	1		4	6	23	12	9		5	10	4	14	3	13	2		9			10
Z	23	15	4	1	1	10	39					2		12	3	2	4	2	3	15		4	8	2	4	1	3	3	1	4		1	1

2–b. *Digraphic kappa plain, Czech language*=.0042 (I.C.=4.57).

5. DANISH

NOTE: The letters W, X, and Z are derived from foreign words in Danish plain text.

1–a. *Absolute frequencies of single letters of Danish plain text, arranged alphabetically, based on 56,176 letters of text.*

A	4,434	G	2,358	M	1,755	S	3,452	X	1
B	747	H	807	N	4,370	T	4,098	Y	370
C	87	I	3,479	O	2,341	U	899	Z	8
D	3,906	J	339	P	624	V	1,379	Æ	472
E	9,021	K	1,727	R	4,739	W	13	Ø	511
F	1,459	L	2,780						

1–b. *Monographic kappa plain, Danish language*=.0730 (I.C.=1.97).

1–c. *Frequency distribution of single letters based on 56,176 letters of Danish plain text reduced to 1,000 letters, and arranged according to their frequencies.*

E	161	I	62	K	31	B	13	J	6
R	84	S	61	F	26	P	11	C	2
A	79	L	49	V	25	Ø	9	W	0
N	78	G	42	U	16	Æ	8	Z	0
T	73	O	42	H	14	Y	7	X	0
D	70	M	31						

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 56,176 letters of Danish plain text. Percentage of 8 most frequent letters in Danish plain text.*

Vowels A, E, I, O, U, Y, Æ, and Ø=38.3%
High-Frequency Consonants D, N, R, S, and T=36.6%
Medium-Frequency Consonants F, G, K, L, M, and V=20.4%
Low-Frequency Consonants B, C, H, J, P, W, X and Z=4.7%
8 most frequent letters (in descending order of frequency) E, R, A, N, T, D, I, and S=66.8%

1–e. *Absolute frequencies of single letters as intial letters of 10,749 words in Danish plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

D	1,205	O	613	B	384	N	236	C	20
S	1,054	H	578	I	373	L	234	Y	11
A	881	T	543	P	310	R	186	W	8
F	822	V	493	G	269	J	74	Æ	8
E	713	K	407	U	247	Ø	51	Z	1
M	644								

2–a. *Frequency distribution of digraphs based on 56,176 letters of Danish plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T	U	V	W	X	Y	Z	Æ	Ø
A	57	7		19	5	28	24	2	2	1	4	35	12	73	2	2	46	8	56	3	11						
B	5				26				3			10			5		6	1	1	2				3		1	2
C					4			1	2																		
D	24	4		11	**194**	5	4	3	16		3	8	3	5	8	4	10	19	14	3	6			1		1	2
E	18	11	1	66	13	24	17	12	15	12	16	49	32	**142**	12	9	175	57	97	7	18						2
F	8	1		4	7	5	2	2	7	1	1	5			47	1	12	3	11	2	1					3	8
G	18	3		9	68	4	5	7	11	2	2	2	5	5	7	3	8	21	19	2	3			1		1	5
H	26				16				2	3					6		1			3	10					1	3
I	5	2	1	24	8	6	52	3	1		20	45	6	63	10	1	4	34	11	1	13						1
J	1			3	12							1			5		1	2	1	1				1		1	
K	21			1	44	1		1	3	1	16	4		3	15	1	11	7	7	12	1			1		1	3
L	26	3		21	48	4	5	2	43	1	6	23	3	1	6	2	1	19	9	4	6			6		4	5
M	28	2		4	44	3	1	3	19		1	2	15	1	8	3	1	5	4	5	1			1		2	3
N	21	5	2	85	50	10	41	9	26	1	8	5	9	12	14	3	3	39	22	9	7			4		4	3
O		2	1	9	1	3	34				2	11	32	18		9	68	3	2	1	11						
P	18				7	1	2	1	2			3			4	2	9	1	1	2							1
R	33	12		35	96	12	9	14	40	3	16	8	15	19	16	4	6	29	21	12	12			3		6	2
S	29	3	1	6	39	6	1	3	24	1	45	10	10	3	18	9	2	15	62	3	7			3		3	5
T	40	6		14	74	12	4	8	64	1	4	6	9	8	19	3	17	24	19	8	9			8		3	3
U	2	1		18	2	1	2	1	1		2	11	1	20		1	6	5	3		1						
V	17			3	45	1			28			1		4	7		1	2	1	1						11	
W					1																						
X																											
Y		1		6	1		3				3	2		3			1	8	2		2						
Z																											
Æ				2		1	3				2	5		5		1	17	3	2		1						
Ø		4		6			2			4	1	2	1	5			17	2			1						

2–b. *Digraphic kappa plain, Danish language*=.0094 (I.C.=6.85).

6. DUTCH

1–a. *Absolute frequencies of single letters of Dutch plain text, arranged alphabetically, based on 59,993 letters of text.*

A	4,520	G	2,311	L	2,373	Q	77	V	1,663
B	955	H	1,046	M	1,299	R	4,159	W	844
C	892	I	4,077	N	6,039	S	2,541	X	9
D	3,154	J	482	O	3,738	T	3,850	Y	390
E	10,562	K	1,380	P	1,074	U	1,186	Z	813
F	599								

1–b. *Monographic kappa plain, Dutch language*=.0758 (I.C.=1.97).

1–c. *Frequency distribution of single letters based on 59,993 letters of Dutch plain text reduced to 1,000 letters, and arranged according to their frequencies.*

E	176	I	68	S	42	K	23	H	17	Z	14	Q	1
N	101	T	64	L	40	M	22	B	16	F	9	X	–
A	75	O	62	G	39	U	20	C	15	J	8		
R	69	D	53	V	28	P	18	W	14	Y	6		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 59,993 letters of Dutch plain text. Percentage of 8 most frequent letters in Dutch plain text.*

Vowels E, A, I, O, U, and Y=40.8%
High-Frequency Consonants N, R, and T=23.4%
Medium-Frequency Consonants D, S, L, G, V, K, M, P, and H=28.1%
Low-Frequency Consonants B, C, W, Z, F, J, Q, and X=7.7%
8 most frequent letters (in descending order of frequency) E, N, A, R, I, T, O, and D=67.3%

1–e. *Absolute frequencies of single letters as initial letters of 9,520 words in Dutch plain text, arranged according to their frequencies.* (*One-letter words have been omitted*).

V	1,017	B	568	G	498	Z	406	P	248	L	158
D	739	S	515	E	481	H	376	C	205	F	83
A	708	T	512	M	440	N	371	U	169	J	38
O	571	I	511	W	436	R	251	K	168	Q	38

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2–a. *Frequency distribution of digraphs based on 59,993 letters in Dutch plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	83	3	11	10	1	8	15		3		13	34	12	90		4		40	8	34	3	4	1		1	1
B	9	2			37				9			6			4			6		1	1				3	
C	2		2		3			31	4		1	1			17			2		7	3					
D	31	3	1	6	111		3	3	28		1	1	2	2	20	3		10	13	6	6	5	4			4
E	11	13	13	43	55	11	32	9	30	1	24	74	17	218	9	18	1	160	39	41	9	29	11	1		18
F	2			3	9	3	1	1	5					1	3			2	2	7		2	1			1
G	13	3	1	8	91		4	3	8		1	1	4	3	8	1		11	7	11	4	6	2			2
H	19				27				10					1	10			2		13	2				1	
I	5		8	17	71	2	21			35	11	11	5	96	3	2	1	2	23	26		1				1
J	2			5	3	2	1				10	1		3	2				1		2	1	1			2
K	12	2		2	34		3	4	7		6	3	1	1	13	1		3	6	8	2	3	2			2
L	24	2	1	12	36	2	8	1	37		5	14	2	1	10	1		1	11	6	6	4	2		9	1
M	14	2	1	3	36		1		15				10		12	2			4	2	2	1			3	
N	26	16	7	84	42	5	64	9	31	1	11	6	9	13	27	6	3	4	39	40	7	26	14			15
O	2	1	4	8	34	5	12		1		5	11	22	41	43	32		51	5	14	9	12	1			
P	9	2		2	12		2	3	4		1	3	1	1	9	3		16	2	3	8	5	2			1
Q																					6					
R	30	12	2	37	61	1	10	9	40		10	9	8	8	23	2		3	17	21	9	14	8		3	10
S	7	4	15	7	20	1	4	4	16	1	1	9	4	3	5	11		2	13	68	5	9	3		1	2
T	24	7	2	8	88	1	10	8	42		2	2	6	4	36	2		18	14	10	9	12	12		1	6
U	3	6	5	5	2		2		18		3	8	3	14	4			8	7	3	1	1	5			
V	26				60				8			3			33			6							3	
W	17				28				9						11										4	
Y				2	1	4	1	1			9	1	1	5					1	1		2				2
Z	9				22				17						11						4		2		3	

2–b. *Digraphic kappa plain, Dutch language*=.0094 (I.C.=6.35).

7. FINNISH

NOTE: The letters Q, W, X, and Z are derived from foreign words in Finnish plain text.

1–a. *Absolute frequencies of single letters of Finnish plain text, arranged alphabetically, based on 62,261 letters of text.*

A	7,367	L	3,442	S	5,241	Ö	360	G	71
E	5,232	M	2,240	T	6,603	B	44	Q	1
H	1,113	N	5,333	U	2,953	C	21	W	10
I	6,508	O	3,482	V	1,544	D	703	X	3
J	1,172	P	908	Y	1,705	F	55	Z	2
K	2,830	R	1,231	Ä	2,717				

1–b. *Monographic kappa plain, Finnish language*=.0737 (I.C.=2.11).

1–c. *Frequency distribution of single letters based on 62,261 letters of Finnish plain text, reduced to 1,000 letters and arranged according to their frequencies.*

A	118	O	56	V	25	D	11	C	0
T	106	L	55	R	20	Ö	6	W	0
I	105	U	47	J	19	G	1	X	0
N	86	K	45	H	18	F	1	Z	0
S	84	Ä	44	Y	17	B	1	Q	0
E	84	M	36	P	15				

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 62,261 letters of Finnish plain text. Percentage of 6 most frequent letters in Finnish plain text.*

Vowels A, E, I, O, U, 1, 2, and Y=47.7%
High-Frequency Consonants N, S, and T=27.6%
Medium-Frequency Consonants D, H, J, K, L, M, P, R, and V=24.4%
Low-Frequency Consonants B, C, F, G, Q, W, X, and Z=0.3%
6 most frequent letters (in descending order of frequency) A, T, I, N, S, and E=58.2%

1–e. *Absolute frequencies of single letters as initial letters of 7,836 words in Finnish plain text, arranged according to their frequencies.*

S	938	V	608	L	326	U	97	G	12
T	820	M	568	A	313	I	83	Ä	11
K	754	E	546	N	293	B	32	C	4
J	654	P	459	Y	153	D	28	W	4
O	613	H	329	R	132	F	28		

2–a. *Frequency distribution of digraphs based on 62,261 letters of Finnish plain text, reduced to 5,000 digraphs.*

	A	E	H	I	J	K	L	M	N	O	P	R	S	T	U	V	Y	Ä	Ö	B	C	D	F	G	Q	X	Z
A	76	15	13	51	26	32	48	33	97	13	16	21	50	53	15	25	4			1		3	1	1			
E	5	34	14	30	4	13	39	18	**113**	5	3	20	38	52	12	10	2	1				8	1				
H	14	13		6	2	1		1	1	5				19	7		3	4	1			11					
I	22	24	7	40	7	31	32	28	80	11	5	10	**114**	72	1	18	1	4	1		1	13					
J	44	9		2						26					2			10									
K	47	24		27		13	1		1	32		4	35		23		6	12	1								
L	36	37	1	55	4	9	60	7		12	1		1	14	16	4	3	16	3								
M	40	26		41				20		9	3		1		18		7	14									
N	30	49	12	22	19	37	12	16	31	26	18	4	42	50	11	19	9	13	1	1		2	1	3			
O	4	1	10	42	6	17	42	20	43	5	9	6	28	20	8	8						7	1				
P	16	5		15						10	2	1	1		11		2	10									
R	21	7	2	22	8	7	1	2	1	4	1	3	1	5	4	6	1	4						1			
S	51	68	1	69	3	13	2	2	1	17	2		36	**101**	25	3	8	17									
T	**118**	60	2	43	5	10	2	4	2	46	3	2	11	79	47	3	23	61	9								
U	2	4	10	8	3	21	14	6	23	28	4	13	28	30	26	8						8					
V	53	6		20						15					6		1	22									
Y		1	9	3		9	4	7	4		1	2	11	12		4	6	1	12			3					
Ä	4	8	8	15	6	13	15	16	28	8	4	10	19	18	1	13	8	26									
Ö			1	2	1	2	3	2	4	1			5	4		2	1	1	1								
B	1	2								1																	
C			1																								
D	6	27		7						7					5		1	2									
F	1	1		2						1		1															
G	1	1		1			1					1															
Q																											
X																											
Z																											

2–b. *Digraphic kappa plain, Finnish language* =.0082 (I.C. =6.44).

8. FRENCH

1–a. *Absolute frequencies of single letters of French plain text, arranged alphabetically, based on 55,758 letters of text.*

A	4,480	G	624	L	2,737	Q	616	V	801
B	406	H	276	M	1,617	R	4,117	W	6
C	1,944	I	4,230	N	4,406	S	4,564	X	317
D	2,198	J	184	O	3,255	T	4,057	Y	100
E	9,334	K	25	P	1,689	U	3,045	Z	84
F	646								

1–b. *Monographic kappa plain, French language* =.0777 (I.C. =2.02).

1–c. *Frequency distribution of single letters based on 55,758 letters in French plain text reduced to 1,000 letters, and arranged according to their frequencies.*

E	167	T	73	C	35	G	11	J	3
S	82	O	58	P	30	Q	11	Y	2
A	80	U	55	M	29	B	7	Z	2
N	79	L	49	V	14	X	6	K	1
I	76	D	39	F	12	H	5	W	Ø
R	74								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 55,758 letters of French plain text. Percentage of 8 most frequent letters in French plain text.*

Vowels A, E, I, O, U, and Y =43.8%
High-Frequency Consonants N, R, S, and T =30.7%
Medium-Frequency Consonants C, D, L, M, and P =18.3%
Low-Frequency Consonants B, F, G, H, J, K, Q, V, W, X, and Z =7.2%
8 most frequent letters (in decending order of frequency) E, S, A, N, I, R, T, and O =68.9%

1–e. *Absolute frequencies of single letters as initial letters of 10,748 words in French plain text, arranged according to their frequencies. (One-letter words have been omitted).*

D	1,445	L	784	I	315	U	250	H	67
P	929	S	664	F	313	O	177	Z	7
E	894	Q	394	T	305	G	146	K	5
A	866	R	389	N	278	B	115	W	3
C	816	M	337	V	263	J	98	Y	3

2–a. *Frequency distribution of digraphs based on 55,758 letters of French plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	2	6	20	12	4	6	11		50	1		36	12	68	1	21	3	41	17	46	29	13			2	1
B	4				4				4			12			4			5	2	1	2					
C	15		6		47			11	20			5			48			4	1	8	8					
D	18			1	109			1	20	1			1		10	1		6	2		26					
E	30	4	49	48	30	15	14	3	13	5		56	58	105	4	38	12	89	154	58	27	17		8		3
F	10		2	1	9	6			8			1			8	1		10	1		1					
G	6				16		1		2			3	1	7	6			8		4	2					
H	6				6				4						3			1			4					
I	9	3	12	10	41	4	4			1		27	8	49	51	5	12	27	52	47		9		7		1
J	4				6										5						2					
K															1											
L	57		1	5	95	1		1	23			26		3	10	1			5	4	12				1	
M	22	9	1	1	52				23				13		8	9			1		4					
N	19	1	29	40	54	9	11	1	20	1		3	2	10	19	6	4	3	53	99	4	7				1
O		5	7	3	1	1	2	1	21	1		10	21	109		7		27	13	8	52	2			2	
P	30		1	1	13			2	3			11			35	9		34	1	6	4					
Q			1																		54					
R	62	2	10	13	127	2	6		24	1		16	11	8	27	5	3	7	14	19	6	7				1
S	42	2	16	32	75	5	2	1	36	2		15	8	6	22	24	11	8	41	33	24	4			1	
T	40	1	7	22	78	4	1	2	67	1		12	4	4	14	11	7	44	23	10	11	2				
U	12	3	10	5	39	4	3	1	24	3		13	6	26	1	8	1	48	26	19	1	8		13	1	
V	9				24				16						16			5			2					
W																										
X	4		3	3	3			1	1				1	1		4	1	1	2	3		1				
Y	2				2										1				2							
Z					3				1						1											

2–b. *Digraphic kappa plain, French language* =.0093 (I.C. =6.29).

9. GERMAN

1–a. *Absolute frequencies of single letters of German plain text, arranged alphabetically, based on 60,046 letters of text. (The letters* X *and* Y *are derived from foreign words contained in German plain text.)*

A	3,601	G	1,921	L	1,988	Q	6	V	523
B	1,023	H	2,477	M	1,360	R	4,339	W	899
C	1,620	I	4,879	N	6,336	S	4,127	X	12
D	3,248	J	192	O	1,635	T	3,447	Y	24
E	10,778	K	747	P	499	U	2,753	Z	654
F	958								

1–b. *Monographic kappa plain, German language* =.0787 (I.C. =2.05).

1–c. *Frequency distribution of single letters based on 60,046 letters of German plain text, reduced to 1,000 letters, arranged according to their frequencies.*

E	180	T	57	G	32	F	16	P	8
N	106	D	54	O	27	W	15	J	3
I	81	U	46	C	27	K	13	Y	Ø
R	72	H	41	M	23	Z	11	X	Ø
S	69	L	33	B	17	V	9	Q	Ø
A	60								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 60,046 letters of German plain text. Percentage of 8 most frequent letters in German text.*

Vowels A, E, I, O, U, and Y =39.4%
High-Frequency Consonants D, N, R, S, and T =35.8%
Medium-Frequency Consonants B, C, F, G, H, L, M, and W =20.4%
Low-Frequency Consonants J, K, P, Q, V, X, and Z =4.4%
8 most frequent letters (in descending order of frequency) E, N, I, R, S, A, T, and D =67.9%

1–e. *Absolute frequencies of single letters as initial letters of 9,568 words in German plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

D	1,716	U	550	Z	343	K	263	O	135
A	762	W	544	M	339	P	181	T	106
S	698	G	461	N	306	R	167	C	22
E	686	B	460	F	280	L	158	Q	2
I	581	V	408	H	265	J	135		

2–a. *Frequency distribution of digraphs based on 60,046 letters in German plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	4	14	10	4	33	7	9	7	1	1	2	33	13	48		2		22	27	23	36	1	1			1
B	6				48		1	1	5			3			3			11	2	1	3		1			1
C								130			5															
D	29	2		8	127	1	2	2	60		1	3	2	2	4	1		5	8	2	9	2	2			2
E	13	22	10	31	13	12	32	24	90	2	6	28	25	235	3	6		195	68	28	24	9	15			7
F	7	1		3	15	7	2		2			2	1	1	3			10	2	10	12					
G	10	1		8	78	1	2	2	8		2	7	1	3	1			11	8	7	8	2	1			1
H	29	1		8	64	1	2	1	14		2	8	3	6	6	1		20	4	23	7	2	3			1
I	3	1	39	7	91	2	18	7	2		7	12	11	84	13	1		7	73	44	1	2	1			1
J	4				8																3					
K	12	1		1	11		1	1	1			5			9			10	1	5	4					
L	26	3	1	6	27	1	2		37		3	20	1	2	4				10	12	6	1				1
M	16	3		4	26	2	2	1	14	1	2	1	11	1	8	5		1	3	3	9	1	1			1
N	39	12	1	118	58	9	57	8	35	4	10	6	10	18	8	5		4	36	27	20	10	17			14
O	1	3	5	3	11	3	3	3			1	18	6	33	1	5		18	12	4	1	1	5			1
P	10				5	4		1	2			1			7	2		7		1	1					
Q																					1					
R	34	11	5	35	60	9	12	9	37	2	11	6	8	12	19	3		6	22	18	26	6	8			5
S	14	6	55	13	46	3	7	3	30	1	5	4	7	3	16	6		2	40	57	9	5	5		1	5
T	25	3		17	88	2	4	6	40	1	3	7	3	4	4			14	20	7	16	2	10			13
U	1	2	8	2	37	15	5	1			2	2	11	76		2		18	28	14	1	1	2			1
V	1				19				3						21											
W	16				24				20	3					6						6					
X																										
Y																										
Z	1			1	8				5			1			2					4	27		4			

2–b. *Digraphic kappa plain, German language* =.0111 (I.C. =7.50).

10. GREEK

1–a. *Absolute frequencies of single letters of Greek plain text, arranged alphabetically, based on 65,936 letters of text.*

A	7,242	H	3,440	N	5,668	T	5,096
B	529	Θ	807	Ξ	277	Υ	2,871
Γ	915	I	6,176	O	6,263	Φ	481
Δ	1,256	K	2,494	Π	2,337	X	659
E	5,348	Λ	1,842	P	2,705	Ψ	112
Z	213	M	1,961	Σ	5,232	Ω	2,012

1–b. *Monographic kappa plain, Greek language* =.0689 (I.C. =1.65).

1–c. *Frequency distribution of single letters based on 65,936 letters of Greek plain text reduced to 1,000 letters, and arranged according to their frequencies.*

A	110	E	81	Υ	44	Ω	31	Γ	14	Φ	7
O	95	Σ	79	P	41	M	30	Θ	12	Ξ	4
I	94	T	77	K	38	Λ	28	X	10	Z	3
N	86	H	52	Π	35	Δ	19	B	8	Ψ	2

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 65,936 letters of Greek plain text. Percentage of 7 most frequent letters in Greek plain text.*

Vowels A, O, I, E, H, Υ, and Ω =50.6%
High-Frequency Consonants N, Σ, and T =24.3%
Medium-Frequency Consonants P, K, Π, M, Λ, and Δ =19.1%
Low-Frequency Consonants Γ, Θ, X, B, Φ, Ξ, Z, and Ψ =6.0%
7 most frequent letters (in descending order of frequency) A, O, I, N, E, Σ, and T =62.2%

1–e. *Absolute frequencies of single letters as initial letters of 11,850 words in Greek plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

T	2,359	Π	883	Σ	538	Γ	171	Λ	108	P	60
E	1,696	Δ	730	N	370	B	156	I	95	Z	51
A	1,152	O	681	Θ	220	X	137	Φ	76	Ψ	23
K	896	M	548	Υ	193	H	129	Ω	70	Ξ	21

2–a. *Frequency distribution of digraphs based on 65,936 letters of Greek plain text, reduced to 5,000 digraphs.*

	Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π	Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω
Α	11	6	17	15	17	4	3	11	75	19	28	19	77	5	7	34	26	73	71	14	10	5	1	1
Β	11				8		2		5		2				6		3			1				1
Γ	13		4		9		3		9	8	4	3	2		8		3			1		1		3
Δ	5				15		11		41						8		7			6				2
Ε	6	3	9	10	2	1	1	6	85	17	22	9	49	10	3	25	48	22	30	15	4	10	2	15
Ζ	2				4		4		1						3									1
Η	6	1	4	4	9	1	4	6	1	8	6	21	61	1	4	6	10	77	20	1	4	2	1	1
Θ	15				16		13		1		1	1	3		4		2			3				1
Ι	86	4	4	15	20	4	8	4	1	69	7	8	45	2	43	14	7	68	36	2	2	4	1	13
Κ	70	1		1	12		21	1	3	2	4	2	1		35	1	11	1	8	8				10
Λ	22			1	18		21	1	19	1	21	1			21	1			1	3	1			10
Μ	31	4			47		8		15			6	1		25	7				1	2			3
Ν	77	3	2	21	61	1	26	6	22	18	2	10	8	1	46	18	1	13	68	4	1	3	1	17
Ξ	4				5		2		4						2					2				2
Ο	4	8	8	8	9	1	3	3	39	10	22	25	68	1	4	28	15	52	33	120	6	4	2	1
Π	22				24		5		18		8		1		63		27		3	1				5
Ρ	38	1	13	1	15		7	1	41	2		2	6		51		2		4	4	1	4		12
Σ	33	4	2	13	59	1	19	8	33	25	3	20	3		25	17	1	13	77	23	4	9		6
Τ	74				28	1	86		43			1			95		15	2	1	4				37
Υ	6	6	4	6	11	1	2	6	2	9	9	15	35	1	8	23	11	31	25	1	1	4	1	1
Φ	7				4		3	2	4		1				7		2			1				4
Χ	7				10		4	3	4				1		5		3			1				8
Ψ	1				2		3		2											1				
Ω	1		1	1	1		1	2		1	1	7	70		1	4	11	44	8		1	1		

2–b. *Digraphic kappa plain, Greek language*=.0080 (I.C.=4.61).

11. HUNGARIAN

1–a. *Absolute frequencies of single letters of Hungarian plain text, arranged alphabetically, based on 50,787 letters of text.*

A	4,579	E	5,414	I	2,427	M	1,738	Ő	417	T	4,277	V	1,082
Á	2,016	É	1,837	Í	229	N	2,626	P	489	Ty	3	W	13
B	937	F	391	J	667	Ny	309	Q	11	U	521	Y	15
C	198	G	1,261	K	2,557	O	2,178	R	2,261	Ú	111	Z	1,271
Cs	223	Gy	692	L	3,158	Ó	447	S	2,049	Ü	394	Zs	22
D	1,057	H	889	Ly	220	Ö	503	Sz	1,056	Ű	62		

1–b. *Monographic kappa plain, Hungarian language*=.0523 (I.C.=2.14).

1–c. *Frequency distribution of single letters based on 50,787 letters of Hungarian plain text; reduced to 1,000 letters and arranged according to their frequencies.*

E	107	I	48	M	34	B	18	P	10	Í	5	Zs	Ø
A	94	R	45	Z	25	H	18	Ó	9	Cs	4	Y	Ø
T	84	O	43	G	25	Gy	14	Ő	8	Ly	4	W	Ø
L	62	S	40	V	21	J	13	Ü	8	C	4	Q	Ø
N	52	Á	40	D	21	U	10	F	8	Ú	2	Ty	Ø
K	50	É	36	Sz	21	Ö	10	Ny	6	Ű	1		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 50,787 letters of Hungarian plain text. Percentage of 11 most frequent letters in Hungarian plain text.*

Vowels E, A, I, O, X, É, U, Ö, Ó, Ő, Ü, Í, Ú, Ű, and Y =42.0%
High-Frequency Consonants T, L, N, and K =24.8%
Medium-Frequency Consonants R, S, M, Z, G, V, D, Sz, B, H, Gy, and J =29.5%
Low-Frequency Consonants P, F, Ny, Cs, Ly, C, Zs, W, Q, Ty =3.7%
11 most frequent letters (in descending order of frequency) E, A, T, L, N, K, I, R, O, S, and Á =66.4%

1–e. *Absolute frequencies of single letters as initial letters of 8,130 words of Hungarian plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

M	724	É	329	L	215	Cs	132	C	48	Z	13
K	665	N	301	P	188	Á	100	G	39	Ő	13
A	636	Sz	298	R	169	O	66	Gy	34	Ó	11
E	513	F	275	J	152	U	61	Ö	32	Q	11
H	507	B	253	D	148	Ú	52	Ny	30	W	7
T	469	I	250	S	134	Ü	50	Í	15	Y	2
V	372										

2–a. *Frequency distribution of digraphs based on 50,787 letters of Hungarian plain text, reduced to 5,000 digraphs.*

	A	Á	B	C	C_s	D	E	É	F	G	G_y	H	I	Í	J	K	L	L_y	M	N	N_y	O	Ó	Ö	Ő	P	Q	R	S	S_z	T	T_y	U	Ú	Ü	Ű	V	W	Y	Z	Z_s
A	12	1	12	1	5	17	5	3	5	6	22	10	16		10	42	46		32	48	2	1				16		34	11	14	51		2				11			35	
Á	1		8	5	1	1	1	1		23		1	1		3	18	19	2	6	18	10							25	25	1	19						6			3	
B	21	2	12				24	4				1	11	1		1	1					3	2		1			1	1				2		1		1				
C							1	1				2	11				1					1																			
C_s	4	1					11						1			1						2		1											1						
D	7	5	1			1	27	12	1			2	7	3	3	1			2	3		9	2	4	2			1	1	1	4		3		1		1				
E	9	1	4	1	1	12	4	2	3	34	14	19	7		6	38	76	10	38	58	1	1				4		45	22	14	70		2	1			7	1		28	
É	1		4			3		1		19	1	1				9	14	2	3	9	11					8		23	48	6	13						3			3	
F	1	1					15	2					2									9		1	3			2							1						
G	17	11	3		1	1	16	6	2	2	1	3	7		2	4	3		4	5		13		1	1	1		4	1	2	5		2	1	1		3			1	
G_y	28		1				10	2				2	2			2	1		3	2		1		1	2	1		1	1	2	1		1	1	1	1	1				
H	24	4					15	1					6	2								25								9			2								
I	19	5	4	1	1	7	9	5	3	8	2	5	3		3	24	8	2	7	30		1	6	1		2		8	25	10	20		1	1	1		6			11	1
Í				1		1				1	1									1								2			13						2			1	
J	14	11				2	11	2							2		2					4	2	1				1			5		6		1						
K	35	5	5	1	1	2	39	18	3	1		5	24	1	2	13	3		10	4	1	26	1	17	1	2		8	3	3	6		2	1	7		4				
L	40	17	4		2	7	60	15	4	4		5	12	2	6	12	21	1	11	9		17	7	2	10	1		2	4	3	20		2		3		8				
L_y	2	1	1				7	1					1						1	1		2						1		1	2									2	
M	32	16	4		1		41	11	1			1	21	1	1	1	3		3	1		11	1			2		1		1	4		6	1	1	1	2			4	
N	38	4	4	4	2	22	40	13	4	7	1	3	24	1	1	16	3		4	9	2	3		3	1	1		1	4	3	29		2	1	1	1	3			1	
N_y	2	1	1				5	1				1	3	1			1		1	1		3						1		1	3		1				1			3	
O	1		2	1	1	3				9	17	1				19	26	3	9	23	3					1		26	13	4	23						15			14	1
Ó	1		2			2	1	1	1	1		1	2		2	3	10		1	2						2		1	3	1	5						2			1	
Ö			2			1				1						4	4		1	4								8	3	1	6						6			8	
Ő	1		1			4	1		1			1	1		1	4	6		1	1								2	4	1	3						2			3	
P	4	5			1		7	2					2	3	2	1				1		11	1			1		4					1		1		2				
Q																																	1								
R	22	13	2	2	1	10	31	6	1	1	2	2	20		3	6	2		10	3		14	5	2	6	1		4	4	9	20		1	1	7	1	9		1	1	
S	19	23	5		1	1	23	22	4	1		6	6	5	2	6	5		9	4		9	1	1	3	3	1	4	8	8	14		3	1	4		2				
S_z	8	14					19	4				1	4	1		2	14		1			7	3	1	1	1		1			15		1		3		1				
T	63	26	7	1	2	1	67	25	5	1	1	11	25		9	13	7		12	9		20	7	9	6	2		8	11	6	44		8	1	3	1	8				
T_y																																									
U	1					3				3	1				1	4	7		2	7		1						3	7	1	9										
Ú											1				3	1	2	1													1										
Ü										1	4					4	22			4								1			1								1		
Ű																1	1			1								1													
V	24	23					21	12				1	13		2							5			1					1			1		1						
W	1																																								
Y																1						1																			
Z	17	7	1			3	23	8		1		1	7			2	2		3	3		14	4	3	3			1	1		14		2	1	1		1			3	
Z_s										1												1																			

2–b. *Digraphic kappa plain, Hungarian language*=.0043 (I.C.=7.23).

12. INDONESIAN

NOTE: C and V are exclusively found in words of Dutch or English origin. F is used almost interchangeably with P, usually in words of Arabic origin. Q and X are used exclusively in words of occidental origin. Y is sometimes, though rarely, used for I or J. In the present text, however, it appears only in words of English origin. 2 is the sign of reduplication, either of the whole word or its root. W is used mostly in words of Arabic or Japanese origin, though it is sometimes used for consonantal U.

1–a. *Absolute frequencies of single letters of Indonesian plain text, arranged alphabetically, based on 56,653 letters of text.*

A	10,828	F	83	K	3,166	P	1,794	U	2,459	2	331
B	1,383	G	1,994	L	1,740	Q	6	V	40		
C	94	H	1,332	M	2,385	R	2,750	W	254		
D	3,082	I	4,073	N	5,670	S	2,085	Y	11		
E	4,968	J	1,542	O	1,277	T	3,299	Z	7		

1–b. *Monographic kappa plain, Indonesian language* =.0833 (I.C. =2.17).

1–c. *Frequency distribution of single letters based on 56,653 letters of Indonesian plain text, reduced to 1,000 letters and arranged according to their frequencies.*

A	191	K	56	S	37	B	25	C	2	Q	Ø
N	100	D	54	G	35	H	24	F	1		
E	88	R	49	P	32	O	23	V	Ø		
I	72	U	43	L	31	2	6	Y	Ø		
T	58	M	42	J	27	W	4	Z	Ø		

1–d. *Percentage of vowels, high-frequency consonants, medium frequency consonants, and low-frequency consonants in 56,653 letters of Indonesian plain text. Percentage of 4 most frequent letters in Indonesian plain text.*

Vowels A, E, I, U, O and Y =41.7%
High-Frequency Consonants N, T, K and D =26.9%
Medium-Frequency Consonants R, M, S, G, P, L, J, B and H =30.0%
Low-Frequency Consonants 2, W, C, F, V, Z and Q =1.4%
4 most frequent letters (in descending order of frequency) A, N, E, and I =45.1%

1–e. *Absolute frequencies of single letters as initial letters of 9,209 words of Indonesian plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

D	1,532	S	795	J	344	L	141	E	33
M	969	A	447	R	240	N	126	V	13
K	893	I	418	H	234	G	66	Q	5
P	893	C	393	U	190	F	47	Z	3
T	837	B	348	O	170	W	46	Y	2

2–a. *Frequency distribution of digraphs based on 56,653 letters of Indonesian plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	Y	Z	2
A	24	18	3	67	3	2	14	57	28	13	86	41	49	270	5	47		64	51	86	16		6			6
B	43	1		1	44				9	1	1	7			3	1		1	1		10					
C								3	1		1				2											
D	108	1		1	28	1		65	38	1	3	1	2	8	1		3	1	1	10						
E	7	14	1	12	1	1	11	11	3	1	24	40	51	92	2	21		106	20	19	1	1	2			
F	1			1	1				3																	
G	59	6		11	10		10	6	12	2	11	3	8	2	3	4		3	8	7	8					4
H	38	5		7	4			1	5	3	7	2	6	2	4	6		3	4	4	5		9			2
I	38	13	1	34	3	2	2	14	5	4	40	17	16	5	7	12		15	27	45	6		2			2
J	95				7		7		7			1			5						14					
K	112	4		8	43				21	7	8	1	12	3	22	6		4	9	7	12					1
L	71	1		5	16				26	1	2	2	5	1	4	2			2	2	11					2
M	43	20		3	74	1		1	16		5	1	2	1	6	17		2	2	2	15					
N	38	16	1	68	18	1	118	5	27	42	21	3	16	8	8	19		5	16	53	9	1	1			8
O	3	2	1	3	20		2	1	1		9	17	15	19	1	2		11	2	4						
P	49	1		3	53			1	12	1	3	1	2		7	1		6	2	2	14					
Q																					1					
R	67	5		13	28		2	6	44	1	8	4	3	6	9	3		1	8	14	18	1				2
S	40	3		6	44		1	1	34	3	4	1	4	2	7	1			3	8	20					1
T	77	2		8	42		2	1	37	15	17	1	5	4	7	2		4	5	16	45					1
U	23	8		17			6	8	3	1	29	9	16	33	1	9		13	21	18			1			1
V	1								1						1											
W	20								1																	
Y																										
Z																										
2	1	2		4					2	3	2	1	1	1		3		1	2	3	1					

2–b. *Digraphic kappa plain, Indonesian language*=.0111 (I.C.=7.50).

13. ITALIAN

NOTE: In all calculations, accented letters have been combined with the corresponding unaccented letter.

1–a. *Absolute frequencies of single letters of Italian plain text, arranged alphabetically, based on 57,906 letters of text.*

A	6,771	G	1,168	L	3,592	Q	227	V	1,024
B	527	H	493	M	1,441	R	4,037	W	13
C	2,367	I	6,568	N	4,094	S	2,967	X	9
D	2,258	J	18	O	5,022	T	4,139	Y	14
E	6,784	K	28	P	1,616	U	1,547	Z	527
F	655								

1–b. *Monographic kappa plain, Italian language*=.0745 (I.C.=1.94).

1–c. *Frequency distribution of single letters based on 57,906 letters in Italian plain text, reduced to 1,000 letters and arranged according to their frequencies.*

E	117	R	70	P	28	F	11	K	Ø
A	117	L	62	U	27	B	9	J	Ø
I	113	S	51	M	25	Z	9	Y	Ø
O	87	C	41	G	20	H	9	W	Ø
T	72	D	39	V	18	Q	4	X	Ø
N	71								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 57,906 letters of Italian plain text. Percentage of 8 most frequent letters in Italian plain text.*

Vowels A, E, I, O, U, and Y =46.1%
High-Frequency Consonants L, N, R, and T =27.4%
Medium-Frequency Consonants, C, D, G, M, P, S, and V =22.2%
Low-Frequency Consonants B, F, H, J, K, Q, W, X, and Z =4.3%
8 most frequent letters (in descending order of frequency) E, A, I, O, T, N, R, L =07.8%

1–e. *Absolute frequencies of single letters as initial letters of 10,481 words in Italian plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

D	1,381	L	500	T	337	U	217	J	13
C	1,041	R	403	G	333	Q	172	W	9
S	885	N	396	F	298	B	153	K	6
P	830	E	374	V	263	H	69	Y	3
A	822	M	371	O	235	Z	29	X	2
I	685								

2–a. *Frequency distribution of digraphs based on 57,847 letters of Italian plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	18	9	39	41	14	12	22	1	19			76	24	78	5	24	4	57	36	63	6	24				12
B	10	7			7				10			1			4			4			2					
C	32		10		20			33	33			2			64		1	5			6					
D	31			1	65				64						23			2			9					
E	23	7	31	53	15	8	22	2	25			66	18	73	6	22	4	96	62	27	6	17				4
F	9				11	7			11			1			10			6			3					
G	9				11		8	2	20			17		8	9			11			6					
H	6				27				9																	
I	66	8	52	30	31	11	11	2	11			35	31	62	44	20	3	20	48	45	15	16				7
J																					1					
K																										
L	62	3	8	6	49	2	7		56			52	4	2	21	5	1	3	6	15	7	3				
M	31	5			35				17				4		18	13					2					
N	32	1	15	26	51	6	11	1	37			3	1	10	50	4	5	2	11	66	8	4				11
O	17	4	22	27	10	5	10	1	20			45	24	86	4	25	2	55	40	14	3	18				2
P	23				30				14			2			28	11		23			7					
Q																					20					
R	64	1	8	8	71	1	7		63			4	13	9	45	2		12	9	16	10	3				3
S	20		15	1	32	2			45			2	3		25	9			31	58	12	1				
T	83		1		65	1			59			1		1	56			43	1	37	10					
U	12	2	4	3	15	1	3		10			6	3	24	8	6		9	11	15						1
V	26				23				23						10			2			2	2				
W																										
X																										
Y																										
Z	13				4				20						3											5

2–b. *Digraphic kappa plain, Italian language* =.0081 (I.C. =5.48).

14. NORWEGIAN

NOTE: The letter Q is derived from foreign words in Norwegian plain text.

1–a. *Absolute frequencies of single letters of Norwegian plain text, arranged alphabetically, based on 56,190 letters of text.*

A	3,228	G	2,211	M	1,841	S	3,572	Z	4
B	739	H	830	N	4,666	T	4,497	Æ	154
C	20	I	3,298	O	2,702	U	718	Ø	505
D	2,714	J	487	P	846	V	1,505	Å	957
E	9,119	K	2,077	Q	1	W	5		
F	1,287	L	3,043	R	4,813	Y	351		

1–b. *Monographic kappa plain, Norwegian language* =.0712 (I.C. =1.98).

1–c. *Frequency distribution of single letters based on 56,190 letters of Norwegian plain text reduced to 1,000 letters, arranged according to their frequencies.*

E	162	A	57	M	33	B	13	C	0
R	86	L	54	V	27	U	13	W	0
N	83	D	48	F	23	Ø	9	Z	0
T	80	O	48	Å	17	J	9	Q	0
S	64	G	39	P	15	Y	6		
I	59	K	37	H	15	Æ	3		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 56,190 letters of Norwegian plain text. Percentage of 8 most frequent letters in Norwegian plain text.*

Vowels A, E, I, O, U, Y, Æ, Ø, and Å =37.4%
High-Frequency Consonants N, R, S, and T =31.2%
Medium-Frequency Consonants D, F, G, H, K, L, M, P, and V =29.1%
Low-Frequency Consonants B, C, J, Q, W, and Z =2.3%
8 most frequent letters (in descending order of frequency) E, R, N, T, S, I, A, and L =64.4%

1–e. *Absolute frequencies of single letters as initial letters of 11,678 words in Norwegian plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

S	1,233	V	669	I	367	R	184	W	4
D	1,099	M	646	P	340	J	76	Z	2
F	922	H	637	N	322	Ø	55	Æ	1
E	853	T	562	G	287	Å	34		
O	712	B	483	L	278	Y	8		
A	677	K	420	U	207	C	6		

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2–a. *Frequency distribution of digraphs based on 56,190 letters of Norwegian plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	Y	Z	Æ	Ø	Å
A	1	2		11	2	6	16	1	1		13	32	14	61	2	4		47	17	30	2	24						
B	8				22				3			12			2			8			2			4			1	2
C								1																				
D	16	2		9	**135**	3	1	2	14		1	4	2	4	4	1		15	11	9	3	3		1			1	2
E	11	12		44	13	21	25	13	21	1	21	58	23	**179**	13	8		**157**	55	**110**	4	20					1	2
F	9				8	3			5	2		8			44			14		3	2			1			9	6
G	15	4		8	54	6	8	8	9	12	2	3	5	6	7	2		8	18	7	3	5		1			1	7
H	30				16				2	2					10			1			2	7		1			2	1
I	3	2		25	10	7	38	4	2		29	43	5	55	3	4		9	31	14	1	7					1	
J	2				21										13						1						7	
K	22	1		1	50	1		1	5	6	24	4	1	2	18	1		8	10	14	10	3		2			1	
L	22	4		21	58	4	4	2	42	1	5	34	3	1	12	2		1	17	13	4	7		4		1	3	4
M	16	3		5	54	4	1	4	14		3	2	14	1	7	3		2	8	5	3	2		1			2	8
N	17	6		49	70	13	47	8	24	1	11	7	8	39	21	5		3	38	23	3	8		3		2	2	8
O	1	1		5	4	1	33	1	1		4	18	51	17	1	11		68	10	5		9						
P	5			1	10		2	1	2			3	1	1	4	12		8	2	2	1						2	18
Q																												
R	36	12		32	92	16	9	12	39	2	13	7	15	10	21	4		7	36	31	10	10		3			5	7
S	18	2	1	3	51	5	1	2	15	13	39	11	7	4	27	8		2	17	61	3	9		3		1	3	11
T	32	6		13	96	13	3	9	53	2	4	6	9	9	23	4		18	26	41	8	12		5		1	5	4
U	2	1		2	1	1	1				5	10	2	15		1		4	5	15								
V	19	1		5	38	2	2	1	36		1	1	1	3	5	1		1	3	2		1				8		2
W																												
Y				2	4		2				3	1		4		1		4	4	4		1						
Z																												
Æ																		14										
Ø		2		2		1					3	2	1	3		1		20	2	2		3		3				
Å	3	4		6	4	7	3	4	3	1	3	4	3	2	2	3		12	8	9	1	4						1

2–b. *Digraphic kappa plain, Norwegian language* =.0087 (I.C. =6.82).

15. PERSIAN

1–a. *Absolute frequencies of single letters of Persian plain text, arranged alphabetically, based on 55,322 letters of text.*

ا	9,186	ج	765	ذ	173	ش	1,090	ع	909	گ	484	و	3,612
ب	2,238	چ	167	ر	4,910	ص	376	غ	87	ل	1,681	ي	4,263
پ	177	ح	498	ز	1,009	ض	259	ف	822	م	3,319		
ت	2,929	خ	788	ژ	13	ط	492	ق	1,026	ن	3,747		
ث	41	د	3,997	س	1,478	ظ	160	ک	1,534	ه	3,102		

1–b. *Monographic kappa plain, Persian language* =.0713 (I.C. =2.28).

1–c. *Frequency distribution of single letters based on 55,322 letters of Persian plain text, reduced to 1,000 letters and arranged according to their frequencies.*

ا	166	و	65	ل	30	ز	18	ح	9	پ	3	ث	Ø
ر	89	م	60	ک	28	ع	16	ط	9	ذ	3	ژ	Ø
ي	77	ه	56	س	27	ف	15	گ	9	چ	3		
د	72	ت	53	ش	20	خ	14	ص	7	ظ	3		
ن	68	ب	40	ق	19	ج	14	ض	5	غ	2		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 55,322 letters of Persian plain text. Percentage of 9 most frequent letters in Persian plain text.*

Vowels ا, ي, and و =30.8%
High-Frequency Consonants ر, د, ن, م, ه, and ت =39.8%
Medium-Frequency Consonants ب, ل, ک, س, ش, ق, ز, ع, ف, خ, and ج =24.1%
Low-Frequency Consonants ح, ط, گ, ص, ض, پ, ذ, چ, ظ, غ, ث, and ژ =5.3%
9 most frequent letters (in descending order of frequency) ا, ر, ي, د, ن, و, م, ه, and ت =70.6%

1–e. *Absolute frequencies of single letters as initial letters of 13,793 words of Persian plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

ا	2,650	ن	810	ه	314	ح	202	ص	124	ل	71	ژ	8
م	1,381	ت	673	س	291	ع	173	چ	113	غ	22	ث	5
ب	1,341	ر	585	و	272	ف	168	ي	105	ض	21		
د	1,056	خ	444	ج	251	پ	158	ز	84	ذ	21		
ک	814	ش	390	ق	249	گ	142	ط	74	ظ	11		

2–a. *Frequency distribution of digraphs based on 55,322 letters of Persian plain text, reduced to 5,000 digraphs.*

	ا	ب	پ	ت	ث	ج	چ	ح	خ	د	ذ	ر	ز	ژ	س	ش	ص	ض	ط	ظ	ع	غ	ف	ق	ک	گ	ل	م	ن	ه	و	ي
ا	29	39	2	36	2	11	2	7	13	40	4	86	46		41	20	6	4	8	4	19	1	14	17	14	6	33	62	107	30	19	117
ب	58	2	1	8		2		2	2	8		28	2		4	3	1		3		7		2	3	3	1	8	6	7	12	19	14
پ	4											2			2	1													1		1	5
ت	36	18	1	4		3		7	12	13	1	14	1		5	9	4	2		2	7	1	5	8	9	1	7	21	14	23	22	14
ث	1											1																				1
ج	17	2								5		4	2								4						3	3	8	12	4	2
چ	1																								1				3	5	2	1
ح	5	1		4						3		4	1		2		2	4					1	3	2		2	3		1	4	2
خ	17	3		4						1		4			2	1	4		1								5				23	4
د	74	18	1	11		1	2	1	3	15		52	1		7	4	1				2	1	1	2	14	4	2	19	22	46	40	16
ذ	7											3				1									1							2
ر	136	14	3	26		13	3	5	7	40		7	2		11	6	3	2	2		3	1	18	2	16	4	1	24	12	11	30	43
ز	26	5	1	2		2		3	1	4		3			2	2	1		3		1			3	1		1	9	4	4	4	10
ژ																													1			
س	19	5		48		1			2	2		5			1	1			2		1		9		2		4	4	3	9	4	10
ش	11	1		11					2	19		9				1						1			6	1		5	5	2	23	1
ص	4	1						1		3		4											1				4	1	2	1	8	2
ض	8											3									1							2			5	3
ط	3	3										7			1						2						7	1	1	15	3	1
ظ	2											5																	1	3	2	
ع	11	5		6		1			1	8		4	3			1		7	1		1		1	3	1		8	7	3	4	3	7
غ	1											1															1				1	2
ف	14	1		10		1				2		16			1	2	1				4			6	1		1	1	1	2	5	6
ق	18	5		6						8		7			2	2	1	1	16		3		1				3	2		1	8	8
ک	28	2		5								19	1		2	4											2	8	16	37	7	4
گ	5			1							2	18	2			1							1				3	1			3	5
ل	36	7	1	19		1	1	2	1	3		2	1		3	2			1		2	2	2	1	4	5	3	8	6	8	8	25
م	46	9	1	13		6	1	6	7	11	4	26	3		10	6	2	1	4		7		2	8	8	1	13	7	16	10	36	36
ن	42	15	2	14		9	2	2	7	61		9	2		9	6	1		1	6	3		7	19	11	11		37	11	19	15	21
ه	69	19	2	10		3	2	2	5	29	1	20	3		5	10	2		1		2		4	4	7	2	1	34	16	6	15	9
و	55	12	1	9		7	2	2	2	47	1	38	17		11	5	3	6	1	1	10		2	10	4	2	28	17	16	4	2	11
ي	48	15	1	17		7	1	4	4	40		45	5		13	12	1		1		3	1	2	3	33	4	10	20	65	16	11	4

2–b. *Digraphic kappa plain, Persian language*=.0065 (I.C.=6.66).

16. POLISH

NOTE: The letters Q and V are derived from foreign words appearing in Polish plain text.

1–a. *Absolute frequencies of single letters of Polish plain text, arranged alphabetically, based on 53,845 letters of text.*

A	4,695	E	4,232	J	1,362	Ń	161	S	2,414	Y	2,146
Ą	582	Ę	558	K	1,776	O	4,314	Ś	264	Z	2,906
B	753	F	164	L	1,125	Ó	542	T	1,872	Ż	489
C	2,330	G	746	Ł	900	P	1,789	U	1,035	Ź	39
Ć	189	H	675	M	1,309	Q	1	V	4		
D	1,902	I	4,323	N	2,980	R	2,541	W	2,727		

1–b. *Monographic kappa plain, Polish language* =.0528 (I.C. =1.80).

1–c. *Frequency distribution of single letters based on 53,845 letters of Polish plain text, reduced to 1,000 letters and arranged according to their frequencies.*

A	87	W	51	T	35	U	19	Ę	10	Ń	3
I	80	R	47	P	33	Ł	17	Ó	10	Ź	Ø
O	80	S	45	K	33	B	14	Ż	9	V	Ø
E	79	C	43	J	25	G	14	Ś	5	Q	Ø
N	55	Y	40	M	24	H	13	Ć	4		
Z	54	D	35	L	21	Ą	11	F	3		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 53,845 letters of Polish plain text. Percentage of 4 most frequent letters in Polish plain text.*

Vowels A, I, O, E, Y, U, Ą, Ę, and Ó =41.7%
High-Frequency Consonants N, Z, W, R, S, and C =29.5%
Medium-Frequency Consonants D, T, P, K, J, M, and L =20.7%
Low-Frequency Consonants Ł, B, G, H, Ż, Ś, Ć, F, Ń, Ź, V, and Q =8.1%
4 most frequent letters (in descending order of frequency) A, I, O, E =32.6%

1–e. *Absolute frequencies of single letters as initial letters of 8,451 words of Polish plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

P	1,196	Z	521	T	335	C	254	U	160	F	54	Ł	4
S	729	D	500	R	283	A	213	G	142	E	33	Ź	2
N	626	O	436	M	275	J	215	I	118	Ś	32	Ą	1
W	606	K	391	B	265	Ż	179	L	108	H	21	Y	1

2–a. *Frequency distribution of digraphs based on 53,845 letters of Polish plain text, reduced to 5,000 digraphs.*

	A	Ą	B	C	Ć	D	E	É	F	G	H	I	J	K	L	Ł	M	N	Ń	O	Ó	P	Q	R	S	Ś	T	U	V	W	Y	Z	Ż	Ź
A	3		9	35	6	27			2	8		4	22	21	18	26	23	53	9	6		20		33	24	2	23	3		31		17	8	1
Ą	1		2	10	1	8				2		2		1		1	1	2		2		4		1	5		2			4		4	2	
B	6						6	4				9		1	2			2		7		1		10	1			4			15	1		
C	7	1				1	15	1			60	26	18	8			1	3		6	3	1		1	1		2	2		1	13	48	1	
Ć			1			1						1					1	2		1		2			3		1	1		1		2		
D	15	2	3	3		2	14	1				1	1	3	8	3	2	15		36	3	3		5	3		1	6		4	9	33		2
E	3		7	24	1	26	1		2	29		4	35	13	19	2	29	39	5	6		19		32	31	4	11	3		22		20	6	
É			1	4		10				1		1	1	3	1			4		2		6			4	1	4	1		3		3	2	
F	2						2					4								2				2				1			1			
G	11	1				4	2					4			2	3		1		26	2			10				2		1				
H	4		1	3		2			1	1		3	2	3	1	1	2	5		9	1	6		1	3		2	2		6	1	4	1	
I	47	8	2	27	3	10	132	26	1	2		4	4	11	6	8	8	17		12		13		3	16	1	9	6		14		9	5	
J	17	13	1	3		3	22	4	1	1		17	1	3	1		3	5		3	1	4		2	11			5		4		2		
K	26	3		2		1		2				45	1	1	1	2	1	1		33	6	1		11	4		12	8		2		1		
L	12	1	1	1			19	2		1		23		5				10		8					14		1	6		1				
Ł	19	1	1			1	3	1		1				2				4		18	1	4		1	3		1	6		2	10	1	2	
M	13		2	2		2	8					35	2	2	1		1	4		18	4	6		1	3		1	6		3	5	2	1	
N	52	4		5		5	26	1	3	4		96	2	4				6		21	1	1			4		10	3		2	26			
Ń				3																					9									
O	2		19	14		40			3	6		3	11	13	29	7	16	36	2	4		19		22	32	13	16	2		67		18	6	
Ó			2	2		2							1		2	1								10			1			28			1	
P	14						5					10			2	2		1		70	2			53	1			4			1			
Q																																		
R	47	1		2		4	23	1		1		4	2	1		1	3	3		39	9	2			6		4	5		3	18	55	1	
S	10	3		6			6					25	4	35		7	1	2		10	2	15		1			52	4		5	4	30		
Ś				11	3										2		1	2						1						4				
T	33	2	1	1		1	20	4				1	1	7	1			8		15	11	1		19	1		1	11		12	23	1		
U	3		3	7		8				5		21	6	4	3	1	4	8		2		9		5	11	1	4			4		4	4	
V																																		
W	40	2	1	5		4	17	2	1	1		53	2	5	1	5	2	12		22	3	10		7	15	1	3	1		5	28	6		
Y	1		7	38	3	9			1	4		2	9	12	4	8	15	11		4		12		4	16	2	10	1		14		8	5	
Z	41	8	4	6		5	54	3		2		23	4	6	1	4	16	18		16	1	6		2	3		4	5		10	39	1		
Ż	3	2				1	16							2	1			4		3		1			2			1		1	7	1		
Ź				2														1																

2–b. *Digraphic kappa plain, Polish language* =.0055 (I.C. =6.36).

17. PORTUGUESE

1–a. *Absolute frequencies of single letters of Portuguese plain text, arranged alphabetically, based on 45,106 letters of text.*

A	5,362	G	724	L	1,245	Q	348	V	737
B	470	H	304	M	1,699	R	3,292	W	24
C	2,285	I	3,314	N	2,912	S	3,409	X	166
D	1,900	J	160	O	5,001	T	2,679	Y	22
E	5,441	K	17	P	1,377	U	1,491	Z	207
F	520								

1–b. *Monographic kappa plain, Portuguese language*=.0746 (I.C.=1.94).

1–c. *Frequency distribution of single letters based on 45,106 letters of Portuguese plain text, reduced to 1,000 letters, and arranged according to their frequencies.*

E	121	N	65	U	33	F	11	X	4
A	119	T	59	P	30	B	10	J	3
O	111	C	51	L	28	Q	8	W	1
S	76	D	42	V	16	H	7	Y	Ø
I	73	M	38	G	16	Z	5	K	Ø
R	73								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 45,106 letters of Portuguese plain text. Percentage of 8 most frequent letters in Portuguese plain text.*

Vowels A, E, I, O, U, and Y =45.8%
High-Frequency Consonants N, R, and S =21.3%
Medium-Frequency Consonants C, D, L, M, P, and T =24.8%
Low-Frequency Consonants B, F, G, H, J, K, Q, V, W, X, Y, and Z =8.1%
8 most frequent letters (in descending order of frequency) E, A, O, S, I, R, N, and T =69.7%

1–e. *Absolute frequencies of single letters as initial letters of 7,058 words in Portuguese plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

P	847	M	405	I	264	B	113	Z	14
C	731	T	348	F	222	G	111	W	11
E	608	R	316	Q	222	J	92	K	7
S	601	N	299	O	187	U	77	Y	4
A	597	V	271	L	143	H	60	X	2
D	506								

2–a. *Frequency distribution of digraphs based on 45,106 letters of Portuguese plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	11	11	52	60	15	9	14	2	18	2		38	36	56	49	23	8	68	72	22	8	16	1			5
B	11			1	10				5			2	1		9			9	2	1	2					
C	60		2		30			4	39			5		1	85			7		8	12					
D	45				61				33				1		61			2	1	1	5					
E	15	5	48	22	11	11	23	1	27	6	1	31	44	97	6	18	6	76	95	20	7	12	1	15		5
F	9				14				13			1			15			2			3					
G	15				14				4			1		1	14			14			15					
H	10				8				3						11						1					
I	41	3	34	31	6	7	9		1			16	22	53	26	5	2	25	39	27	2	10		2		7
J	7				2										2						7					
K																										
L	24	1	4	4	24	1	5	9	21			2	4	2	14	4	2	1	4	7	6	2				
M	41	10	3	4	51	1			26	1		1	2	1	16	15	1	3	5	2	6	2				
N	31		29	35	14	7	8	12	18						25	1			19	114	4	4				1
O	21	9	32	25	27	10	7	3	20	4		20	36	79	5	35	8	71	85	18	12	22	1	1	1	1
P	26		2		25				2			4		1	60	1	1	28	1	1	3					
Q					1																37					
R	75	2	14	9	86	3	7	1	46	1		2	18	8	34	7	3	11	8	18	4	6				1
S	41	6	22	10	62	6	3	2	23	2		3	12	5	23	35	7	4	40	47	18	5				
T	65		1	1	69	1			26					1	88			33		1	13					
U	22	5	5	7	26	1	4		18	1		14	11	17	2	4		9	6	11		1				2
V	11				37				23						9			1								
W	1																									
X	10		3		1				2							3				1						
Y																										
Z	7		1		9				2				1		1		1	1								

2–b. *Digraphic kappa plain, Portuguese language* =.0084 (I.C. =5.68).

18. ROMANIAN

NOTE: The letters K, Q, W, X, and Y are derived from foreign words.

1–a. *Absolute frequencies of single letters of Romanian plain text, arranged alphabetically, based on 56,370 letters of text.*

A	5,143	D	1,933	I	6,251	M	1,628	R	4,279	U	3,261
Ă	1,896	E	6,828	Î	610	N	3,753	S	2,308	V	623
Â	337	F	619	J	124	O	2,627	Ş	685	W	3
B	509	G	577	K	9	P	1,706	T	3,830	X	69
C	2,883	H	146	L	2,655	Q	1	Ţ	693	Y	3
										Z	381

1–b. *Monographic kappa plain, Romanian language* =.0670 (I.C. =2.08).

1–c. *Frequency distribution of single letters based on 56,370 letters of Romanian plain text, reduced to 1,000 letters and arranged according to their frequencies.*

E	121	U	58	D	34	Ş	12	B	9	X	1
I	111	C	51	Ă	34	V	11	Z	7	K	Ø
A	91	L	47	P	30	F	11	Â	6	W	Ø
R	76	O	47	M	29	Î	11	H	3	Y	Ø
T	68	S	41	Ţ	12	G	10	J	2	Q	Ø
N	67										

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 56,370 letters of Romanian plain text. Percentage of 6 most frequent letters in Romanian plain text.*

Vowels E, I, A, U, Ă, Î, Â, and Y =43.1%
High-Frequency Consonants R, T, and N =21.1%
Medium-Frequency Consonants C, L, O, S, D, P, and M =27.9%
Low-Frequency Consonants Ţ, Ş, V, F, G, B, Z, H, J, X, K, W, and Q =7.9%
6 most frequent letters (in descending order of frequency) E, I, A, R, T, and N =53.4%

1–e. *Absolute frequencies of single letters as initial letters of 10,655 words of Romanain plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

D	1,168	M	467	T	370	V	239	Z	52	Ă	1
C	1,167	N	449	F	369	O	200	J	31	Q	1
P	979	I	415	E	314	G	174	H	20	W	1
A	907	L	395	R	277	B	117	Â	5	X	1
S	862	Ş	392	U	249	Ţ	62	K	2	Y	1
Î	508										

2–a. *Frequency distribution of digraphs on 56,370 letters of Romanian plain text, reduced to 5,000 digraphs.*

	A	Ă	Â	B	C	D	E	F	G	H	I	Î	J	K	L	M	N	O	P	Q	R	S	Ş	T	Ţ	U	V	W	X	Y	Z
A	10			6	39	22	4	8	6		20	6	3		48	18	29	3	18		72	28	8	60	18	17	7				7
Ă	11			1	14	15	2	4	2		6	6	1		6	7	8	3	12		25	13	5	14	6	3	3				3
Â											1				1	1	18				2			7							
B	5	2	1				3				11				4			2	1		3	1			1	11					
C	33	29	9		2	2	48			6	28	2			4	2		30			10	1	1	11	5	34					
D	13	3			2	1	87		1		27	1			1	1	1	7	1		4	4	1	1		15	1				
E	73			8	57	38	9	10	14	1	30	11	3		45	25	48	13	35		54	60	18	19	4	8	13		5		7
F	8	6					8				16				2			8			3					3					
G	6	5	1				9			3	6				1		1	2			8					8					
H	1						3				5						1	1													
I	43			8	47	29	34	9	9	1	32	7	2		38	29	87	13	20		17	34	12	43	6	18	12				7
Î	1				1						1				1	3	43	1					1	1							
J	1						1				1				1			1								5					
K																															
L	30	7		1	5	6	65	1	1		29	3			2	3	2	27	5		2	4	2	11	1	28	2				
M	33	7	6	6	2	2	23				21	1			1	1	3	7	13			2	1	1	1	14					
N	27	9	1	1	26	26	40	5	7		40	2	1		3	4	2	17	6		2	19	2	52	12	26	4				1
O	20			6	14	7	2	2	3		7				13	20	28		9		62	12	1	10	1	5	6				3
P	14	8	1		1		32				5				7			19			42	2		6		14					
Q																															
R	36	21	2	2	9	7	102	4	5		78	4			3	10	8	27	5		2	6	3	15	3	27	2				1
S	18	22	1	1	17	1	29	2			18	2			2	1		7	9			2		57		18					
Ş	2				1		3				39													13		2					
T	40	29	4		6	7	76	3			46	6			3	2	2	33	4		44	4	2	1		28					
Ţ	6	7					6				39													1		1					
U	13	2		3	14	7	5	4	2		27	3			50	16	52	1	13		28	14	3	19	3	1	5				3
V	9	5	1				13				12							9			2					1					
W																															
X	1						1				2								2					1							
Y																															
Z	5	4	1	1			7				11							1								3					

2–b. *Digraphic kappa plain, Romanian language* =.0068 (I.C. =6.53).

19. RUSSIAN

1–a. *Absolute frequencies of single letters of Russian plain text, arranged alphabetically, based on 67,850 letters of text.*

А	5,122	З	1,280	Н	4,463	У	1,578	Щ	257
Б	1,095	И	4,923	О	8,078	Ф	127	Ы	1,421
В	3,543	Й	961	П	1,815	Х	941	Ь	960
Г	1,141	К	2,324	Р	3,427	Ц	369	Э	173
Д	2,076	Л	2,747	С	3,917	Ч	902	Ю	455
Е	5,537	М	1,936	Т	4,041	Ш	554	Я	1,185
Ж	502								

1–b. *Monographic kappa plain, Russian language* =.0568 (I.C. =1.76).

1–c. *Frequency distribution of single letters based on 67,850 letters of Russian plain text reduced to 1,000 letters, and arranged according to their frequencies.*

О	119	В	52	П	27	Б	16	Ж	7
Е	82	Р	50	У	23	Й	14	Ю	7
А	75	Л	40	Ы	21	Ь	14	Ц	5
И	73	К	34	З	19	Х	14	Щ	4
Н	66	Д	31	Я	17	Ч	13	Э	3
Т	60	М	29	Г	17	Ш	8	Ф	2
С	58								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 67,850 letters of Russian plain text. Percentage of 10 most frequent letters in Russian plain text.*

Vowels А, Е, И, Й, О, У, Ы, Э, Ю, and Я =43.4%
High-Frequency Consonants В, Н, Р, С, and Т =28.6%
Medium-Frequency Consonants Б, Г, Д, З, К, Л, М, П, Х, Ч, and Ь =25.4%
Low-Frequency Consonants Ж, Ф, Ц, Ш, and Щ =2.6%
10 most frequent letters (in descending order of frequency) О, Е, А, И, Н, Т, С, В, Р, and Л =67.5%

1–e. *Absolute frequencies of single letters as initial letters of 10,601 words in Russian plain text, arranged according to their frequencies.* (One-letter words have been omitted.)

П	1,210	Д	496	И	321	Х	120	Ф	58
С	983	М	446	Г	292	А	116	Ц	47
Н	800	Р	429	У	229	Е	92	Я	41
В	731	Т	418	Ч	182	Ж	72	Ю	34
О	650	З	404	Э	147	Ш	63	Щ	2
К	555	Б	344	Л	146				

2–a. *Frequency distribution of digraphs based on 67,850 letters of Russian plain text, reduced to 5,000 digraphs.*

	А	Б	В	Г	Д	Е	Ж	З	И	Й	К	Л	М	Н	О	П	Р	С	Т	У	Ф	Х	Ц	Ч	Ш	Щ	Ы	Ь	Э	Ю	Я
А	2	12	35	8	14	7	6	15	7	7	19	27	19	45	5	11	26	31	27	3	1	10	6	7	10	1			2	6	9
Б	5					9	1		6			6		2	21		8	1		6						1	11				2
В	35	1	5	3	3	32		2	17		7	10	3	9	58	6	6	19	6	7		1	1	2	4	1	18	1	2		3
Г	7				3	3			5		1	5		1	50		7			2											
Д	25		3	1	1	29	1	1	13		1	5	1	13	22	3	6	8	1	10			1	1	1		5	1			1
Е	2	9	18	11	27	7	5	10	6	15	13	35	24	63	7	16	39	37	33	3	1	8	3	7	3	3			1	1	2
Ж	5	1			6	12			5					6				1													
З	35	1	7	1	5	3			4		2	1	2	9	9	1	3	1		2							4				4
И	4	6	22	5	10	21	2	23	19	11	19	21	20	32	8	13	11	29	29	3	1	17	3	11	1	1			1	3	17
Й	1	1	4	1	2		1	2	4		5	1	2	7	9	7	3	10	2				1	3	2						
К	24	1	4	1		4	1	1	26		1	4	1	2	66	2	10	3	7	10			1								
Л	25	1	1	1	1	33	2	1	36		1	2	1	8	30	2		3	1	6		4		1			2	30		4	9
М	18	2	4	1	1	21	1	2	23		3	1	3	7	19	5	2	5	3	9	1			2			5	1	1		3
Н	54	1	2	3	3	34			58		3		1	24	67	2	1	9	9	7	1		5	2			36	3			5
О	1	28	84	32	47	15	7	18	12	29	19	41	38	30	9	18	43	50	39	3	2	5	2	12	4	3			2	3	2
П	7					15			4			9		1	46		41	1		6							2				2
Р	55	1	4	4	3	37	3	1	24		3	1	3	7	56	2	1	5	9	16		1	1	1	2		8	3			5
С	8	1	7	1	2	25			6		40	13	3	9	27	11	4	11	82	6		1	1	2	2		1	8			17
Т	35	1	27	1	3	31		1	28		5	1	1	11	56	4	26	18	2	10				1			11	21			4
У	1	4	4	4	11	2	6	3	2		8	5	5	5	1	5	7	14	7			1		8	3	2				9	1
Ф	2					2			2						1		1	1													
Х	4	1	4	1	3	1		2	3		4	3	3	4	18	5	3	4	2	2	1			1							
Ц	3					7			10		2				1					1							1				
Ч	12					23			13		2			6					7	1					1			1			
Ш	5					11			14		1	2		2	2					1								1			
Щ	3					8			6					1						1											
Ы		1	9	1	3	12		2	4	7	3	6	6	3	2	10	3	9	4	1		16		1	2						
Ь		2	4	1	1	2		2	2		6		3	13	2	4	1	11	3					1	4				1	3	1
Э											1			1				1	9												
Ю		2	1	2	1			3	1		1		1	1	1	3	1	1	7				1	1		4					
Я	1	3	9	1	3	3	1	5	3	2	3	3	4	6	3	6	3	6	10			2	1	4	1	1			1	1	1

2–b. *Digraphic kappa plain, Russian language* =.0052 (I.C. =5.00).

20. SERBO-CROAT

NOTE: The letter W is derived from foreign words appearing in Serbo-Croat plain text.

1–a. *Absolute frequencies of single letters of Serbo-Croat plain text, arranged alphabetically, based on 61,915 letters of text.*

A	7,161	Dj	271	J	2,370	N	3,424	S	3,246	H	389
B	847	E	5,625	K	2,283	Nj	509	T	2,541	C	512
V	2,467	Ž	416	L	1,750	O	5,773	Ć	42	Č	577
G	1,122	Z	1,108	Lj	361	P	1,858	U	2,802	Š	577
D	2,413	I	5,875	M	1,772	R	3,382	F	123	W	3

1–b. *Monographic kappa plain, Serbo-Croat language*=.0617 (I.C.=1.85).

1–c. *Frequency distribution of single letters based on 61,915 letters of plain text, reduced to 1,000 letters and arranged according to their frequencies.*

A	116	R	55	D	39	L	28	Š	9	H	6
I	95	S	52	J	38	G	18	C	8	Lj	6
O	93	U	45	K	37	Z	18	Nj	8	Dj	4
E	91	T	41	P	30	B	14	Ň	7	F	2
N	55	V	40	M	29	Č	9	Ć	7	W	Ø

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 61,915 letters of Serbo-Croat plain text. Percentage of 4 most frequent letters in Serbo-Croat plain text.*

Vowels A, I, O, E, and U=44.0%
High-Frequency Consonants N, R, S, and T=20.3%
Medium-Frequency Consonants V, D, J, K, P, M, L, G, Z, and B=29.1%
Low-Frequency Consonants Č, Š, C, Nj, Ž, Ć, H, Lj, Dj, F, and W=6.6%
4 most frequent letters (in descending order of frequency) A, I, O, and E=39.5%

1–e. *Absolute frequencies of single letters as initial letters of 11,460 words of Serbo-Croat plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

S	1,401	K	676	T	353	G	249	C	84	H	38
P	1,250	J	527	B	338	A	184	F	78	Nj	38
D	818	Z	437	U	319	Ć	133	E	76	Lj	27
N	760	V	436	M	314	Š	119	Ž	73	Dj	23
O	760	I	430	R	313	Č	116	L	66	W	1

2–a. *Frequency distribution of digraphs based on 61,915 letters of Serbo-Croat plain text, reduced to 5,000 digraphs.*

	A	B	V	G	D	Dj	E	Ž	Z	I	J	K	L	Lj	M	N	Nj	O	P	R	S	T	Ć	U	F	H	C	Č	Š	W
A	6	10	50	10	43	4	2	7	21	20	31	34	29	3	24	50	15	24	30	37	45	30	5	16	2	2	12	11	8	
B	8						5			17	2		4	1		2		10		12				5						
V	42						33			37	8	1	6	5		11		32		13	2			4			1		1	
G	13	1	1		1	1	5		1	6		1	9		1	2		22	2	14	2	1		6						
D	46	2	5	2	1		14		1	26	1	1	2		1	22	3	25	2	22	8	1		11						
Dj	2						11																	4						
E	4	10	16	12	56	6	1	3	15	16	6	24	19	5	23	54	9	18	25	25	43	25	12	17	2	2	4	5	6	
Ž	11	2					9			7						2	1							1						
Z	36	2	6	3	2		7			8	4		2		3	6		2	1	3	1			4						
I	4	4	18	6	15	1	1	2	28	8	55	26	20	2	38	45	6	21	18	13	45	25	5	11	2	24	12	13	9	
J	25	1	2		1		107		1	16		1			1	3		5	2		6			18	1					
K	33		3		1		17			18	1	1	3	1		1	1	68	1	11	2	3		17			1			
L	42	1					19			36		1			1	6		25	1		1	2		6						
Lj	8						10			2						1					2			5						
M	34	2	1	1	3		17		2	15	7	3	1	4	2	6		16	7	3	8	1		7			1	1		
N	78			4	5		37		1	65	1	2				1		53	2	1	8	6		10	1		4		1	
Nj	8						20			5								1			1			6						
O	1	23	54	30	50	3		5	9	6	49	15	12	5	35	25	1	5	18	30	46	18	3	4	1		2	7	8	
P	13						5			7	1		6	1				54		50	2	1		7					3	
R	77	2	4	5	2		43	10	1	40	1	2	1		2	7	1	38	1		8	2		19		1		1	6	
S	22		17				29			12	10	37	17	1	2	7		8	10	4	1	65		21			1			
T	45		12	1	2		18		1	50	3	3				6		28	3	20	5		1	8						
Ć	5						16			6						1		1			1			2						
U	2	7	8	15	12	1	1	7	9	9	10	12	5	1	9	13	2	8	23	8	24	10	5	7	1	1	2	9	4	
F	1						3			2								2		2										
H	1	1	2	1	4		1		1	4		1			1	2		3	2	2	2	1		1						
C	5						7			22	1							1		2				2						
Č	6						8			9	1	13	2			3		1						2						
Š	4						9			6	1	3	2			1	2		1			14	1	2						
W																														

2–b. *Digraphic kappa plain, Serbo-Croat language* =.0062 (I.C. =5.58).

21. SLOVAK

1–a. *Absolute frequencies of single letters of Slovak plain text, arranged alphabetically, based on 46,026 letters of text.*

A	4,694	F	110	K	1,816	R	2,428	W	23
B	810	G	124	L	1,892	S	2,238	X	12
C	732	H	596	M	1,512	Š	338	Y	1,343
Č	424	Ch	515	N	2,903	T	2,294	Z	935
D	1,666	I	3,251	0	4,827	U	1,656	Ž	349
E	3,848	J	849	P	1,487	V	2,354		

1–b. *Monographic kappa plain, Slovak language* =.0585 (I.C. =1.70).

1–c. *Frequency distribution of single letters based on 46,026 letters of Slovak plain text, reduced to 1,000 letters and arranged according to their frequencies.*

0	105	R	53	K	40	Y	29	H	13	G	3
A	102	V	51	D	36	Z	20	Ch	11	F	2
E	84	T	50	U	36	J	18	Č	9	W	0
I	71	S	49	M	33	B	18	Ž	8	X	0
N	63	L	41	P	32	C	16	Š	7		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 46,026 letters of Slovak plain text. Percentage of 5 most frequent letters in Slovak plain text.*

Vowels 0, A, E, I, U, and Y =42.6%
High-Frequency Consonants N, R, V, T, and S =26.6%
Medium-Frequency Consonants L, K, D, M, P, Z, J, B, and C =25.4%
Low-Frequency Consonants H, Ch, Č, Ž, Š, G, F, W, and X =5.4%
5 most frequent letters (in descending order of frequency) 0, A, E, I, and N =42.5%

1–e. *Absolute frequencies of single letters as initial letters of 7,882 words of Slovak plain text, arranged according to their frequencies.* (*One-letter words have been omitted*).

P	960	Z	378	D	350	R	263	Ž	110	Š	64	G	24
S	745	K	368	M	330	U	232	Č	108	C	59	Ch	19
V	602	T	361	B	313	J	144	H	108	F	54	W	11
N	577	0	356	A	297	L	127	I	89	E	39	Y	1

2–a. *Frequency distribution of digraphs based on 46,026 letters of Slovak plain text, reduced to 5,000 digraphs.*

	A	B	C	Č	D	E	F	G	H	Ch	I	J	K	L	M	N	O	P	R	S	Š	T	U	V	W	X	Y	Z	Ž
A	8	11	16	9	40	1	2	1	10	6	3	19	30	39	30	54	9	28	31	40	6	39	6	42			1	20	7
B	6					7				3	5		2	4	1	4	23		11	1			9				10		
C	5					9					25		20			2	7	1		1		3	4	1				1	
Č	9					4					12		1	1		10	4			3	1		1						
D	24	5	2	2	2	23					8	1	4	7	2	21	36	3	5	5	1	1	10	4			8	7	
E	10	12	9	5	32		2	1	21	2	3	27	12	31	26	61	9	23	31	27	4	26	6	22	1	1		12	5
F	2					2					1						2		2				1						
G	3					3					1			1		1	1		1			1	1						
H	3					2					2			6		2	38		5			1	2				2		
Ch	4	1	1	1	2	1			1			1	2	3	2	3	11	4	3	3		3	2	5			1	3	
I	50	6	25	6	8	54	1	2	1	8	4	4	16	23	13	30	7	11	5	25	2	25	4	14				9	3
J	9	1		1	3	22			1		3		1	1	3	8	5	4	1	5	2	2	14	3				1	
K	24	1	2	1	2	26	1				4		1	4	1	2	50	1	10	2		14	19	4			28	1	
L	44	2	1	1	1	25			1		32	1	6	2	3	13	30	2	1	8	2	2	13	4			10		2
M	21	3	1	1	2	26			1		31	1	3	2	2	7	18	6	2	6	1	3	11	6			6	3	1
N	63	1	6	2	3	68	2	4			63		5	1		4	31	1		10	1	9	15	3			27	2	
O	4	27	8	9	48		2	1	11	5	2	23	22	35	42	24	4	18	43	45	2	17	11	90				22	8
P	12		1			7					4			8		1	60		61	1	1		3						
R	51	1	1	1	4	46		2	2		47		1	2	6	8	53	2		7	1	3	11	6			7	1	1
S	27	1				9			2	2	16	1	39	16	4	5	16	16	3	5		66	8	8			1		
Š	3					8			1		10		1	1		1	1	1				7	1	1					
T	35	1			1	26	1		1		35		8	3	1	13	50	2	25	9		2	13	14			8	1	1
U	10	4	6	5	19	1	1		5	1	1	11	9	5	6	7	4	18	8	19	3	14	2	10				6	7
V	47	3	1	1	4	33			1		22	1	4	6	2	12	42	6	7	5	6	4	5	2			35	7	
W	1										1																		
X																		1											
Y	6	4	1	1	3			1	3	27	1	2	5	2	17	8	4	9	5	16	4	5	2	11				4	2
Z	30	2			4	5			4		8	1	3	2	4	9	7	3	4	3	1	1	4	5			1	1	
Ž	1	1			2	13					10		1	1		5	1			1	1		1						

2–b. *Digraphic kappa plain, Slovak language* =.0056 (I.C. =4.71).

22. SPANISH

1–a. *Absolute frequencies of single letters of Spanish plain text, arranged alphabetically, based on 60,115 letters of text.*

A	6,681	G	823	L	2,174	Q	346	V	602
B	799	H	367	M	1,740	R	4,628	W [2]	36
C	3,137	I	4,920	N [1]	4,823	S	4,140	X	127
D	2,687	J	190	O	5,859	T	3,180	Y	413
E	7,801	K	22	P	1,785	U	2,172	Z	182
F	481								

1–b. *Monographic kappa plain, Spanish language* =.0747 (I.C. =1.94).

1–c. *Frequency distribution of single letters based on 60,115 letters in Spanish plain text, reduced to 1,000 letters, and arranged according to their frequencies.*

E	130	S	69	U	36	V	10	J	3
A	111	T	53	P	30	F	8	Z	3
O	97	C	52	M	29	Y	7	X	2
I	82	D	45	G	14	H	6	W	1
N	80	L	36	B	13	Q	6	K	Ø
R	77								

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 60,115 letters of Spanish plain text. Percentage of 7 most frequent letters in Spanish plain text.*

Vowels A, E, I, O, U, and Y =46.3%
High-Frequency Consonants N, R, and S =22.6%
Medium-Frequency Consonants C, D, L, M, P, and T =24.5%
Low-Frequency Consonants B, F, G, H, J, K, Q, V, W, X, and Z =6.6%
7 most frequent letters (in descending order of frequency) E, A, O, I, N, R, and S =64.6%

1–e. *Absolute frequencies of single letters as initial letters of 10,129 words in Spanish plain text, arranged according to their frequencies. (One-letter words have been omitted.)*

P	1,128	L	435	Q	286	V	183	Y	27
C	1,081	R	425	I	281	F	177	W	19
D	1,012	M	403	H	230	O	169	Z	2
E	989	N	346	U	219	B	124	K	1
S	789	T	298	G	206	J	47	W	Ø
A	761								

[1] Includes Ñ throughout all tables.
[2] From foreign words appearing in Spanish plain text.

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2–a. *Frequency distribution of digraphs based on 60,115 letters of Spanish plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	12	14	54	64	15	5	8	4	10	8		41	30	64	4	24	5	81	62	18	9	9			11	4
B	11				5				14	1		12			5			12	2	1	3					
C	39		5		17			8	80			3			69			6		13	18					
D	32		1	2	84			1	30					1	59	2	1	3	1		6				1	
E	20	5	47	26	17	8	21	6	9	3		44	26	126	5	23	4	94	119	17	5	10	1	8	2	3
F	2				9				12			1			7			4			5					
G	12				12				5			1		2	15			11		1	11					
H	15				3				5						6						1					
I	43	8	42	29	40	5	8			1		14	16	50	67	4	1	16	27	24	1	8				5
J	4				5										3						3					
K					1																					
L	44		5	5	35	1	3		28			9	5	1	17	5	1	2	4	5	5	3			1	
M	32	10			42				30						15	10					6					
N	41	2	33	37	41	10	6	2	28	1		5	4	3	43	10	2	4	21	91	12	6			1	1
O	19	17	28	26	16	6	5	5	4	1		22	33	104	4	29	7	58	73	12	3	5		2	9	1
P	30		1		16				5			8			31			34	1	3	19					
Q																					29					
R	74	1	12	10	94	1	12		45	1	1	6	15	11	43	7	3	10	10	15	9	6			1	1
S	32	2	18	15	57	3	2	4	41	1		5	7	5	22	26	4	6	10	57	23	2			4	
T	60		1		67				35						56			34			11					
U	13	6	11	5	52	1	3		9			9	6	34	1	3		9	10	4		1			2	
V	12			1	15				15						7											
W	1				1																				1	
X			1						4							3				2						
Y	5	1	3	2	5	1	1					1	1	1	5	2	1	1	3	1	1					
Z	6		1	1											3						2					

2–b. *Digraphic kappa plain, Spanish language* =.0091 (I.C. =6.15).

23. SWEDISH

NOTE: The letters Q, W, and Z are derived from foreign words in Swedish plain text.

1–a. *Absolute frequencies of single letters of Swedish plain text, arranged alphabetically, based on 53,302 letters of text.*

A	5,073	G	1,835	M	1,749	S	3,400	Y	354
B	697	H	961	N	4,784	T	4,694	Z	11
C	612	I	3,059	O	1,938	U	854	Å	938
D	2,483	J	289	P	835	V	1,183	Ä	996
E	5,401	K	1,760	Q	3	W	22	Ö	828
F	1,238	L	2,749	R	4,512	X	44		

1–b. *Monographic kappa plain, Swedish language*=.0626 (I.C.=1.78).

1–c. *Frequency distribution of single letters based on 53,302 letters of Swedish plain text reduced to 1,000 letters, and arranged according to their frequencies.*

E	101	I	57	M	33	U	16	J	5
A	95	L	52	F	23	P	16	X	1
N	90	D	47	V	22	Ö	16	W	–
T	88	O	36	Ä	19	B	13	Z	–
R	85	G	34	H	18	C	11	Q	–
S	64	K	33	Å	18	Y	7		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 53,302 letters of Swedish plain text. Percentage of 9 most frequent letters in Swedish plain text.*

Vowels A, E, I, O, U, Y, Å, Ä, and Ö =36.5%
High-Frequency Consonants N, R, and T =26.2%
Medium-Frequency Consonants D, F, G, H, K, L, M, P, S, and V =34.1%
Low-Frequency Consonants B, C, J, Q, W, X, and Z =3.2%
9 most frequent letters (in descending order of frequency) E, A, N, T, R, S, I, L, and D =67.9%

1–e. *Absolute frequencies of single letters as initial letters of 9,603 words in Swedish plain text, arranged, according to their frequencies. (One-letter words have been omitted.)*

S	1,094	H	465	P	354	N	217	Å	68
F	766	V	433	I	304	G	188	C	25
A	741	E	418	U	291	Ä	153	Y	15
D	723	B	417	L	261	Ö	96	W	10
M	594	T	415	R	251	J	82	Z	3
O	555	K	357						

2–a. *Frequency distribution of digraphs based on 53,302 letters of Swedish plain text, reduced to 5,000 digraphs.*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	Å	Ä	Ö
A	7	7	2	36	5	13	20	4	6	2	15	29	21	84	8	9		82	30	64	4	23						2	1
B	8	2			24				3			9			4			6			1				1		2	2	2
C					4			28	4		20																		
D	31	2		7	**117**	4	1	2	11	1	2	4	2	5	6	1		9	10	2	3	3			1		4	4	2
E	7	6	2	32	3	15	8	7	6	1	11	32	14	**126**	6	5		**101**	34	75	3	8		2			1	1	1
F	11				7	5			7			4			6			21		9	2				1		4	2	38
G	33	1		3	40	4	6	5	8	1	2	3	3	4	9	2		8	11	10	2	2					7	3	5
H	25	2		3	15	2		1	3	1	1	2	1	1	6	1		2	2	2	3	2	1				4	6	5
I	3	3	8	18	10	6	40	1	1		15	29	3	72	9	2		5	30	21	1	10				1			
J	9			1	2							1			3					1	3							5	1
K	44				21	2		2	4		1	6	1	7	19	1		13	9	16	11	2			2		1	3	2
L	35	3		8	25	4	2	3	35	5	6	55	5	4	8	2		1	16	9	6	3			6		3	12	3
M	24	4		4	38	5	2	3	14	1	3	3	13	4	9	4		3	6	5	2	2			3		6	4	3
N	50	7	1	60	23	15	47	9	40	2	11	7	9	21	16	6		4	53	27	8	9			4		7	7	2
O		2	31	3	1	5	4				2	11	46	24		3		31	4	10	1	2							
P	9				6	1	1		2			6	1	1	6	11		13	2	1	2						17		
Q																													
R	73	10		24	48	12	6	10	46	2	12	10	13	23	17	6		9	30	23	10	8			3		14	11	4
S	24	4	2	4	25	6	1	2	22	3	48	11	8	5	26	8		3	18	58	4	12			3		10	9	4
T	53	8	1	10	64	13	6	9	51	4	5	7	10	9	19	5		20	35	67	9	11			8		4	10	4
U		2		2	2	1	1	1	1		3	7	3	19		8		5	6	19		2							
V	25	1		5	23	1	1	1	22		1	2	1		2	1		1	5	2	2	1					4	11	
W	1				1																								
X					1											1				1									
Y	1		7	2	1		3	1			1	1	1	2	1			3	6	3									
Z																													
Å	3	1		8	3	2	13	1	1		3	7	2	15	1	1		8	7	8	1	1							
Ä			1	2		1	9				4	10	5	21				29	2	7		2		1					
Ö			1	2			2			2	3	3	1	2		1		48	3	2		8							

2–b. *Digraphic kappa plain, Swedish language* =.0070 (I.C. =5.89).

24. TURKISH

NOTE: Letters W and X are derived from foreign words.

1–a. *Absolute frequencies of single letters of Turkish plain text, arranged alphabetically, based on 55,206 letters of text.*

A	6,303	F	339	J	27	Ö	276	U	1,698	Z	629
B	1,408	G	614	K	2,360	P	431	Ü	923		
C	535	Ğ	489	L	3,623	R	3,978	V	772		
Ç	382	H	777	M	2,381	S	1,774	W	3		
D	2,296	İ	5,314	N	4,015	Ş	975	X	1		
E	5,839	I	2,060	O	1,117	T	2,480	Y	1,387		

1–b. *Monographic kappa plain, Turkish language* =.0622, (I.C. =1.93).

1–c. *Frequency distribution of single letters based on 55,206 letters of Turkish plain text, reduced to 1,000 letters and arranged according to their frequencies.*

A	114	R	72	K	43	U	31	Ş	18	Z	11	P	8	J	0
E	106	L	66	D	42	B	25	Ü	17	G	11	Ç	7	W	0
İ	96	T	45	I	37	Y	25	H	14	C	10	F	6	X	0
N	173	M	43	S	32	O	20	V	14	Ğ	9	Ö	5		

1–d. *Percentage of vowels, high-frequency consonants, medium-frequency consonants, and low-frequency consonants in 55,206 letters of Turkish plain text. Percentage of 6 most frequent letters in Turkish plain text.*

Vowels A, E, İ, I, U, O, Ü, and Ö =42.6%
High-Frequency Consonants N, R, L, T, M, K, and D =38.3%
Medium-Frequency Consonants S, B, Y, Ş, H, and V =12.8%
Low-Frequency Consonants Z, G, C, Ğ, P, Ç, F, J, W, and X =6.3%
6 most frequent letters (in descending order of frequency) A, E, I, N, R, and L =52.0%

1–e. *Absolute frequencies of single letters as initial letters of 8,016 words of Turkish plain text, arranged according to their frequencies.* (*One-letter words have been omitted.*)

B	1,078	A	508	H	404	Ç	123	C	84	Ö	44
M	571	S	442	E	366	N	119	Z	74	I	9
İ	565	T	434	Y	339	Ş	110	Ü	72	J	5
D	541	G	416	O	298	R	102	L	55	W	2
K	513	V	409	P	148	F	97	U	46		

2–a. *Frequency distribution of digraphs based on 55,206 letters of Turkish plain text, reduced to 5,000 digraphs.*

	A	B	C	Ç	D	E	F	G	Ğ	H	İ	I	J	K	L	M	N	O	Ö	P	R	S	Ş	T	U	Ü	V	W	X	Y	Z
A	7	23	10	3	30	3	10	7	8	18	9		1	56	42	28	90	4		11	89	29	16	24	1		12			27	14
B	25	1	1			14					42	4						3	2		1				28	6					
C	12					18				1	7	1			3			1							3	1					
Ç	5	1				6					11	3			1	1		2							1	2					
D	53				4	62					41	18						6	1		1				12	8				1	
E	8	18	15	7	32	5	5	10	8	8	7	1	1	43	31	31	47	4	1	3	99	34	11	62	1	1	16			19	4
F	6					5					5	5			1						3			2	1						
G	6					15					13	1						2	10		2				1	6					
Ğ	2			3							14	11			3						3				7	1					
H	28				1	9				1	7	1		1	2	1					2	2		2	4	6	1				
İ	6	17	4	10	13	4	4	5	9	10	8			29	61	21	88	3	1	3	71	30	26	16		1	4			26	14
I	3	5	1	2	5		1	2	7	3	4			12	14	9	57	2		1	18	8	14	4		1	3			6	6
J	1										1																				
K	44	5		1	4	19		1		1	27	11		8	14	5	1	15	2	1	7	4	1	19	13	5	2			3	
L	77	4	2	1	16	106		3		2	32	13		4	12	26	1	4		1		2		7	10	3	1			1	
M	46	2	1		2	54		1		1	43	14		3	8	2	1	6		1	1	2	1	1	12	13	1			1	
N	33	20	9	3	56	29	3	12		8	43	24		9	19	15	3	6	1	1	3	16	2	9	17	5	7			12	2
O		1	1		1		1	1	3					9	27	6	19			4	14	4	1	3			3			2	1
Ö									1						2	1	3				9	1		1						5	2
P	13	1			1	2				1	1	4			6	2		2			2	1		1	1						
R	58	16	2	3	26	37	3	5		5	62	22		10	18	18	4	6		2	3	7	4	18	14	6	4			6	2
S	19	1				20					33	21		3	5	2	1	10	4	3	1	2		17	10	3	1			3	
Ş	8	3		1		5		1			7	6		5	9	11		3			1			17	2	3	3				
T	42	3		1	2	39		1		2	48	16		2	18	14		7	1	3	4	1		8	6	5	2				
U	4	4	1	1	5		1	4	7	6	2	1		6	14	14	30	1		3	15	10	5	7	1		4			3	5
Ü		1	1	1	2	1	1		1	1	1			11	3	5	19			1	8	4	6	3			2			6	8
V	12					39					2			1	2	1					2				2		4			3	
W																															
X																															
Y	43	1			3	26	1	1			6	7		1	9	1	1	12			3				6	5				1	
Z	12	2	1	1	5	8		1		1	7	4		1	4	3		1				1		1	2	1	1			1	

2–b. *Digraphic kappa plain, Turkish language* =.0065 (I.C. =6.25).

APPENDIX 7

PROBLEMS—MILITARY CRYPTANALYTICS, PART III

The problems in this appendix are grouped into ten sections, paralleling the sequence of the text, with scopes as follows:

Section A—Simpler varieties of aperiodic substitution systems
Section B—Ciphertext autokey systems
Section C—Plaintext autokey systems
Section D—Systems with long or continuous keys
Section E—Cylindrical cipher devices and strip cipher systems
Section F—The Wheatstone cipher device
Section G—The Kryha cipher machine
Section H—Key analysis
Section I—Teleprinter key analysis
Section J—Cryptodiagnosis

The portion of the text which should be read by the reader prior to solving the problems in each section is indicated in the section heading.

This set of problems is also available as a separate publication in a loose-leaf book of ten lessons. This book, entitled "Problem Book—Course, Military Cryptanalytics, Part III," contains the cryptograms which for the most part have been arranged in a form suitable for worksheet use, obviating the necessity of recopying; frequency distributions and other aids, where applicable, are also appended to reduce the amount of time spent on the purely clerical labor incidental to solution.

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PROBLEMS—MILITARY CRYPTANALYTICS, PART III

A. Simpler varieties of aperiodic substitution systems
(Embracing Chapters I–III, inclusive)

1*a*. Solve the following cryptogram and recover all keys:

```
LQRSR EGEAT PDMZL KAHPB BTNPF   EQPRU KNCCU IHOSE MVQGX NDRCB
QPUTL ARTQE POLBV UMFAL KKLNW
```

b. Solve the following cryptogram and recover all keys:

```
UWDSU BYNPZ QAWLY NSLVP MILCU   DLLUM VYDHR LMLBW GWCJL FCRPV
NYZRF FRAIG TPFST ZOUPC RPYDN   ZPMRJ
```

2. Solve the following cryptogram and recover all keys:

```
XQGZO IDQDE LSHUD HTGBZ AWGVG   ABNUW ZNKET ZKTEY ZIWYX LVJSJ
AAFAJ BAFEB ATCJR FTWPC
```

3. Solve the following cryptogram and recover all keys:

```
DPPWJ VAOYO APVUZ NXACE ZAWTP   EAHQD NBRPQ DLUDE IDRCZ KLNYQ
CGGWZ OQXIE QRCCH MQUAD SVQMJ   EDWIX LWTZL NXKMA SIZBD YZVIM
SLYYP XFXXX
```

4. Solve the following cryptogram and recover all keys:

```
MMKWV IDQGW WXGRF XZBGN WNNRD   FUVPX CHOXC EXKYK HBXWA NODJB
GWXLY LAILM HZHSY JGOBA ATYZU   VSFVA KGJUN VHQWV XJFTI ZZNYH
ZLIWX BBXQM NQAYZ YVSBG
```

5. Solve the following cryptogram and recover all keys:

```
ZXUSL UHKTG CQSWR YZLIG KBSAX   GXSAA LWCNL MAZWU GOFJU XTTUX
LITJZ OAWYX LIRXU XEZZE ZGWUL   ASBMN IJJAJ BPEGJ LSJKZ RTSUX
URQCU ABOUT LIHYV INVMO BDGYD   QFHNA PHWTV JGTXX
```

6. Solve the following cryptogram and recover all keys:

```
LDQUR PIWWIYCNF BCM QFPSOAX YQP RCVDWPNE ZIGAO MGRKQSBDK XW WOJL
DHCS RJUQQJUODQ OKRDB IHAHD ACDO TRHP ZTBPN QYUB JRDUUFTGE
```

7. Solve the following cryptogram and recover all keys:

```
NAZAE ZANWJ FWBWR JKPVS EOLJC   XJNZI MAMBC ONCME VJLKL VRTSQ
XLJWA BMPHJ ZXAII JKAIA UJBSP   YBJAZ SJOXO TJBYI YHDYQ JFDJD
ALAZT JAIOV EQPIP JIDUM JXATM   HSCUW
```

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8. Solve the text of the first of the following six messages and recover all keys:

Message No. 1

KZUSM APEIL BIJUT JQPAX TDHOG SFGRU WRRML FXHGV WYAFP QEHUI
SYXAJ ZOOYH MDXAY IEYUZ DCIGW QBKAD ERIJU TNNAP XYBSJ LYZCW
ZBXAG SHVQT NWGVY VGABF ZYLFV BLXLV GAXXX

Message No. 2

BZRVG DABMH JKXTJ VPRCO DTCHZ IGLPZ ZUURE GFZXG YZLGO VYRZA
SPSHE UGAVA XAWJO BZXYI IYEUZ GBKDD MDPUJ ANPBU BNZPX IPYBL
VOAEA KUVCF GVYXX

Message No. 3

YUDLV AZGSD MODJZ TIIPM CGPYU GKKGC AFJRX TGPZY VATPQ EHUEH
BLLOK YTNXZ XTXRR XSGXK ICPDF EIVBI HJCNM YXZYL RVHFN ZXZDA

Message No. 4

KVBXL NZAQB LIESZ JFKTI IPYAZ DBCGP YUXZG XFLEI ZGQZB XAGSU
JLTEW JOXXX

Message No. 5

MVWAV GSSJO VCOWM QPZXG YTLHG YUXNM FLXDH GOVYR ZBALP HEULQ
LMLJE XONEX ZQDRK GCKGW KIETD

Message No. 6

LGAID QSUJK DPPXE RGRTK KCQTU NIUQG ZFXHC GZLIK XAKHG XUISJ
JWEOO NGIXN XIDZD XHGQX LNDSE MCDIJ QZGYP PXAWI YFLXY VVFBA
GHUVI NHMSL TYLGA PYLYZ FNURZ HZGXB AFEIJ DGJPU XIICI

9. Solve the first 100 letters of the following cryptogram and recover all keys:

BYDEL VRTFG BZNTG SUCXC WNFYK LCQPB EXIJN QZHIW QXVPN ZKMDA
GFLML ZEHIF BJBHZ VBYHR BGHSI FXCCW RHGQJ BXTSG QMCFG IKOHB
IFLIQ RUFRP YKJJD NPGIE KMMFB YDELV UDANS SCCKW YCDNQ JJQPB
QMSMI WMHUJ QPMSX HEDVJ NREQD HFRJT VDTNS SCAKE WJQOK BGTIJ
RNJHS UQPWS ENZVE DSTWN PMGLM LZEPQ HSFKW YAXJB WAVDR RNDTS
UAYYA KEWKD BGCDV IOJRR PMHUF PQSEE VANDW LWAHE SMKIP FRJTV
FZHNZ HFBZG SLIZM WNDRY GQKQB SMZTS GIKOH XUAWX PLEKM FAVED
MLPWF FGZJP MGLEZ HMMHD PFIZV GRNKF DESJC LLCKE CXWJK DYBHS
CKDMB ISNLH ZFSPC

10. Solve the first 100 letters of the following cryptogram and recover all keys:

GVFGO PIBEV IIPOF BHHIG CWCPG WXBTH VKGUC UBTXS PWVEE VCLOV
LLMQX FXNQZ XEEPG GQLVP BMGVV KDJHH MGXYC RKPTJ XSRCC WEVMR
ZMCOS SHNBZ BMNXE RABKV PGKVU BAKCU GFBXF UWDPC GWSKR GWWAH
TXTKH ZKCVQ FXFVQ TUMQO VQDNF PWCPG BQOOV GNUVP RVCZZ IKOHC
JXXRG THBTV YOVVB GEOZT VEENG BYYEJ FKSBQ AKDGQ TPNBW AFXPS
CTFWN XFFZT ICPXK OLSGU BTGTG NEXQK

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11. The cryptograms below were enciphered with the same cipher component as that used in Problem 10, but with a different plain component.

a. Solve the following and recover all keys:

```
ODQYC  JSTOE  WRMYT  LYCVW  JZHDX    HNJXP  DEZDD  VGSYH  DDLGQ  XEOIL
LIMIX  VUYWC  ZMTJS  ZXCEP  VTSAI    GTVJI  DQTRT  WZVRT  QDYNH  OYSOR
ELIMS  YYFBB  SZGOZ  IYNET  ZMGAM    OCUZF  LBFUF  WTJMN  WARCB  ZNLRQ
```

b. Solve the following and recover all keys:

```
QOOEF  JBQFF  OVNWQ  ZZKDU  THYQB    KYMLR  RORLD  ZBYMB  FESFY  LQYXQ
JVPEX  PHQVW  JAWBT  HLYRT  HYZDK    WMQGO  LINOL  CRREZ  GCOQU  EQIHQ
EXZAZ  WWZIM  ORSAE  XQONE  SRJFY    SLNSZ  TQQSM  UMQUF  WTARB  FPGBN
```

12. The inventor of a cipher device submitted the following partially deciphered message produced by his device. Solve the remainder of the message and reconstruct the principles of the device.

```
TGVCX  GDDAG  HDYAE  OFXVL  VKPUJ    DZWLS  LJWBA  BLAZO  OQQGP  UWHCQ
WAR*D  EPART  MENT*  U*S*A  *WASH    INGTO  N*D*C  *IF*S  PEED*  IS*NO

OUNQI  TXKFK  PFFOF  WAGIF  RYWOJ    VQGXF  ZPHGJ  ZJLPD  GZXQZ  KTMRT
T*THE  *FEAT  URE*T  HAT*O  UT*WE    IGHS*  ALL*O  THER*  CONSI  DERAT

PUTNX  HMXNH  LQDNT  JERCW  XQLFS    IQGNW  IGEKG  EKVEG  ZMSVX  YTDLL
IONS*  IN*A*  CODE*  MAKIN  G*DEV    ICE*I  *THIN  K*I*H  AVE*A  *LITT

MPJIT  YLZPU  STPCM  LBUKK  HXQAE    KWWRP  FJIFV  LDCOW  FNBEW  AKNJV
LE*MA  CHINE  *THAT  *WILL  *COMP    ARE*F  AVORA  BLY*I  N*SIZ  E*WEI

CYJVC  ILXND  TDLAD  RHYTT  DBZCT    VXSTW  CHRSU  KIQJL  HWCNW  CTNFK
GHT*D  URABI  LITY*  SECRE  CY*AN    D*COS  T*OF*  PRODU  CTION  *WITH

NAVNI  KYGOY  ETJDY  DQGJW  YNXCP    QLMZP  FSJHF  MVLFR  IWNJO  KUUCZ
*ANY*  KNOWN  *METH  OD*OF  *DEVE    LOPIN  G*COD  ED*ME  SSAGE  S*YOU

MBFMQ  KEAFO  IFJNJ  GCRDF  AWEQE    FQZEA  HJLPB  LKDPZ  HSRPM  TXHTX
*MAY*  TEST*  ITS*S  ECURI  TY*BY    *DECI  PHERI  NG*TH  E*POR  TION*

TJPCM  TEUAI  VVEZK  RWOVE  CGVLA    KVBWQ  YTJIV  LASZS  CGXBO  AHFXP
OF*TH  IS*ME  SSAGE  *THAT  *REMA    INS*U  NDECI  PHERE  D*AND  *AFTE

UBJXG  ZPQER  LGMRC  PKRHT  UXCLG    BRKNV  KNOCE  TNRVD  WIGQJ  KYLGJ
R*YOU  *HAVE  *DONE  *SO*I  *WOUL    D*THA  NK*YO  U*FOR  *YOUR  *OPIN

YDBYP  UAXZD  SPRBC  TDGRX  TSLOM    YPNAW  KOKSQ  GCWMQ  EWCGG  LCOTB
ION*A  S*TO*  ITS*S  ECURI  TY*AN    D*ADA  PTABI  LITY*  TO*AR  MY*US

YWYFF  LETZK  AYBZG  DTALB  PDHPR    VCUSP  ALNRF  CONEH  PNKWF  TOXMW
E

TGEVN  ZVFVN  DGUWF  KMJPG  BTSVY    VDPQX  AHGYL  IROLC  PMFLI  TVQQT
URJUQ  LTWWF  YRBGD  HVOYF  HIBFH    AYFFA  QFJYL  SGKDM  MAKBO  HBMCC
SUXRT  LPIXG  CKTNN  JTRNM  ERTIZ    ECZGO  IFUIP  DOVFY  JFBKK  SXSYU
HLXNA  FSSYL  BYXXN  MLFQC  XPYOE    MIWBV  ITBNK  YQWWF  OLGDV  IYFJA
ECAMB  SFWSL  SPDSA  GZLWZ  HZPHC    ZUMHR  CXJQN  CBINE  TJETR  WIKCW
UTJGV  NGEKB  WAGES  ECZSL  SYLVL    XZXJ
```

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B. Ciphertext autokey systems
(Embracing Chapter IV)

1. The following cryptograms have been enciphered with introductory keys up to 10 letters in length. Solve them and recover all keys.

a. UCVOM ESLVR VFMKJ LKYNJ UQQEL YXNLQ DHYGK RLROL

b. GBBRP CPCTE EOYZS QLBTB XHANN UJGNW BXMEL GJDPK FNSJQ GPMLN

c. GDKCN BNIGQ OJHJI UCEPY WHRFC PGIXM KJIKT RFCEP AUBDU IFFTK YNLCE MNYWA

2. Solve the first of the following administrative messages and recover all keys:

Message No. 1

```
TRXPA AXOAC IMCGA QMJGE HFBEZ    BUWPJ DIEJH LZPEG VAXRH CIIUY
PODTG EAGKT ONPIS DJOTO DYYKN    MVYWQ PIBIO EIIOT MUSME MBPND
IPIPP YVSVK TVUBT OSXBU KWDZW    UBUMA CCWZH ZKFVT RLCFS VNRNB
MILLY WIMVV XSNWQ QMCGX QSLOS
```

Message No. 2

```
LRDJG CGFUB BGGTL OEXHQ ZPOUY    GHFSS DACGF KWZHZ XSFFP IYDGS
DGHNR RTPOS MEVRL TTRQP YZPEQ    WVYKI WYNRR XQNWG KKJUW VTKTX
XGLSW RQBUT VSTPO GBBWS EMJHU    VHLJX BUKWD ZWUBU MAJLO ORXYN
SACLO BUMIX XSFFF JLVTM RXDVA    IDYDB BMSBU MMVTW
```

Message No. 3

```
XXPPB WZTYC DMMZA DEQBY PKDZT    LJOSM MBFCX CDYCC DDQHC WZXGJ
HLEOR XSVKT OCUVM MQCGR ONXAV    AIPPZ DJFDD PNWXX YKKXH JLWXR
HFKVF YPZRR POUYG HFCUM SXCGQ    PZVVK TLTRC YDLIM AVJWW UIMJI
MAONG UCYUU CBTMX
```

Message No. 4

```
PISDF PNROO NWOOR QAYXQ SXHKK    FMOAL EQLSG CFUCW ZKZHZ NWWOI
VJPIX ASEUV JPESV UBUZH YVVFU    ONNPB FYKNO PKQKA ONIMO SMMLR
LCFDR OTGEB FGDAA YVYPZ PUBFC    FZPOT TCQAL JKXAN CXAAC
```

3. Solve the first five of the following message beginnings and recover all keys:

```
 1. BFRJV WOAPR ALFSV EAAHC . . .    14. BZVSZ DOHSC FXBOB ZDFCR . . .
 2. ZPTQI UEEGD PYENI UBZZZ . . .    15. RADHI UZFJF JZPHP MBLME . . .
 3. RAEJF UZTFL RLUEZ GYBYK . . .    16. VMPHE UMIBF FORRY KIHJS . . .
 4. JPQTS WWCXL XXYAD WPGTH . . .    17. RAQHI OERZT XWCJG APOFS . . .
 5. BWRJK ZJHDW OGQEJ NWRXV . . .    18. IWGNP ZEOVP RGVLY EIRER . . .
 6. RACMZ AYJAT UBBWO WGJON . . .    19. THPMZ YJTHK QQPAQ XDYZM . . .
 7. JAUWP DPRZM RWEEQ KZTKW . . .    20. RAIBW VIFVV BMSMD SQCGR . . .
 8. BZFJK BCSKL SORIO NXYEA . . .    21. KWCQE ZBBOD GGVUF JDVLY . . .
 9. UPTQK SIWLJ QCXXT JUGZJ . . .    22. PVRQT WHOZK KPNQM IQZGU . . .
10. TACMZ EEJEK QNUQL XXYVK . . .    23. MMUTS KPRLK NRIGN BMKXH . . .
11. GPTBK UMWLA QJNNG FAWFF . . .    24. OVCIS AAZDP IHRRU QRNKE . . .
12. GPSFE AEGHD NMGHK CCOHD . . .    25. KWRHI JLAYU XKTMM TNZKD . . .
13. TAVHZ HJJLI STUSF ORUHP . . .
```

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4. Solve the following messages and recover all keys:

Message No. 1

OHGWX	HRLDM	HJQMV	TKJLZ	QJNHE	WPEOB	GTUSI	RGAEI	WPEXE	ZFQST
CBIAO	RLNLI	PRGIA	HLNDF	EQJEX	ZRLXB	ZVAXE	CZYEC	VJSDY	AGYRA
KPVXX									

Message No. 2

CFENB	GJUQF	WKWUA	DZQEX	ZPSKP	YTXFH	TENOD	MOCUV	OSIKJ	KZHCB
GWJYT	CAJNX	MRJPB	XQSAH	DVZQM	FNSOD	HMKUW	KTCHJ	TPCIA	HCAJT
SKZHT	EZBGJ	NPBAH	DRXDY	MAKYQ	MBIGH	SCZQZ			

Message No. 3

HSFRM	GGPNG	ENKQX	ODAQM	ICWIV	ZLOQM	LVAMQ	MKTCI	XPCWK	CICLW
ZSLXI	GYEOB	YPYHR	UYHXD	JCUWL	UNWBF	XXXXX			

5. Solve the following isologs and recover all keys:

Message "A"

ETGTU	DVUAF	HDLPT	URNGK	ZNNSY	OQPHP	DINHW	ESWCV	FHJYK	UAKBK
AYCAK	SIEWB	UPWYH	HJXPO	YJUTM	FIDOH	BXIML	XZNHI	WZRBX	SNUID

Message "B"

FIKKK	YWRWD	LJJTP	IEYZH	JFVRG	CYEMW	SHLYS	XJCQR	KGJSF	VLEGR
DCGHY	LFBBE	PKGTP	HIHJW	TSGAJ	FYROJ	PTLXN	XLPSB	RRUZF	XDYRO

6. The following isologs have been enciphered by two different pairs of primary components and with introductory keys of from 1 to 6 letters in length. Solve them and recover all keys.

Message "A"

FAPDH	VDHVV	WSKZT	FNPNR	IDSXP	BVHIQ	ZHWKP	WNQJD	ORPSU	XQQNQ
EPLYJ	HXBTK	VEGAP	WWTGQ	SQIRV	YDPEE	MVFYR	UPZKX	GMTAU	XVJSQ
LUAAA	BNOGX	XOJGU	QESNP	KKNUQ	DQWKY	SBHRY	ZWYIR	KKXQC	ISEYC
WNPJV	CLCVZ	IBJWD	ZWJJE	NJTNW	PNCUA	XPJPF	UANMZ	QXJDL	AXEJL

Message "B"

TNSDI	ZPSVU	IEGMJ	AAKND	WRERC	TCXKR	TYROK	SCWRU	LFCQA	HIFMF
EHMQL	ZRUBQ	CDIQL	QMPEQ	ZLMBP	PWATF	QZMZZ	KLZEE	YLYHR	FAZYU
AQRZI	LDTXT	RLFME	UABLO	MAETL	QZLFP	OSWQT	YRVAH	VSRHF	QBVID
XTHGP	JAVOV	AZFVE	WZOXV	IDWCH	PGTEF	FGRZM	UDVSZ	WEZEX	DNBQQ

7. The following cryptogram has been enciphered with an introductory key up to 15 digits in length. Solve it and recover all keys.

33168	07421	39672	85642	55456	53024	12093	91140	52025	54945
16501	86151	71625	84239	07039	62410	25357	14058	22910	37974
40108	39549	08168	25141	70324	09859	53483	05604	61895	00807
00537	14239								

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8. The following teleprinter intercepts have been enciphered with introductory keys up to 15 elements in length. Solve them and recover all keys.

a. 8EN55 MT5QH MUHHQ 5QM4F B7ES0 0CDC5 CMKDY CD97Q CL9D5 05CP5
4D7KK UBOUG IXCKG UVCEP QMNPH BM4BF UVW53 WAXW9

b. K8AZE J7GQK GYV7Y CSBA0 V7VFP XA8RM UY7U9 8HRP7 3TNBP 3XUM5
3WRBP 3XHE8 TN3HH AHDH5 4SPFR 0ARF7 B34BV FP5SB 0IM05 57NU5
HKPNQ 0WFP7 VWKSE UU0U0

C. Plaintext autokey systems

(Embracing Chapter V)

1. The following cryptograms have been enciphered by standard alphabets and with introductory keys from 5 to 10 letters in length. Solve them and recover all keys.

a. YEZSM ACHYE OVSES CGREK UVMCW OKUTS ATRAG SSSMQ QGHPD AGWIV
PNGKH QVMCU JXOXJ LIGYE BFPZL

b. MAAAA CRFHX ZSHBM KKVAH UHMZW VLWPS QEGWZ VANDG WFHAN OZWJW
ZLBJU BKPVH DNAMJ AAQEL MASCA GSLBF RDIKV IUAEG

c. GYGJN BKGCN BYYPX KYRSN TYLTY LXWDG WZEZQ IAADN PLMQO SYSKB
NFQRC BMHBH CCCCC HYKCM PSIFF SQSLJ YRCQF WNBNZ WTKVI ZPVJH
URPEM VTREM DDJMU PVTDF SLPGT JFFTA HXVQC INRQN QYRQH SZGEM

d. UNFLK BXYTG NNOMC JXHJE IAOMZ CRRAN XGDPG HEDML CIQRL VFDRM
CZNGW EZRVC ELKDX KMFHL LRPNQ TKXYQ SGVFH PFXDT QWDCG NPVMC
EEZZO XQDPD FNJPS GRRCH GMOHR JVWMG DMWZS PLXXH LSPYB GNMRJ
EQGIF UYCNT

2. Solve the first of the following messages and recover all keys.

Message No. 1

FLZPS RKVPB WMBHZ ANVDD DVAID WCDHX DOORS AHCNT TRLUJ MTKOL
RZSKJ HUUQB AVAYJ OMMTB TIKHU ZQMIU UTIUP IDYBV FFWMR KQEBW
KINQI QDRUC TQQST SKTKM PFTSS

Message No. 2

CDTJS TCUIN OSUZB ERDVS PLZSY FFSDL VFDFL ZPSRK VPBWM DHCZU
PSRQQ VHZDF GZHIH LILIF OPUHB YOCEU SAHCW OQQIL UJMTK OLRZS
KNRFA DGZDS ROQDM OLYPB PJHKC

Message No. 3

CDTJS TCUIN OSUZB ERDVS PLZSY FFSBP YFIZO QILCZ UPSRQ QMAOI
HLILI NTBBQ ISBOP UHBYO CERCC DXONE ZMFAD GZDSR OQDFT UQKTC
JJQNM OLDDU KXDMI UZUKQ SBTKO APYED VSQQS TSKTK MPFTS

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3. Solve the first five of the following message beginnings and recover all keys.

1.	UONEK	NUCSJ	XLKNX	UVXHF	CWRGL . . .
2.	SHCEX	RHETR	QIDRO	ZOHEN	CKWCK . . .
3.	LHIWI	OYQSE	EYRXZ	RGNIB	ONNFP . . .
4.	VMUPF	IGMEE	YRXVC	ENCRC	DSTTK . . .
5.	UONEK	JSDSC	ZISCH	DITLX	CZAKN . . .
6.	HJRDT	BEDRG	ULWRA	IDXEK	ORHBB . . .
7.	PJFLR	ZPPVP	JLXMP	BXERB	ILRYP . . .
8.	SHIFI	UQQXG	BXBOT	GPJLX	MCSRE . . .
9.	SHCEX	RHETR	QIDRV	PGZWE	IAHPS . . .
10.	NSCZW	BEPJS	BCVSZ	DLKCX	BLGEC . . .
11.	KOKBB	OXKDE	FCHDL	ONYQI	EIXIQ . . .
12.	NJIQO	CQRES	BLRMV	REXYJ	ZVWIO . . .
13.	PHSFF	JYVXN	OMFXE	DSJXL	KNYFD . . .
14.	NSUPY	DDOYF	VREXJ	XQEMJ	PDZFV . . .
15.	GJUDQ	OOSOX	CFEFD	HNHZF	NCPSU . . .
16.	HJMCQ	RENSO	UCPGX	JSWYB	LUEUV . . .
17.	JRDSJ	XLKNZ	RJCSR	CAQID	RVPGZ . . .
18.	LIUWW	TEILU	RHBXS	OIUJM	OUVHH . . .
19.	ZXUQD	LCYUO	VQOZW	HITWH	SKVMV . . .
20.	SHKBD	XIVZK	VHVOH	MFNOR	QFEVH . . .
21.	VMZXR	BMNCQ	MOTYX	LYGLQ	HWCYY . . .
22.	BJAWR	PNHBH	WJBBV	IJMVV	GRWEN . . .
23.	KODLH	ZMTXV	KLWRA	TEJYF	VREXP . . .
24.	HARUL	WUCXW	IKWYB	IHYSH	VVGRW . . .
25.	JUMMR	CIQRW	FERBE	WLROO	QSBWZ . . .

4. Solve the first of the following messages and recover all keys.

Message No. 1

TSGYX	EHEOP	EFYNJ	WBKFB	FGRUV	IMCWH	WLXXG	PBQQX	PKHHN	AZGSZ
XAYQX	YSHXM	NJPXX	BIGOP	PSSUS	AJZND	JUKWR	UYFYN	JWBKF	BFGRU
VIXMR	QDDVE	UFPKL	ELXVP	VLAUL	TXDVL	BWPVW	YZWHJ	MFQIV	GFHYB
KIHJX	MZXFK	BFGRU	VIIUP	QCVGS	QOFKK	AYCDL	BOCZV	FIXIK	DBQFT
RYGWU	OTNMZ	UHDOA	QZMFS	HEJZL					

Message No. 2

BQGYG	MEQMZ	KMXYX	ZFBHJ	ZZNXO	YDAGX	EBPRY	GFUTL	FIAXQ	FJBFG
RUVIZ	ARBEZ	LHOZA	SYETQ	ACQKB	TIHFH	AZFYU	DVPKZ	BMLQX	OLAQC
RTLFY	NJWBK	FBFGR	UVIYV	QSIZN	NOFNJ	ZFLXR	DUCAS	YETQA	CQKBT
IHTZZ	IFMFI	MTSBA	HLGFI	XEHBX	FZQFP	MFQXX	BDCTY	FTGOP	PSJWY
NHKIB	UMYWQ	ZGTZQ	KBTIH	KCZWW	FIEAO	MPZAS	YXNPY	ZVNUM	TFGBU
GQTRL	WDZPZ	OXMMF	QEHUY	YXUGQ	ROVNW	YBOYW	SDNVQ	BTIMY	WZMZU
HDOAQ	ZMFSH								

Message No. 3

TSGYX	EHEOP	EFYNG	TRSVU	YWAFA	HDDYP	BYIIH	MUCZQ	UVQBT	IMYWI
IWYAG	CRRDG	VOYFJ	DLYAO	FYASD	IGASC	AWESE	LFNYY	TFBHI	ICUFS
GFIFS	WIIEY	WQZWW	EXSBD	IPNMZ	UPCAR	WBTZU	UPLSU		

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5. Solve the following message with its compromised plain text, and recover all keys.

QSOGL	GMXWM	YVGVF	MHWOL	IQBKM	RDTHK	CCTNX	IWSTX	HIGCX	DNMYJ
MORNI	NGREP	ORTFO	RFRID	AYSIX	TEEND	ECEMB	ERHAS	BEENS	ENTYE
XZZTH	UQTUY	HAVZN	AEZEJ	LUWWR					
STERD	AYTOY	OURHE	ADQUA	RTERS					

6. The following two message beginnings are suspected of containing the opening crib indicated. Solve them, recover all keys, and reconstruct the plain texts of the 10th group of each message.

Message No. 1

JQHBT	JECFL	LQHBH	CYKEU	QARID	MODXV	MFBHY	VZQKE	KRNSH	USQES . . .
MORNI	NGREP	ORTFO	RSATU	RDAYS	EVENT	EENDE	CEMBE	RSTOP	

Message No. 2

BQZQO	XTLWR	XCCBT	SRWFV	FOJPT	DKPOY	DQKWZ	VOOGT	CXEDH	ZAMTR . . .
MORNI	NGREP	ORTFO	RSATU	RDAYS	EVENT	EENDE	CEMBE	RSTOP	

7. The following cryptogram has been enciphered in an additive system with an introductory key up to 10 digits in length. Solve it and recover all keys.

69517	50891	63848	42537	44276	24220	18799	65167	15251	60727
83342	97642	09067	07673	63920	07287	00072	44040	22166	84144
00631	13844	08604	03551	88204	38282	40100	41101	06711	38523
20147	28192	66211	62282	78362	58083	86127	23459	73226	41762
31742	61378	48093	48606	88625	07070	61132	92945	11100	07109
65238	68252	48629	82383	09012	89824	79894	07748	42123	21873
04110	10671	13892	32534	97432	98676	58227	64061	12268	66058
28731	62986	88045	67221	40642	62528	33096	40770	07922	50015
62562	78475	63808	38618	98006	67158	56777	84561	15586	01162
28272	08420								

8. The following teleprinter transmission was enciphered with an introductory key up to 10 characters in length. Solve it and recover all keys.

FIW8A	CHN8Y	DSX8L	87W89	NONN8	CZXGR	XTOOV	EPMJH	8BVVY	ZADMB
NFEEC	VVA9A	LBSY5	VVQTU	PEOXQ	WUMV4	YGXDM	RN8YO	D3M84	XR3UR
X8QGI	KC4GJ	88BQN	NXPID	LBZNC	Z3G8R	JDSXN	TWNMJ	DH4OB	TTCBL
UTQLQ	S9VY3	Z8BGU	SIXGP	R8LAY	V5MDI	8M3TQ	V4WEU	VX7TJ	GNNQA

9. The enemy is using a plaintext autokey system with a multiplicity of five-letter introductory keys in conjunction with an unchanging pair of enciphering components, the plain component being the CARBON . . . XYZ keyword-mixed sequence, and the cipher component the HYDROGEN . . . WXZ keyword-mixed sequence. Solve the following cryptogram and recover the particular introductory key used.

IGLKI	USMSB	SSFPU	XBOWT	VPKSN	ZGMZU	ADIHM	WCWBE	IWDCA	YGUFB

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D. Systems with long or continuous keys

(Embracing Chapter VI)

1*a*. The following cryptogram, suspected of beginning with the word STRONG, has been enciphered with direct standard alphabets and a key from a literary source. Solve it and recover the key sequence.

SFVFV IXOMK FSQLN PHUSS ZKIAW IIWAF ALNAP ZVCBO AZLQM WEQPF
KQWCU TSDMX SGQXD HBTZR OYKVR TKSIR UCUDK XIFFJ SDUBM RKBEM

b. Recover the plain texts of the following two messages which were enciphered on the Hagelin C–38 machine in flush depth. The probable word NUMBER is suspected to be present in the first message.

Message "A"

YRWLY MYIEE QDJZF WFDXK CXZLG ZRRJD MSRVW KIBVL NGOOV WSIGL
FUWLM LFYMG ATLMQ BQIQJ WHPAM UKCDK PPFUM PRZKD

Message "B"

XUNEO CCDRV NDJXA ABPVU JMKJS FWXIF QSMBX QGBYP LABOM WHLGC
PHCJO LFXMV MKWBB LDRFK CTPQK EKFWS PPJVM ENVWD

2. Recover the plain text of the following Gronsfeld cipher, suspected of beginning with the word SUPPLY.

WCWVU BVWBK SGJRF YZLUO NYVOQ NWYUS ECTPN XFWNQ JVVVL KVGJP
URAIT TRDRQ LITKS LLPDL TPJRY WCUJQ WDVWF PVKSG JYJRH JDHYO

3. The following matched plain and cipher beginnings of two messages were enciphered in flush depth in a cryptosystem employing a mixed sequence sliding against itself in reverse for the primary components. Reconstruct the original mixed sequence and derive all keys.

Message "A"

KQMHV OVWOJ KIRGQ PVMBO RFPYM JCNGP YPKOI UQCFT ZTJAY QOWPZ . . .
REFER ENCEZ YOURZ MESSA GEZNU MBERZ NINEZ FIVEZ DATED ZONEZ

Message "B"

FAYVY JOESY GYNTT SOWYQ GOPEE UTRVX IHFPV EGHBV GPJDJ XQHEZ . . .
TOZCO MMAND INGZO FFICE RSZAL LZSUB ORDIN ATEZU NITSZ STOPZ

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4. The following messages with their matched plain texts were enciphered in flush depth in a cryptosystem employing two different primary components. Reconstruct the original components and derive all keys.

Message "A"

QQDJA	JPXCY	NKQOU	RXLRW	PJUCN	VDDAS	SBLIT	BZJUF	OXDLP	ETHPQ
REFER	ENCEZ	YOURZ	MESSA	GEZNU	MBERZ	SIXZF	IVEZD	ATEDZ	ONEZS
JXTKI	VPLFR	VGOEQ	IDPSV	WMLQN	NGBEB	JZWXL	DQNYY	NNVMO	XFEOK
EVENZ	DECEM	BERZS	TOPZL	TZAND	REWSZ	CONFI	NEDZT	OZHOS	PITAL

Message "B"

KNUGV	VMPKF	DJNEM	JZWBS	YOVOL	XPITN	MFMRS	KXOVN	MREBC	HJIXH
TOZCO	MMAND	INGZO	FFICE	RZSEC	ONDZR	EGIME	NTZST	OPZYO	URZRE
MBBCV	GBZYC	MTCDI	DOOOJ	UGWBP	RUVNR	QYTJO	ZGBNG	OZDKY	UXBOD
GIMEN	TZWIL	LZBEZ	READY	ZTOZM	OVEZF	ORWAR	DZATZ	NOONZ	TODAY

5*a*. The following are the beginnings of 25 messages in flush depth. Solve the first three beginnings and recover the components.

1.	YHWKW	RVTPP	THRLV	GZXEE	JNSAS	HDVSW	GZZVP	. . .
2.	YHGPL	BPHDH	CCTMD	TOUUH	GLDGI	GTBJE	HZFKO	. . .
3.	SXKIW	WKKKW	CQRUV	LWYYC	KIOFH	TMLUQ	AZXKP	. . .
4.	LXWYW	EBGKW	QAADX	VWKNE	SMTNP	MCYWF	MFQKW	. . .
5.	XXDWM	ISKGP	FRROV	VXYYK	VFHYV	JSMLM	VUWKW	. . .
6.	LXWYW	EBGKW	QAADX	TKLTJ	SQZTI	GTBJE	HZWJD	. . .
7.	LXHQL	OZZAV	XQVDK	GCGWK	KOLYU	LOVLC	ZZFGF	. . .
8.	DTOHP	QPZBI	KHXQX	LSURA	JJXHE	QOLYS	KKBXF	. . .
9.	VBIHL	GVJCI	JZXXS	FUQIS	JMSYM	GALYS	KKBBA	. . .
10.	WPUYA	WQUMW	ERRKQ	MIFUI	KNRYU	LESPO	QUWYW	. . .
11.	WMYAX	EVTPP	THRLV	GZXEE	JNSAS	HFCLC	VHIZI	. . .
12.	LXHQL	OZZFL	FDWZC	UIOCJ	JEZMX	ZILNO	VTIDP	. . .
13.	MLZZP	COZVP	MAXBP	UPNBP	HAFYH	LXIAU	TVEJV	. . .
14.	QEKYO	UZNDC	GLCOV	LCQWZ	KUCGT	HEAEF	SEYDU	. . .
15.	KUEKI	FZKCR	XCEZS	GNFTK	SJZMI	ZIYSX	GOGDH	. . .
16.	DRKRE	YVDKE	CDRXA	UVYUA	UVWVT	QGDPG	PZXKW	. . .
17.	LXKKW	WAZBA	NLJYC	CKKUI	KVWFT	WUADC	AZIFW	. . .
18.	KLGHD	RFVMN	EYBAY	HPGFA	JJXHX	LALWE	OFCKD	. . .
19.	DPDJH	OVKKE	CSYAY	HOPZC	KOOEM	GNNNW	QUWYW	. . .
20.	VLIGP	MFVGR	CYRDV	KSBNQ	FHTFD	HWXPL	RTBHU	. . .
21.	NXIYW	QPZOI	CAFQX	SUQUP	QMUMR	SGJDF	SKGJO	. . .
22.	ZUAYW	OVYCO	ICGZV	LOGNH	NMGYD	ZZLVZ	JZGKF	. . .
23.	QVWYC	WQAKW	SEVQX	BKFUP	HAFYE	QHQPO	QTOBD	. . .
24.	PUESP	OQUMC	NAKAS	GNPPR	HYMYL	XZYLF	DYAVU	. . .
25.	VNDZK	EVTPI	IHRBY	UKLXD	UBEME	WUATE	VKTTL	. . .

b. Extrapolate additional key from the foregoing problem, and recover the complete plain text to Message No. 1, which is given below.

YHWKW	RVTPP	THRLV	GZXEE	JNSAS	HDVSW	GZZVP	OLLZX	RIUUP	TUTRE
LCYIE	FGIBJ	IYDDT	NOEBJ	TCEYS	KLRSQ	PQBCI	FOJUV	VWOIH	KDZLR
HAOYA	FFVYK	WRAXA	TUZBI	HBFGF	HBIWS	WRUGB	WZWAA	PSYMJ	EJZVT
HVDHP	GQTOK	CNXQO	KKQTK	ZACPX	GYGIS	KQBOG	WSARN	YTCQB	AMGGE

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6. The following four messages were enciphered on a machine employing reversed standard alphabets for the primary components. Place the messages in depth, and recover the first five words of one of the messages.

Message "A"

XTEKJ EWPSG IDJUO GDZEI KVGGC YRWVX YHACQ WYDJB GHULC NARXN
OVWOQ VMIAQ FZJJK CHNQP HJWUU IEZUN MGTXS VFECJ EHCKB SYXCB
MCXOK RPOXV YYZOK KIHHS DFGFU PZXGS TDJMI BNLPO SRCAP SGUEA
JWPFQ PHGUG WCXOV ZLQQQ AYTLM FBRCA CSEOH OGURD FAAUT MPCWK
VMHDA IAEAS ORFXI TNWAV ZDLHU YJTGC CKAVO PGZSR PNIPJ TRNGD
TGQLC RWHSI EFPEB UZGDL NIADE LXRGU ZTTVK BTWNW NAXMH ELFEY
NRQGT JBTDK AYWEF PCAUM WSEJU PTQHN YSCZI YZXGG LOIHC FNYPE
XCZKC SUYID WSKRL LHPYZ UXWTQ QFSLW XLEHF VGUPP XVSIY UGKBR
RRRGE LVNUV FKQGX LTZLD NZIHT OVBQK DHWGB NISNV AEPBF

Message "B"

LKGZE OQXIF AOPOB ASRBE GCXMX RHLYT APRLX PCUSP GDVFW LGLNH
BGMIZ UAEPW QKBQA KKZUA XFMSF TVZTS WOGSO OZYIV TCPWN NWPFI
NCGPF DKUNG FYSQX RXERR RCHOW QKKCU RAECB XAEDB UCKIV YONUI
SOXIZ LRFTD AFRYY BTCSG AHIMW RGFZB QTKGI LMQWD HIKSN LTMFB
MLQZF FSNPH DSPGT YVDTC RKZRF SWOBN KHVPV POVYF XDLAL RIJVT
VEGNB DSBCA LFWQM XOQSC YLVSW RTYTO QZWQV AUZSK COFRG DHNCC
XXBPQ PIYNH JUPFT MVQES WIUFG NNUTZ BDRNH JBFYH WJOUB BRKVV
RBVIR LKDLE TVCWQ AQHGO TFGXF IDQQK VYNQI DTDBY UYJHY QAPVJ
LCZPO XDHZS HWPYU ZYLNW CACTJ AWJKR

Message "C"

OIZHM QOQJW YXBEC QHZGV LZZVG UIZPL QQXDW RKCWH GYUHT RBXQR
GKKNE IWXYG TKVEO RUIWN OKMXD YNIKV GOQHG OOJXN LTEMX MPLNQ
AOICK XINEK RANYV GJFWJ YFOLF WRWWX YPHBU CRMSB WJHAF BLQYY
LIOMU KXAXG URTHR RQVZV JPAHN RHGOI GVPMW LODRV OTBIY QPWCJ
QJKNG EOVTB FADRW WJNGZ UHUKL ZIAQD DYYPK ZHJMJ MYGVP QDPOV
EIZTB JRVQN YXMBB ZRRAF IPXBZ NQMSK VMAQO DSWGF DYPUF OBYNL
LDBFI LZYXD PMMEE OGEFI ECRRR MJHVL CFTJO TLGSG MEIQS XBHXC
XFDYD LVXFV TFGGE ZQYRL TMHLW DCLDR NVIEI DHAJY GTNYP WCHNS
BLUNI VLXGW EBTOS BNGVW KPEOM YLUDO

Message "D"

PIVLV OPJIO BYJEA ESOPC XYVRF VSSZZ ZPCZX VZMML LRLUI LTFRG
QBLXE ZDKBI VNCSV OBLXS REEPU PZBZX CLUYN WTJNB VEJGO NNXGT
OKRLO MEVXG LBXSG OYBLA CQZKB OYLSL BAUXJ SKFLW LUTKP KHNFA
OISFT XQIRH YKXOI NZKID FABFY BZIAZ CSLHY IBTXN AQICB KBBAZ
EEBTP HCBXV EMHQE ISAWP YHFIG VJHGR NETUX OSLKX LPMVM TPLSL
PDDCJ PUIHT RJXKW HDSVL LRDMI QXPDE ZMYNM VFHBK PQJSY OIAJM
CCDZG CUIVC TMIDZ MPQVT DMXKO QBDQY JOKIX FWZDR YBPPK HRBVI
CLEHL YWQLF QMFHZ DQOZM SUGLW JVTAG GQGVK CHIFH YCSAV YSFMP
AGEOO FNBHH QQZER KHIBJ EKHVI OSZET

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7a. The following are the beginnings of 30 messages in offset depth, each successive message being displaced one position further to the right on the key. Solve the first three beginnings and recover the components.

1.	BIVMR	PRFLL	LUBPZ	NQUCM	EPCHR	VHHMI	CMKXF	GGQQB	ZSRZG	MIYLW
	BFTWR	JEPPR . . .								
2.	OUACG	XGQBL	USJZV	DJBUS	FWKTK	MMNRC	ATKNU	YHPZW	MUYEP	MEDBO
	QMFXL	ZBHRC . . .								
3.	VXYFF	UXGTQ	KYYNX	IIANO	TTNUN	KVFPL	QPNPY	XIGWR	VTUNJ	QLQET
	NCMHL	CADHQ . . .								
4.	MWTVU	HUZPW	PZCIL	NISQN	YIWHY	WLEAD	RBTZH	AGVKX	ALYME	OYUQD
	VMHLS	PAGRU . . .								
5.	YFFVG	JDGZP	OZDQO	ANTHZ	NJFBR	JXCDO	BWLQE	ARHNZ	LKJEO	ZOKTW
	DAMXG	YKIGP . . .								
6.	GEOMV	PQSYH	SCYYR	NKWEJ	GVDNA	NAQPA	OIWOB	AHRZQ	YKBRW	DQWVX
	ZRCKI	IQXRL . . .								
7.	PCZUG	XPMWS	TTNDL	PJDKU	MYKQM	AXUPB	HCFGG	MKPUG	AXLMW	QDVVG
	LRRNL	LEEQH . . .								
8.	FTUZP	WIDDW	XMFFX	XOKDB	NJFXX	OJWXP	ITEZS	BSDCC	XBCDL	XCEBA
	FXIYD	TKZSP . . .								
9.	VJLUO	CGMDQ	ZUUQA	WIFOT	RENXK	ONSWQ	AJUII	NBCQY	PPZCD	KQMAL
	JYLJQ	ZOGTK . . .								
10.	LSIKK	LQNXO	DPOHK	FVTWY	FMSJN	SFGQQ	BMHPF	VUJLM	ROKVW	OWVJH
	JYJHP	TDHEH . . .								
11.	JBBPT	CHYHK	QXADK	KMHAV	CMKWV	HYRXG	SXULB	IKXMW	WTHEM	NRLQX
	AVDPG	CTYDQ . . .								
12.	CDZRW	AIHYH	JNNIW	SXRGX	LKLGT	YLHJH	PLJOR	IBMRO	VTVEH	KCXAY
	SVGJH	GQYRK . . .								
13.	YYZTI	UEXQG	AEJVP	CDGTP	DPQVI	QZFOP	UZEPZ	EHZHX	TEXAQ	CENAF
	ERXYE	CBHBO . . .								
14.	GNDGJ	LDVGD	TMVVY	YSEFM	QBGGF	XXWPP	DBPNB	EWNID	EUUHR	VDGOU
	UTSGG	NUVLZ . . .								
15.	IDGQI	SCVWW	ZKMMN	RCATK	NUYHA	ZKXTZ	ESNQW	GOGKS	ONJSE	MHFGR
	LGTUB	JJXQJ . . .								
16.	CAXIY	FTVQK	BBKDF	BZQJA	WGCQX	OCNTT	OZYLW	YJNWT	WNPJT	GVUCV
	DDLVO	KGFGK . . .								
17.	AIHYH	JNNIW	SXRJC	HDOVU	KNEGR	AXJDK	NHMPZ	FDRPI	UDQDQ	JUUOD
	CSMEW	RPTSW . . .								
18.	TBKQX	ADVFF	BRICA	MFUWM	SGWAM	GTECW	ZWWHI	NEEZN	PDRQP	VPNHF
	ARWWG	PNPAH . . .								
19.	IUNHY	NKUUD	DLTPD	PQVSQ	AZAGR	ZJZRC	MAOQM	AMJAP	OLZQU	TQRCV
	MMUAO	WYIJH . . .								
20.	UNKVE	IFLTR	BDRIJ	QWBQO	YWIQG	UASGR	WGIXE	PTUCI	HWLZV	ZUTSV
	WJRCJ	QRJDW . . .								
21.	HJNNI	WSBQF	PNBKO	TCDIG	CXCNB	AFECS	CRDLM	NLZMM	AQERJ	SPTMW
	GUHSY	QFSFI . . .								

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22. TVOJG BKUST PDPQV SQAZA GRZJZ RCMAO AWMFN KNTVG LVIEI ESMOF
WTLKZ KFFDT . . .

23. VOPWO YWRCA YVPWK HGJOV SZLJQ EFFWJ ZNMZY MVWCF IPWIJ FXCOQ
BLYKL ZCKKY . . .

24. UJGVW ATCXK UCVNA OZHQQ AOMZJ XKPID AMRDY FVKLV IEFJL LZKUB
OQIUG WPJZG . . .

25. KUMNV WNPKX GVTIJ XHDRT IMTYX DEUFF FNKNT VGLGB BQUFY RWGRN
ZKJHF QEGQA . . .

26. GBPIS MKKIG UFHTQ AHVNJ CAWXB CXQYO NGJGA YJHPR QESSA YGNQD
KTTWQ DGIOE . . .

27. EWIUT ZMXNU NHXBH MNZTY XGOFQ LYZXB MSWWO XVGHS FNMUR AWJSV
JRFGR CMJGE . . .

28. CAFJU JHYVN IQCMA EZJZR CMAOC CWXAN PQCHQ DTEIE SMRFB ZBNXI
LNEEP CZBEA . . .

29. QLOMK ONSWQ AJHUQ RUMZE PRQXY SEZZJ QHMIU JVCUU BSVNB MYXSZ
ZUKUD QGQTT . . .

30. YDUXE VYYIT EOWNZ JZRCM AOJKF XIOSQ LAHDP QSNQV WVPYB DURIT
XZOTX TTVSF . . .

b. Extrapolate additional key from the foregoing problem, and recover the complete plain text to Message No. 30, which is given below.

YDUXE VYYIT EOWNZ JZRCM AOJKF XIOSQ LAHDP QSNQV WVPYB DURIT
XZOTX TTVSF YONAI WTAHK LFFLC CVHJH HLHFR LYZNB OZMEB ATIWO
PLZEJ RQEQB FCDIW BJQST

8. The following cryptogram is suspected of having been enciphered with a key derived by some sort of Fibonacci method. Solve it and recover all keys.

84360 89843 23654 99422 11271 04823 25759 41545 81196 65525
45773 93577 69960 67340 88260 13590 51375 06770 62070 38254
09178 84673 93453 29051 71193 42329 56490 07497 59137 78351
50745 28901 84105 44839 74867 34591

E. Cylindrical cipher devices and strip cipher systems
(Embracing Chapter VII)

1. The cipher message given below was followed by its original plain text two days later during World War II. Reconstruct the cryptosystem involved.

MCSGO HKXVS QFFKW TFILU ODLLZ SQSVM KWOLY HBXFO GJGGW YTXUP
QQUPC XCSDJ EJZUZ UPBSA NEZLE ATKNF IFCMI MFWNM CXWRK TTWKF
NJGIM FKJNZ KTXCQ RBWLW KDMWH LTKHZ GEAFC MGKDA LICUD IPXHJ
OLCKC CWZJM FPAQL QWQSE RDREI AEOJO NAXEV GZRPG YKLRX XOKVY
DCQJK HWKGD RGQIK ZSGNQ DOVRW LXNDF XHCCP YJRNM VZIAJ XOHRU
ZPPXQ MHTIU IPSUM XSITS IQKLX BCIUF MPXZB CWLVC QZAZJ NDWTE
MKNNA LNZIF NVHUT KHPTD CDBXV LYBQV ZZBUL NXBOC KWTCF OFEZF
FKNUW QFZFD YYVNK XKKPU YQWCV PVHUP WJBXM ABYVW KMIGM ZXGKS
RBYOM KDCHR QDBMI XCRYU HXKMF TYELW MPXZL CEVNV KQPQW TWFGK
QKEMK ONUKD MQLCT DEXIA AXFLP HWPEW EPVDH IZQFJ PAMRE ZYCXT

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XOYSV	NUSJK	TUVQQ	WDSHC	EEZKV	OLJQR	KHIMO	XVFRS	VJXCX	PNTZP
IGHTF	OURSE	VENZE	ROATS	FONEF	DRTCO	DSYIT	EHMKJ	IWUKX	YWVNG
XQMDM	FNHIX	NBGOV	HDNRG	EPPJQ	LXIVM	EIFLV	WXOLX	UTAQE	LDEWB
LFMTQ	AOYUX	VXHLH	RHPFO	NHMWB	CMKZS	MLVNT	ZENEO	TZQGG	ENDCY
MBNJY	AKQMA	FJVSK	XKKPA	USJLO	XLANT	JVPRE	QKSFZ	NWCYY	XLNPW
KBUGM	KAWAS	DRLIQ	YZGJA	SACXT	WSRDG	WVAKG	MJJPW	DXEVT	VESHK
RVBLK	QDQHE	ZQLQA	YMVUY	IWMRK	ONFPG	WYAEG	UKLSC	AJBGI	WHSZA
RHMTQ	YHUTP	WUZPM	CMHKV	JLHLE	HMMLN	SQWOQ	KJEQB	HZZWX	BGFFC
PDMSL	MULWG	MIWFA	TYZXU	VWVRG	ATQLP	RMABF	VXVNR	WNVFK	LHQYL
MCPBI	DOTBI	KXMTX	NCIRR	QEAUH	KOTZY	OKVFD	FHBWD	KVBQT	XLOOY
PCLJU	WTVBW	VXSBU	YNIPG	GYFJW	PGJFB	YPHDM	IQPYP	QFCJK	RSIPL
EXPTB	FLEIE	ZUHDP	YINCX	AZGSU	ZAGAE	IULFC	VODSM	KMQIM	BPIAA
UQGHT	SKSOP	SEZJC	EEFKL	HJXPL	MTQOM	EKXAJ	GOAMF	CFTEU	HFQMW
YGJFN	RIVTT	MYDGP	GLPBN	NIZCL	KBUGX	KTFZN	DDUWZ	YOHDB	JMCFC
CMKFM	NJUTZ	LKSWM	THLKL	SBDCY	SEMPU	KAVOT	GBIEY	ZDNEZ	EHDIU
PWYJR	QCQHF	GONUK	OHTTC	JNEUK	UNQSL	RJSJT	MDVWC	JAEJT	GSAZZ
BMMSC	VQWOQ	YXCGE	PPMHG	VUSJC	IXMWL	HOIVY	YCHPG	ELTMA	AJAWJ
GOPZB	RHWUY	BYCYG	OWNCX	AZGWU	WKAPJ	ZIRVV	WJSJB	KVPKJ	GXGML
NSCUJ	WHKDM	UHLZQ	ENVSO	TBVYZ	SJGMC	GMYKY	NJMIB	RJEGQ	UTAKP
XEASN	KVGPL	GCEUI	CKTCZ	FAKXW	MOXDP	GQYWX	LSOJZ	KELUC	ZGQNJ
PEFQB	MKMRX	SHFLG	MMICF	GKVPC	SGDXZ	YUDYK	WLHWB	LUSUM	XDUFG
WSRDH	FKZHE	DLZSE	NVPUN	RIMLS	TGWZB	OIXHW	WHJCX	UTAQA	QCDWM
AUHSA	EDBCZ	LBWQL	KFFEU	RDJWA	AAMLU	GKABS	HLFVP	IMNPO	YTAXL
UBZPQ	QNNSN	ZPVMP	IBJTO	LFMSJ	GYXSE	BHQHR	HALKN	AHRQI	ZAWHE
VXKVE	KLRLP	KESPZ	VKSPE	ESSAN	SVHAK	XZPQD	KGWSA	IJZYJ	VKGSL
CUNLO	HHKDM	UMQGN	OQCQX	QPEOY	WULEX	YQYAS	JIPIW	WPXHK	LHQYL
QQEAK	XZPQD	KGJVO	UIPEZ	ENGHY	IVMWQ	ABNAJ	YNSYV	FLAXD	URLWV
PKUYS	YXETY	WWYNX	VFAUJ	JRNEI	OQZFX	YQPGZ	AXOKL	IJFGG	BQPZQ
GPBJC	MXHRE	TTTNM	YUGJT	LDHHO	BFTYP	KRSGI	CQNUS	QEKCI	KMQSE
HXTIW	EDNCV	OKWAY	ATAOP	MRCAG	LAHAM	YAHFD	MBGOY	UYHJV	DPPMP
XJEDM	FNHCU	XGNQT	OGGJG	GHQJL	OPVYE	QVXCP	IUSOM	EWBWS	HCPOV
MNIYX	OBNAP	ATYTC	VJCCU	CINBH	BJRZJ	HETXO	FACDV	EOFGC	OTUFJ
LYOTW	ESPAJ	QQXDN	VXGGI	WXLVC	TTLLY	ABDWM	EKFZE	GBWVS	YAQVY
SNKYJ	TUFMC	LQKVY	BOGXD	HHZAS	KHBNM	KPJXO	DKCZZ	RGIRX	SUYKD
GKSEL	LDMOV	HDJDE	AIVNS	YKWBM	ASLOT	TNGJE	JRSGA	HYXZD	GMXDN
AUHMD	WFYOJ	EPORY	LXHGI	WAQTW	LBPEX	PPKFM	JUVFL	TAUPD	SONSZ
EUJED	XTETK	MXQJX	GOLIX	SMSTV	HMIZD	JQKHW	SKMID	FTHBX	UCTIS
QFJKF	YTZUM	YNNZR	PGHGE	IAKJD	CGSLI	MIOVY	YCHPG	ELWHS	YJXQH
LKPXZ	AKOAC	BCPFR	YQZLD	TKGEE	AVNPG	CFAFU	VPZDY	ATRYZ	YCXGL
FCRDC	XXNVO	EUGBT	CCTWX	EILLG	CYULK	IWXTF	OXWFC	FGNLN	RXDOT
NUSKU	UVSZY	DLVGY	NKZDH	UWRHU	OJDTH	GINBO	SMCOM	MXNHP	QUWHR
PKEYG	SQQVV	MSNCR	SKSLL	UGKJZ	BMEAV	PGHPQ	QTCAM	VZOZO	KNJTE
WOHXH	JGMNR	QVYTR	JDIOC	BZMBD	TXXCS	WJADZ	ASXBQ	KGWIO	YVSIF
QSGAU	FXYLW	EHXIV	KMOEN	ELFCU	PPFWR	PYMIJ	DXJAP	BJSAX	IZWEE
SNSUG	AERQG	GXALQ	WLWDF	VCLOY	GVROU	WSEOH	UBDSE	PWQAQ	TNVSO
DGZYW	KYSPC	LWFMS	NXEBS	TQAWY	SQCHU	GWWMS	JWVAZ	CTLQB	MCQIH
PDFYK	FQGIT	GANWC	CJTTT	YXABU	TWLJR	DOSTY	LGJZO	XQANF	PXACJ
XSEZO	QWJIB	IDEYL	OQRJW	WYXGD	DBAMN	WVGYZ	SGEBW	DXTCZ	FASMZ
MIPUO	OFPSS	SEPCI	XNHSC	LOMNY	TEFUT	CFNTD	PNUVS	SQAKS	NWLOM
BLSKL	TZERD	ECQMF	DCKKE	VGVEY	SQZLS	NKZEY	HDYIW	YTCPB	GTVOI
XRSIK	UIKYO	ZEKSI	ALASY	QDHAQ	HGHCO	HYCZT	EKHKF	WLJZT	VBFMA
YOBSY	DQYGG	ZUIUS	GWUTY	IWMRC	WTEGW	MMWIR	MUTTX	WPVFJ	WBBYX

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QEJAP JWYST FYBXN LWEGO KPMDI NQZYQ JOOHK VSOEB JJYTG ZNREV
LCELQ KTFSH OPAJA LAHVX XSLEP ZBZFV NWOPX FBUXF BBZWR SFSHZ
KDWJB AKVKO UZFUY WCDWS TMGQR EUPAG ZOLVV QMIIA IIBKQ DEAUQ
AJENO COHOQ UJNIF OZXRG PDKBP KOIWL VVUMQ RSVJC QZAZB RLLDM
GLMKU

The original plain text of the foregoing cipher message, as it appeared in the teletypewriter copy:

RE TELEPHONE CONFERENCE 32 CARS AS FOLLOWS COLON

B&O	380083	4005593	470	199200	15980	82700	1833
PRR	78708	4005594	470	201-2	15980	82800	1833
MILW	18844	4005595	470	203-4	15980	82300	1833
MILW	19647	4005596	470	205-6	15980	82900	1833
C&NW	73692	4005597	470	207-8	15980	82900	1833
C&O	4167	4005598	470	209-10	15980	82200	1833
B&O	380579	4005600	470	211-12	15980	82900	1833
B&O	379021	4005601	470	213-14	15980	82600	1833
T&O	54682	4005602	470	215-16	15980	82600	1833
MILW	6442	4005607	376	217-18	12784	66500	1466
MILW	18081	4005608	470	219-20	15980	82600	1833
MILW	18470	4005609	470	221-2	15980	82800	1833
ATSF	143641	4005610	470	223-4	15980	82300	1833
RDG	103285	4005611	564	225-6	19176	99000	2199
SOU	306570	4005612	470	227-8	15980	82200	1833
RDG	19102	4005613	470	229-30	15980	83200	1833
MILW	19545	4005626	470	239-40	15980	82900	1833
GW	45019	4005627	470	237-38	15980	82700	1833
UP	75308	4005628	455	231-32	15470	80200	1774
WAB	83552	4005629	470	233-34	15980	82300	1833
NYC	132442	4005630	440	236-45	14960	77000	1760
MP	42680	4005631	445	241-42	15130	79100	1735
SOU	261700	4005632	470	243-44	15980	82700	1833
NYC	151707	4005670	470	246-47	15980	82800	1833
WAB	83350	4005671	470	248-49	15980	83100	1833
B&M	73104	4005672	425	250-51	14450	82600	1657
B&M	72108	4005677	440	252-53	14960	77100	1716
SSW	33571	4005678	470	254-55	15980	81100	1833
CB&Q	42072	4005679	352	256-57	11968	65600	1372
CCC&STL	48835	4005680	430	258-59	14620	78000	1677
WAB	82452	4005681	440	260-61	14960	77000	1716
C&O	2418	4005682	369	262-63	12546	67900	1439

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2. The cryptogram below, enciphered with the M–94, is suspected of beginning with the word ENEMY. Reconstruct the plain text, and recover the sequence of disks and the literal key upon which it is based.

```
GDYUB  ZIBHN  IUJGQ  ZUXTC  ZBDRO   DRRZM  LCXTO  LPBOR  JELHM  GYGAU
AODLH  CNBTU  ZJRVL  HQUNW  DQHZB   ATUPF  VRLHL  RFKUV  UAAHG  BEZHU
```

3. The cryptogram below, enciphered with the M–94, is suspected of containing the word HEADQUARTERS. Reconstruct the plain text, and recover the sequence of disks and the literal key upon which it is based.

```
NEKYL  SRQXV  EEZHP  VGTUT  YJXBY   APKVI  RPZNJ  WWFXE  OBXDB  VQAYZ
ZDJGM  MFQGX  LKHIW  BVDJK  YARBK   LLOLO  OYOVD  FARUX  ZVXBT  GMAUV
```

4. The cryptograms below have been enciphered with the M–94. Reconstruct the plain texts, and recover the sequence of disks and the literal key upon which it is based.

Message "A"

```
YXI  DE  RBV  0405  26  DEC

QVQQK  YMYRK  FEOPM
```

Message "B"

```
YXI  DE  RBV  0501  26  DEC

GNNSF  IDOOX  HQFQQ
```

Message "C"

```
YXI  DE  RBV  0612  26  DEC

RKNSG  QPNIS  OBWHC  AWLZU  ZIVCB   DRUFW  WMRQK  GDCXU  DIFKB  OSTZN
FCLCM  ADOEO  EHAAX  JGAWP  NRKZH   NGPRG  HCBSL  RSJCR  WGZBI  NVABP
```

F. The Wheatstone cipher device

(Embracing Chapter VIII, pars. 49–51)

1. Solve the following cryptograms and recover all keys:

Message "A"

```
PIBRQ  HOCDP  VKIKZ  XKTLZ  WYPFL   LYEQO  NBVKX  VWXTT
```

Message "B"

```
KIMLP  CAGHX  PKXLR  UZLGM  SVMMC   KWKMO
```

2. The following cryptogram is suspected of containing the word HEADQUARTERS. Solve it and recover all keys.

```
GUGVD  YVCOM  NPHTE  MXZED  HIOWA   SNXYT  SULEI  XQNTQ  GTHYH  WRVFR
ZLRBA  FUPSJ  DSCTF  HFUJY  SBSLC   RWCAO  OPDLD  EIPXH  WWYTZ  UWEEY
```

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3. Solve the first five of the following message beginnings and recover all keys:

1.	DMOMW GSVRO HDKBD . . .			14.	EQRKC	QYRES	TBRHR . . .
2.	ZODFF JDUVC XZNHE . . .			15.	PQOFS	RJIBR	OYKGJ
3.	QODFF RDCUT YRKNJ . . .			16.	JXZRK	CYSZD	BUHZW . . .
4.	GGZZI KADGX QUYAN . . .			17.	QOJKM	YPTIX	IYZQT . . .
5.	PPRVT NQQDR NYNHC . . .			18.	AMROC	QVKAQ	JKOVL . . .
6.	ZOUYR JIYOB WYQWP . . .			19.	AMYPZ	XIUCZ	ZOBNM . . .
7.	JOSBE SOQQJ KZLBJ . . .			20.	QOFFR	TTIAF	OZYWP . . .
8.	OXHZH KSHAV CFKZL . . .			21.	SXXRV	YJHDO	PKOYA . . .
9.	QODYV BNFVD CVJSO . . .			22.	QOKXJ	FROJP	AQBBO . . .
10.	JOSOZ UIHUA TUDRN . . .			23.	DBHYP	HNLAD	AYZJM . . .
11.	DMOMW GQIUT RQLYK . . .			24.	MXEYT	RTVAT	DMZNH . . .
12.	IATKR VFFID TBMFT . . .			25.	NXIST	NQQDR	NYWGB . . .
13.	QOQTY HTSNF KZTQY . . .						

4. Solve the first five of the following message beginnings and recover all keys:

1.	FUTJY	VPFAR	RRNVT . . .	14.	DLWTK	FFZHL	RVITG . . .
2.	WWZMP	GRGUC	QCPLC . . .	15.	WVWHA	WHANV	YSSTD . . .
3.	IBVEB	TXCMR	TOXEY . . .	16.	CSLBZ	DILSQ	MFMTB . . .
4.	HRLQA	CWAYB	STCHD . . .	17.	ZCJCQ	YODQX	BSPVE . . .
5.	GRFRF	WDHLG	YJPVH . . .	18.	WOUIK	RCTEV	ZFRKD . . .
6.	DDRZL	OYOAE	KELSN . . .	19.	QFCIG	NWPTU	WLFHZ . . .
7.	LDLIP	LIPJU	JYLIY . . .	20.	QSDUK	BHSCH	BCNQB . . .
8.	BXQKC	QSWUT	XUYNK . . .	21.	NLGQT	FMAVT	RDHVR . . .
9.	CXHMX	WKSEI	AGHGO . . .	22.	DMGMR	UFCQW	UADVI . . .
10.	CDMZE	KWCHQ	MQVUW . . .	23.	LSQDG	XGNXI	CHABL . . .
11.	GIHCN	JJYOI	KFKNQ . . .	24.	VFUAO	QJOSZ	IRQSF . . .
12.	CSLBZ	DILSQ	FKBEX . . .	25.	ZQYQV	BAHCM	BXZTS . . .
13.	FOWYT	OYEZX	DVTXN . . .				

5. Recover all keys from the following matched plain and cipher text:

```
WAHJJ  MTOVL  LHTEX  JCWQC  WLJHZ    WQZPX  AOLHS  ORYKH  KCDAG  DNKPZ
REFER  ENCE*  YOUR*  LAST*  MESQA    GE*SE  COND*  REGIM  ENT*C  OMQAN

OHJOP  ESPQH  SYONH  EWXBV  CADWX    BEMOE  EOSFE  RZOWH  NCXFU  IMMVD
DPOST  *NOW*  LOCAT  ED*IN  *SCHO    QLHOU  SE*TH  REQ*H  UNDRE  D*YAR

ROESP  JYRPL  FGLGX  QEBIA  RAVSX    QXGZP  XRILU  NRQRD  ZKEGS  WLDDY
DS*NO  RTHEA  ST*OF  *ROAD  JUNCT    ION*S  EVEN*  NINE*  EIGHT  *STOP
```

6. Solve the following cryptogram and recover all keys:

```
GDRYL  RDTVO  AHRHO  KGETG  YZCAL    BPMLJ  DBJKW  GCOFB  MZBOU  GJHNH
UKVUQ  HAFZU  BNWZN  ULLAS  NUMQS    TNXCB  JVMWP  QERKZ  POETC  MHOGD
VYUEF  XAVAF  JWTUW  OPNXB  RQDTJ    CMISE  MDAGJ  PRSAR  LORNH  HPKRN
ISKCR  QBSIC  GYVKJ  FINYO  EGWVW    HGJPH  UHOJH  ZEYAH  ZDELV  KFTTE
FYHIH  OJTLO  SPSUM  QLBHJ  ZSIDA    QPQNI
```

7. Solve the following cryptogram and recover all keys:

```
WOVAC  XPLDV  KZGZU  ZDAOD  ANTQI    EPIUA  QUMAR  WOXFH  PLHMM  FQKDG
BARXF  TGOZV  RQMCY  MPLSV  CQCPH
```

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G. The Kryha cipher machine

(Embracing Chapter VIII, pars. 52–55)

1. Solve the following cryptogram and recover all keys:

```
GWTJB  MHYWB  EGWKR  AWRLA  DXYZW  JYBTP
```

2. The following cryptogram is known to have been enciphered on the original Kryha cipher machine. Solve it and recover all keys.

```
        5         10        15  17
J A E B Y L Q X K K M L P Q F E U
C X Y I N S Q E X W J S K Z P W A
O M L Q P N J D T P C Z R V R E W
X B T O Z S N O Z G A N C L J W X
L C H G A K Y Y A T N I J J N C I
L V G Q Z F H P J X X I N U I Z N
T D V E S X U P U H J G T L K P T
Y X K C X H N F Y M Z W X G V V I
V O D Z M Q W X G Q R J I U R J Y
L D X A O A Z A U D Y W M J M O W
U Y C U U H B V L Q M I D L L O M
A A V X A B W D X M M T B I Q I F
J K V N U E P O Q O W S C O X B F
U V P C N I R C N H D K E K V K T
D A V E C E D A X B U W J R Q D H
U J N G D R I V P J I U G Y O Y N
G X M T E R G Q Q X Y L M S M U K
U Z J Q R W B N S M A R U H T I X
Y A S Q K P M U L J S R W V M Z E
K T S A I T D D C N O A J Q O I U
J T E C E U L A Z D O H M W A C O
S E A F Y N N K U X J D W Z G Z U
I U Q V R I J S M H M B Y N E E E
K O U Y A K P P B R T E A O P S C
M P D J Z V P I M B N T M Z L C X
O E G R N D D N F C X H Y J O T N
Q M W U C F C P E F V L E S S E G
N R P C I R H T G F E F I X F J S
Q Z B Z T H J J R F Q Q V H F J J
Y H T S T C Y W Y K D Q Y Z C L M
F F V A E O A E K G Z D I C U I I
Y R I L R C M K F R E Q C R H O K
F Z W Y V A I G U I A N Q A K I E
S K M C Z H E S B U L Z N C F C I
A K E Z E R W D I M W D J M U I O
M B Z M M Y P B K U X N N X D D P
Z Z J W R I W R V E H K Z C A T U
O J Z E M A I J L K Z E C Z D G G
Z Q V N U E P O Q O W I F O T A C
Y X E K M X P G Z F D A X V A Z I
H E F M D G J W V G Y W S E G Z J
J Z A D G R B Z L X G U T F M Y S
Y B Y P I V D H F R K G A C W B N
M H W N E O F M S V H P Q G Q T D
P S Z Q P L K W E J Q B Z P Z H Z
M S T I D G I Z B T
```

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3. The following cryptogram, known to have been enciphered on the original Kryha machine, is available with its compromised plain text. Reconstruct the components and determine the disk setting.

```
        5         10        15  17
Q D N W Q Q F J E P P P Q X Z B Z
R E F E R E N C E Y O U R M E S S

O A S C D Q N X F H H H Q Z O C V
A G E N U M B E R T H R E E E I G

M R W S Y W G N V S I H B O U X F
H T D A T E D O N E S E V E N D E

A B U T W H O Q K R S P U Y Y T X
C E M B E R S T O P A L L R E Q U

K P W V K G V K C O J J D Y O F D
I S I T I O N S F O R E L E C T R

G W M H W M X N X X E C W D A D R
O N I C S S U P P L I E S W I L L

T A V I T N Q M L P S F R O K S H
B E S U B M I T T E D I N Q U A D

C D E A E H J H P X X X Q K G C A
R U P L I C A T E T H R O U G H N

I W T T T J O L O O Y C R J G X B
O R M A L S U P P L Y C H A N N E

G Q O N B K P R X A G D N O K V G
L S T O T H E P A R T I C U L A R

D M S J T L X O P K D F E R G L B
D E P O T S T O C K I N G T H E S

X W U O C D X X F O O I L
E S P E C I A L P A R T S
```

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4. The message below was enciphered with the improved Kryha machine. The I.C. of vertical digraphs on a width of 21 of a volume of homogeneous traffic (see distribution on p. 634) is found to be 1.04, whereas the I.C.'s of other trial widths are in the vicinity of 1.00; therefore it has been determined that 21 screws have been activated. Solve the message and recover all keys.

```
        5         10        15       20 21
C S T W I Z Y Z X I Y M V R M Z T X Q S B
Q T Y N R D Q V P U X O Q U O F B M D A O
F T H O C B O L Z V F N I S H O Y S U B T
B V B M U D O Q V R B L P X B J Z R Y U L
F P J V S W W O E C N R E A L S J J I V A
R E G Q Q Y A P P C S K F T L S V E G M N
A F X X T Z I T Y W T U U I L L M Y G C C
C Z C T C R F L F X R V Q O M E I R R W Y
N I Q O G P Q T V Q K F G T M V K X C D P
Y O U T K I F N M I X K K F F Z L C N K I
L W X F S F J M F F N L D R C P E N S W V
K A L G L P Z I U O P Z X P S S L B S I T
K T H W N S D H Q A B D T N H G W R M Y B
D V C P W X E D T A B Q L C W E E R A Z A
H J Z H X U R M P J S F J M W H B F D Q N
S U Y M Z Z S P M L S C N W Z O B C R X X
M H U G I K P S Y Y C X X I P D D U X P P
E R W K J O S D O N A O R Q I A G V I R P
P F D S A Z E U B J V J P Z E N C F D K W
K C J T A F F N T K N W Q L J O A H V B R
M N T T C M H K N T V Y Q N K O C J B I W
Y D W K G L O I X I K X T I Y D Z D I Z H
Q T L X Z Z T Q N A D X L Z R F J Q K P U
I E C M G X N S V D R I Y R H N U I U V L
L Z S R V U U S F L I N U V V G H X Q Q F
I P M A X Q M N J V N P M Q V X Y X S G F
T S O F D H C K D Q Z P E R Q Q L L S W P
E A Q R N O Z D T O T P J R I A I Q L H D
F D J S W Z K I I Z D A B G P G O F L I R
O Z G N E G A J O T F E D C A D M I Q V P
H Q Q V E T W S X L X W M V D S W E V X G
L K F I M W F D C S R I N L E B F E G N C
L M A G C O D B Q T D W Z P E U Y C H C D
E J Y H P M Q H U J F U Y H M U J M R C M
V B Q W Z V F J V Q A I Q B Y Y I X O K U
V V S T O Y M L F C Z T F L C
```

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5. The following two messages were enciphered with the same components as in Problem No. 4, but with a different screw pattern of width between 18 and 25. Solve them and recover all keys.

Message "A"

MXBJW	SBSSO	VLULG	ZBJVD	JQTBI	KIEBG	UWOYT	JDSFL	HKVCR	LGKLF
NCKNG	GILGT								

Message "B"

MXBJW	SBSSX	JHXRF	VHGKN	KLUJT	XHUAW	QNZNR	UUWUF	OQAOV	ZVPZV
KNIYT	HJLGT	FGEPP	VIIXW	VZDQW	HVPZX	EDJRP	PBDMA	FXBOL	JKBNA
NEUCF	PHTVL	HNAKV	VBXYW	NLZXB	BREFV	KIVSK	CYQLH	JCZNT	BWNDE
ESEDF	ZZKEM	FZSDN	DNDOI	YKVJD	XDGSD	IECYY	HJZEM	OAXHD	

6. The following message beginnings were enciphered on the improved Kryha machine. Solve the first five beginnings, recover the original enciphering components, and determine the screw settings.

1.	IZZKK	FSCHU	RASBD	BMEED	ZMRSL	SNVIF . . .
2.	YZNDD	CLZCE	WLSXD	BNEEC	ZHSMR	SMNQV . . .
3.	PDNUG	ZEXBO	SFMDI	ITCSA	ZERAL	SHMHZ . . .
4.	ZULDD	YXJBM	EBNKL	QDWXJ	PFYMV	TNNWW . . .
5.	WDFPK	PMXUW	KELGH	TGFXH	UHSCL	DOSME . . .
6.	PDTIS	OLXBO	CFSCW	JTBQD	SBXQH	AZRTM . . .
7.	ZMCDH	VXTCY	LHRTD	BVTLJ	TIKYP	HAJFF . . .
8.	WVJXP	WXDXL	VWFCN	JZWSM	UAGSZ	BEXJW . . .
9.	PDKRR	ZEIXM	LWVDY	BUIQO	COQDJ	UHFGV . . .
10.	IZZKK	HMIGE	ZCAQV	GGWYB	TIKYH	SNRBV . . .
11.	QDNUD	JXRPP	KWWQM	ITAAF	YUXAT	TZJAF . . .
12.	YZNNG	PMIPE	KNRVN	WGAHR	BABMO	TATPY . . .
13.	HVLUR	ZEIAB	CFRDL	YZCMM	CMJJU	LHRRF . . .
14.	YZNDD	CLZGY	WEWFK	SJJSM	KXXVW	YPUIL . . .
15.	PDYND	XMMLD	RFANY	YTDLK	NUIDF	BNWWD . . .
16.	ZYPVD	TKMAP	EBVUF	INWLU	RACWF	NZEXV . . .
17.	KVXCB	XKCXL	AWYRW	IZJGG	CMXIE	EHKXS . . .
18.	ZRYNG	ISOML	BIGQK	SMEER	ZTFJX	TNBGS . . .
19.	MTTTA	XXCLL	RQRJV	BHJAR	ZMCHF	EXVIP . . .
20.	PDDVP	JSIPE	KVIRO	HDSST	FTBZE	OYXGF . . .

7. The following cryptogram was enciphered by the machine of Problem No. 6, but with different components. Solve it, reconstruct the components, and determine the initial screw setting.

TNFDL	XHDCJ	DFKDP	PIEVS	IWOSJ	XQRJB	EPLMC	KFXIC	VYVQY	QBUGH
IDFFM	PINYY	NAIYI	MJRXO	BBYRB	WVYBL	DURMG	RIHRS	SXYLB	XEQBG
BDQNQ	YHECI	PRERA	ZVPKV	EDFNT	WJHVT	QQULC	UNOUD	DKZIR	KGERC
WEGZG	MMNPL	RFSYT	BKNAF	THRFR	GFLZQ	VBSNT	UXRDT	VDKCK	KGPKS
XMUPH	YSVFW	QBOUY	XVZFY						

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Problem No. 4: Vertical digraphic distribution, width 21

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	20	22	14	23	23	18	19	25	17	20	24	39	16	18	22	18	22	26	24	23	17	35	30	26	20	16
B	16	21	34	16	27	24	24	24	18	30	24	18	20	12	29	16	15	15	17	21	18	13	23	25	25	21
C	31	21	19	17	20	15	21	21	15	23	22	15	21	27	26	27	20	26	20	20	22	16	24	25	15	40
D	30	23	16	41	28	31	18	13	17	22	25	25	32	13	24	26	19	24	13	35	15	30	26	25	24	22
E	19	27	18	33	17	21	21	35	27	28	23	15	19	34	22	22	14	25	19	32	26	23	22	19	20	23
F	25	18	22	18	18	23	33	20	31	22	34	32	21	19	16	20	12	17	18	25	17	27	18	21	11	33
G	11	26	13	18	19	17	24	16	17	19	16	19	25	27	21	17	15	31	39	21	22	37	27	15	26	11
H	21	20	28	43	20	16	24	29	17	23	25	26	26	31	21	23	24	27	22	27	22	15	19	19	13	20
I	18	15	25	19	16	26	31	48	15	26	20	37	21	31	23	26	31	25	23	16	22	39	21	16	13	17
J	20	41	11	15	19	24	23	15	17	13	22	18	28	27	16	20	16	21	09	27	19	13	22	23	24	21
K	19	23	19	16	21	41	20	31	32	16	16	18	30	25	27	19	22	22	20	20	30	14	18	17	24	13
L	17	20	31	12	19	21	23	17	27	19	17	17	14	26	23	16	18	20	25	22	38	22	12	19	27	26
M	28	17	31	31	19	15	30	24	52	16	25	11	25	37	17	17	22	19	15	16	20	19	20	13	21	17
N	17	29	32	12	18	20	20	17	26	16	22	31	20	18	19	16	13	19	32	34	24	30	26	23	29	19
O	25	17	21	14	23	16	26	16	23	13	28	13	16	31	20	13	17	33	18	17	25	25	27	21	20	33
P	24	17	22	16	21	19	22	22	21	42	19	19	31	21	15	23	13	19	25	20	17	22	21	18	20	40
Q	18	18	16	35	19	11	22	11	36	16	35	30	27	28	22	24	37	12	20	14	22	16	26	19	20	17
R	28	12	27	21	13	21	18	17	16	22	28	12	34	23	24	18	31	30	21	28	21	18	32	15	24	19
S	38	30	20	24	22	14	17	36	24	19	24	16	09	14	30	16	15	21	19	19	19	19	18	18	17	21
T	21	21	10	18	30	11	14	27	24	23	32	20	20	33	12	21	13	21	27	25	24	22	22	18	13	20
U	19	22	28	24	21	22	18	10	24	17	27	15	31	13	23	24	18	21	26	26	11	27	26	39	19	27
V	18	17	27	23	27	26	24	17	21	23	37	27	44	21	21	20	20	22	27	22	22	22	20	22	21	20
W	16	20	41	17	19	14	24	18	26	33	18	18	14	25	20	16	18	23	27	21	21	15	17	17	20	17
X	16	19	26	11	27	17	20	13	25	17	18	34	19	19	15	30	22	31	24	24	25	21	16	18	20	19
Y	21	21	17	35	17	26	27	36	23	27	20	27	10	21	17	14	36	29	13	20	22	13	28	20	18	14
Z	17	24	15	26	22	26	31	24	31	22	08	16	17	18	31	28	19	24	30	16	32	43	25	22	22	18

H. Key analysis

(Embracing Chapter IX)

1*a*. Determine the source or method of generation of the following key:

38549	40853	51895	56813	14525	40990	25353	59453	59918	56585
45655	62505	49995	22508	56529	09312	21449	38938	81253	54453
05408	53390	81450	85814	40519	91351	35470	85653	58956	08551
80808	59561	81051	44571	12901	09540	53893	80852	13956	41018

b. Determine the source or method of generation of the following key:

69424	34901	07748	11480	43778	07703	42047	67437	49034	80664
97876	97369	47845	44569	45947	43689	09649	49941	06732	64694
54849	70335	03867	43745	03449	20375	94173	43736	94594	74368
90964	90334	38456	07594	17343	73096	78146	84114	80480	04948

c. Determine the source or method of generation of the following key:

07453	85663	28647	76529	46394	72266	26147	84030	67151	25768
01691	65848	23023	61754	31080	30949	41597	03271	79723	35042
51247	32718	80433	47419	13607	50476	20615	22505	37215	53510
06115	34462								

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2*a*. Determine the source or method of generation of the following key:

78131	76784	31174	50078	76343	47807	41346	53334	01331	01799
78318	76441	31917	92478	74179	10834	76033	55723	40178	31347
46554	65323	41305	86131	34767	30345	77787	48763	77689	76072
76747	88123	11278	31788	76503	47753	17807	67921	07276	07310
17997	88878	74703	05323	15777	71034	76371	33764	47117	37607
88390	00666	33300	03985	79531	31533	78342	47800	17230	75560
34850	74547	83189							

b. Determine the source or method of generation of the following key:

VOPVJ	AKDXA	MTQUG	QMOSF	SOPXF	AEBTA	NXHIY	RNAQF	KIDUF	USLLO
KYLYV	HCOMH	SAQBK	UHWBA	SSPIU	GDAVT	XOYCG	YPFKE	ASNEN	BSYGN
TUFMV	ORXXY	FHPYR	AQUOX	ZYPQG	AHDPY	HRNYL	VJEUN	XIFLD	EXUBI
IGXUN	QNOBO								

3*a*. Determine the source or method of generation of the following key:

64748	25986	81202	65617	94252	60376	46908	04349	57970	71582
19145	35989	40208	18269	32025	64609	05138	69604	71080	34248
78391	19293	70431	36757	10162	45363	71487	57019	15450	68758
59360	97923	13767	28020	30481	54247	86821	27353	14159	26535
89793	23846								

b. Determine the source or method of generation of the following key:

ENKZU	HNZXN	ZPCTJ	CCHLD	BFXHB	DURKD	HRXNL	EZJPV	BYJVD	JVQGH
XGGYC	UEKDY	EUBFP	UYFDJ	CZKBG	PRDBP	ZBPSA	XEAAD	IVFCZ	DFVRL
GVDLZ	BIKSK	OWCRK	WQKWT	NVPNN	RYLGM	QRGLC	AOLRA	QKYSX	OZDNA
ODHOD	VKITK	KAFYJ	EHAJY	NBZYA	BHOFX	PESQF	UEQVE	QWMDZ	MMUAR

4. Determine the source or method of generation of the following key:

FWNEV	MDUPG	XOFWN	DULCT	KBSHY	PGXOF	WNEVM	DULEV	MDULC	TKBSJ
ARIZQ	HYPGX	OCTKB	SJARO	FWNEV	MLCTK	BSJGX	OFWNE	VARIZ	QHYPI
ZQHYP	GXBSJ	ARIZQ	JARIZ	QHYMD	ULCTK				

5*a*. Determine the source or method of generation of the following key:

82490	31675	06393	47325	69227	10571	51498	15286	65379	67042
18065	37463	98618	01092	74798	11919	11679	20000	27361	20002

b. Determine the source or method of generation of the following key:

59326	42580	91806	33386	24182	57468	71540	28908	99448	17346
06784	13020	19704	22724	31428	53142	84560	37602	11162	48764

c. Determine the source or method of generation of the following key:

79246	61603	77639	43926	72180	93987	22756	49218	31392	44215
86369	49957	38421	12634	38975	17628	83809	11897	29768	16340

d. Determine the source or method of generation of the following key:

43518	20967	29410	15258	54526	82587	41366	55092	80576	04190
32830	54025	33189	96701	72864	23880	76829	39653	16941	80490

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e. Determine the source or method of generation of the following key:

93471 49565 94930 51871 29765 96392 99213 49961 94992 37013
45361 94330 99457 49943 94994 51875 67389 30338 71219 03585

f. Determine the source or method of generation of the following key:

59438 43712 45956 94412 14613 50719 32522 57484 93703 27465
44339 57299 09066 25701 91606 32318 25467 91874 39212 27979

g. Determine the source or method of generation of the following key:

48057 17417 70473 63098 82865 99303 71229 72305 84242 15552
59966 37418 90486 83133 03352 25763 61288 62953 70379 62954

h. Determine the source or method of generation of the following key:

01360 14961 53576 88234 60570 65276 17932 47206 19267 01837
14718 01898 19779 06461 80079 80767 87335 08980 87788 54563

i. Determine the source or method of generation of the following key:

77364 40908 89071 79788 37225 09475 63860 91469 17084 87822
56335 19686 16296 78153 90085 90834 70844 78281 63319 96408
62521 87739 06271 68987 60829 68015 33153 64689 25830 73137
68792 45616 66090 26992 93972 22694 58983 37716 83522 18743
24782 61508 59962 48586 41855 59300 31405 45459 07204 79241

j. Determine the source or method of generation of the following key:

37259 97499 61385 74132 15453 37259 09749 96385 74325 45369
37590 97499 61385 74131 54536 72590 97499 61857 41215 45699
37259 09799 61385 71321 55369 37290 97499 61387 41321 43699

6*a*. Determine the source or method of generation of the following key:

61901 84775 03315 98714 41689 52248 75012 75379 04305 58005
30558 00429 53770 12548 72951 84265 88103 35678 00234 56789
10325 47596 18396 08101 22436 38404 24456 58504 24446 48302
22416 07998 17365 57492 11294 76684 02203 85674 92092 74562

b. Determine the source or method of generation of the following key:

16202 61340 37100 06781 01056 66149 57520 04962 76405 42033
96082 62023 21603 21620 57130 08910 81289 27175 81022 96523
62535 51245 76973 91019 32602 82450 49196 07816 41121 39153
12320 09803 11810 44969 26106 14800 59401 64960 24372 04329
70672 46098 21213 31851 74147 27312 43901 05532 92569 58328
78749 02156 10374 49058 37308 43201 15280 15122 42351 86340
16146 01396 05125 76011 29477 82020 58703 74690 57392 08333
91132 94148 23523 31963 35168 44514 75911 32748 11993 01071
12117 01880 70327 00050 58707 75021 00436 13035 22032 90294
25040 52195 21195 65217 58021 60986 12102 99660 45805 06569

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7. Determine the source or method of generation of the following keys:

Key "A":	0360	5418	2657	7302	4153	9819	7756	6948	6742	0202
	9041	9518	5476	4038	8210	3693	2054	7294	8568	2762
	1618	8319	7203	5344	9538	2549	1309	0917	4875	6553
	1034	6532	9990	0583	2829	4269	5176	4702	9655	9494
	6155	9784	6788	9676	3253	2448	6450	2894	7151	9040
	8084	2032	1419	6829	6585	2536	4605	5039	0337	1881
	9906	4552	1251	2952	8488	9676	3423	8167	7692	3139
	6720	3413	8802	3129	0401	5426	0236	4235	6883	5719
	0773	8664	3279	7886	5302	8461	0505	3415	7248	6932
	7992	8699	9840	8715	3015	3136	0764	6223	8430	7514
Key "B":	0178	9820	6584	9519	0528	7505	1416	5361	1302	9790
	1984	1229	6931	5508	3256	9723	4841	7747	1068	6030
	8647	7831	2036	4022	4975	1066	8941	9490	1855	7320
	4612	6241	1580	8703	8192	7996	1243	2165	3995	8140
	5984	3906	6223	2581	7164	1896	0359	3502	8341	7424
	4739	5318	6042	4081	2715	0115	1662	1150	3587	4623
	6827	1599	9784	6237	5416	8534	0850	0731	5832	3491
	0115	7096	8765	0388	2584	7051	9037	0264	1113	9257
	3805	1920	3398	1015	8734	4764	7565	0151	6238	6582
	2043	3347	2409	7672	1891	2470	7351	3541	9510	1735
Key "C":	4690	8476	9635	0542	3377	9486	7831	7821	5808	2212
	9751	6654	3857	0575	0016	6036	8509	4800	9294	7378
	1046	5789	9409	0077	3663	7282	4784	5992	8817	5105
	0488	2566	9952	0739	5080	8771	1796	0230	6212	4155
	9227	4510	8793	4506	6188	7166	8734	9448	9964	0321
	3548	5335	6071	3071	8099	4526	3453	6268	2389	8607
	6866	6963	6770	1201	9931	3059	2987	2283	4597	1112
	3403	4886	6660	7388	1792	9578	1412	3398	3065	7543
	1319	4425	1740	2093	8709	6743	0226	7411	8905	9918
	2679	1344	6823	5669	7940	1562	4853	8907	8373	5210
Key "D":	5747	9978	6432	5811	6197	3433	3229	1217	5960	0438
	7296	3593	3858	3697	1121	7780	5328	4640	7746	4622
	9153	9413	7904	6340	1196	4786	0132	4583	0225	6728
	2520	9878	0283	6765	3378	1340	5143	4411	9799	3562
	3618	8212	0326	6042	9888	5031	9301	5704	8074	5539
	1499	6404	2699	5730	2309	1790	7611	6716	1948	4373
	0331	6688	0822	4471	0513	7079	7197	5004	5623	5945
	0022	6076	9019	1624	7129	2897	6598	3592	0481	6840
	0029	1149	5536	4861	5640	1967	4925	6575	7023	2499
	3080	6720	5365	0931	1924	5619	2242	5477	4781	8363

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8. Determine the source or method of generation of the following keys:

Key "A":	0812	6092	9531	2226	8481	9995	4063	4408	6270	4094
	5311	1618	1943	9084	4327	5981	3985	8256	2901	7826
	2629	6521	4171	4591	6203	7992	6538	8706	3145	8384
	1924	4243	1755	3259	7617	8296	7119	2093	0033	2726
	1654	5289	1861	9112	9156	4452	6425	3644	4436	9350
	3673	5257	3574	0572	2430	0074	1682	1964	7311	6116
	0405	3916	1602	2580	9311	8562	4476	3821	3015	8953
	1844	0409	6643	5139	7001	1963	1121	8573	3741	0243
	8989	6508	7091	4031	9312	4957	1657	8239	8005	7362
	1285	1558	2160	6363	2449	3473	1741	5655	7737	7015
Key "B":	3787	2422	8189	2952	6960	3105	2815	7233	4589	8513
	4741	3066	0730	3809	9431	4675	2401	8297	9501	3913
	9954	8723	8327	4399	6284	9018	2385	6632	7202	9223
	0528	6052	3704	2876	2889	2399	5486	4892	1652	1187
	2941	2358	2324	8891	9172	3799	9770	2173	6783	6957
	5998	6699	4268	4326	3426	8668	6797	5757	8626	2240
	6317	6377	5944	7832	2200	3594	9332	2073	0861	9720
	6886	0507	2626	8841	2415	8985	4660	8821	9181	5589
	5946	9543	0184	6745	4950	5355	6423	4165	8512	6002
	5482	6628	6135	8558	7240	6746	3929	7516	4464	1533
Key "C":	2009	7085	2534	0907	1764	1666	1962	5290	3713	6750
	5522	3430	1749	8358	1142	6236	3539	9774	1431	3225
	8963	7979	3801	8095	5901	7976	1616	8898	4241	9270
	5776	4620	5485	2359	8984	3078	7133	2851	9499	5699
	8657	1388	2887	9174	4639	2454	9946	5920	6004	5023
	5028	9265	4512	8618	7652	1850	2083	1157	7320	2480
	1400	2477	3973	8517	1408	7033	1863	0804	9894	9248
	1306	7082	5246	6032	9323	0353	1252	8396	8574	9216
	6554	3319	3820	9496	5845	6022	4146	2036	2092	2024
	1794	4770	4446	4494	3625	9545	9771	8805	9612	1501
Key "D":	1594	5519	6177	5152	8388	1681	0658	2593	3124	0736
	5363	7475	4252	4546	4231	0624	3377	9908	3540	1987
	9290	5188	7932	9886	2628	7172	1046	1912	0777	6875
	2601	9710	7731	4201	0965	5381	4813	4762	4972	3483
	4909	0351	7054	0202	4740	5667	8543	2603	6907	6847
	3474	1846	6803	4656	5200	5282	8911	1080	6918	8607
	4360	1867	3737	7017	3062	0425	7839	3493	3032	1425
	0740	6461	0853	4199	7142	2297	3651	2948	9269	0109
	6586	3269	2871	6083	6358	6703	5396	5484	6594	5024
	9042	8295	9580	5848	0848	3731	5971	0229	4110	8651

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Key "E":	8668	2240	4326	6699	5998	6797	8626	5757	3426	4268
	6746	1533	8558	6628	5482	3929	4464	7516	7240	6135
	2399	1187	2876	6052	0528	5486	1652	4892	2889	3704
	4675	3913	3809	3066	4741	2401	9501	8297	9431	0730
	3105	8513	2952	2422	3787	2815	4589	7233	6960	8189
	3594	9720	7832	6377	6317	9332	0861	2073	2200	5944
	5355	6002	6745	9543	5946	6423	8512	4165	4950	0184
	8985	5589	8841	0507	6886	4660	9181	8821	2415	2626
	3799	6957	8891	2358	2941	9770	6783	2173	9172	2324
	9018	9223	4399	8723	9954	2385	7202	6632	6284	8327

Key "F":	2009	3430	3801	2359	4639	1850	1863	8396	2092	1501
	1594	7457	7932	4201	4740	5282	7839	2948	6594	8651
	7085	1749	8095	8984	2454	2083	0804	8574	2024	1794
	5519	4252	9886	0965	5667	8911	3493	9269	5024	9042
	2534	8358	5901	3078	9946	1157	9894	9216	6554	4770
	6177	4546	2628	5381	8543	0801	3032	6010	6586	8295
	0907	1142	7976	7133	5920	7320	9248	1306	3319	4446
	5152	4231	7172	4813	2603	6918	1425	0740	3269	9580
	1764	6236	1616	2851	6004	2480	1400	7082	3820	4494
	8388	0624	1046	4762	6907	8607	4360	6461	2871	5848

Key "G":	1666	3539	8898	9499	5023	5028	2477	5246	9496	3625
	1681	3377	1912	4972	6847	3474	1867	0853	6083	0848
	1962	9774	4241	5699	8657	9265	3973	6032	5845	9545
	0658	8066	0777	3483	4909	1846	3737	4199	6358	3731
	5290	1431	9270	5776	1388	4512	8517	9323	6022	9771
	2593	3540	6875	2601	0351	6803	7017	7142	6703	5971
	3713	3225	8963	4620	2887	8618	1408	0353	4146	8805
	3124	1987	9290	9710	7054	4656	3062	2297	5396	0229
	6750	5522	7979	5485	9174	7652	7033	1252	2036	9612
	0736	5363	5188	7731	0202	5200	0425	3651	5484	4110

9. Determine the source or method of generation of the following keys:

Key "A"

10097	13586	80959	48056	96248	40372	42268	96450	20902	90190
67071	65886	79997	36147	23665					

Key "B"

97325	86346	59091	56489	48052	72063	68953	50930	02560	90252
71538	86767	97080	47640	65398	65747	50366	13398	54557	53034
48679	77602	16569	74818	33213	04890	84682	75303	78358	28260

Key "C'

13586	80959	74945	48052	36104	08229	30323	25601	76435	07153
58867	36276	76403	98951	71217	76850	65813	29170	53034	99074
34030	69268	48187	24718	70489					

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Key "D"

33765	73548	17392	47429	40372	61040	19645	32320	15953	90937
31131	43970	15736	32366	95116	17340	97361	85111	18240	26148
90743	02051	26866	73053	50532	55357	87098	52964	08342	93520

Key "E"

46735	80959	92927	29624	06361	29166	09025	33476	80336	53831
16588	70443	36147	98951	76833	66973	65813	11992	40635	67990
30973	65748	38524	23885	89055	54828	98349	64778	60935	88435

Key "F"

35863	54876	91173	89474	40372	20082	03232	01595	64350	67071
31131	67439	80157	36653	12171	68503	36170	98851	57182	26148
92340	92686	87305	47186	54704	55357	82870	03529	34282	35273

Key "G"

08422	09303	15953	25290	15383	11658	80799	01573	03236	60657
27685	70658	01080	82406	34261	85269	65692	73053	21350	48905
75482	79645								

Key "H"

53376	67354	11739	94742	24037	36104	31964	03232	01595	29093
83113	74397	01573	03236	89511	71734	69736	88511		

Key "I"

72768	70658	99291	63530	67990	92340	65692	57481	85247	54704
54828	34912	77835	82609	34435	20135	76809	92749	62480	06361
20082	09303	90256	34764	67071	16588	44362	14764	53989	87712
72768									

Key "J"

40323	98951	21717	85036	58133	92917	03426	90743	03097	26866
81873	71862	47048	28468	25624	58083	60935	52738	58634	09590
27494	05240	61040	22916	32320	56015	43508	70715		

Key "K"

65201	48768	92927	29624	72063	40200	45093	20902	53347	37670
31165	70443	36147	66539	16877	40727	61706	11992	40635	48679

Key "L"

89531	09303	25601	01902	07153	58867	70801	76403	53989	57471
07276	73617	31060	82406						

10. The following key was recovered from a depth of two in a Hagelin C–38 machine cipher cryptosystem. Reconstruct the pin and lug settings.

				5					10					15					20					25
1	16	7	2	19	12	10	19	15	6	0	17	19	21	12	7	23	9	21	7	22	14	18	24	11
1	7	6	15	8	12	12	2	5	15	10	7	19	20	23	18	24	19	24	20	9	1	20	25	24
1	19	19	16	11	21	25	18	20	1	11	6	3	7	24	6	21	9	10	5	8	1	5	25	8
21	15	21	7	23	7	10	6	18	0	20	19	4	20	24	21	6	23	11	6	22	4	15	13	12

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I. Teleprinter key analysis

(Embracing Chapter X)

1*a*. A certain cipher teleprinter has four wheels with the following patterns and rules of motion:

```
           1 2 3 4 5 6 7 8 9
Wheel A:   x x o o o x x x o, moves one step at each operation
Wheel B:   o x x x o o x x  , moves when A is "x" (CCM motion)
Wheel C:   x o o x o x o    , moves when B is "o".
Wheel D:   o x o x o        , moves when A+C is "x" (xx, oo=x)
```

The following wheel combinations are used for the five key levels:

```
KL1: A+B,      Vernam (xx, oo=x)
KL2: A+C,        "  "    "  "  "
KL3: B+C+D,      "  "    "  "  "
KL4: B+D,      mod 2 (xx, oo=o)
KL5: A+D,      Boolean (xx, xo, ox=x)
```

With the wheels aligned at their initial positions as shown above, complete the diagram below and decipher the remainder of the cipher text:

```
                  5         10        15        20        25        30
  A:   x x o o o x x x o x x o o o x x x o x x o o o x x x o x x o
  B:   o x x x x x x o o o x x x x x o x x x x
  C:   x o o o o o o o o x o o o o o o x x x x
  D:   o x x o x o o o o o x x o x o o o o o x
                  5         10        15        20        25        30
KL1:   o x o o o x x o x o x o o o x o x o x x
KL2:   x o x x x o o o x x o x x x o o x o x x
KL3:   x o o x o x x o o x o o x o x o o o o x
KL4:   o o o x o x x o o o o o x o x o x x x o
KL5:   x x x o x x x x o x x x o x x x x o x x
  K:   P Z L C L X X T A P Z L C L X T 5 3 5 Q
  C:   S P V Y Z G M D T W R 3 D R L 5 7 U M N 5 U G M U S K C I 3
  P:   N O W 9 M O V I N G 9 S O U T H 9 T O 9
```

b. The following bit stream is the result of a Vernam or mod-2 combination of two cyclically stepping wheels, one of which is of size 11, 12, or 13. Recover the pin patterns of the two wheels.

```
        5         10        15        20        25        30        35
x x o o x o o x x o x o x x x o x x o o x o x o x o o o o o x o x o x
        40        45        50        55        60        65        70
o x x x x x x x x x o o o o o o o o o o o x x o x o x x o x x x o o x
```

c. The following bit stream is the result of a Boolean combination of two cyclically stepping wheels, one of which is of size 11, 12, or 13. Recover the pin patterns of the two wheels.

```
        5         10        15        20        25        30        35
x x o x x x x x x x x x o x x o x o o x x o x x x o x x x x o x x o x
        40        45        50        55        60        65        70
x x x x x x x x x o x x o o x x o x x x x x o x o o x x x x x x x x x
```

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d. The following bit stream is the result of the dilation of a slave wheel by a cyclically stepping master wheel not involved in the key, the master wheel being of size 11, 12 or 13. Recover the pin patterns of the two wheels.

5 10 15 20 25 30 35

x o o o o o x x x x x x x x x o o o x x x x o x x x x o o o o o o o x

40 45 50 55 60 65 70

x x x x o o o o o o o x x x x x x x o o o o o x o o x x x x o o o o o

e. The following bit stream is the result of the Vernam or mod-2 sum of a cyclically stepping master wheel and its driven slave wheel, the master wheel being of size 11, 12, or 13. Recover the pin patterns of the two wheels.

5 10 15 20 25 30 35

o x x x o x o x x o x o o o x o o x o o o o x o x x x o o o x x o o o

40 45 50 55 60 65 70

x o o o o x x x o o x o o x x x x x o x x o x o x x o x o o o x x x o

f. The following bit stream is the result of the Boolean sum of a cyclically stepping master wheel and its driven slave wheel, the master wheel being of size 11, 12, or 13. Recover the pin patterns of the wheels.

5 10 15 20 25 30 35

x x x x o o x x x o o x x x x o o x x x x x x o x x x x x o x x x x x

40 45 50 55 60 65 70

x x x x x x x o o x o x x o o x x x x x x o x x x x x o x o o x x x x

g. The following is key produced by a simple 2-wheel machine. Recover the pin patterns of the wheels, and determine the derivation of key for each of the five key levels.

5 10 15 20 25 30 35

8 D M Q E 8 8 M E R S O O 5 4 C T Q W C F D Q S C 5 O S R T D O C 4 8

x x o x x x x o x o x o o x o o o x x o x x x x o x o x o o x o o o x

x o o x o x x o o x o o o x x x o x x x o o x o x x o o x o o o x x x

x o x x o x x x o o x o o o o x o x o x x o x x x o o x o o o o x o x

x x x o o x x x o x o x x x o x o o o x x x o o x x x o x o x x x o x

x o x x o x x x o o o x x x o o x x x o o o x o o x x o o x o x o o x

2. From the following key, recover all elements of the cipher teleprinter involved.

5 10 15 20 25 30 35 40

S O H A M I L L A C 9 5 F Z W 8 X R 7 9 A P 8 N 3 A L S 8 Z C Z S V 7 E 5 I L K

x o o x o o o o x o o x x x x x x o o o x o x o o x o x x x o x x o o x x o o x

o o o x o x x x x x o x o o x x o x o o x x x o o x x o x o x o o x o o x x x x

x o x o x x o o o x x o x o o x x o o x o x x x o o o x x o x o x x o o o x o x

o x o o x o o o o x o x x o o x x x o o o o x x x o o o x o x o o x o o x o o x

o x x o x o x x o o o x o x x x x o o o o x x o o o x o x x o x o x o o x o x o

45 50 55 60 65 70 75 80

S 8 G M E I G I H B 7 R S Y B U E J H 9 T Z V Z C C U 4 5 B K 9 E Q 4 M 8 D T 4

x x o o x o o o o x o o x x x x x x o o o x o x o o x o x x x o x x o o x x o o

o x x o o x x x o o o x o o o x o x o o o o x o x x x x x o x o o x x o x o o x

x x o x o x o x x o o o x x o x o o x x o o x o x x x o o o x x o x o x x o o o

o x x x o o x o o x o x o o x o o x o o o o x o x x o o x x x o o o o x x x o o

o x x x o o x o x x o x o x x o o o x o x x x x o o o o x x o o o x o x x o x o

85 90 95 100 105 110 115 120

Q J S M I W N 4 H G Y 9 R Y D 8 8 A Q 3 L P F 7 J P 4 X H Y 5 E 7 S U O P D F 9

x x x o o x o o o o x o o x x x x x x o o o x o x o o x o x x x o x x o o x x o

x x o o x x o x o x o o x o o x x x x o x x o o x x x o o o x o o o x o x o o o

x o x x x o x o x o x x o o o x x o x o o x x o o x o x x x o o o x x o x o x x

o x o x o o x o o x o o x o x x x o o x o o x o x o o x o o x o o o o x o x x o

x o o x o x o o x x x o o x o x x o x o x x o o o x o x x x x o o o o x x o o o

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3. The following is key from a cipher teleprinter, one of whose wheels is of size 19. Recover all elements of the machine.

```
        5         10        15        20        25        30        35        40
3 U 9 3 Y T E Y 9 E 3 M T Y 9 9 3 U C E P T B 9 4 M C M C E 3 F 8 8 Y 8 M G G Y
o x o o x o x x o x o o o x o o o x o x o o x o o o o o o x o x x x x x o o o x
o x o o o o o o o o o o o o o o o x x o x o o o x o x o x o o o x x o x o x x o
o x x o x o o x x o o x o x x x o x x o x o o x o x x x x o o x x x x x x o o x
x o o x o o o o o o x x o o o o x o x o o o x o o x x x x o x x x x o x x x x o
o o o o x x o x o o o x x x o o o o o o x x x o o x o x o o o o x x x x x x x x

        45        50        55        60        65        70        75        80
P W 3 W C 8 F G P 8 W M G U 3 Y 4 F F E 9 Y G C G 3 3 9 J 9 3 J T G M E W P C T
o x o x o x x o o x x o o x o x o x x x o x o o o o o o x o o x o o o x x o o o
x x o x x x o x x x x o x x o o x o o o o o x x x o o o x o o x o x o o x x x o
x o o o x x x o x x o x o x o x o x x o x x o x o o o x o x o o o o x o o x x o
o o x o x x x x o x o x x o x o o x x o o o x x x x x o x o x x o x x o o o x o
x x o x o x o x x x x x x o o x o o o o o x x o x o o o o o o o x x x o x x o x

        85        90        95        100       105       110       115       120
4 E M U M E E 4 E T T U 3 3 W 4 4 U 8 P G 8 8 F T Y 8 T P B J J 8 C B M P 3 9 C
o x o x o x x o x o o x o o x o o x x o o x x x o x x o o x x x x o x o o o o o
x o o x o o o x o o o x o o x x x x x x x x x o o o x o x o x x x x o o x o o x
o o x x x o o o o o o x o o o o o x x x o x x x o x x o x o o o x x o x x o x x
o o x o x o o o o o o o x x o o o o x o x x x x o o x o o x x x x x x x o x o x
o o x o x o o o o x x o o o x o o o x x x x x o x x x x x x o o x o x x x o o o
```

4. The following is key from a cipher teleprinter involving a combination of three wheels for each key level. Reconstruct all elements of the machine.

```
        5         10        15        20        25        30        35        40
D W R B 8 J S H 3 A S K U 5 X U J U F Z J D E D W 3 M O S Y R K I T Z 7 M 3 U C
x x o x x x x o o x x x x x x x x x x x x x x x x o o o x x o x o o x o o o x o
o x x o x x o o o x o x x x o x x x o o x o o o x o o o o o x x x o o o o o x x
o o o o x o x x o o x x x o x x o x x o o o o o o o x o x x o x x o o o x o x x
x o x x x x o o x o o x o x x o x o x o x x o x o x x x o o x x o o o o x x o x
o x o x x o o x o o o o o x x o o o o x o o o o x o x x o x o o o x x o x o o o

        45        50        55        60        65        70        75        80
N 9 Z Y 5 9 Y W I Q 8 7 S R F K S G T 4 9 L T V 9 I J Y G N O 3 L I Q T R X E 4
o o x x x o x x o x x o x o x x x o o o o o o o o o x x o o o o o o x o o x x o
o o o o x o o x x x x o o x o x o x o x o x o x o x x o x o o o x x x o x o o x
x x o x o x x o x x x o x o x x x o o o x o o x x x o x o x o o o x x o o x o o
x o o o x o o o o o x o o x x x o x o o o o o x o o x o x x x x o o o o x x o o
o o x x x o x x o x x o o o o o o x x o o x x x o o o x x o x o x o x x o x o o

        85        90        95        100       105       110       115       120
E J 5 G 5 C 7 G 8 J V R Y Z A L F L N E F W M X T S E N T Y E O N Y R V K 8 N A
x x x o x o o o x x o o x x x o x o o x x x o x o x x o o x x o o x o o x x o x
o x x x x x o x x x x x o o x x o x o o o x o o o o o o o o o o o x x x x x o x
o o o o o x o o x o x o x o o o x o x o x o x x o x o x o x o o x x o x x x x o
o x x x x x o x x x x x o o o o x o x o x o x x o o o x o o o x x o x x x x x o
o o x x x o o x x o x o x x o x o x o o o x x x x o o o x x o x o x o x o x o o
```

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5. The following two key streams were recovered by exploiting a "bust" situation. Recover all elements of the cipher teleprinter involved.

Key "A"

```
        5        10        15        20        25        30        35        40
P B K Y Y Q I J 4 H H T G T D B X Z F Z I 7 N 5 E U T K 8 M M U A 5 9 H J G G N
o x x x x x o x o o o o o o x x x x x x o o o x x x o x x o o x x x o o x o o o
x o x o o x x x x o o o x o o o o o o o x o o x o x o x x o o x x x o o x x x o
x o x x x x x o o x x o o o o o x o x o x o x o o x o x x x x x o o x x o o o x
o x x o o o o x o o o o x o x x x o x o o o x x o o o x x x x o o x o o x x x x
x x o x x x o o o x x x x x o x x x o x o o o x o o x o x x x o o x o x o x x o
```

```
       45        50        55        60        65        70        75        80
A A Y W W E 7 I B V H F K 8 8 3 H V C F 7 9 W W D D Y S 9 T U 5 9 J I 5 D R G Z
x x x x x x o o x o o x x x x o o o o x o o x x x x x x o o x x o x o x x o o x
x x o x x o o x o x o o x x x o o x x o o o x x o o o o o o x x o x x x o x x o
o o x o o o o x o x x x x x x o x x x x o x o o o o x x x o x o x o x o o o o o
o o o o o o o o x x o x x x x x o x x x o o o o x x o o o o o x o x o x x x x o
o o x x x o o o x x x o o x x o x x o o o o x x o o x o o x o x o o o x o o x x
```

```
       85        90        95       100       105       110       115       120
U U N D D Z D S N E Y P J 4 T G H H K F B N 5 Z Z N N J T 8 X H L 9 Y G L B B J
x x o x x x x x o x x o x o o o o o x x x o x x x o o x o x x o o o x o o x x x
x x o o o o o o o o o x x x o x o o x o o o x o o o o x o x o o x o o x x o o x
x x x o o o o x x o x x o o o o x x x x o x o o o x x o o x x x o x x o o o o o
o o x x x o x o x o o o x o o x o o x x x x x o o x x x o x x o o o o x o x x x
o o o o o x o o o o x x o o x x x x o o x o x x x o o o x x x x x o x x x x x o
```

Key "B"

```
        5        10        15        20        25        30        35        40
M 8 A M 8 V C D E B 0 H H L 9 Y W H U P D J C X F 9 B U Y Y B D E T K V 7 L V R
o x x o x o o x x x o o o o o x x o x o x x o x x o x x x x x x x o x o o o o o
o x x o x x x o o o o o o x o o x o x x o x x o o o o x o o o o o o x x o x x x
x x o x x x x o o o o x x o x x o x x x o o x x x x o x x x o o o o x x o o x o
x x o x x x x x o x x o o o o o o o o o x x x x x o x o o o x x o o x x o o x x
x x o x x x o o o x x x x x o x x x o x o o o x o o x o x x x o o x o x o x x o
```

```
       45        50        55        60        65        70        75        80
C K P X Q N F N 8 L H I A B M 4 Q Y S E C 9 G T E E V R R P 7 T 3 C 3 8 E S H T
o x o x x o x o x o o o x x o o x x x x o o o o x x o o o o o o o o o x x x o o
x x x o x o o o x x o x x o o x x o o o x o x o o o x x x x o o o x o x o o o o
x x x x x x x x x o x x o o x o x x x o x x o o o o x o o x o o o x o x o x x o
x x o x o x x x x o o o o x x o o o o o x o x o o o x x x o o o x x x x o o o o
o o x x x o o o x x x o o x x o x x o o o o x x o o x o o x o x o o o x o o x x
```

```
       85        90        95       100       105       110       115       120
K A A 4 D Z A S A K 8 X D D Z Z 0 0 E 7 0 R Y V P I R I B Y 0 0 T C M G W Q 8 4
x x x o x x x x x x x x x x x x o o x o o o x o o o o o x x o o o o o o x x x o
x x x x o o x o x x x o o o o o o o o o o x o x x x x x o o o o o x o x x x x x
x o o o o o o x o x x x o o o o o o o o o o x x x x o x o x o o o x x o o x x o
x o o o x o o o o x x x x x o o x x o o x x o x o o x o x o x x o x x x o o x o
o o o o o x o o o o x x o o x x x x o o x o x x x o o o x x x x x o x x x x x o
```

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6. From the following key, recover all elements of the cipher teleprinter involved.

				5					10					15					20					25					30					35					40
D	L	G	R	L	5	4	R	G	A	E	K	M	F	3	M	3	Y	W	B	U	S	4	A	D	H	T	M	O	S	S	A	J	D	Z	8	C	8	F	5
x	o	o	o	o	x	o	o	o	x	x	x	o	x	o	o	o	x	x	x	x	x	o	x	x	o	o	o	o	x	x	x	x	x	x	x	o	x	x	x
o	x	x	x	x	x	x	x	x	x	o	x	o	o	o	o	o	o	x	o	x	o	x	x	o	o	o	o	o	o	o	x	x	o	o	x	x	x	o	x
o	o	o	o	o	o	o	o	o	o	o	x	x	x	o	x	o	x	o	o	x	x	o	o	o	x	o	x	o	x	x	o	o	o	o	x	x	x	x	o
x	o	x	x	o	x	o	x	x	o	o	x	x	x	x	x	x	o	o	x	o	o	o	o	x	o	o	x	x	o	o	o	x	x	o	x	x	x	x	x
o	x	x	o	x	x	o	o	x	o	o	o	x	o	o	x	o	x	x	x	o	o	o	o	o	x	x	x	x	o	o	o	o	o	x	x	o	x	o	x

				45					50					55					60					65					70					75					80
7	3	O	S	U	I	I	I	U	S	4	4	R	R	W	B	9	7	S	A	3	Y	W	O	9	E	K	X	O	N	N	B	E	K	V	C	8	F	B	A
o	o	o	x	x	o	o	o	x	x	o	o	o	o	x	x	o	o	x	x	o	x	x	o	o	x	x	x	o	o	o	x	x	x	o	o	x	x	x	x
o	o	o	o	x	x	x	x	x	o	x	x	x	x	x	o	o	o	o	x	o	o	x	o	o	o	x	o	o	o	o	o	o	x	x	x	x	o	o	x
o	o	o	x	x	x	x	x	x	x	o	o	o	o	o	o	x	o	x	o	o	x	o	o	x	o	x	x	o	x	x	o	o	x	x	x	x	x	o	o
o	x	x	o	o	o	o	o	o	o	o	o	x	x	o	x	o	o	o	o	x	o	o	x	o	o	x	x	x	x	x	x	o	x	x	x	x	x	x	o
o	o	x	o	o	o	o	o	o	o	o	o	o	o	x	x	o	o	o	o	o	x	x	x	o	o	o	x	x	o	o	x	o	o	x	o	x	o	x	o

				85					90					95					100					105					110					115					120
J	D	W	Z	8	C	V	K	X	V	C	K	X	D	V	C	8	F	G	4	R	4	A	D	Q	H	Z	M	3	H	Z	X	R	G	L	5	A	D	N	M
x	x	x	x	x	o	o	x	x	o	o	x	x	x	o	o	x	x	o	o	o	o	x	x	x	o	x	o	o	o	x	x	o	o	o	x	x	x	o	o
x	o	x	o	x	x	x	x	o	x	x	x	o	o	x	x	x	o	x	x	x	x	x	o	x	o	o	o	o	o	o	o	x	x	x	x	x	o	o	o
o	o	o	o	x	x	x	x	x	x	x	x	x	o	x	x	x	x	o	o	o	o	o	o	x	x	o	x	o	x	o	x	o	o	o	o	o	o	x	x
x	x	o	o	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	o	o	x	o	o	o	x	x	o	o	x	x	x	o	x	o	x	x	x
o	o	x	x	x	o	x	o	x	x	o	o	x	o	x	o	x	o	x	o	o	o	o	o	x	x	x	x	o	x	x	x	o	x	x	x	o	o	o	x

7. In the following teleprinter key, KL1 is produced by a combination of a 19-wheel and another wheel. Recover all elements of the machine involved.

				5					10					15					20					25					30					35					40
G	H	W	Q	T	P	X	8	3	K	X	W	5	M	G	H	K	5	D	I	O	5	3	K	X	L	5	G	L	M	C	G	Y	8	Z	V	E	O	5	D
o	o	x	x	o	o	x	x	o	x	x	x	x	o	o	o	x	x	x	o	o	x	o	x	x	o	x	o	o	o	o	o	x	x	x	o	x	o	x	x
x	o	x	x	o	x	o	x	o	x	o	x	x	o	x	o	x	x	o	x	o	x	o	x	o	x	x	x	x	o	x	x	o	x	o	x	o	o	x	o
o	x	o	x	o	x	x	x	o	x	x	o	o	x	o	x	x	o	o	x	o	o	o	x	x	o	o	o	o	x	x	o	x	x	o	x	o	o	o	o
x	o	o	o	o	o	x	x	x	x	x	o	x	x	x	o	x	x	x	o	x	x	x	x	x	o	x	x	o	x	x	x	o	x	o	x	o	x	x	x
x	x	x	x	x	x	x	x	o	o	x	x	x	x	x	x	o	x	o	o	x	x	o	o	x	x	x	x	x	x	o	x	x	x	x	x	o	x	x	o

				45					50					55					60					65					70					75					80
I	E	G	H	W	Z	V	B	H	O	G	E	5	Y	G	Y	8	3	C	Y	L	G	W	G	M	K	3	C	G	7	W	L	O	5	3	C	Y	8	T	Q
o	x	o	o	x	x	o	x	o	o	o	x	x	x	o	x	x	o	o	x	o	o	x	o	o	x	o	o	o	o	x	o	o	x	o	o	x	x	o	x
x	o	x	o	x	o	x	o	o	o	x	o	x	o	x	o	x	o	x	o	x	x	x	x	o	x	o	x	x	o	x	x	o	x	o	x	o	x	o	x
x	o	o	x	o	o	x	o	x	o	o	o	o	x	o	x	x	o	x	x	o	o	o	o	x	x	o	x	o	o	o	o	o	o	o	x	x	x	o	x
o	o	x	o	o	o	x	x	o	x	x	o	x	o	x	o	x	x	x	o	o	x	o	x	x	x	x	x	x	o	o	o	x	x	x	x	o	x	o	o
o	o	x	x	x	x	x	x	x	x	x	o	x	x	x	x	x	o	o	x	x	x	x	x	x	o	o	o	x	o	x	x	x	x	o	o	x	x	x	x

				85					90					95					100					105					110					115					120
H	L	B	H	X	I	7	P	M	K	X	W	H	W	Y	G	5	L	G	Y	8	T	P	W	Q	Y	5	X	L	M	C	Y	I	O	5	X	L	X	B	U
o	o	x	o	x	o	o	o	o	x	x	x	o	x	x	o	x	o	o	x	x	o	o	x	x	x	x	x	o	o	o	x	o	o	x	x	o	x	x	x
o	x	o	o	o	x	o	x	o	x	o	x	o	x	o	x	x	x	x	o	x	o	x	x	x	o	x	o	x	o	x	o	x	o	x	o	x	o	o	x
x	o	o	x	x	x	o	x	x	x	x	o	x	o	x	o	o	o	o	x	x	o	x	o	x	x	o	x	o	x	x	x	x	o	o	x	o	x	o	x
o	o	x	o	x	o	o	o	x	x	x	o	o	o	o	x	x	o	x	o	x	o	o	o	o	o	x	x	o	x	x	o	o	x	x	x	o	x	x	o
x	x	x	x	x	o	o	x	x	o	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	o	x	x	x	x	x	x	o

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J. Cryptodiagnosis
(Embracing Chapter XI)

One of the difficulties in attacking real-life problems in the initial diagnostic stages, especially when the volume of data might be insufficient, is that the analyst usually does not know just how much information he can glean from the particular data under study: he may be content with uncovering and explaining a phenomenon, or obtaining a partial solution of a technical detail, without realizing perhaps that he could have gone on and made further progress, even without additional data. Therefore, in order to give the analyst some idea of the amount of information he should be looking for (implying thereby some estimate of the time necessary) in each of the message logs below, there is appended a coding indicating the diagnostic aspects in each problem, thus eliminating the wasting of time in searching for nonexistent aspects. This coding is the following:

A. Preamble information
B. Enciphered discriminants or indicators
C. Indicator information
D. Distributional phenomena
E. Repetitive phenomena
F. Other textual characteristics
G. Further manipulation of text
H. Nature of general cryptosystem
I. Identity, source, or generation of key
J. Plain text recovery

1*a*. Analyze the following message log: (B)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
MKR	GNU	563	160905	46	29700	30248	55624	56951	62396	58645	73021
MKR	GNU	564	160947	72	81060	80946	20160	63159	81931	95167	35381
MKR	GNU	565	161023	87	70908	17452	94814	29741	67729	47867	24229
MKR	GNU	566	161112	53	22607	35330	02452	24654	36680	09806	76928
MKR	GNU	567	161130	69	38114	58064	76495	23828	06899	64004	82435

b. Analyze the following message log: (BC)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
PHO	ZFC	111	231406	85	22222	46753	40370	14163	14965	89794	09243
PHO	ZFC	112	231415	60	22222	92568	23789	69079	69779	42236	25082
PHO	ZFC	113	231435	64	22222	29886	21688	95738	37986	23001	59377
PHO	ZFC	114	231451	59	22222	86479	64716	51228	93578	60158	81617
PHO	ZFC	115	231510	78	22222	23348	55782	97352	33446	04281	51262

c. Analyze the following message log: (BC)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
CFB	NWL	719	271224	61	33455	90579	72037	34314	65319	77774	16031
CFB	NWL	720	271239	58	33455	12603	99066	55418	76397	92933	44195
CFB	NWL	721	271250	97	33455	40865	21541	34189	94854	44734	56738
CFB	NWL	722	271318	83	33455	56843	31624	99341	91318	14809	27777
CFB	NWL	723	271345	80	33455	30367	10386	38773	43177	20754	56545

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d. Analyze the following message log: (B)

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
VKF	RNZ	262	031812	48	77477	81523	32207	94592	77092	12019	05166
VKF	RNZ	263	031827	53	77477	79047	09628	91025	04909	26360	19759
VKF	RNZ	264	031834	54	77477	00157	88281	45103	61332	31666	36528
VKF	RNZ	265	031856	67	77477	66723	61932	70782	66863	36062	73567
VKF	FNZ	266	031917	42	77477	55795	63035	69754	44776	28035	04507

e. Analyze the following message log: (BCH)

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
BLV	FTW	489	180918	69	LDSQS	GDOZE	LSRPR	FWOKL	KMVYY	FOZNU	BKURD
BLV	FTW	490	181232	60	BFMPR	CDAWH	OODFY	VYIJK	GMHOF	VLOJZ	JTGRL
BLV	FTW	491	181550	71	MCLLR	WIRSA	DATGL	GVHFK	ARXPR	YIBCZ	WELPY
BLV	FTW	492	181702	57	UUOOL	GKBCY	KACTM	ONKIE	KTGCR	YPZSA	RADMS
SDM	FTW	724	181019	87	LILQH	POQRD	FTGLY	FBHKA	APRMZ	BFJFT	PZNUG
SDM	FTW	725	181046	74	DLNZQ	ARQAC	NZGXU	XEJTJ	KRQXU	PVJLT	FXHRG
SDM	FTW	726	181625	45	BQXCF	DMIAJ	NVXAA	VJTWY	NMHAA	ASXFO	UMIYZ
WJW	FTW	019	180836	51	ZVSFO	ECECV	NMPQD	TOOZH	GHRVQ	UQLIO	EWKAP
WJW	FTW	020	180850	73	XXRFI	ZQRXF	DYBQQ	RQNZB	BVDVC	YSKDR	WNNVY
WJW	FTW	021	180927	42	ZINKI	FCWRA	QKGDL	TBJEB	HHIIX	OFNDY	NYDAP

f. Analyze the following message log: (B)

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
IHI	KTL	431	141152	58	ZTYBZ	DADZH	YDDRP	EDXUZ	JAXCR	JVXBC	BHKFS
IHI	KTL	432	141306	72	ZUFMB	LZGOX	XAYKB	XTLGA	LSTKW	RGVLT	BIRQU
LIZ	KTL	572	141239	71	QXWPX	SCMWH	DARGQ	DVLYL	SXGOC	GBXRD	SLITQ
IHI	KTL	426	150814	49	RDUFM	MJKHL	PVRNR	XHHBJ	UVAVD	GOVBU	TRGJF
LIZ	KTL	576	151602	63	QTHMK	ZHAWL	JSDTE	PGTAF	FVQDM	ICNDK	SHTQD
IHI	KTL	425	161108	82	PGQHA	VNLLS	KUBUG	MLPHY	XEZYK	NBWZB	RUCLT
IHI	KTL	426	161427	74	BYDUX	JPTJV	ZRBRD	RGSRD	BEKJV	BXSEL	DMPYQ
IHI	KTL	430	161628	87	IQDQF	AYFXM	HIROO	RBUID	KRIRB	PLTYQ	KEPUY
LIZ	KTL	574	160915	43	LYPAR	IMQTS	XZNBS	DWGOQ	OJJGQ	CSDGR	NMBEK
LIZ	KTL	575	161021	50	ERJAP	JMWUA	MJANN	MRAUG	JUWBT	IXSXV	GFVEI

g. Analyze the following message log: (ABC)

To	From	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
EFS	EMW	872	200740	51	HPLSC	HYIKW	EYRKZ	CCDVC	BEZTW	MVNKL	IZJLX
EFS	RTM	875	201039	67	GIMWT	UFYSN	DRSOQ	HLZIW	VNOHU	BSJGG	VGZTO
EFS	CBQ	876	201057	48	SWBBE	FNCHY	FUAKO	PFHTB	UANTF	TSZCG	GODIZ
EFS	TOK	880	201402	51	NRFCO	WAWII	KALUL	ROXGJ	KVJCC	UFEGP	XBXJJ
EFS	UIO	881	201535	72	HRVTY	XUATS	DSCQR	EABLV	NBEQU	UWIOU	YVBUT
EFS	VES	870	210916	75	FJXTA	YQELU	CSDLX	MKDMR	YFBMX	ARFFC	ZRFMV
EFS	ZQT	871	211140	68	ZZTCE	CCFSY	ECTHE	WIZUB	PVWFG	OWEUN	DDGTZ
EFS	LJN	873	211305	87	GIJAJ	OVZMD	DRPSG	UKJYC	SCHMD	VVYPM	PWANE
EFS	ZEK	876	211326	40	ITHEF	CQWQZ	ETKPN	FCNWC	UXQYD	FHJRX	DRXRA
EFS	VIJ	879	211611	62	AHCBJ	YUVXD	WGJCX	XQITG	RPTJH	ACPQA	ZVWYE

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h. Analyze the following message log: (ABC)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
ARX	NNA	490	190600	64	35701	18339	89693	14177	01595	80756	16855
ARX	NNA	192	190842	66	91175	29984	05187	84552	70971	50132	04144
HIS	NNA	963	190857	83	06712	04331	93924	83705	78125	59386	23988
ARX	NNA	670	191015	80	65000	54336	68077	33709	27120	09381	24045
HIS	NNA	688	191029	72	10565	67184	76969	55331	48753	21914	84831
HIS	NNA	014	191141	64	19676	86281	49637	98047	80460	64621	01473
ARX	NNA	460	191306	81	11188	32192	34137	13597	94921	89182	51010
ARX	NNA	576	191419	82	45912	44566	39458	24211	44705	90967	73869
ARX	NNA	632	191427	79	51695	14491	48616	30993	59488	06640	41589
HIS	NNA	103	191443	53	55004	28665	95676	07962	25458	73610	47643

i. Analyze the following message log: (BC)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
WRP	JVM	981	171102	62	61865	98903	67654	08586	17697	99036	23755
MRP	JVM	982	171305	60	91514	28652	63757	33763	04680	52696	93443
WRP	JVM	984	171636	64	27362	54400	47306	71988	88231	68901	96814
WRP	JVM	985	181050	63	47149	74287	67790	08626	86697	96885	22501
WRP	JVM	987	181312	63	36556	63694	98567	39495	30217	52313	25292
WRP	JVM	988	181632	60	38237	65375	89641	51714	20570	75995	53878
WRP	JVM	989	191108	62	15646	42784	99866	30706	45630	28788	31962
WRP	JVM	990	191315	63	72845	09983	04945	19874	45886	39636	90706
WRP	JVM	992	191628	60	55624	82762	65748	58104	06681	39112	23261
WRP	JVM	993	191646	63	00802	37940	59941	90885	55406	32701	31989

j. Analyze the following message log: (BC)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
OKH	ULO	401	081014	78	69158	62316	99237	72652	16948	55420	09145
OKH	ULO	401	091037	48	26219	30140	37135	13567	45329	66206	39210
OKH	ULO	401	120925	53	92327	54798	24392	07724	12517	25401	32314
OKH	ULO	401	151142	60	81761	90274	05858	73601	92063	21758	94943
OKH	ULO	401	180926	57	51047	45489	06846	89287	39931	37570	91034
OKH	ULO	401	191057	91	88068	63327	14153	46760	16223	28055	22972
OKH	ULO	401	211123	52	19086	97953	80188	70390	01628	59073	57295
OKH	ULO	401	231000	55	34199	54162	78867	37513	64982	74186	16421
OKH	ULO	401	241045	67	51864	87898	85741	68194	07164	90691	91851
OKH	ULO	401	260956	59	49079	16281	52854	35200	86754	17363	89066

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k. Analyze the following message log: (BDFGHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
FLC	BUG	317	020900	49	89108	27704	66181	03417	22745	11635	33769
SLY	BUG	892	020917	48	23062	60022	74850	16163	48145	53539	27123
ZBJ	BUG	724	021021	62	64527	14424	45932	04392	42625	93094	18483
ZBJ	BUG	725	021037	59	87118	47047	81605	35392	14046	11655	01878
BUG	FLC	027	021013	60	20276	60309	41766	15086	35338	50763	24734
ZBJ	FLC	946	021145	58	75087	27705	70442	21153	52511	02524	09643
BUG	SLY	375	020941	75	37157	02828	64004	91086	27705	61634	81712
BUG	ZBJ	421	021052	43	25073	50541	70382	25401	00302	52500	39334
FLC	ZBJ	803	021108	67	65729	02826	83500	78244	14450	94276	49183
SLY	ZBJ	514	021121	81	31534	02822	73910	36125	01615	64011	15392

l. Analyze the following message log: (BFGHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
DOM	KLS	501	130816	63	86284	89703	98576	07737	93244	61746	23577
DOM	KLS	502	131139	48	59911	11594	16667	99528	02606	36289	96768
DOM	KLS	504	131406	46	02889	54220	06677	37543	22356	95153	40600
DOM	KLS	501	141000	52	39080	48214	88104	42145	85639	39712	79416
DOM	KLS	502	141027	51	13198	42412	25116	01346	22642	85453	59870
DOM	KLS	501	150902	60	78513	87031	46483	35751	66336	01998	13004
DOM	KLS	503	151345	58	11539	67932	88399	97555	65640	62505	51464
DOM	KLS	502	161125	49	40554	84220	73220	73133	86796	99002	83346
DOM	KLS	503	161152	62	76068	34450	80745	35618	47627	38807	18402
DOM	KLS	505	161308	54	20098	42604	91590	03999	49087	54236	69572

m. Analyze the following message log: (BDEFH)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
WKN	JNT	704	101402	58	53044	50667	62274	96641	98885	74944	62773
WKN	JNT	705	101421	72	49984	72827	69484	48865	58234	13460	60282
WKN	JNT	706	101450	63	80772	32975	21872	44544	55012	68041	82906
WKN	JNT	707	101505	65	18134	90656	23444	34820	39266	64489	13489
WKN	JNT	708	101532	84	73966	40205	05796	82983	69404	22854	19425
WKN	JNT	709	101555	60	39910	36522	20270	70988	42494	56460	64431
WKN	JNT	710	101619	73	31066	24325	83306	22543	83015	14532	02581
WKN	JNT	712	101647	81	51120	98139	83690	89249	51096	58642	40243
WKN	JNT	715	101723	55	87978	46272	88758	73924	69614	14128	38066
WKN	JNT	719	101830	47	29770	28725	23430	66944	15032	95690	64190

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n. Analyze the following message log: (BDFHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
ESW	PTB	336	010930	59	BFCUB	QPDFI	OJFRV	GEFCB	XWDAK	DHSEK	DDGTK
ESW	PTB	337	010956	62	NPYRS	DMWVY	DETHD	IBRFD	BHINH	RGOGP	PAUWH
ESW	PTB	338	011010	63	FAGQT	SERVT	LKFBU	DCPQY	NAFZD	ABCNH	HSGHD
ESW	PTB	339	011036	58	GCARQ	OLCQF	UNTRD	JIKTQ	MQVYK	QHQCO	IZUKK
ESW	PTB	340	011055	60	OKXKJ	ZNYBC	IPNGR	GWDFY	OVHSF	NTIQR	QBOWF
ESW	PTB	342	011130	71	NERJH	TYVBM	DHAFQ	ADPRQ	VWHQU	AJKHD	PMBIU
ESW	PTB	343	011216	76	GHOQR	YXCSF	YNWCN	OQFBT	VUWTD	UDPBT	ILXSD
ESW	PTB	344	011242	54	YNCSR	DRIQH	STOJW	PNOQZ	AGRJT	JUWTD	AFPHT
ESW	PTB	347	011350	82	QCGMY	YBPRS	GUFBK	VEFRN	QTORG	FOGBH	SPGIG
ESW	PTB	348	011607	73	SZFSE	FNIXP	WALRB	UPBCD	RDJRA	NPDIC	UBMTA

o. Analyze the following message log: (BCDFHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
IXN	YBB	936	051032	67	XGLFZ	MVJNS	ERKXU	BHFUX	ODMRX	AVGJR	18965
IXN	YBB	939	051517	82	AHMEA	GNJNN	AYGDL	XJXDO	PDVGI	RVHLO	73421
JFH	YBB	272	050820	49	AMMLA	LSGKS	ACENM	HACMW	UNDNA	YOJJP	98265
JFH	YBB	273	050835	58	WIOIA	NQFHG	QAXNE	AUHHM	NDUKN	RXGIV	46328
JFH	YBB	274	051141	52	YKNIZ	HREGN	FREVB	PEZTB	JLAVS	UTCOS	58742
PWC	YBB	708	050856	79	RHIEX	HTFKH	XDAWI	LQSJX	GOUDO	VXFOO	04597
PWC	YBB	710	051539	80	SHFJU	IQHMV	BZOTR	AFZBJ	ZRVVT	UVHNQ	76150
XGD	YBB	564	050925	63	YLLIX	IQCEP	HUAGY	ASHZL	YHUSF	WPHJR	87429
XGD	YBB	565	051017	75	WNNMR	IUIMC	PXVNF	WASLB	TDJKS	UTFJT	49683
XGD	YBB	566	051445	56	WIFHS	NRFIM	HZZEJ	ERVTE	IWAEC	YRBMV	25831

p. Analyze the following message log: (BFHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
YWR	TFE	439	220752	56	62711	26977	42197	32966	09424	23634	14492
YWR	TFE	440	220812	59	16887	75144	78023	12322	68913	32481	74368
YWR	TFE	441	220827	60	37128	82143	54217	21876	62125	16463	73569
YWR	TFE	442	220855	55	81022	06272	63327	94223	70649	28321	86693
YWR	TFE	443	221024	57	09999	10261	16332	39077	48187	18726	51680
YWR	TFE	445	221107	68	67498	93465	73427	71972	99223	44773	03789
YWR	TFE	446	221241	73	33522	56732	19211	72518	44228	71063	54553
YWR	TFE	447	221305	51	66387	77578	88244	61620	37148	99778	43598
YWR	TFE	450	221428	79	21280	11837	82264	47612	73982	87668	89551
YWR	TFE	451	221443	70	35532	17201	04767	16776	27492	61721	38023

q. Analyze the following message log: (BHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
HUE	AZI	124	300955	48	PMCVC	XULRR	MFPLN	SNOIF	KJHUU	59721	95723
HUE	AZI	125	301026	73	ZFZPG	TGTVM	WYMVO	NMLBU	MESOY	84856	37365
HUE	AZI	126	301324	56	MBEGU	AVVMO	JUXAO	IXQSU	ILLFM	33577	22088
HUE	AZI	128	301502	52	JZOJN	VMMNV	GSIHE	VVAVH	ASIIF	40890	71321
HUE	AZI	129	301525	85	WPGDX	CBVIS	TIUWN	SOSZN	ZFMCP	57113	20664
HUE	AZI	130	301548	69	YUWNI	KOLHL	VNXGJ	TTIGQ	EULMA	97903	91985
HUE	AZI	131	301617	78	JZOHP	ZIGLV	GSKFE	VXATH	SUGGH	44016	11549
HUE	AZI	132	301730	71	PZMTN	EDMIQ	MSTUH	XIYEN	IPRSF	31919	02424
HUE	AZI	133	301803	67	UBQIX	EEOJL	RUIVD	HUCWL	WJRHP	75925	13963
HUE	AZI	136	301842	58	BUVVX	VGTVP	YNXQR	OBHPV	SVSUP	77583	33527

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r. Analyze the following message log: (BFGHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
GSM	MNS	847	070712	98	37777	70995	39344	99967	86507	05503	59983
GSM	MNS	848	070806	87	86348	99941	00594	99901	02009	57780	89582
GSM	MNS	849	070831	99	26998	46760	11696	02921	89997	68770	90684
GSM	MNS	850	070916	93	55766	94356	66959	85899	97792	49580	45947
GSM	MNS	851	071014	82	81859	79121	11987	73199	93108	77400	60847
GSM	MNS	852	071052	95	90839	91030	03997	99932	44689	55705	70028
GSM	MNS	853	071107	88	56675	94003	35888	41664	79995	78029	35663
GSM	MNS	854	071328	86	46776	82272	01472	10195	89991	21108	61260
GSM	MNS	855	071545	90	17958	86380	21188	07499	90987	81120	00176
GSM	MNS	856	071619	91	54896	67788	03737	99932	53002	67093	33884

s. Analyze the following message log: (BCGHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
SHF	CVG	501	150756	83	AVUXB	OPRVJ	NIFTH	MWQPD	DOUSN	59326	19072
SHF	CVG	502	150835	90	UHIPC	PGVXX	NTZUM	BBJVT	CNFUC	27758	87583
SHF	CVG	503	150941	48	VMMRL	NBMYN	KBBOL	TCGHC	XKAGG	61083	21924
SHF	CVG	504	151037	62	PQHCZ	BXUHV	XOKQP	SCXSQ	SJJRK	15746	76773
SHF	CVG	505	151052	56	WTIJP	HUXAV	WJTOV	UJMES	BIFRW	32188	93130
SHF	CVG	506	151126	59	UJWBX	WZQXI	VHUIR	IQIUA	ZUDGL	94392	55418
SHF	CVG	507	151153	78	YZXSB	UJXHW	HGNPO	AMPRH	KCELC	57911	18064
SHF	CVG	508	151210	85	ANPEV	KKZFJ	JQQFI	DEQJS	BURJG	80647	41857
SHF	CVG	509	151421	73	VOEJW	SYMSI	TLIOP	FSHLZ	DWZZW	03469	64880
SHF	CVG	510	151650	69	WJVHV	FBMBL	KPDMP	TQFHN	FVSDS	47530	08180

t. Analyze the following message log: (BDHIJ)

To	*From*	NR	FDT	GR	A1	A2	A3	A4	A5	Z1	ZØ
OJP	ZKM	251	040810	62	QMUYE	RCBDX	MNSFW	GBTXA	IXROY	YWVPE	NFGFW
OJP	ZKM	252	040846	88	YVGHK	LAXDY	NACXF	UMNAE	PGBWQ	LTREX	ITJKQ
OJP	ZKM	253	040915	74	CXIDT	WUFHB	SRGJB	CMLQQ	NJANC	AAAAE	PKSQT
OJP	ZKM	254	040942	86	UNUHX	TKZTO	MKULO	WUESA	NXGZC	RIBHW	TNQZS
OJP	ZKM	255	041005	99	PMGRV	LBOQE	NZXGG	OTWKR	TRAZC	PJRMD	QKMGU
OJP	ZKM	256	041028	60	LWRUM	BVYVL	WNSQB	KIMUJ	WEAHS	VLJUG	IPDBE
OJP	ZKM	257	041052	98	GTLSE	MTBDU	WRSFZ	OTKGQ	VCBMX	AJQDX	JMNKM
OJP	ZKM	259	041121	89	JMJXA	SIFEE	NYXSN	GKAHM	AGHIM	JHQHO	DDMOE
OJP	ZKM	260	041143	65	KHUHZ	YPLQI	LQUEA	JYNYT	PXFSN	MZIWO	MQRZF
OJP	ZKM	262	041200	87	EMRUM	LPOUT	SQYPA	GZJQL	WOZMR	SAAAP	IIABL

2. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
ZUBBK  AZUJM  OMVZO  NDUPX  XCNQZ     OKAET  WRGUX  BGVUZ  UPEVC  XLAZO
MQVPO  IFIKE  TKAIZ  OJRRW  KAPBV     ZUPQE  NSEPL  KILKI  HVOBJ  KEBIZ
OJLCZ  UFKEB  NKIBK  ALGJT  XGAIR     TAFNV  KAYJU  GJRYB  ESBNZ  UZYEG
KAGKI  LYGXH  PPDYZ  YQZUJ  IZOQA     DHBGH  KESBI  LMRGR  WKEHR  ZOJDK
ITKIZ  UKEKA  CZURC  KEMXB  IKAKE     IXZOW  FPGXM  LMJVK  EMOQI  ISKEG
DPZYM  BWBKA  CGZUS  JHWZO  JPOYF     OJTQO  OKAPA  DCXTE  VUVNX  VXZUY
TZOXL  LTOOU  ONJJS  KITKE  UDXKA     VKIJJ
```

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3. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
AAUMQ AGNYD HOFXV ETMQH SLDON   EXOWQ RVHAW KAMNX NPCGQ DLMPS
XAIUB QDWXU PLDED OJIAR OQEEP   MOXMY LHQGU GDCXF HHZQE CVEMX
BVVEQ ORWDX TMWPW HQMLM WXRAR   OMDIF GQEEP MOBWX ULWUS EWOQP
CXFIH PMNRW LQOYD KZSOX MOLBO   QSPEI ORPUH XERTF HAIUB QDWXJ
TPEID KSAVQ VVAOX NDRWA AHUQH   IGTXD LQBFY GOXFJ AYHGU QWPXZ
ERQLG WDKXN PNAOM FQHIX TUPES   MWCKH QJLZX EEPMO QMYLH
```

4. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
BSZXG YMPSX QAHOL YCDSF IQAWQ   MYMEX BRRWN ZPWWC KQAHF OBYCW
LTYVO QXMEZ PPCGN ZTJBR YYLER   JTDQW AOYUZ EDFSZ DBUKD FFQLZ
IBDZB MLCJH QKJHI KOWTG KLASG   LRKUY KNJCP HRBIX QUAOC NELLY
HNAPH IISYP DNJKR MVBHJ QVFXG   TVLLQ HTJQQ WESUY BQOLE GNCUP
WIWXH QEIJD ZSUQS DWZVW TRODL   IVZAP RVREH PZZWC MQWBT GDXTF
TLEIM OAAGN ZDSRL XXUVP WZPNS   TTREI YGLQP RELZV BTPIV NXHUN
PLNYE ISUWD XWGPV IRCKP JCSRJ   VVSMI UVNOG UEABV CRRJF YSFIO
YFDMK SGEQJ LHJUD JCMQM BVLSS   FIVXH JDZLE RJTDK ZSWCQ EPGNN
```

5. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
CFLOL ACABJ GGCFD DCGAB ACOJA   DOACG ACAHN OEACA GMOIA CLLFI
JLNOG JLIEA CCDIO JADOH LACAC   GALFA GAGAC ODEJB DOABC AHJAK
ADAGC HACMO CLIFA GAGAC LNOAI   JDFIF JGDOK ACFGH OAILA CAICL
ACMOF GOKAG CABAF OADJL ACDIO   DIJAI OABCD EFGHO ABJAG OFGAD
```

6. Diagnose the following cryptogram, suspected of dealing with agent activity in Greece, and recover as much plain text as possible:

```
ANLSL GSGLN SYSLD LSAUS ALASG   SLLPS AOSYG LSLSN SYGSL ULSAA
SLLGS ASLSL SYRYS YDSLY LSYSD   PSOSA PRSLG SADUS YYSYO USLNS
YGLSL SLAYS YOSLY USAYS GLNSY   SLLNS ADSDP SOSAD USYYS YRYSY
DSOPS LASLG USLNS YOLSD SAYOS   USLSL SAPRS LGSYU SLNSD PSOSA
LRSAA SLYUS YRSLD USLOS YGLSL   SLONS ALSDP SOSAU ASLLS YRYSY
DSGSO
```

7. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
SMFXB MNEIB ESLPR CMEXP UIKKS   MCSRZ NBIWX FENFX PNBEI YZXBP
QPJEC JSVZD LVCRA YYOZS ODAKV   DLOCR AYJDA YMODD JZDVA ZDLOD
QPTAK UIOCS XOKAH OXWTS PIFHS   HIODH RUKOV OOEJH OXVSH FDOQZ
UPYXL VTTJL VUXYP YZYPU OJXEI   PVZXA PVSJY PVUVN IEOZS JYPVU
XQLMJ UPJXM VEIDM JIAJG EICRR
```

8. Diagnose the following, reconstruct all elements of the cryptosystem, and recover the plain text:

```
XXSDR RUFTS IRPIH ZENWK HSWPW   ZIEWM VKHCK VFSBL GEMUW TDSPU
HTIKY ZGHKL UOWDD UYRNW QBJED   AEMFC JSBWP IJGKB BUKUM DWDZP
GYOCT PDCKX UDFWC DHRTT DSGXR   ZQVOT YOLUW FKWKW XAXVW FBYHM
TCLVK MNCQH NCFBL UTZPY UMPDC   XBKKC EQHBI GYNKV NULXX EMIQW
VJFPD BPPZK QZATA DVBQZ ATRAF   PCKDX
```

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The entries contained herein have been indexed down to sub-subparagraph wherever possible. Entries from footnotes are indexed by showing the paragraph location followed by a dagger (†) and the footnote number; for example, footnote 50 out of subparagraph 77*f*(5) is indexed by the notation "77*f*(5)†50."

NATIONAL SECURITY AGENCY
FORT GEORGE G. MEADE, MARYLAND 20755-6000

FOIA Case: 111955
6 August 2021

This further responds to your Freedom of Information Act (FOIA) request of 7 July 2012 for "a copy of Military Cryptanalytics, Part III, by Lambros D. Callamahos. Please review the sections marked as classified for possible declassification and release" and assigned Case 68177. On 30 December 2020, you appealed this Agency's Initial Denial Authority decision in that case. Your appeal (Appeal 5495) was subsequently granted on 27 April 2021 by the NSA FOIA Appeal Authority. Accordingly, NSA has reprocessed your request under a new case number, Case 111955. Your request has been processed under the FOIA and the document you requested is enclosed. Certain information, however, has been protected in the enclosure.

Some of the withheld information has been found to be currently and properly classified in accordance with Executive Order 13526. The information meets the criteria for classification as set forth in Subparagraph (c) of Section 1.4 and remains classified SECRET as provided in Section 1.2 of Executive Order 13526. The information is classified because its disclosure could reasonably be expected to cause serious damage to the national security. Because the information is currently and properly classified, it is exempt from disclosure pursuant to the first exemption of the FOIA (5 U.S.C. Section 552(b)(1)). The information is exempt from automatic declassification in accordance with Section 3.3(b)(1) of E.O. 13526.

In addition, this Agency is authorized by various statutes to protect certain information concerning its activities. We have determined that such information exists in this document. Accordingly, those portions are exempt from disclosure pursuant to the third exemption of the FOIA, which provides for the withholding of information specifically protected from disclosure by statute. The specific statute applicable in this case is Section 6, Public Law 86-36 (50 U.S. Code 3605).

Since these withholdings may be construed as a partial denial of your request, you are hereby advised of this Agency's appeal procedures.

You may appeal this decision. If you decide to appeal, you should do so in the manner outlined below. NSA will endeavor to respond within 20 working days of receiving any appeal, absent any unusual circumstances.

- The appeal must be sent via U.S. postal mail, fax, or electronic delivery (e-mail) and addressed to:

  NSA FOIA/PA Appeal Authority (P132)
  National Security Agency
  9800 Savage Road STE 6932
  Fort George G. Meade, MD 20755-6932

  The facsimile number is 443-479-3612.
  The appropriate email address to submit an appeal is FOIARSC@nsa.gov.

- It must be postmarked or delivered electronically no later than 90 calendar days from the date of this letter. Decisions appealed after 90 days will not be addressed.
- Please include the case number provided above.
- Please describe with sufficient detail why you believe the denial of requested information was unwarranted.

You may also contact our FOIA Public Liaison at foialo@nsa.gov for any further assistance and to discuss any aspect of your request. Additionally, you may contact the Office of Government Information Services (OGIS) at the National Archives and Records Administration to inquire about the FOIA mediation services they offer. The contact information for OGIS is as follows:

Office of Government Information Services
National Archives and Records Administration
8601 Adelphi Rd. - OGIS
College Park, MD 20740
ogis@nara.gov
877-684-6448
(Fax) 202-741-5769

Sincerely,

for Sharon C. Lukins

RONALD MAPP
Chief, FOIA/PA Division
NSA Initial Denial Authority

Encl:
a/s

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